STEAM POWER

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PREFACE

The study of the scientific principles involved in the application of Steam Power to industrial purposes should be approached through the objective realities of a Steam Plant. The formulated laws which focus the slowly gathered knowledge of energy transformation are more easily understood after some general knowledge has been obtained of the practical working of a Steam Plant. The plan of the following work is therefore to describe a Steam Plant first, then to explain how the cost and the magnitude of the power which it develops can be measured, and afterwards to consider, in as good order as possible, the various practical and theoretical considerations which arise in connection with the Thermodynamics and the Dynamics of the plant.

Chapter I. is preliminary. It is arranged to aid a student who is at work in a steam laboratory. A steam plant is described, the measurement of its power is discussed, the laws of thermodynamics which condition its thermal efficiency are stated, and then, leaving objective realities, the Heat Stream flowing through the plant is imagined in a series of temperature falls, and two heat energy streams are drawn from experimental data. This way of graphically representing the Heat-Flow appeared in an introductory note to the Report of the Institution of Civil Engineers Committee, published in 1896. The note was written by Captain Sankey. The study of this Report led me to the idea of analysing the action of a steam plant into three interacting but independent circuits. Accordingly, in the course of the chapter, the Heat-carrying Media circulating through the plant are analysed into three interacting circuits. The first of these is the Heating Circuit, in which furnace gas is compelled to circulate by the chimney or the element which is producing the draught; the second is the Motive Power Circuit, in which water stuff is made to circulate by the feed pump; the third is the Cooling Circuit, in which condensing water is circulated by the circulating pump. The analysis of the Heat-carrying Media in this way has been found by the author helpful in teaching. A general perspective of the action of the plant is presented at the outset, and afterwards as each element is studied in detail the true

relation of each part to the whole is kept in view. Moreover, the idealized action of the three circuits reduces to a Carnot engine.

After the first, the chapters may be grouped in three divisions. The heat-flow through the plant is considered in detail in the first division, consisting of Chapters II., III., IV., V., and VI. Attention is concentrated on the Heating Circuit in Chapter II., and on the Cooling Circuit in Chapter V. The principles of Thermodynamics are developed in Chapter III., and the reader is led up to the calculation of the properties of steam recorded in the Steam Tables from Callendar's characteristic equation for steam. Finally the principles are used to establish the Rankine ideal standard of performance for a steam engine. The actual performance of a steam engine is compared with this ideal standard in Chapter IV., and the reduction of the difference between the actual and the ideal performance by expansive working, by compound expansion, and by the use of superheated steam are discussed and illustrated. The interaction of the three circuits one on the other is specially considered in Chapter VI., and the results of the heat energy flow through a plant are shown graphically in a Characteristic Energy Diagram. The diagram is drawn for a locomotive, because a locomotive is a plant complete in itself, and experimental data over a wide range of speed and load are available for individual engines.

The second division, comprising Chapters VII., VIII., IX., and X., relates to the dynamics of the steam engine treated mainly in connection with the reciprocating engine. The Governor and the Flywheel (Chapter VII.) are elements specially belonging to the stationary engine, where the load is arbitrary and independent of the speed. The treatment of governors in the text was suggested by a valuable paper, "Les Regulateurs à force Centrifugal," by L. Rith, given to the author by Mr. J. B. Peace, of Emmanuel College, Cambridge. Chapter VIII. is devoted to problems relating to cases where the load on a plant is a function of the speed, as in the case of marine and locomotive engines. These problems are technical aspects of the general problem of Motion in a Resisting Medium. The motion of a train is selected for illustrating the general principles involved and incidentally the action of a locomotive is specially considered in relation to load and speed. The general principles of the chapter apply to any type of tractor. The Characteristic Dynamical Diagram for the motion of a train was first published by the Author in a Paper contributed to the *Proceedings of the Institution of Mechanical Engineers* in 1912. The diagram is a general graphic solution of the problem of motion in a resisting medium, and may be applied to many practical cases besides the motion of a train. Chapter IX. relates to the Balancing of Engines by the methods published by the author in a paper contributed to the *Transactions of the Institution of Naval Architects* in 1902. The subject is treated in fuller detail in subsequent papers, and in the author's treatise on "The Balancing of Engines,"

published in 1907. Chapter X. relates to Valve Gears. The treatment is sufficiently extended to give the reader a general knowledge of the principles of steam distribution by automatic and mechanical means, together with a knowledge of how to analyse by means of a valve diagram the distribution effected by a gear and how to make a preliminary design of a gear to effect a given distribution. The subjects are treated in greater detail and with more illustration in the author's treatise, "Valve and Valve-Gear Mechanisms".

Chapters XI. and XII. fall in the third division, which is reserved for the special consideration of Steam Turbines. As a preliminary to the actual study of the turbine, the flow of steam is separately considered in Chapter XI. In Chapter XII. the general principles involved in the action of a steam turbine are developed, first in connection with single turbine pairs and afterwards in connection with chains of turbine pairs. The idea of a turbine pair prevents confusion between the number of rows of blades and guides, and it keeps in mind an essential feature of a steam turbine, namely, that it is a compound turbine.

In recent years the Science of Thermodynamics has been enriched in many directions by the researches of Professor H. L. Callendar. In particular, the application of the principles of thermodynamics to the Callendar Characteristic Equation of steam enables the properties of steam to be calculated, and only a few experimental results are needed to fix the values of the constants in the integrated equations. The agreement between the calculated values of these properties and the experimental values found by Regnault and more recent workers is close and mutually confirmatory.

The Thermodynamics in Chapter III. is developed so as to include part of Callendar's researches relating to steam. The Steam Tables in the Appendix were calculated by Professor Callendar from the expressions given in this chapter, and I am greatly indebted to Professor Callendar for his permission to use these tables in this book.[1]

Professor Callendar has recently made an important contribution to the theory of the flow of steam through nozzles, the main results of which are included in Chapter XI.

The researches of Callendar and Nicolson on the condensation of steam in the engine cylinder go to the root of things, and an account of these researches is included in Chapter V. The laboratory method described for the determination of the whereabouts of the quantity of steam missing from the steam supplied to the cylinder is based on these investigations.

The author's researches on the science of the locomotive are

[1] Extended Steam Tables, calculated by Professor Callendar, in terms of the lb.-calorie, the British thermal unit and the kilogram-calorie, are obtainable separately.

included in this book, though in a somewhat scattered form. In a paper entitled "The Economical Working of Locomotives," contributed to the *Proceedings of the Institution of Civil Engineers* in 1906, the author suggested that the relation between the mean pressure in a locomotive cylinder of standard design and the piston speed was linear in form, basing the suggestion on the published experiments made on the testing plant at Purdue University by Professor Goss and on the results of some experiments made on the Chicago and North-Western Railroad. An examination of the data recorded in "High Steam Pressures in Locomotive Service," by Professor Goss, and of the data obtained at the locomotive trials at the St. Louis Exhibition and recorded in "Locomotive Tests and Exhibits," published by the Pennsylvania Railroad System, to whom the author is indebted for a copy of this valuable collection of experimental data, goes far to establish the linear law suggested, and for practical purposes of design converts the suggestion into an established fact. The study of the data also revealed that there is to a first approximation a linear relation between the rate of heat transmission across the boiler heating surface and the indicated horse-power, a relation which is practically independent of speed and cut-off. This relation is shown in Fig. 94, page 328, a diagram which was first published in *Engineering*, August 19, 1910. The records enable the working of a locomotive to be probed still deeper, and they enable the conditions of the heat-flow through the engine to be followed in detail through the whole range of its action. The curves showing these conditions, together with the I.H.P. line, constitute the Characteristic Energy Diagram already mentioned above. A Characteristic Energy Diagram is published in *Engineering*, August 19, 1910, for each of the locomotives tested at the St. Louis Exhibition and for some of the trials made by Professor Goss on the engine "Schenectady," fourteen in all. The generalization connecting the horse-power and the piston speed, and the generalization connecting the horse-power and the heat transmission, place the science of the locomotive on a firm experimental basis.

Attention may be drawn to a few new methods. The design and the action of a governor of almost any type can easily be analysed by the semi-graphical method developed in Section 111. Two completely worked examples of the diagram are given on pages 394 and 397. The method of calculating the volume for a given power and speed of the cylinders of either a simple or a compound engine, given on pages 285 to 298, may be found useful. The expression for the volume, eq. (1), page 285, shows in a simple and direct manner the extent to which the volume is influenced by the ratios of expansion and compression and the clearance factor. The circumstances of the motion of a flywheel are directly deduced from the curve of crank effort by the method due to Professor Henrici, Section 104.

A new chart is described in Section 57, page 202. It is constructed by plotting along oblique co-ordinates the total energy of one pound of steam against the temperature. This new diagram has many advantages, though for some purposes it is not quite so convenient as the Total Energy-Entropy diagram (the Mollier Diagram), the outstanding feature of which is that a reversible adiabatic change is represented by a straight line. The new diagram shows at a glance the advantages of a good vacuum and the advantage of using superheated steam, and also how little thermodynamic advantage is gained by the use of high steam pressures along with highly superheated steam. One of the minor advantages of the new diagram is that solutions of most practical questions can be obtained by its aid, together with the available work of a Rankine engine, without the necessity of specific reference to the troublesome conception of entropy.

Two large diagrams are placed in the covers of this book. One, the new diagram obtained by the oblique plotting of the Total Energy against the temperature; the other, a Total Energy-Entropy diagram obtained by plotting the total energy against the entropy. Both diagrams were plotted from the data in the Steam Tables in the Appendix.

The unit of heat used throughout this book is the lb.-calorie. It is one-hundredth part of the quantity of heat required to raise one pound of water from 0 to 100° C. The corresponding value of Joule's equivalent is 1400 ft.-pounds. The British thermal unit is five-ninths of the lb.-calorie. Modern research is nearly always expressed in terms of the Centigrade scale, and the advantage of adopting the lb.-calorie as the unit of heat is, that the same figures stand for total energy and the related quantities whether the heat unit is the lb.-calorie or the kilogram-calorie, or the gram-calorie.

The unit of force and the unit of mass employed generally are those usually used by engineers, namely, the unit of force is the weight of one pound, and the unit of mass is such that the number of mass units in a body is obtained by dividing the weight of the body by g. In balancing problems which are mainly concerned with mass relations the unit of mass is taken equal to the mass of a quantity of matter weighing one pound. No difficulty will be found in employing whichever system of units is the more convenient in any particular case.

Sections 45 to 48 inclusive may be omitted without break in continuity. Chapters XI. and XII. may be read after Chapter V. The Dynamics of the Steam Engine, beginning at Chapter VII., may be studied concurrently with the Thermodynamics in Chapters III. and IV.

A few examples are added at the end of the book which may prove of use in helping students to test their progress, and in helping teachers to frame others more suitable for their purposes.

I thank my friends and the firms who have so kindly helped me with data. Acknowledgment is made in the text in the places where the data are used. I thank Mr. A. Cruickshank for help in regard to the preparation of some of the drawings. I am also greatly indebted to Mr. E. E. D. Witchell for the care with which he has read the proofs and for his helpful suggestions. In a book involving so much numerical work errors are inevitable. It is hoped that they are few.

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STEAM POWER

CHAPTER I

General Action of a Steam Plant; its Efficiency and the General Principles Regarding the Flow of Heat Energy

1. **General Description of a Steam Plant.**—A steam plant in its complete form consists essentially of a **boiler** for producing steam; an **engine** for doing work by means of the steam produced; and a **condenser** for reducing the pressure of the steam after its action in the engine to a pressure as far as possible below the pressure at which it is formed in the boiler.

The action of the plant is continuous. If the engine works against a steady load a definite weight of water is pumped into the boiler per minute, an equal weight of steam is produced per minute, and an equal weight of steam is condensed per minute; and the steam flowing through the engine cylinder is able to do a definite amount of work per minute because of the pressure difference established by the boiler and the condenser.

The essential parts of a steam plant are shown in diagrammatic form in Fig. 1. This diagram is not drawn to scale. The diagram shows the mutual interdependence of the various parts of the plant. The **boiler** is connected to the engine by the **steam pipe**; the engine is connected to the condenser by the **exhaust pipe**, and the condenser is connected to the boiler by the **air pump**, the **hot well**, the **feed pump**, and the **feed pipe**.

The heat produced by the combustion of fuel on the grate of the furnace raises the temperature of the furnace gas sufficiently high to determine a rapid flow of heat across the heating surface to the water in the boiler. The **heating surface** of the boiler indicated in Fig. 1 consists of the main flue together with the return tubes, two only of which are shown.

One pound of good coal properly burned will furnish about 8000 lb.-cals. of heat energy, of which about 70 per cent. may be transferred across the heating surface to the water in the boiler, the remainder escaping, partly in the chimney gas, partly by radiation, and in other ways.

The heat flowing across the heating surface boils the water, and the maximum pressure of the steam produced is fixed by the load W

on the **safety valve.** The safety valve prevents the pressure in the boiler increasing beyond the limit fixed by the load W, because if this pressure is only slightly exceeded the valve lifts, and allows steam to escape into the atmosphere.

Every pound of steam requires for its production a definite quantity of heat, a quantity which depends upon the pressure at which the steam is produced, and upon the temperature of the feed water. Thus, if the safety valve is loaded so that steam is produced at a pressure of 200 lbs. per square inch absolute, 1 lb. of steam formed at this pressure from water at, say, 50° C., absorbs 620 lb.-cals. of heat energy.

The steam flows from the boiler through the **boiler stop valve** into the **steam pipe,** and through the **engine stop valve** to the **valve chest** C, from which it is admitted to the cylinder by the valve S or S_1. In Fig. 1 the valve S is shown open and the valve S_1 is shown closed. The steam admitted through S exerts a pressure on the piston P, and pushes it along the cylinder, and by means of the connecting rod and the crank L this pressure is made to turn the crank shaft against the resistance which the plant is designed to overcome. The **back pressure** on the other side of the piston, which forms part of the resistance against which the steam pressure works, is the pressure to which the steam is reduced when, at the end of a working stroke, the cylinder is put in communication with the condenser by the exhaust valve E or E_1. In Fig. 1 the right exhaust valve E_1 is open, so that the pressure on the right side of the piston corresponds to the pressure in the condenser.

As the piston works to and fro, the two ends of the cylinder are placed alternately in communication with the condenser by the **exhaust valves** E and E_1, whilst steam from the boiler is admitted to alternate ends by the **steam valves** S_1 and S. The opening and closing of these valves at the proper time is done automatically by means of the **valve gear mechanism.** The detailed consideration of valve gear mechanisms and steam valves is reserved for Chapter X. A detailed drawing of the kind of valve designed for distributing the steam to a cylinder when four separate valves are used is shown in Fig. 170.

The condensation of steam is brought about in the condenser by means of cold water circulating through the tubes around which the exhaust steam circulates when the way to the condenser is open through either exhaust valve E or E_1. Four tubes only are shown in Fig. 1. Condensation is effected at a temperature somewhat above the temperature of the circulating water, because without a temperature difference the heat which must be abstracted from the steam to condense it cannot flow across the cooling surface of the condenser tubes into the circulating water. If condensation takes place at 38° C., the pressure is reduced to about 1 lb. per square inch absolute.

Circulating water, as cold as possible, is pumped into the condenser by the **circulating pump,** shown in Fig. 1 as a centrifugal

pump, and the water, getting hotter as it passes through the tubes because of the heat flowing into it across the cooling surface from the stream which is being condensed, is discharged into a **cooling pond** to cool down before again being circulated through the condenser. In Fig. 1 the arched culvert of the sump is supposed to lead to a cooling pond. The discharge pipe from the circulating pump is shown discharging into the sump (Fig. 1) for convenience only. Actually the pipe would lead away through the culvert, and the water would be discharged directly into the air like a fountain, so that the jets and drops into which the water breaks as it falls may mingle with the air, and so facilitate the transfer of heat from the water to the air. The lowest temperature to which the water can be reduced is ultimately fixed by the air temperature.

The condensed steam is pumped out of the condenser by the **air pump**, so called because in addition to pumping condensed steam it pumps the air, which is liberated from the water and the steam. If left to accumulate in the condenser this air would increase the pressure and thus increase the back pressure. The air pump in Fig. 1 is shown connected to the tail rod of the piston by means of a bell crank lever or L-bob. The upward stroke of the air-pump bucket draws water and air from the condenser through the foot valve, and at the same time forces the charge above the bucket valve through the delivery pipe into the hot well. During the upward stroke the bucket valve is kept closed by the weight of the water above it. The bucket valve opens and the foot valve closes immediately the air-pump bucket begins its downward stroke. During this stroke the bucket merely passes through the water drawn in during the up stroke. This kind of pump, therefore, sucks both the water and the air from the condenser and discharges the charge of water and air above the bucket during the up stroke, but does nothing during the down stroke. The water is pumped by the feed pump from the hot well into the boiler again, against the boiler pressure.

The method of driving neither the circulating pump nor the feed pump is shown in Fig. 1. Sometimes the circulating and the air pumps are both connected mechanically with the engine. Very often, however, the air and circulating pumps are separately driven units, the steam for driving them being taken from the boiler direct. The feed pump is generally separately driven. However the driving is done, the energy is obtained from the boiler, and the steam supplied to these auxiliaries must be included in any measurement of the steam required by the plant per unit of work done.

The action of the plant is conditioned by three well-marked **pressure stages**, namely, the pressure in the boiler, the pressure in the condenser, and the atmospheric pressure against which the air pump discharges the condensed steam from the hot well. The highest and the lowest pressure stages of a steam plant are shown on the respective pressure gauges fixed on the boiler and on the condenser. The pressure given by the boiler pressure gauge is the pressure above that of the atmosphere, so that the reading

must be increased by the atmospheric pressure in order to obtain the actual pressure of steam in the boiler. The term "**gauge pressure**" is often used to distinguish the pressure shown by the gauge from the real pressure, and to remove all ambiguity the real pressure is often called the "**absolute pressure**". In what follows the term "pressure," unqualified, will be used for the actual or so-called absolute pressure. A pressure read from a gauge will be called a gauge-pressure. The gauge on the condenser shows the pressure below that of the atmosphere in pounds per square inch or in the equivalent number of inches of mercury, and it is called a vacuum gauge. The term "**vacuum**" means a negative pressure taking the atmospheric pressure as the datum. The actual pressure in the condenser is therefore found by subtracting the reading of the vacuum gauge from the atmospheric pressure, both the reading and the atmospheric pressure being expressed either in pounds per square inch or in inches of mercury. Neither the pressure in the boiler nor in the condenser, therefore, can be found from the gauge readings alone; an observation must be made on the height of the barometer to find the atmospheric pressure. The barometer reading in inches multiplied by 0·49 gives the pressure in pounds per square inch with sufficient accuracy for ordinary purposes.

In the boiler the steam is in contact with the water from which it is produced, and in the condenser it is in contact with the water to which it is reduced, and in these circumstances there is a definite temperature corresponding to each pressure. Corresponding values of the temperature and pressure of steam in contact with water are shown in the steam tables, columns 1 and 2, page 738.

The temperature of the furnace gas, which varies from the high temperature just over the fuel bed, where the combustion is most intense, to the temperature at which the gas passes into the chimney, is at all points higher than the temperature of the steam in the boiler; and the temperature of the condensing water, which is lowest at entry to the condenser and highest at exit, is lower than the temperature at which the steam is condensed.

These different temperature levels may be regarded as forming a series of **temperature falls** which determine a flow of heat from the furnace gas to the cooling pond, where the condensing water cools to its lowest temperature. The action of the plant is determined by the flow of heat energy through it from the furnace to the cooling pond. If there were no losses of heat energy during the flow due to radiation, conduction, or leak, the quantity of heat energy taken up by the cooling water would be less than the quantity of heat energy received from the boiler by the exact equivalent of the work done in the cylinder. For every 1400 ft.-lbs. of work done in the cylinder, 1 lb.-cal. disappears from the heat energy flow. For example, in the ideal conditions assumed, if one pound of steam receives 670 lb.-cals. in the boiler and rejects 600 lb.-cals. in the condenser, the difference, namely, 70 lb.-cals., is transformed into work in the cylinder.

The study of the conditions in which the greatest amount of work

can be obtained from a given flow of heat energy through a heat engine has developed into the modern science of thermodynamics.

A general consideration of the action of the plant shows that heat energy is associated with some heat-carrying medium. In the furnace the heat-carrying medium is the furnace gas. In the boiler, engine, and condenser, it is the steam. On the cooling side of the condenser it is the condensing water. These media are not at rest. Each flows along a definite path in the plant. The flow of the furnace gas is maintained by the chimney; of the steam in the boiler, engine, and condenser, by the feed pump, and the flow of cooling water through the condenser, by the circulating pump. The heat-carrying media may thus be analysed into three circuits mutually interlaced but each distinct and individual in character.

The heat supply to the engine is originated by the combustion of fuel at the grate, and this combustion is caused by the flow of air through the fuel bed; and the air, after passing through the bed, becomes changed to furnace gas. This hot furnace gas passing through the boiler flues loses the greater part of its heat to the water in the boiler, and the gas, considerably cooled, passes out into the air again. There is thus a circulation of gas, carrying heat from the furnace to the water in the boiler, and the stream may therefore be appropriately called the **heating circuit**. It is an open circuit in the sense that air is drawn into the furnace from an inexhaustible supply, and the air, increased by the relatively small quantity of gas into which the fuel is changed by combustion, is discharged as furnace gas from the chimney top.

The second circuit may be regarded as beginning in the hot well. Following the journey of a pound of water through the plant, the feed pump draws it from the hot well and forces it along the delivery pipe and into the boiler. Here, imagining that the pound of water still retains its individuality in the mass of boiling water surrounding it, it is gradually turned into steam by the heat energy received from the heating circuit. It then passes along into the steam pipe, and after making its way through the stop valve, the valve chest, and steam admission valve, finds itself in the cylinder exerting a pressure on the piston. The piston yields, and in yielding allows the steam to do work, and the pound of steam loses heat energy equivalent to the work done. After pushing the piston to the end of its stroke it finds a way of escape through the open exhaust valve and passes through the exhaust pipe into the condenser, where, rushing into contact with the condenser tubes through which cold water is circulating, it loses heat energy and is again reduced to a state of water and falls to the bottom of the condenser. There it feels the suction of the air pump, and passing along through the foot valve into the space under the bucket it is forced through the bucket on the return stroke of the pump and finally tumbles into the hot well. Here it is caught up by the feed pump and sent round the same circuit again.

The net result of the journey of a pound of water round the circuit is that it has received a quantity of heat from the heating circuit at

a high temperature and has carried this to the cylinder, w converted into work, and then the steam passes on and delivers the remainder to the cooling circuit at a low temperature.

Every time a pound of water is driven into the circuit by the feed pump the burden of heat which it receives in the boiler from the heating circuit converts it into steam, and thus enables it by virtue of the elastic properties of steam to convert part of the heat which it carries into work by the mechanical displacement of a piston.

This circuit may therefore be called appropriately the **motive-power circuit.** The water material which flows round the circuit is called the **working agent.**

The steam pipes and the cylinder are clothed or **lagged** with some non-conducting material to prevent the escape of heat from the working agent. There are many kinds of non-conducting material in use for this purpose, amongst which may be mentioned mica in its various forms, magnesia and asbestos, and these substances are generally held in position by an outer clothing of wood or thin lagging steel. Sometimes wood or even steel lagging is used alone, the effect of which is to clothe the surfaces with an air jacket, air of itself being a good non-conductor if it is prevented from circulating. Frequently the cylinder is further protected with a **steam jacket,** as indicated in Fig. 1. The steam jacket is an annular space round the cylinder barrel fed with steam by a small steam pipe, J, from the main steam pipe. The water condensed in the jacket is drained away through a **steam trap.** Sometimes the ends of the cylinder are jacketed also, and occasionally the condensed steam is drained back into the boiler, so that there is a continuous circulation of steam through the jacket. Unless the jacket is properly drained it does more harm than good, because the condensed water accumulates and the jacket tends to become a water jacket, accelerating instead of retarding the flow of heat from the working agent. In a recent type of engine the ends of the cylinder only are jacketed, and the steam passes through the jackets on its way to the admission valves. The steam jacket was first applied by Watt in 1763.

The third circuit, the **cooling circuit,** is the path followed by the water from the cooling pond, through the circulating pump, through the condenser tubes, and back into the cooling pond again, where the heat taken up in its passage through the condenser is gradually dissipated and the water cooled. The temperature of the cooling pond is the lowest temperature in the series of temperature falls through the plant.

Thus the work done by a steam plant is the result of the mutual action of the heat-carrying media in the heating circuit, the motive power circuit, and in the cooling circuit, and the mutual action determines a flow of heat energy from the furnace gas to the cooling pond, which is in part transformed into work in the cylinder through the elastic properties of steam called forth by the heat which it is made to carry.

A quantitative example of a heat stream through an engine,

together with the corresponding heat flow in the media flowing in the three circuits, is considered in detail below.

In addition to the essential parts of a steam plant shown in full lines in Fig. 1, parts are sometimes added with the object of increasing the general efficiency of the plant.

One addition which is often made when there is room for it is an **economiser or feed water heater.** The temperature of the water in the hot well is considerably below the temperature of water in the boiler. The temperature of the chimney gas is considerably higher than the temperature of the water in the hot well. There is, therefore, the possibility of establishing a temperature fall between the hot chimney gas and the cold feed water, and so by means of heat obtained from the chimney gas bringing the temperature of the feed water nearer the temperature of the water in the boiler. This temperature fall is arranged in the **economiser or feed water heater,** the general arrangement of which is indicated by dotted lines in Fig. 1. As will be seen, a bank of pipes through which the feed water is forced by the feed pump to the boiler is placed in the path of the chimney gas. To bring the economiser into action it is only necessary to place two dampers, A and B, in the position across the chimney indicated by dotted lines. The furnace gas is then guided into the economiser chamber from which it escapes at the top into the chimney above the damper B.

Again, the furnace gas in the combustion chamber at the end of the flue tube is much hotter than the steam in the boiler and in the steam pipe. The steam in the steam pipe may therefore be dried from moisture carried along mechanically from the boiler, and in addition may be **superheated,** that is, it may be raised above the temperature at which the steam is produced in the boiler, by carrying the steam pipe through the hot combustion chamber as indicated by dotted lines in Fig. 1, and so causing some of the heat in the furnace gas to flow directly into the steam passing through the steam pipe. The arrangement of tubes, valves, and receivers, which in practice correspond to carrying the steam pipe through the furnace, is called a **superheater.** In some cases the steam pipe is led into a superheater which has its own furnace and grate. That is to say, a separately fired superheater is added to the plant.

A superheater to be of any value must be placed in a hot part of the furnace, otherwise the temperature fall between the gas and the steam will not be sufficiently great to establish a heat flow of sufficient magnitude to effectually superheat the steam. Superheating steam in this way does not increase its pressure, but only raises its temperature above the temperature of formation.

It will be clear from the diagram, Fig. 1, that the oil introduced into the cylinder for the purpose of lubrication ultimately finds its way into the condenser, and that it is pumped out along with the condensed steam into the hot well, and that unless precautions are taken it is pumped by the feed pump into the boiler. Oil finding its way into the boiler spreads over the heating surface and reduces the

efficiency of the boiler and tends to allow the plates to be overheated. Every precaution should be taken therefore to prevent the introduction of oil into the boiler with the feed water. **Oil filters** are added to the plant to strain off the oil from the condensed steam before it is drawn into the feed pump.

Plant for the softening of water before it is used in the boiler is sometimes added, and in the case of marine engines **evaporating plant** is often fixed in order to distil fresh water from sea water so as to make up for the inevitable loss of fresh water circulating in the motive-power circuit owing to leaks, steam blowing from the safety valves, and other losses. Sea water should never be used in the motive-power circuit, otherwise there is a gradual accumulation of salt in the boiler with consequent danger of overheating. In emergencies, however, if sea water must be used, the water in the boiler must be blown out at intervals so that the salt never increases beyond the proportion of $\frac{3}{33}$ part by weight. On the other hand, the sea furnishes an inexhaustible supply of circulating water for the cooling circuit, and is in fact a cooling pond of constant temperature. The sea temperature is for the marine engine the lowest possible temperature in the fall established by the steam plant fitted in the ship.

The **power** of a steam plant is the rate at which it can do work. Work is measured in foot-pounds, and the unit of power might be defined as one foot-pound per second. The arbitrary horse-power unit introduced by James Watt is, however, universally used by engineers in connection with engines and motors of all kinds.

One horse-power is equal to 550 ft.-lbs. of work done per second, or 33,000 ft.-lbs. per minute, or 1,980,000 ft.-lbs. per hour.

Thus, if an engine works at the rate of 10 H.P., it does work at the rate of 5500 ft.-lbs. per second.

The term "Horse-Power" of an engine may mean either Indicated Horse-Power or Brake Horse-Power.

The Indicated Horse-Power (I.H.P.) is the power developed by the steam in the cylinder, and the name is derived from the Indicator, the instrument used to obtain a diagram showing the pressure-volume cycle through which the steam passes in the cylinder during a double stroke of the piston.

The Brake Horse-Power (B.H.P.) is the power transmitted through the crank shaft of the engine, and is in fact the power which the engine as a whole can develop to drive machinery external to itself. The name is derived from the Brake, the apparatus used in the process of measuring the crank-shaft horse-power.

The difference between the I.H.P. and the B.H.P. is the power used to overcome the resistance of the engine machinery. The **mechanical efficiency** of the engine is the ratio between the brake and the indicated horse-powers. The mechanical efficiency depends upon the load on the engine. At full load, it usually varies between 80 and 90 per cent., according to the type of engine, and even 90 per cent. is sometimes exceeded.

The unit quantity of work, namely, a foot-pound, is inconveniently small in practice, and engineers use a much larger unit expressed in terms of an engine working at the rate of one horse-power for one hour. The definite quantity of work done in these circumstances is 1,980,000 ft.-lbs. The name of the unit is the Horse-power-hour. Hence the term H.P.-hour stands for the definite quantity of work 1,980,000 ft.-lbs.

The performance of a steam plant is shown by a statement of the quantities of fuel and water supplied to the plant per horse-power-hour. The basis of comparison is generally the indicated horse-power, although the brake horse-power is sometimes used.

For example the performance of a plant is indicated by the following statement:—

Coal fired per I.H.P.-hour	2 lbs.
Water fed to boiler per I.H.P.-hour	18 ,,
Water pumped through condenser per I.H.P.-hour	540 ,,

The cost of the unit of energy, namely, the I.H.P.-hour, is partly obtained by converting the above quantities into money. To find the complete cost, allowance must be made for the interest on the capital cost of the plant, the depreciation of the plant, wages and superintendence, oil and grease used, rates and taxes and general charges of the station.

Another unit of energy is also largely used in connection with electrical undertakings, namely, the kilowatt-hour, more commonly called a Board of Trade unit. The meaning of the term is that an engine or dynamo working at the rate of 1 kilowatt for one hour does the definite quantity of work 3,600,000 joules, equivalent to 2,655,403 ft.-lbs. Or, put in another way, 1 kilowatt-hour is equivalent to 1·341 H.P.-hours. In practice it is often sufficiently accurate to consider that the horse-power is equal to three-quarters of a kilowatt, the true fraction being 0·746.

Before considering in detail the methods by which power is measured, it is convenient to briefly distinguish the main types into which steam plants are divided.

The condenser is not an indispensable part of a steam plant, and in many cases it is omitted, as in locomotives where the provision of condensing water is impracticable, or in engines fixed in places where a supply of cold condensing water is not easily obtained.

In these circumstances the steam is discharged from the cylinder into the atmosphere, so that the lowest pressure to which the steam can fall in a **non-condensing** plant is fixed by the pressure of the atmosphere, that is about 15 lbs. per square inch. Hence the essential difference between non-condensing and condensing steam engines is that in non-condensing engines the back pressure cannot be lower than about 15 lbs. per square inch, whilst in condensing engines it can be reduced to the region of 1 lb. per square inch. The lower limit is determined by the lowest temperature at which the cooling water can be supplied to the condenser.

Again, there is the distinction between **single-acting** and **double-acting** engines. In the single-acting engine, the boiler steam is admitted to act on one side of the piston only; the other side is permanently in connection with the atmosphere or with the condenser, according as the engine is of the non-condensing or the condensing type. Every alternate stroke is therefore an idle stroke because after the boiler steam has pushed the piston along the cylinder to the end of the stroke, the exhaust valve is opened, and during the return of the piston both sides of it are in communication with either the atmosphere or the condenser. In a double-acting engine, the boiler steam is admitted at every alternate stroke to opposite sides of the piston, so that every stroke is a working stroke.

In general, the supply of steam from the boiler is cut off by the admission valve when the piston has been pushed along only part of its stroke. During this part of the stroke the pressure is approximately constant, but slightly below the boiler pressure. After the supply of steam is cut off, the steam still goes on pushing the piston, but its pressure falls as the volume increases, and the size of the cylinder is proportioned so that at the end of the stroke the pressure has fallen nearly, but not quite, to the back pressure. When the exhaust valve is opened there is therefore a sudden drop of pressure from the pressure at the end of expansion to the back pressure in the exhaust pipe, which is generally slightly above the pressure in the condenser. For example, steam produced in the boiler at 100 lbs. per square inch may, during admission in the cylinder, be reduced to 90 lbs. per square inch, owing to the throttling action of the pipes, valves, and passages through which it passes from the boiler to the cylinder. The admission valve to the cylinder may be closed when the piston has been pushed along $\frac{1}{5}$ of its stroke. During the remainder of the stroke the pressure will gradually fall from 90 to about 20 lbs. per square inch, and this will suddenly drop to say 3 lbs. per square inch when the exhaust valve is open. In the early engines the admission valve was kept open during the whole stroke, so that the drop of pressure at the end of the stroke was made from the boiler to the condenser pressure without the intermediate process of expansion. There is therefore a distinction between **non-expansive** and **expansive** working. The economic advantage of expansive working was discovered by James Watt in 1769. With few exceptions, all steam engines are now worked expansively, and these exceptions apply to special cases where the economy of steam is immaterial in relation to mechanical convenience obtained by non-expansive working, as in the small steam engines used to operate steering gears or to control reversing motions, or to work feed pumps.

The reduction of the pressure by expansion from the boiler pressure to the condenser pressure may be made in a series of stages, each stage being carried out in a separate cylinder. For example, the volume of the cylinder to which the steam is admitted from the boiler may be so proportioned that at the end of the stroke the

pressure falls to a value midway between the boiler and the condenser pressure. Instead of completing the reduction of the pressure by a sudden drop to the back pressure, the steam is led into a second cylinder, where it falls in pressure as it pushes the piston along in the second cylinder, and the volume of this cylinder is so proportioned that at the end of the stroke the steam pressure has fallen to a point somewhat above the back pressure. An engine arranged to work in this way is called a **compound engine**, and the steam is said to be reduced in pressure by **compound expansion.** The cylinder into which the steam is admitted from the boiler is called the **high-pressure cylinder**, and the cylinder from which it flows into the condenser is the **low-pressure cylinder.** Sometimes three cylinders in series are arranged to allow of three-stage expansion. The middle cylinder is called an **intermediate cylinder,** since it is not directly connected either with the boiler or with the condenser, but receives its steam supply from the high-pressure cylinder and exhausts into the low-pressure cylinder. An engine in which the expansion is carried on in three stages is called a **triple expansion engine.** Sometimes **quadruple expansion engines** are built, in which the reduction of pressure from the boiler to the condenser pressure is made in four stages. The advantages of expansive working will be fully discussed below.

A **simple engine** is one in which the expansion of the steam is carried out in one cylinder only.

The reader is recommended to examine an actual steam plant and identify, and sketch in his note-book, all the details indicated in Fig. 1, and to make out the general type to which the engine belongs and if possible to note the various pressure stages in the working of the plant.

2. The Indicator Diagram and the Indicated Horse-power.—The rate at which steam is doing work in the cylinder of an engine is determined by means of the **indicator.** An indicator of the kind manufactured by the Crosby Steam Gauge and Valve Company, is shown in Fig. 2. It is connected by the union U to a cock screwed in at one end of the cylinder. A piston P, usually ½ inch diameter, is exposed to the action of the steam by turning on the indicator cock, and its movement is controlled by the action of a spring S applied above it. (The steam pressure in the cylinder acting on the small piston compresses the spring, and as the compression of the spring is proportional to the pressure acting upon it, the displacement of the piston is also proportional to the change of steam pressure in the cylinder.) The actual displacement of the piston of the indicator is kept small, but it is multiplied by the linkage attached to the piston rod so that the point in the linkage which carries the pencil *p* also moves proportionally with the change of steam pressure in the cylinder. A paper is wound round a drum D and, for the moment assuming the drum to be at rest as the engine works, the pencil rises and falls as the pressure in the cylinder rises and falls

and it traces a vertical line on the paper the highest point of which gives the highest pressure in the cylinder and the lowest point the lowest pressure. The scale of the spring is stamped on it. The stamping, 30, means that 1 inch movement of the pencil corresponds to a change of pressure in the cylinder of 30 lbs. per square inch.

When taking the indicator diagram the drum is connected to the cross head of the engine through a cord C attached to a reducing linkage designed so that the angular velocity of the drum is proportional to the linear velocity of the piston without sensible error.

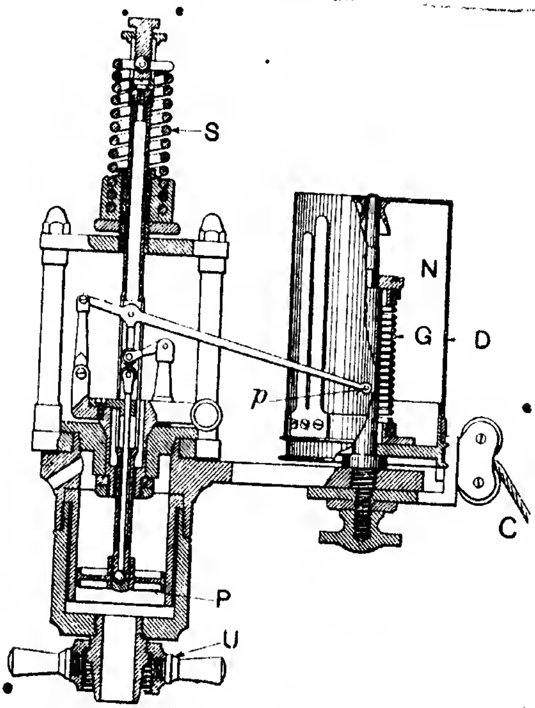


FIG. 2.—The indicator.

A spring G inside the drum keeps the connecting cord in tension, and the amount of tension can be regulated by turning the nut N. One form of reducing linkage is shown in Fig. 3. A long rod AB, free to oscillate about the fixed centre A, is connected to the cross head by the radius rod BC. The cord from the indicator is attached to the rod AB at D. Hence the velocity ratio between the cord and the cross head is $\frac{AD}{AB}$, although this is not quite constant during the stroke but varies slightly with the angle which the rod AB makes with its mean position. Linkages giving a constant velocity

ratio can be made and are sometimes used. They are generally more complicated than the simple form illustrated in the figure. A general description of several of these reducing mechanisms will be found in "Mechanism," by Dunkerley. Tension on the cord connecting the point D with the drum is produced, as mentioned above, by the spring in the indicator drum. The tension in the cord is variable because in addition to overcoming the resistance of the spring G, it has to overcome the variable resistance of the drum to acceleration. The variable tension causes the cord to stretch and shorten during a stroke, and this variation in length introduces an error in the proportionality between the motion of the cross head and the drum, even when the reducing mechanism is itself mathe-

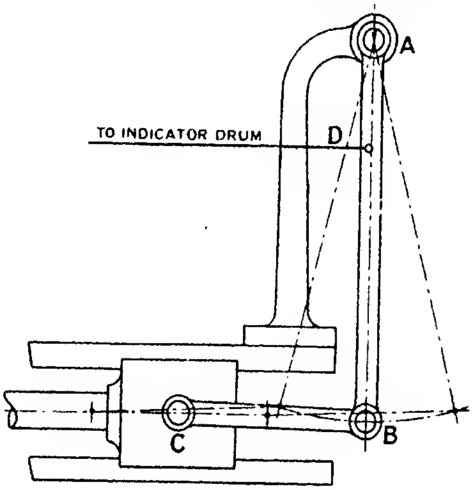


FIG. 3.—Reducing linkage.

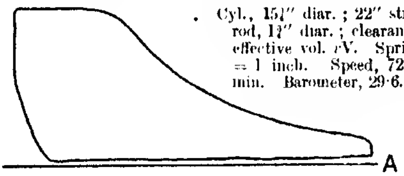
matically accurate. Steel wire connections should be used where possible to avoid errors by stretching.[1]

The vertical movement of the pencil over the oscillating drum traces out a closed curve, the **indicator diagram**, a typical example of which is shown in Fig. 4. After the diagram has been taken, the pencil is for the moment withdrawn from contact with the paper, and the piston of the indicator is put into communication with the atmosphere, the passage to the engine cylinder being at the same time shut off, by means of the two-way indicator cocks. The pencil is now pressed on the paper and a line AA is drawn, Fig. 4. This is called the **atmospheric line**; it represents on the diagram the pressure of the atmosphere.

[1] The errors of the indicator are considered in a paper by Osborne Reynolds and H. W. Brightmore, *Proc. Inst. C.E.*, 1896, vol. 83.

The closed curve of the diagram represents the cycle of pressure and volume changes through which the steam passes during a double stroke of the piston. The points of the stroke at which the valves open or close can be found from the diagram, and the area enclosed by the curve is proportional to the work done by the steam on one side of the piston during one double stroke. The shape of the curve shows if the steam ports and passages are properly designed to prevent undue throttling of the steam pressure, and also gives a good deal of information about the working of the engine. These matters

Fig. 4.—Indicator diagram.



Cyl., 15¼″ diar. ; 22″ stroke. Tail rod, 1¾″ diar. ; clearance, 7·1% of effective vol. *r*V. Spring, 30 lbs. = 1 inch. Speed, 72 revs. per min. Barometer, 29·6.

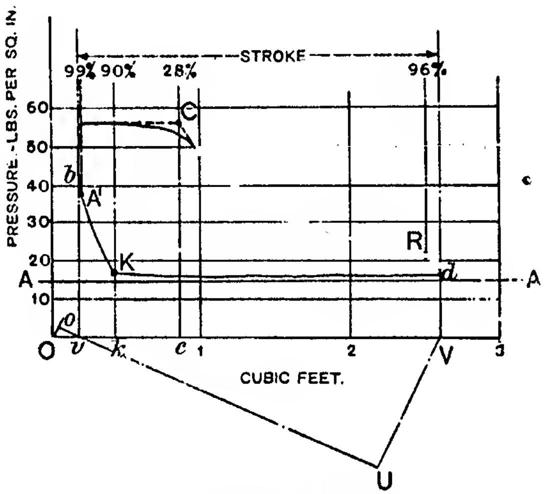


Fig. 5.—Indicator diagram calibrated.

will be brought out by considering in order: (*a*) the **calibration** of an indicator diagram; (*b*) the determination from the diagram of the **events of the cycle**, an event being the opening or closing of a valve; (*c*) the **ratio of expansion**; and (*d*) the determination of the **mean pressure** corresponding to the diagram.

(a) *The Calibration of an Indicator Diagram.*

When axes are added to the diagram so that the pressure and volume of the steam in the cylinder corresponding to every point on the curve can be read off, the diagram is said to be calibrated. The diagram of Fig. 4 is shown calibrated in Fig. 5. The calibration is done in the following way.

Draw a line OV parallel to the atmospheric line AA at a distance below it corresponding to the atmospheric pressure observed at the time the indicator diagram was taken, measuring the pressure to the scale of the indicator spring. The atmospheric pressure in pounds per square inch is found with sufficient accuracy by multiplying the height of the mercury barometer in inches by 0·49. The line OV is the volume axis, and is a line of zero pressure.

Draw tangents vb and Vd to the ends of the indicator diagram at right angles to the atmospheric line. Then the length vV along the volume axis between these tangents represents the effective volume of the cylinder.

The effective volume of the cylinder is the area of the piston less the area of the piston rod, multiplied by the stroke. With the data given on the diagram Fig. 4, the effective volume is 2·44 cubic feet. Hence the distance vV represents 2·44 cubic feet and the volume scale is fixed, but not its origin; because the point v represents the volume of the steam in the cylinder when the piston is just beginning its stroke, that is when the piston is at a dead point. The volume enclosed between the piston in this position and the cylinder cover, including the steam passages right up to the admission valve, is called the **clearance space** of the cylinder. The origin of the volume scale is to the left of the point v at a distance corresponding to the clearance volume measured on the volume scale just determined.

The volume of the clearance space is computed from the drawings of the cylinder. In some cylinders it can be measured experimentally by noting the quantity of water required to fill up the clearance space. Having found the clearance volume, take the point O to the left of v so that Ov represents the clearance volume to the scale on which vV represents the effective volume.

A convenient way to fix O is as follows:—

Through the point v draw a line vU inclined at any angle to vV; set off vU and vo to represent respectively the effective volume and the clearance volume, using any convenient scale for the purpose; join UV and draw oO parallel to UV: then O is the position of the origin.

A line through O at right angles to the atmospheric line is the pressure axis, along which the pressure scale is fixed by the scale of the spring with which the indicator diagram is taken.

A rectangular network of lines parallel to the axes may be added corresponding to equal intervals of pressure, and equal intervals of volume, as shown in the figure.

The pressure and volume corresponding to any point on the curve may now be read off. For example, at the point K the pressure is 17·5 lbs. per square inch absolute and the volume is 0·41 of a cubic foot.

If the engine is double acting there is a similar pressure-volume cycle belonging to the other end of the cylinder, the diagram of which is obtained by means of a second indicator. Sometimes the two diagrams are taken by one indicator which is placed alternately in communication with the two ends by means of connecting pipes, the connection to the ends and to the atmosphere being controlled by a three-way cock. It is preferable, however, to use two indicators, one placed at each end, in order to avoid the throttling action of the connecting pipes.

The two P.V. cycles in a double-acting engine may be distinguished by naming the cycle in which the steam pressure acts to push the piston rod out of the cylinder, the **outstroke cycle**, and the cycle in which the pressure acts to push the piston in the direction in which the piston rod is drawn into the cylinder, the **instroke cycle**.

(*b*) *Events of the Cycle.*

There are four points on the diagram, namely, A′, C, R, and K, marking four events in the distribution of steam to the cylinder during the pressure-volume cycle represented by the diagram. The steam valve opens at A′ just before the piston comes to rest at the dead point, and remains open until the piston has in the next stroke moved to the volume C. This cut-off point is not very sharply defined on the diagram, but it can be found pretty nearly by continuing the expansion curve upwards to meet a horizontal drawn through the point of maximum admission pressure as indicated by dotted lines. From C onwards no fresh steam is admitted to the cylinder, and providing that there is no leak past the piston or through the valves, the weight shut in is constant. At R the exhaust valve opens and the steam flows out of the cylinder into the atmosphere, or into the condenser, as the case may be, until the piston has come to the position K, where the exhaust valve is closed. A definite weight of steam is left in the cylinder, which, as the piston continues to move towards its dead point, is compressed.

There are thus four definite stages during one P.V. cycle.

(1) The **admission stage**, during which steam flows into the cylinder from the steam chest.

(2) The **expansion stage**, between the closing of the steam valve ("cut off") and the opening of the exhaust valve ("release"), during which the weight of steam shut in is constant, and during which the pressure falls as the volume increases.

(3) The **exhaust period**, between the opening and closing of the exhaust valve.

(4) The **compression period**, between the closing of the exhaust

valve ("compression") and the opening of the steam valve ("admission"). By measurement it will be found that the four events of "admission," "cut off," "release," and "compression," take place at 99, 28, 96, and 90 per cent. of the stroke respectively.

(c) *The Ratio of Expansion.*

The **Ratio of Expansion** is the ratio between the maximum volume which the steam can occupy during the cycle, and the volume which it occupies at the point of cut off. That is to say, it is the ratio OV to O*c*. In Fig. 5 the maximum volume measures 2·62 cubic feet, and the volume at cut off 0·86 of a cubic foot, so that the ratio of expansion is a little over 3.

The **Ratio of Compression** is the ratio between the minimum volume and the volume when the exhaust valve closes, that is, it is the ratio O*v* to O*k*, which in the case of the cycle shown is

$$\frac{0{\cdot}18}{0{\cdot}41} = 0{\cdot}44.$$

(d) *Mean Pressure.—Indicated Horse-power.*

The pressure acting on the piston in its direction of motion at any instant during the stroke from v to V, Fig. 5, is represented by the ordinate to the curve bCd corresponding to the piston position. During the return stroke from V to v the piston is driven against the back pressure, represented by the curve dKb, by pressure acting on the other side of the piston, assisted, when necessary, by energy drawn from the moving parts of the engine. The net work done by the steam on the piston during the P.V. cycle shown by the indicator diagram is represented by the area enclosed by the P.V. curve.

The mean effective pressure during the cycle is the average width of the diagram parallel to the pressure axis measured on the pressure scale. The work done during the cycle is this mean effective pressure multiplied by the effective area of the piston and by the length of the stroke.

The average width of the diagram is found with sufficient accuracy in practice by taking the average of ten strips equally wide, and a grid consisting of ten narrow steel rules joined together like a parallel rule may be used to facilitate the division of the diagram. Fig. 6 shows a diagram divided into ten equal parts. A dotted line is drawn at the middle of each part. The lengths of the dotted lines are written at the base of the diagram, and from these figures it will be seen that the average width is 0·72 in. This multiplied by 30, the scale of the spring, gives 21·6 for the mean effective pressure during the cycle.

The average width is also given by the quotient of the area of the diagram and the length of the diagram. The area can be measured

with a planimeter. The area of the diagram Fig. 6 measures 2·35 sq. ins. with a planimeter, and its length measures 3·27 ins., hence the mean width is by this method 0·719 in., and the corresponding mean pressure is 21·6 lbs. per square inch. This method of finding the mean width is the more convenient, and the more accurate, especially if a planimeter of the sphere and roller type is used. Some planimeters are provided with two points on the bar, which when set to the length of the diagram so adjust the scale of the instrument that the reading gives not the area of the curve but the mean width.

Let p represent the mean pressure in pounds per square inch; and let A represent the area in square inches on which this pressure acts, namely, the area of the cross section of the cylinder less the area of the piston rod, or the tail rod; and let L be the length of the

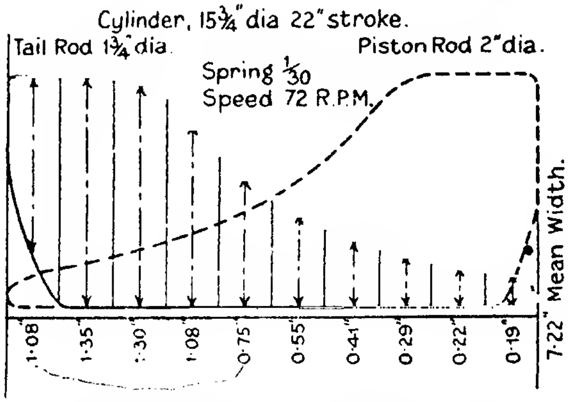


FIG. 6.—Indicator diagram, and determination of mean pressure.

stroke in feet; then the work done during the P.V. cycle represented by the indicator diagram is pLA ft.-lbs.

The rate at which work is done by the engine, or the work done per minute, is found by multiplying this by the numbers of cycles per minute N, and expressed in horse-power the rate of working is

$$\frac{pLAN}{33000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and this is the **Indicated Horse-power.**

In a single-acting engine there is one P.V. cycle per revolution, so that in this case N is the number of revolutions per minute. In a double-acting engine there are two P.V. cycles per revolution, so that if p and A have equal values in each cycle, N is equal to twice the number of revolutions per minute, that is to the number of strokes per minute.

More generally, let p_1 and p_2 be the mean pressures corresponding

to the two cycles of a double-acting engine, and let A_1 and A_2 be the respective effective areas on which these pressures act, then

$$\text{I.H.P.} = \frac{(p_1A_1 + p_2A_2)LN}{33000} \quad . \quad . \quad . \quad . \quad (2)$$

N is here the number of revolutions per minute, and L is the length of the stroke in feet. When the pressures are nearly equal and the areas are nearly equal, the average of the mean pressures multiplied into the sum of the areas may be used instead of the actual sum of the products in the brackets.

The labour of working out the indicated horse-power from the large number of indicator diagrams taken during a trial is reduced by calulating the **cylinder constant**, which is merely the product of the constant factors in the expression for the horse-power. The cylinder constant for a single-acting engine is $\frac{AL}{33000}$, and this multiplied by the number of revolutions per minute and the mean pressure gives the indicated horse-power. A constant like this is found for each end of the cylinder in a double-acting engine, and the indicated horse-power is then calculated for the cycle at each end separately. When the areas are nearly equal and the mean pressures are nearly equal, the one constant $\frac{(A_1 + A_2)L}{33000}$ is calculated, and this, multiplied by the number of revolutions per minute, and the average of the mean pressures at the two ends of the cylinder, gives the indicated horse-power.

The data relating to Fig. 6 are written above the diagram.

From these data, A_1, the effective piston area corresponding to the full line diagram, is 192·7 sq. ins., and the indicated horse-power is

$$\frac{21{\cdot}6 \times 192{\cdot}7 \times 1{\cdot}833 \times 72}{33000} = 16{\cdot}6$$

The dotted diagram, Fig. 6, is the corresponding P.V. cycle from the other end of the cylinder. The mean pressure found by a planimeter is 22·1 lbs. per square inch; the effective piston area is 191·9 sq. ins. The effective piston area is slightly smaller in this cycle because the area of the piston rod exceeds the area of the tail rod. Then

$$\text{I.H.P.} = \frac{22{\cdot}1 \times 191{\cdot}9 \times 1{\cdot}833 \times 72}{33000} = 16{\cdot}9$$

The I.H.P. for the cylinder is thus 33·5.

Applying the approximate rule, the sum of the areas is 384·6. And the average mean pressure is 21·85. Therefore,

$$\text{I.H.P.} = \frac{21{\cdot}85 \times 384{\cdot}6 \times 1{\cdot}833 \times 72}{33000} = 33{\cdot}6$$

The cylinder constant for the P.V. cycle shown in full line is

$$\frac{192{\cdot}7 \times 1{\cdot}833}{33000} = 0{\cdot}0107$$

and for the dotted cycle

$$\frac{191{\cdot}9 \times 1{\cdot}833}{33000} = 0{\cdot}01064$$

or approximately, for the combined cycles

$$\frac{384{\cdot}6 \times 1{\cdot}833}{33000} = 0{\cdot}02136$$

3. **Brake Horse-power.—Dynamometers.**—The rate at which work is done by the crank shaft of an engine against the resistances which oppose its rotation, is measured by the product found by multiplying together the torque exerted by the shaft and its angular velocity. If the torque is measured in pounds-feet, and the angular velocity in radians per second, then the product gives the rate of working in foot-pounds per second. The **Brake Horse-power** of the engine is found by dividing this product by 550. Thus, if a shaft exerts a torque of 1000 lbs.-ft. when the speed is 120 revolutions per minute, that is 4π radians per second, the shaft is transmitting work at the rate of 12,560 ft.-lbs. per second, which is at the rate of 22·9 H.P.

In order to measure the brake horse-power, a dynamometer must be applied to the shaft to measure the torque. Dynamometers are of two kinds, namely, those called **absorption dynamometers**, which apply to the shaft the whole of the external resistance against which it turns; and those called **transmission dynamometers**, which apply only a negligibly small resistance themselves, but enable the torque to be calculated from observations of the elastic deformation produced by the torque in the shaft itself, or in springs forming part of the apparatus.

The general principles underlying the action of a large number of absorption brakes are well illustrated by the Prony brake shown in Fig. 7. The figure is reproduced from Baron Prony's original paper published in *Annales de Chemie et de Physique* in 1821. The brake consists of two symmetrically shaped timber beams clamped to the engine shaft AEBD, the pressure being so regulated by the bolts b and b that a weight W just holds the brake in equilibrium as the engine shaft is driven round against the friction produced by the clamping.

Stops are provided to limit the motion of the brake when the load W fails to keep it from turning.

Let F be the whole frictional resistance produced by the clamping;
- r the common radius of the rubbing surfaces;
- W the force which keeps the brake in equilibrium;
- R the perpendicular distance from the axis of the shaft to the line of action of W;
- ω the angular velocity of the shaft in radians per second.

Then when the adjustments are made so that the engine runs

steadily at uniform speed, and so that the brake is held in equilibrium, clear of the stops, by the force W, the torque T, exerted by the shaft, is equal to the frictional torque Fr acting at the rubbing surfaces, and this is equal to the torque WR which holds the brake in equilibrium. That is

$$T = Fr = WR \quad \ldots \ldots \quad (1)$$

When more than one force is applied to the brake, then quite generally

$$T = Fr = \Sigma WR \quad \ldots \ldots \quad (2)$$

The summation sign includes the moment due to the weight of the brake itself unless it is balanced about the axis of rotation. The rate at which the shaft does work against a friction brake is then in general

$$\omega \Sigma WR \text{ ft.-lbs. per second}$$

or

$$\frac{\omega \Sigma WR}{550} \text{ horse-power} \quad \ldots \ldots \quad (3)$$

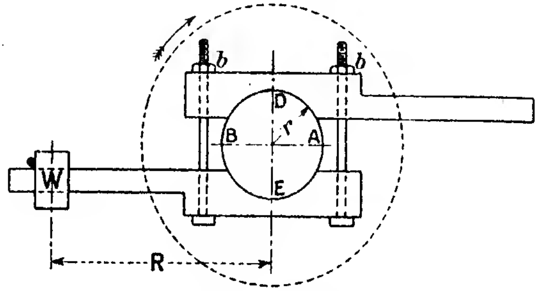


FIG. 7.—Prony brake.

The Prony brake shown in Fig. 7 is inconvenient to use in practice, and in its later forms the wood beams have been replaced, usually by a flexible band lined with wood blocks or some equivalent device to form a brake strap, and this is applied to the flywheel instead of to the shaft. The difficulty of continually adjusting the grip of the brake on the wheel or shaft to allow for variations in the coefficient of friction μ, due to heating or unequal lubrication, led to the design of self-compensating brakes. In this kind of brake any movement of the brake strap from the position of equilibrium causes the grip to be increased or relaxed, and in this way compensates for a decrease or an increase in the value of μ.

The rope brake, invented by Lord Kelvin in 1858, is simple, cheap, and accurate, and in some form or other is generally used when the power to be measured is not great. As will be seen from Fig. 8, the brake consists of a rope band wrapped round the flywheel, and fastened at one end to a spring balance w, the other end carrying a weight W, which hangs free. The ropes are spaced laterally by

the blocks B, B, B, B, which, by means of their projecting flanges, also prevent the ropes slipping sideways. By slackening away the rope to which the spring balance is attached, the weight W may be lowered on to the floor, thus entirely relieving the grip of the rope on the wheel. When the wheel is turning the rope band is held in equilibrium by the forces applied by the freely hanging weight W, the weight of the hooks and shackels attaching W to the rope, and the weight of the vertical part of the rope itself, on the one side, and the pull p shown on the spring balance corrected for the hooks and the part of the rope connecting the spring balance to the brake, on the other side. Hence the frictional torque against which the wheel turns is measured by

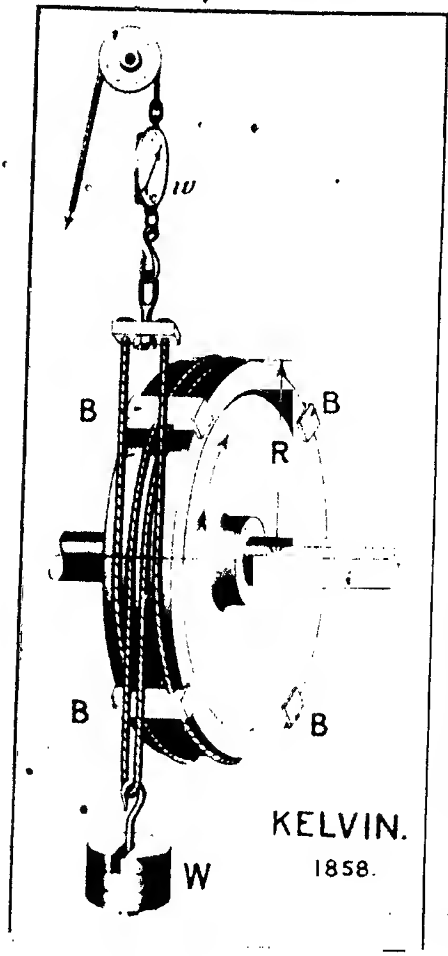


Fig. 8.—Rope brake.

$$W_1R - p_1R_1 = \Sigma WR \quad (4)$$

where W_1 and p_1 are respectively the total hanging weight and the corrected pull on the spring balance. If, as is often the case, R is the radial distance from the centre of the wheel to the centre of the rope, and $R = R_1$ the frictional torque is measured by

$$R(W_1 - p_1) \text{ lb.-ft.}$$

and the brake horse-power is given by

$$\frac{2\pi nR(W_1 - p_1)}{550} \quad . \quad (5)$$

where n is the number of revolutions of the wheel per second. The brake for a small engine should be made of a band of flat webbing ; R then becomes practically equal to the wheel radius.

As the coefficient of friction changes, the rope band turns slightly,

W rising or falling against the action of the spring balance until a stable condition of running is obtained.

The ratio $\frac{W_1}{p_1}$ is given by $e^{\mu\theta}$ where $e = 2{\cdot}718$; μ is the coefficient of friction and θ is the angle, measured in radians subtended at the centre by the arc of contact between the rope and the wheel. This ratio increases rapidly as θ increases, and therefore by making θ large p may be made a small fraction of W, thereby rendering errors of observation of the spring balance negligible. The essential feature of the brake is that the heavy load W hangs free, whilst the arc of contact between the rope and the wheel is sufficiently extended to make the pull on the tail end fastened to the spring balance a small fraction of W.

The work done against the frictional resistance of the brake is converted into heat, and the wheel must be water cooled to prevent overheating and also to ensure uniform conditions of temperature. Engines specially designed for testing are usually provided with a brake wheel having a trough-shaped rim, or else a trough ring is bolted to the flywheel rim. Water trickles continuously into the trough, and the centrifugal action holds the water as an inside lining against the bottom of the trough where it slowly evaporates and thus keeps the wheel rim cool. More constant conditions are obtained if the end of a waste pipe is bent over into the trough and flattened so that it scoops out water from the water lining as the wheel revolves. By this means a constant circulation of water is maintained through the wheel trough and heat can be carried away as fast as it is produced, so that the temperature of the rim is kept constant.

Example.—A rope brake wrapped round a water-cooled flywheel is held in equilibrium by a freely hanging weight of 430 lbs. and a pull on the tail end of 40 lbs. when the speed is 95 revolutions per minute. The perpendicular distance of the line of action of the weight from the centre of the crank shaft is 4·94 feet and the perpendicular distance of the line of pull of the spring balance is 4·5 feet from the centre. Find the brake horse-power.

$$\text{B.H.P.} = \frac{T\omega}{550}.$$

The torque exerted by the shaft is $(430 \times 4{\cdot}94 - 40 \times 4{\cdot}5) = 1944$ lbs.-ft.

The angular velocity is $\frac{95}{60} \times 2\pi = 9{\cdot}94$ radians per second.

Therefore the brake horse-power is $\frac{1944 \times 9{\cdot}94}{550} = 35{\cdot}1$.

The principle of an effective form of water brake invented by W. Froude[1] is shown in Fig. 9. Two similar castings, A and B, are

[1] Froude, W., "On a New Dynamometer," *Proc. Inst. Mech. Eng.*, 1877.

placed face to face on the shaft. One of these is keyed to the shaft whilst the other is free to turn. When placed together there would be formed an annular circumferential ring but for the fact that the ring is divided into a series of pockets by a number of equally spaced vanes inclined at 45 degrees to the vertical plane of rotation. These vanes are seen in the perspective view of the left casting A shown in the figure. When A is held fixed whilst B is rotated, centrifugal action starts the water with which the apparatus is filled into a series of vortices, and the consequent changes of momentum give rise to oblique reactions which are measured by the torque which must be applied to the casting A to hold it still. In

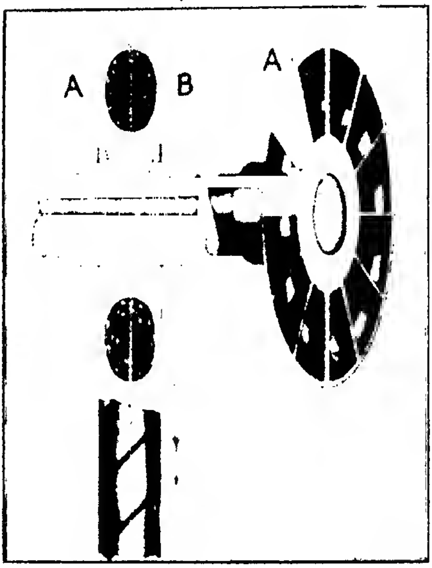


FIG. 9.—Froude brake.

an actual brake a wheel is formed by placing two castings like B back to back and bolting them to the shaft of the engine, or to a connecting shaft. This central wheel is enclosed in a casing formed of two castings like A. The vanes are so arranged that the vortices on the one side of the brake whirl in opposite direction to those on the other, thus balancing the reactions parallel to the axis. Froude used a brake of this kind to measure a torque of 105,566 lbs. at 90 revolutions per minute and the brake itself was only 5 feet diameter. Prof. Reynolds patented a water brake in 1887, using Froude's turbine to obtain the highly resisting spiral vortices in combination with specially arranged air passages to prevent the

accumulation of air brought into the brake by the water flowing through it. A full account of the Reynolds brake[1] is given in the paper quoted below.

In the Alden brake, Fig. 10, resistance to the motion of the crank shaft is obtained by turning a cast-iron disc K keyed to the shaft, against the frictional resistance of two thin copper plates C, C, held in a casing DDDH, which is free to turn on the shaft to which the cast-iron plate or disc is keyed. The pressure between the copper plates and the cast-iron disc K is produced by a regulated flow of water through the casing and the water also serves to carry away the heat developed by friction. The water enters at E and leaves at F. Oil circulates between the rubbing surfaces, entering at O and leaving at Q. Rings R, R are screwed up against the glands S, S to relieve the casing of the water pressure. The torque required to hold the casing still as the disc revolves measures the torque exerted by the shaft to which the disc is keyed. Brakes[2] of this kind have been designed by Prof. Goss to absorb the work done by a locomotive. The four coupled wheels of the engine rest upon similar wheels mounted in a pit, the uppermost part of their circumference being at rail level. The shafts of these wheels are provided with two Alden brakes each. With water in the casing at a pressure of 40 lbs. per square inch, pressing the thin copper plates on to the central disc, the resisting torque is 10,500 lbs.-ft. The diameter of the disc is 4·8″. A similar plant was installed at the St. Louis Exhibition, and it was from this plant that many of the data given below were obtained.

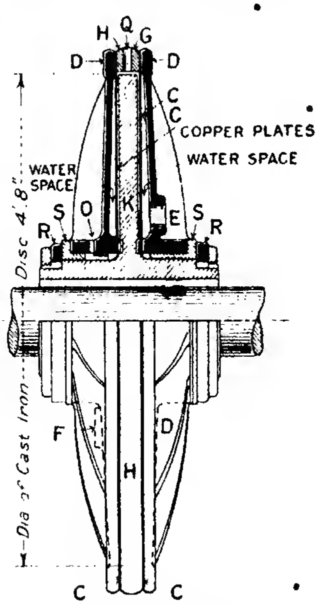


FIG. 10.—Alden brake.

One of the most convenient ways of measuring the brake horsepower is to couple the engine to a dynamo. The dynamo may be regarded as a **transmission dynamometer,** since the work done by the engine is transformed into electrical energy which may be used for lighting or power purposes; or it may be turned into heat again by wasting it in a resistance if it is not wanted. If E is the

[1] Reynolds, O., *Proc. Inst. C.E.*, vol. 99, 167. See also Reynolds and Moorby, "On the Mechanical Equivalent of Heat," *Phil. Trans.*, 1897, 306.

[2] *Amer. Soc. Mech. Eng.*, 1892, vol. 13.

potential difference produced at the brushes, measured in volts, and if A is the total current delivered at the brushes by the machine, measured in amperes, then the horse-power at the brushes is $\frac{EA}{746}$. Allowance must be made for the internal losses of the dynamo to get the shaft horse-power of the engine, but there is no difficulty in doing this. Further information regarding both absorption and transmission dynamometers will be found in an article by the author, entitled "Dynamometers," in the *Encyclopædia Britannica*, 11th edition.

4. Mechanical Efficiency.—The ratio between the brake horse-power and the indicated horse-power is called the mechanical efficiency of the engine.

The difference between the indicated horse-power and the brake horse-power is the horse-power wasted in overcoming the internal resistances of the engine.

If, for example, the indicated horse-power is found to be 95 whilst simultaneously the brake horse-power is 80, the mechanical efficiency is $\frac{80}{95} = 84 \cdot 2$ per cent., and 15 H.P. are required to overcome the engine resistances.

The internal resistance of an engine is caused by the resistance to relative motion at all the slides and joints. The resistance to relative motion contributed by any pair of surfaces depends upon the degree of completeness with which they are lubricated. Lubrication of any slide or journal may be intermittent, as when oil is supplied drop by drop to the parts, or it may be continuous. "Forced lubrication" is the term applied to that system of lubrication in which oil is continuously forced under pressure between the moving surfaces of the slides and journals of the engine.

The torque corresponding to the wasted horse-power may be regarded as the sum of all the resistances in the engine reduced to an equivalent torque on the crank shaft. The equivalent torque is calculated from $\frac{550 \,.\, \text{H.P.}}{\omega}$. For example, returning to the data above and assuming the crank-shaft speed to be 150 revolutions per minute, *i.e.* 15·7 radians per second, the torque corresponding to the 15 H.P. wasted in overcoming the engine resistances is

$$\frac{15 \times 550}{15 \cdot 7} = 526 \text{ lb.-ft.}$$

This may be regarded as the sum of the resisting torques at all the journals and the torques equivalent to the resistance at all the slides of the engine mechanism.

It is found that in general the resisting torque for different loads is made up of a constant part, namely, the torque corresponding to the indicated horse-power when the engine is running without

external load, and a part which is proportional to the external load on the engine.

If T_f is the frictional torque when the whole torque exerted by the engine is T, and T_0 is the torque corresponding to the indicated horse-power at no load, then

$$T_f = T_0 + mT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

approximately represents the law of resistance of the engine. The constant m is generally small, and in many cases it vanishes, showing that, as a first approximation, the frictional torque T_f may be regarded as constant.

If the law expressed in equation (1) is assumed to be correct, then the constants T_0 and m can be calculated from two values of the indicated horse-power together with the corresponding values of the brake horse-power.

For example, corresponding values of the indicated and brake horse-power of a particular engine, are respectively 57 and 47 at a speed of 100 revolutions per minute equal to 10·46 radians per second, and 38 and 30 H.P. at a speed of 90 revolutions per minute equal to 9·42 radians per second. Find the "no load" frictional torque and the constant m of equation (1).

Considering the first pair of values, the horse-power wasted in friction is 10, which, at the given speed, is equivalent to a frictional torque at the crank shaft of 526 lbs.-ft. At the second speed the horse-power wasted is 8, corresponding to a torque of 467. The torques corresponding to the two brake horse-powers are respectively 2470 lbs.-ft. and 2196 lbs.-ft. Therefore

$$526 = T_0 + m2470$$
$$467 = T_0 + m1751$$

From which $T_0 = 322$ lbs.-ft., and $m = 0·082$.

The frictional torque at any load can then be calculated from

$$T_f = 322 + 0·082T$$

The mechanical efficiency of the engine increases as the load on the engine is increased, being, of course, nothing at no load. A convenient expression for the mechanical efficiency for various degrees of loading can be formed as follows:—

Let I be the indicated horse-power when the brake horse-power is B, and let N be the revolutions of the shaft per minute. Let I_0 be the indicated horse-power at no load, and N_0 the corresponding speed. Then assuming that the frictional resistance of the engine varies as in (1)

$$\frac{I}{N} - \frac{B}{N} = \frac{I_0}{N_0} + m\frac{B}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Dividing through by I, and noting that $\frac{B}{I}$ is the mechanical efficiency η

$$\frac{1}{N}(1 - \eta) = \frac{I_0}{N_0 I} + \frac{m\eta}{N}$$

from which
$$\eta = \frac{1 - \frac{I_0 N}{I N_0}}{1 + m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If then the no-load horse-power is known the efficiency can be calculated for any indicated horse-power I at any other speed N, differing not too widely from the no-load speed N_0.

For example, if the indicated horse-power at no load is found to be 9 when the revolutions are 100 per minute, and if m is 0·08, then

$$\eta = \frac{1 - \frac{9N}{100I}}{1·08}$$

From this expression the mechanical efficiency at 50 I.H.P. when the speed is 95 revolutions per minute is

$$\eta = \frac{1 - 0·17}{1·08} = 0·77$$

and this increases to 0·85 if the indicated horse-power increases to 100 without any change in the speed.

The test of the correctness of the assumption of a linear frictional resistance law, as in equation (1), is, that the difference between the indicated horse-power divided by the speed and the brake horse-power divided by the speed plots as a straight line against the brake horse-power divided by the speed.

The mechanical efficiencies of steam engines working at full load with ordinary methods of lubrication vary between 80 and 90 per cent. depending upon the type, construction, workmanship, condition, and efficiency of the lubrication.

Self-lubricating engines of first-class workmanship exceed even 90 per cent. mechanical efficiency at full load. The term "self-lubricating" is applied to engines in which a continuous circulation of oil is maintained through all the rubbing parts by means of a small oil pump driven by the engine itself. The oil falls from the outer boundaries of the bearings into a well below the crank-shaft, in which the suction pipe of the pump is placed. The whole of the working parts are enclosed when such a system of forced lubrication is adopted. Engines with forced lubrication run noiselessly and the wear of the bearings is small.

As an example of the high efficiency which is possible with this kind of engine, the results of a trial made by Sir Alexander B. W. Kennedy on a Belliss engine supplied to the Sunderland Corporation may be quoted.[1]

[1] "Quick-revolution Engines," by Mr. A. Morcom, *Proc. Inst. Mech. Eng.*, 1897, Part 3.

TABLE 1.—TRIALS OF A BELLISS ENGINE.

Mean indicated horse-power.	Mean brake horse-power.	Mechanical efficiency.	Mean speed. Revs. per minute.
217·5	209·5	96·3	364·6
193·6	186·0	96·1	365·0
147·1	140·6	95·2	364·5
102·7	97·0	94·4	363·9
49·8	44·5	89·3	363·8

The torque corresponding to the horse-power can be calculated from

$$T = \frac{33{,}000 \text{ H.P.}}{2\pi N} = 14{\cdot}4 \text{ H.P.}$$

for the speed is so nearly constant that the torque may be taken proportional to the horse-power. In these circumstances the indicated

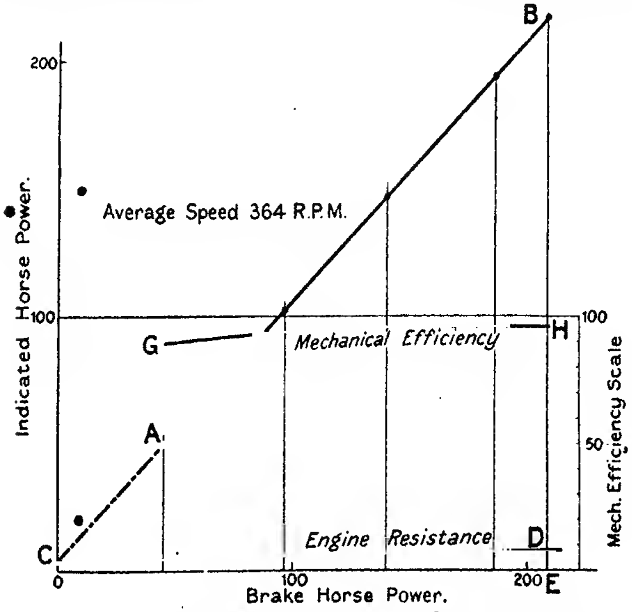


FIG. 11.—Relation between B.H.P. and I.H.P. Belliss engine.

horse-power plotted against the brake horse-power gives a diagram from which the law of resistance of the engine may be found.

Referring to Fig. 11 it will be seen that the points found by plotting the indicated horse-powers, given in Table 1, against the brake horse-powers fall on a straight line AB. This line produced to cut the vertical axis at C determines the no-load indicated

horse-power, and by measurement from the diagram this is found to be 4. The point D on the line CD is found by setting down from B the brake horse-power corresponding to the point B. The line CD slopes upwards, showing that the law of resistance expressed by equation (1) is in this example fulfilled. DE is the part of the indicated horse-power BE which is wasted in overcoming the internal resistance of the engine. The slope of the line CD is 0·019. Then if I_f is the part of the indicated horse-power required to overcome the engine resistance, and B is the corresponding value of the brake horse-power,

$$I_f = 4 + 0{\cdot}019B$$

Multiplying this through by the constant 14·4, the expression corresponding to equation (1) is at once determined. It is

$$T_f = 58 + 0{\cdot}019T$$

where T is the torque exerted by the engine shaft, that is the torque which would be measured on a brake applied to the flywheel, and T_f is the torque equivalent to the internal resistances of the engine reduced to the crank shaft.

The expression for the mechanical efficiency given in equation (3) becomes

$$\eta = \frac{1 - \frac{4}{I}}{1{\cdot}019}$$

in which I is the indicated horse-power.

In Fig. 11 GH is the curve of mechanical efficiency.

In plotting this diagram it is assumed that there is no sensible variation in the speed of the engine. In general a diagram like Fig. 11 can be found by plotting the several powers divided by the revolutions per minute. The quotients found are not the actual torques, but are numbers proportional to the torques.

5. On the Calculation of the Rate at which Heat Energy is Supplied to and Rejected from the Steam, and on the Thermal Efficiency.—In order to calculate the heat energy supplied to the steam it is necessary to know

(1) the quality of the steam in the steam pipe;
(2) the heat supplied from the heating circuit per pound of steam circulating;
(3) the weight of steam flowing along the steam pipe per unit of time.

The quality of steam is described as "**wet**," "**superheated**," or "**dry and saturated**". **Wet steam** is really a mixture of water and steam. The quality is specified by the "**dryness fraction**," that is the proportion of the mixture by weight which is steam. Thus steam of 0·95 dryness is a mixture of 95 per cent. steam and 5 per cent. water.

If the temperature of the steam is higher than the temperature of

evaporation corresponding to the steam pressure, the steam is said to be superheated. Corresponding values of the pressure and the evaporation temperature are given in the steam tables. To ascertain if steam is in the superheated condition, observe its pressure and temperature. Find from the table the evaporation temperature corresponding to the pressure. If it agrees with the observed temperature, the steam is not superheated. If the observed temperature is higher than the tabulated temperature, the steam is superheated.

Steam containing no water and at the temperature of evaporation is said to be dry and saturated. The slightest addition of heat to steam in this condition causes the temperature to rise, but the withdrawal of heat from it at the temperature of evaporation does not alter the temperature, but causes part of the steam to condense.

These points are considered more particularly in connection with the Motive-power Circuit. Meanwhile, assuming that the condition of steam in the steam pipe has been ascertained, the quantity of heat supplied from the heating circuit to produce 1 lb. of steam of the ascertained quality can be found in the following ways.

The properties of dry saturated steam are tabulated in the steam tables given at the end, pages 738 to 744. In these tables heat energy is reckoned "from water at 0° C.". The heat energy carried by 1 lb. of dry saturated steam is shown in Column 6 of Table 1, page 738. Thus 1 lb. of dry saturated steam at a pressure of 100 lbs. per square inch possesses or carries 662 lb.-cals. of heat energy more than it carries as water at 0° C.

If the feed water is supplied at a temperature of t° C., it carries with it into the boiler the heat energy tabulated for t° C. under the heading, "Total Energy of Water". Hence the heat energy which must be supplied by the heating circuit to the working agent in the motive-power circuit is the difference of these two quantities, that is

$$\left.\begin{array}{l}\text{Heat supplied from heating circuit to}\\ \quad\text{form 1 lb. of dry saturated steam}\end{array}\right\} = I_s - I_w = H \quad . \quad . \quad (1)$$

where I_s is the total heat energy of the steam given in Column 6 of Table 1, corresponding to the pressure in the boiler;

and I_w is the total heat of the water given in Column 7 of Table 2, corresponding to the feed-water temperature, which is generally taken equal to the temperature in the hot well.

Thus the heat H supplied from the heating circuit to produce 1 lb. of steam at a pressure of 100 lbs. per square inch from feed water at 50° C. is

$$H = 661{\cdot}8 - 49{\cdot}9 = 611{\cdot}9 \text{ lb.-cals.}$$

If the steam in the pipe has a dryness fraction q, the weight of water per pound of the mixture is $(1-q)$ pounds. And the heating circuit supplies a quantity of heat

$$H = I_s - (1-q)L - I_w \text{ lb.-cals.} \quad . \quad . \quad . \quad . \quad (2)$$

to produce 1 lb. of steam at the pressure corresponding to I_s, from feed water at the temperature corresponding to I_w.

L is the latent heat of the steam corresponding to the energy I_s.

Thus to produce 1 lb. of wet steam with $q = 0{\cdot}95$ at a pressure of 100 lbs. per square inch from feed water at 50° C.

$$661{\cdot}8 - 0{\cdot}05 \times 496 - 49{\cdot}9 = 587{\cdot}1 \text{ lb.-cals.}$$

must be supplied from the heating circuit.

I_s and L for the given pressure are taken from Table 1, and I_w for the feed temperature from Table 2.

Finally, if the steam is superheated at constant pressure, the heat supplied by the heating circuit is

$$I' - I_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where I' is the total heat of steam at constant pressure. Values of I' in terms of the pressure and temperature are given in Table 3. Thus the heat required to form 1 lb. of steam at a pressure of 100 lbs. per square inch, from feed water at 50° C., and then to superheat to 250° C., is, from Tables 3 and 2

$$707{\cdot}6 - 49{\cdot}9 = 657{\cdot}7 \text{ lb.-cals.}$$

A useful diagram is obtained by plotting the total heat I_s of dry saturated steam, the latent heat L, the total heat of water I_w, and the pressure corresponding to the temperature of evaporation : against the temperature of evaporation. Such a diagram is described in Section 57 in connection with Figs. 63, 64, below, and a large diagram of this kind is supplied with the book.

Having ascertained the condition of the steam and the heat supplied by the heating circuit per pound, there remains to find the weight of the working agent which is flowing round the motive-power circuit.

If there were no losses of steam at the safety valve, or by leak, the average rate at which steam flows in any part of the circuit could be found either by measuring the average rate at which the feed pump supplies water to the boiler, or equally by the average rate at which condensed steam is discharged from the air pump. A measurement of the air pump discharge is always to be preferred in computing the supply of steam along the feed pipe to the engine, because the steam lost in the boiler is not in this measurement credited to the engine as it would be if the flow were estimated from the boiler feed.

The rate at which steam flows along the steam pipe of an engine unprovided with a condenser can only be estimated from the average rate at which the feed pump supplies water to the boiler.

Let W be the weight of water discharged by the air pump in t minutes. Then $\frac{W}{t}$ is the flow of steam along the steam pipe per minute. Then the heat energy carried by the steam from the heating circuit to the engine per minute is

$$\frac{HW}{t} \text{ lb.-cals.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Some of the heat supplied by the heating circuit to the motive-power circuit is lost between the boiler and the engine, and in consequence the condition of the steam is not exactly the same when it enters the cylinder as when it left the boiler. Also so far as the engine cylinder is concerned, the temperature of the steam in the exhaust pipe is the temperature at which the condensed steam would appear in the hot well if the condensing plant were perfect. Hence, when considering the net supply of heat to the cylinder alone, the condition of the steam supplied is measured just before the steam passes through the engine stop valve, and the temperature of the feed water is assumed to be equal to the temperature of the steam in the exhaust pipe when reckoning the value of I_w in the above expressions. The value of H found in this way is the proportion of the heat actually supplied by the heating circuit which can properly be credited to the cylinder. The remaining portion is required to balance the losses in the steam pipe and in the condensing plant.

Again, in comparing the results of engines together it is often convenient to assume that the steam passes into the exhaust pipe at the lowest possible pressure in the circumstances, and then to take the temperature corresponding to this pressure as the feed temperature. In the case of non-condensing engines the lowest possible pressure in the exhaust pipe is the atmospheric pressure. The feed temperature to be assumed for reckoning H is then 100° C.

In the case of a condensing engine the lowest pressure is that to which it is practicable to reduce the steam in a condenser. The value assumed by Mr. Willans in his classical series of trials was for comparative purposes fixed at 1·26 lbs. per square inch, corresponding to a temperature of 43·3° C. In condensers used with steam turbines, where it is necessary to obtain as low a pressure as possible, the pressure has been reduced even below 1 lb. per square inch. So that for comparative purposes the feed temperature in condensing engines may be reckoned 40° C.

The actual temperature in the feed tank will always be considerably lower than these assumed temperatures.

The ratio between the rate at which heat is converted into work in the cylinder and the rate at which heat is supplied to the engine is called the thermal efficiency of the engine. The rate at which an engine transforms heat into work is measured by the indicated horse-power, and this multiplied by 33,000, and divided by 1400, the mechanical equivalent of heat, expresses the rate in lb.-cals. per minute.

Thus the rate at which an engine transforms heat energy into mechanical energy is

23·56 I.H.P. lb.-cals. per minute

or 1414 I.H.P. lb.-cals. per hour.

The rate at which heat is supplied to the engine per minute is

$$\frac{WH}{t} \qquad (4)$$

where W is the water condensed in time t, and H, as described above, is the net heat supplied to the cylinder per pound of steam.

It follows that with t in minutes,

$$\text{the thermal efficiency is } \frac{23{\cdot}56 \text{ I.H.P.}t}{\text{WH}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or with t in hours,

$$\text{the thermal efficiency is } \frac{1414 \text{ I.H.P.}t}{\text{WH}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

By way of example, calculate the thermal efficiency of a steam engine in each of the following cases:—

1. The temperature of the feed water is the same as the temperature of steam in the exhaust pipe of the engine, 54° C.
2. The temperature of the feed water is 40° C., the temperature corresponding with the lowest practicable pressure it is commercially desirable to produce in the condenser.
3. The temperature of the feed water is the temperature of the water in the hot well, namely, 30° C.

In each case the engine is supplied with dry saturated steam at a temperature of 200° C. (225 lbs. per square inch), at the rate of 3200 lbs. per hour when the indicated horse-power is 200.

In the first case H = 671 − 54 = 617 lb.-cals per lb.
In the second case H = 671 − 40 = 631 " "
In the third case H = 671 − 30 = 641 " "

The three efficiencies, calculated from expression (6), are then 0·143, 0·140, and 0·138.

The heat energy carried away from the engine by the cooling circuit is computed from observations of the rate at which water circulates through the condenser, and the rise of temperature produced in the circulating water in its passage through the condenser. Let W_c be the weight of water forced through the condenser by the circulating pump in t minutes. Further let t_1 be the temperature of the water as it enters the condenser, and t_2 the temperature as it leaves it. Then the heat energy taken up per pound of flow is $(t_2 - t_1)$ lb.-cals., and the total heat carried away by the cooling circuit is

$$\frac{W_c(t_2 - t_1)}{t} \text{ lb.-cals. per minute.}$$

The way to measure the rate at which heat energy is produced in the heating circuit, and transferred from it to the motive-power circuit, is discussed in Chapter II.

26. The Measurement of the Flow of Water.—The rate at which water is evaporated, and at which steam passes along the steam pipe to the engine, does not necessarily correspond with the rate at which water is supplied to the boiler by the feed pump, because even if the pump worked continuously it is not possible to regulate its speed so that the rate at which it delivers water to the

boiler is exactly equal from instant to instant to the rate at which the engine demands steam from the steam pipe. The difference between the rate of supply and demand is met by the water in the boiler, and this difference is indicated by the changing level of water in the gauge glass.

The average rate at which the feed pump supplies water to the boiler is, however, equal to the average rate at which the steam is supplied to the engine, neglecting losses at the safety valve, injector, and by leak, providing that at the beginning and at the end of the interval of time during which the measurement is made the level of water in the boiler is the same.

There is always some uncertainty about the level of the water

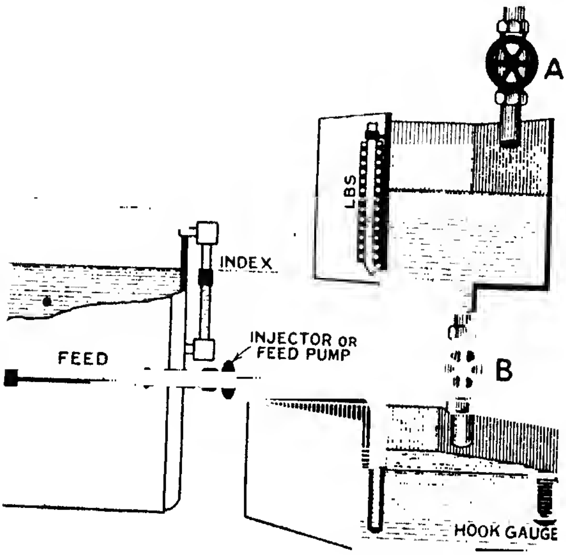


FIG. 12.—Tanks for measurement of feed water.

in the boiler as shown by the gauge glass, because the water is rarely quite still, and moreover a small error in the reading of the gauge glass corresponds to a considerable quantity of water storage, hence the time interval over which the measurement is made should be considerable in order to reduce the initial and the final errors of the gauge glass reading to negligible percentages.

One way of measuring the feed water is to place the suction pipe of the feed pump in a separate feed tank fitted with a pointed gauge, as shown diagrammatically in Fig. 12, where the gauge is shown in the form of a hook riveted to the side of the tank. The object of the point gauge is to fix a definite water level to which the tank can always be filled. The point of the gauge immersed in the water

and looked at from above indicates a definite water level with great accuracy. This feed tank is used in combination with some means of fixing the water level in the boiler, and this is usually accomplished by means of a thin brass sleeve on the gauge glass which can be pushed up to fix any particular water level shown in the glass.

At the beginning of a measurement, the pump is temporarily stopped, and the feed tank is filled up to the level of the point of the gauge. The gauge glass sleeve is then pushed to the level of the water in the glass. The time is noted and then the feed pump may be started when required. The feed tank is kept replenished at intervals during the trial by adding to it weighed quantities of water. A bucket and a weighing machine are required for this purpose. Towards the end of the trial the feed is regulated so that water in the gauge glass is slightly above the original level, and the water in the feed tank is slightly below the original level. Then the feed pump is stopped and the gauge glass is watched carefully, and the trial ends at the instant when the level falls to the original level. The feed tank is then filled by weighed quantities of water until the water reaches the original level as fixed by the point gauge. The total quantity of water added to the feed tank during the trial together with the quantity added after the trial to restore the original water level in the tank is the quantity of water W supplied to the boiler during the time interval t. The average rate at which steam flows along the steam pipe, assuming that there are no losses in the boiler by leak or by escape at the safety valve or injector, is then $\frac{W}{t}$ pounds per minute.

The feed tank may be kept replenished from a calibrated supply tank fixed permanently above it as shown in Fig. 12. This is a convenient laboratory arrangement. The stop valve A is on the water main, and the stop valve B is used to let a known weight of water flow from the upper to the lower tank. The water is here measured by volume by means of the graduated scale of pounds at the side of the upper tank. A better way is to place the supply tank on the platform of a weighing machine, and so weigh each addition to the contents of the feed tank.

The best way of all is to place the feed tank on the platform of a weighing machine, and so weigh directly the water withdrawn from the tank by the feed pump during a noted time interval. This is a method which can of course only be used with relatively small plants, because the feed tank must in this case be large enough to hold sufficient water for at least a three hours' trial.

An accurate measurement of the air pump discharge can be made in quite a short time interval, since it is only necessary to divert the water into a tank and weigh it, or alternatively to measure it volumetrically.

If the discharge is to be measured over a long time interval, two tanks may be arranged side by side, so that the water may be

guided first into one, and then, when this is full, into the other, by means of some deflecting device fixed to the end of the discharge pipe. These two tanks may be calibrated with a scale and gauge glass at the side, or each may be supported on a weighing machine as shown in Fig. 13.

The sizes of the tanks, and the cocks for emptying them, must be so adjusted that there is ample time to make the necessary observations on a full tank, and then to empty it in readiness for re-filling whilst the other tank is filling. The cocks A and B, Fig. 13, may empty the tanks direct into the feed tank.

Another way of measuring the flow, and this is especially useful where the flow is large, is to lead the water over a weir or let it flow through an orifice, and then record the head of water at short intervals during the trial. These records together with the dimensions of the weir or orifice furnish data from which the flow can be calculated with accuracy. This method has the advantage that a

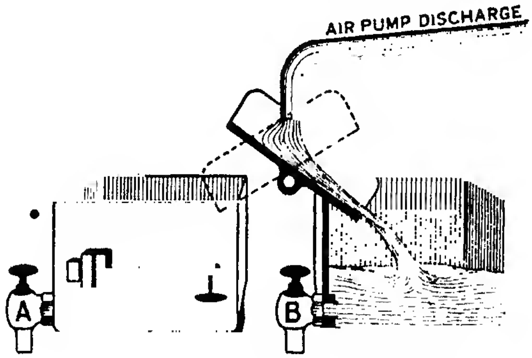


FIG. 13.—Measurement of air pump discharge by twin tanks.

large error of the kind which results from an incorrect tally in the twin tank method is impossible, because the head, which is the only thing under observation, remains very nearly constant over long periods, and therefore any large error of reading is more easily detected.

The discharge over a weir or through an orifice varies approximately as the square root of the head producing the flow, so that a small variation of the discharge is accompanied by a large variation of the head if the head itself is large. For example, the discharge through a small orifice produced by a head of one foot is only doubled if the head increases to four feet. With a weir the head is necessarily small, so that the advantage of an open scale for easy observation is lost. For this reason, it is preferable to measure the flow when possible by means of an orifice of such size in relation to the flow that the head required to produce the flow is large.

In any apparatus of this kind it is necessary to still the water as

it approaches the orifice, and this is usually accomplished by placing a series of screens made of perforated plates or of wire mesh to act as baffle plates. When the water is stilled, the discharge through a sharp-edged circular orifice is given approximately by

$$\text{discharge in pounds per minute} = 98{\cdot}5\, d^2 \sqrt{h} \quad . \quad . \quad (1)$$

Here d is the diameter of the orifice in inches, h, the head in feet, is the vertical distance from the centre of the orifice to the free water level in the tank. An orifice tank devised by the author, and suitable for laboratory use, is shown in Fig. 14. The object aimed at is to bring the stream lines of the water into a condition of parallel flow as they approach the orifice. The water enters the tank by the pipe A, and the discharge is made against a submerged cup-shaped baffle B. The water passes up to and over the rounded lip of the cup and then flows down towards the bottom through the annulus between the edge of the cup and the sides of the tank. The baffle is on the underside gradually reduced in section; the stream of water flowing downwards is consequently gradually enlarged in section and its velocity proportionally reduced. The baffle is finally reduced to a point placed vertically over the orifice, so that the water, as it descends, is enlarged to a stream of constant diameter moving at the central part in parallel stream lines of nearly constant velocity. The expression

$$135ah^{0{\cdot}47}$$

gives the discharge in pounds per minute through an orifice a sq. ins. area when the head is h feet, and the head sufficient to drown the baffle. The expression is valid for orifices up to 0·5 in. diameter when the diameter of the tank itself is 9 ins. The discharge is not affected by variations in the distance of the point of the baffle above the centre of the orifice providing that the baffle is drowned. A suitable distance is about ¼ in.

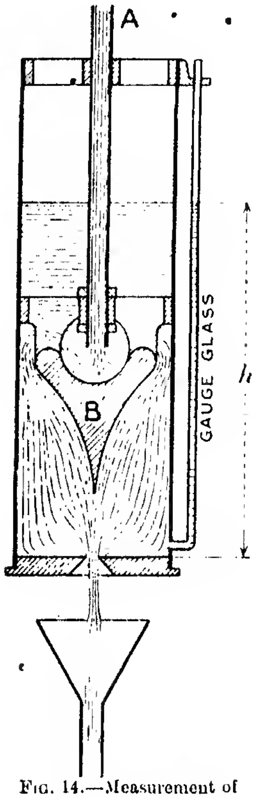


Fig. 14.—Measurement of flow. Orifice tank.

§ 7. Performance Estimated from the Steam Supply. The Willans Line.—A reference to the steam tables shows that the heat carried by a pound of dry saturated steam increases only very slowly with the pressure. A change of pressure from 100 to 200 lbs. results in an increase of the heat carried by only about 1 per cent. The heat

supplied to the engine is thus practically proportional to the steam supply, that is to say, the rate at which feed water is supplied to the boiler, or if preferred, the rate at which steam is condensed in the condenser and discharged by the air pump, measures the rate at which heat energy is supplied to the cylinder to a near approximation.

The weight of steam supplied to the engine per indicated horse-power-hour is a quantity which is widely used by engineers as a record of performance.

The record states the weight of water which must be supplied to the boiler and evaporated therein (neglecting losses) in order to

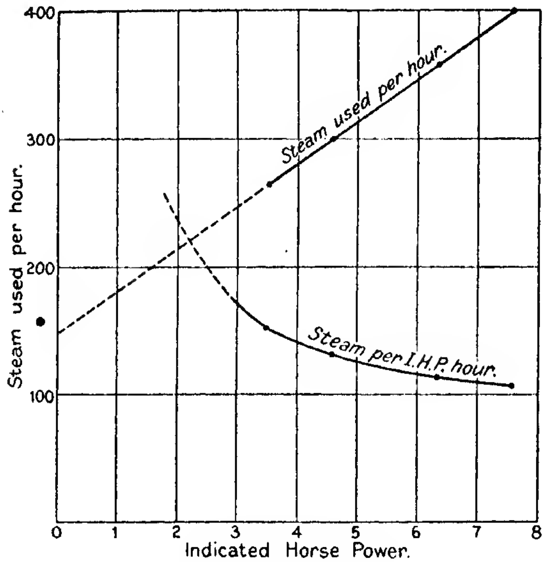


FIG. 15.—Willans line.

obtain 1,980,000 ft.-lbs. of mechanical energy from the engine cylinder.

If, for example, an engine uses 400 lbs. of steam per hour when developing 8 I.H.P., its performance is 50 lbs. of steam per I.H.P.-hour.

If the total steam used per hour is plotted against the indicated horse-power developed, the points so found fall on a straight line when the power of the engine is regulated by reducing the initial pressure, the cut off in the cylinder being maintained at a constant fraction of the stroke. The line drawn through these points cuts the vertical axis above the origin, as will be seen from Fig. 15, where the points plotted represent the results of trials on a small steam

engine in which the cut off was maintained constant at 60 per cent. of the stroke whilst the power was regulated by a throttling governor.

The figure shows that the total steam per hour required by the engine is a linear function of the indicated horse-power of the form

$$S = w + k \cdot \text{I.H.P.} \qquad (1)$$

This is known as Willans' Law, in honour of the discoverer. It was first made known by Willans in his paper, "Steam Engine Trials," published in the *Proceedings of the Institution of Civil Engineers*, April 11, 1893.

The law does not apply to engines in which the regulation of the power is done by varying the cut off while maintaining the initial pressure constant. In these conditions the line is concave upwards.

The performance expressed in pounds of steam per indicated horse-power-hour is, for an engine governed by throttling, given by

$$\text{lbs. of steam per I.H.P.-hour} = \frac{w + k \cdot \text{I.H.P.}}{\text{I.H.P.}} \qquad (2)$$

The constants w and k for a particular engine can be determined from two trials, but it is of course desirable to make a series. The "Willans Line" should always be plotted, even as a trial proceeds, because it enables mistakes of observation to be detected.

The equation to the line in Fig. 15 is

$$S = 147 + 33 \text{ I.H.P.}$$

and the weight of steam required by this engine per indicated horse-power-hour is

$$\frac{147 + 33 \text{ I.H.P.}}{\text{I.H.P.}} \text{ lbs.}$$

At 10 I.H.P., for example, 47·7 lbs. of steam must be supplied to the cylinder per I.H.P.-hour, whilst at 1 I.H.P. the supply must be increased to 180 lbs. per I.H.P.-hour.

The curve corresponding to the performance of this engine is plotted in Fig. 15.

If the Willans Line passes through the origin, the constant w is nothing and the curve showing steam per I.H.P.-hour becomes a straight line parallel to the I.H.P. axis, showing, as is otherwise obvious, that the performance is the same at all powers. The nearer, therefore, the curve can be brought down towards the origin, the smaller is the increased demand for steam per I.H.P.-hour as the load on the engine is decreased.

The engine whose performance is here shown appears to be very wasteful, but the curves give a fair record of the performance of a small non-condensing steam engine.

Records of the performance of larger engines are given in the table on page 263.

8. Principle of the Conservation of Energy.—The Principle of the Conservation of Energy has been tacitly assumed in the previous sections. It is desirable at this stage to consider the principle in some detail, because of its fundamental importance in connection with heat engines.

In its broadest form the Principle of the Conservation of Energy is an assertion, founded on experience sufficiently broad and extended to enable the assertion to carry all the weight of a natural law, that the stock of energy in the universe is constant and that no part of this stock can be destroyed neither can the stock be increased by any known means.

What is true of the universe is equally true of any isolated part of it. Whatever be the stock of energy possessed by the isolated part at the instant of isolation, no part of this stock can be destroyed, neither can the stock be increased, by any process which takes place within the system itself. The system may receive energy from outside and may expend energy against resistance external to it, but whilst receiving energy or expending it, it cannot create or destroy energy.

Imagine, therefore, an isolated system receiving energy and at the same time expending energy against an external resistance. Since the system cannot destroy or create energy, the stock of energy it originally possessed will be changed by an amount exactly equal to the difference between the energy it receives and the energy it expends. When energy is received and expended by a system continuously the idea of time requires to be specifically introduced.

The statement of the Principle of the Conservation of Energy with regard to an isolated system then becomes—

$$\left.\begin{array}{l}\text{Rate at which energy}\\ \text{is received by the}\\ \text{system}\end{array}\right\} = \left\{\begin{array}{l}\text{Rate at which the}\\ \text{stock of energy}\\ \text{possessed by}\\ \text{the system is}\\ \text{changed}\end{array}\right\} + \left\{\begin{array}{l}\text{Rate at which}\\ \text{the system}\\ \text{expends en-}\\ \text{ergy exter-}\\ \text{nally to itself.}\end{array}\right.$$

A steam plant, or any heat engine, may be regarded as an isolated system which receives energy from the combustion of fuel, and utilizes it, partly to change its stock of energy and partly to do work external to itself.

The energy produced by the combustion of fuel is received into the heating circuit of a steam plant at a steady rate, and it is useful to imagine that the energy flows into and through the plant like a stream which as it flows divides into separate channels.

The application of the Principle of the Conservation of Energy enables the assertion to be made that the plant cannot by any action within itself either increase or diminish the volume of this stream flowing past any section of it without storage in an imaginary reservoir. Thus assuming a steady flow with no storage, the energy,

stream may be divided by imagination into certain well-defined streams of energy, the smaller streams dividing again into still smaller streams. This flow may be pictured as continually taking place during the working of the engine like the flow of the Nile river, which on its way to the sea divides itself into the many smaller streams which form its delta.

9. Equivalence of Different Kinds of Energy.—Energy appears in different guises, as for instance mechanical energy, electrical energy, and heat energy. A system receiving energy of one kind may within itself transform it into energy of another kind. The Principle of the Conservation of Energy implies that whatever transformation takes place a quantity of one kind of energy must disappear, or go out of existence, exactly equivalent to the quantity of the new kind into which it is transformed.

Thus, for every unit of mechanical energy spent in producing heat, a definite quantity of heat comes into existence.

The equivalent of the 1 lb.-Fahr. unit of heat in mechanical energy, determined by Joule from a series of experiments begun in 1843, was originally stated to be 772 ft.-lbs. of mechanical energy.

The lb. Fahr. thermal unit, or the British thermal unit, as it is usually called, is defined to be the quantity of heat which will raise 1 lb. of water from 39° to 40° Fahr.

Later experiments have shown that 778 is more nearly correct.

Reckoned in terms of the lb.-calorie heat unit, the equivalent is

1400 ft.-lbs. of mech. energy = 1 mean lb.-cal. unit of heat energy.

The mean lb.-calorie is $\frac{1}{100}$ of the quantity of heat which will raise 1 lb. of water from 0° to 100° C.

The value 1400 for the **mechanical equivalent of heat** is used throughout this book, together with the mean lb.-calorie heat unit. The energy developed by the combustion of fuel may therefore be expressed in thermal units or in mechanical units. Similarly, the work done on the piston of an engine may be expressed either in foot-pounds or in pound-calories.

Equivalent numerical values of mechanical energy, heat energy, and electrical energy are brought together in Table 2 for convenience of reference.

The horizontal lines give the numerical expression for equal quantities of energy when measured mechanically, electrically, or thermally.

TABLE 2.—ENERGY EQUIVALENTS.

Energy expressed as		
Mechanical energy.	Heat energy.	Electrical energy.
Mechanical energy measured in ft.-lbs.	Heat measured in pound-centigrade units, or pound-calories	Electrical energy measured in joules, or watt-seconds, or volt-coulombs.
1 1400 0·737 = $\frac{3}{4}$ app.	$\frac{1}{1400}$ 1 53 × 10^{-5}	1·356 = $\frac{4}{3}$ app. 1899 1
Work measured in horse-power-hours (1 H.P.-hour = 1,980,000 ft.-lbs.)	Pound-calories	Kilowatt-hours (1 kilowatt-hour = 3,600,000 joules)
1 707 × 10^{-6} 1·34 = $\frac{4}{3}$ app.	1414 1 1896	0·745 = $\frac{3}{4}$ app. 527 × 10^{-6} 1

Complete lists of energy equivalents, as well as tables for the conversion of the quantities used in engineering work from one system of units to another, are given in "Ready Reference Tables," by Carl Hering, M.E., Chapman & Hall, Ltd., London.

10. Heat Energy Streams.—In order to illustrate the conception of a heat energy stream the heat energy stream flowing through a compound condensing engine is drawn in Fig. 16, so that the width at any point of the stream, or of the several divisions of it, represents approximately to scale the heat energy flowing, reckoned as a percentage of the heat flow produced by the combustion of fuel in the furnace.

The figures have been deduced from the records of an experiment published in the *Transactions of the American Society of Mechanical Engineers*, vol. 16.

The material part of the steam plant which forms the environment of the streams is indicated diagrammatically, two noticeable features being the lines sloping up from left to right; the upper line represents the heating surface of the boiler and economiser across which heat flows from the heating to the motive-power circuit, and the lower line represents the cooling surface of the condenser across which heat flows from the motive-power circuit to the cooling circuit.

In the actual plant the combustion of the fuel originated a flow of 102,000 lb.-cals. per minute. The actual flow at any point in the stream can be found by multiplying the figure shown on the stream by 1020.

Looking at the flow as a whole, it will be observed that there are small streams returning on to the main stream. The inner circular path represents the heat stream flowing through the steam jacket of

the engine cylinder; the outer represents the steam passing through the feed pump and through the feed-water heater into the boiler. The heat stream diverted by means of the feed-water heater from the furnace gas to the feed water is also indicated.

Starting from the grate with a stream 100 units wide, 5·5 per cent. of it is lost by radiation from the boiler before it divides into two well-defined streams, the upper and smaller stream passing through the feed-water heater and the lower flowing across the heating surface into the boiler. The main stream has now been reduced to 71·7 per cent. of the original stream. Following the upper branch, 0·5 per cent. is lost by the stream on its way to the feed-water heater, and then a division into three branches takes place; the

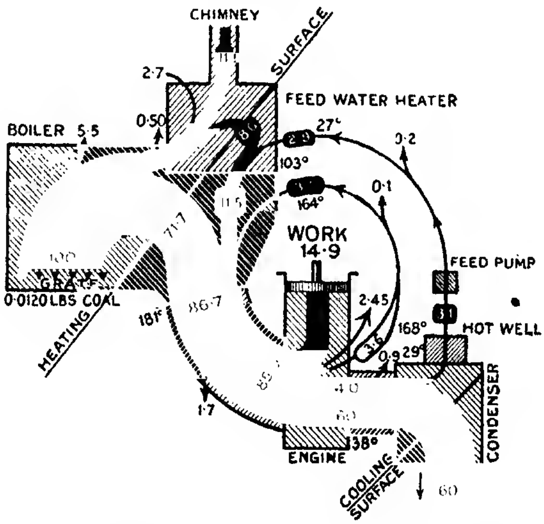


FIG. 16.—Heat energy stream. Compound condensing engine.

small branch to the left represents a loss of 2·7 per cent. in radiation; the middle branch carries away 11 per cent. up the chimney, and the branch to the right, flowing across the heating surface of the feed-water heater, joins with the small flow of heat brought in by the feed water to form a branch whose width is 11·5 per cent. of the original width when it joins the main stream in the boiler. This is again slightly increased by the return from the jacket circulation bringing 3·5 per cent., to the total of 86·7 per cent., the width of the stream which enters the steam pipe. Losing from the steam pipe 1·7 per cent., the main stream enters the cylinder carrying 85 per cent. of the flow originated at the grate. The stream now divides into four branches. The upper branch, representing 14·9 per cent. of the original flow, is annihilated as heat, and

is transformed into mechanical energy by the action of the carrying agent on the piston of the engine. Sixty-four per cent. of the original flow passes across the cooling surface into the circulating water in the condenser, 2·45 per cent. is lost by radiation from the engine, and 3·6 per cent. is carried by the jacket circulation and 3·5 per cent. is delivered back to the boiler. The branch in the condenser divides into two streams, the small upper branch carrying 4 per cent. of the original flow into the hot well, where further on in its flow it is slightly reduced by losses, until finally it arrives at the feed-water heater carrying 2·9 per cent. of the original flow. The lower branch carries away 60 per cent. of the original flow into the cooling pond.

The mechanical work done by the plant is 14·9 per cent. of the heat produced by combustion, therefore the thermal efficiency of the plant as a whole is 14·9 per cent.

The cylinder receives 85 per cent. of the original stream, of which 3·5 + 2·9 per cent. is returned to the boiler, leaving a net supply of 78·6. So that the thermal efficiency of the engine, including the condenser, is $\frac{14 \cdot 9}{78 \cdot 6} = 18 \cdot 9$ per cent.

If the condensing plant and feed pump had been perfect the heat in the steam condensed at a temperature corresponding to the temperature in the exhaust pipe would have been 4 per cent. of the original stream as indicated on the diagram. In which case the net supply of heat to the cylinder is

$$85 - (4 + 3 \cdot 6) = 77 \cdot 4$$

and the thermal efficiency

$$\frac{14 \cdot 9}{77 \cdot 4} = 19 \cdot 2 \text{ per cent.}$$

Considering the boiler alone, its efficiency is 71·7 per cent., because this is the proportion of the total heat produced at the grate which is transferred from the heating to the motive-power circuit. The feed-water heater increases the efficiency of the boiler to 80·3 per cent.

The diagram, Fig. 17, is designed to show the interaction of the three circuits on one another. In this diagram the heat flow is followed along each circuit.

Consider the heating circuit, and imagine that we follow the journey of a particular mass of air from the grate to the chimney. As the air flows through the fuel bed it becomes chemically changed to furnace gas, carrying heat at a high temperature of about 1370° C. say. The gas flows along in contact with the heating surface and loses 71·7 per cent. of the heat which it carries to the colder water in the boiler. The particular mass of furnace gas into which the air has been changed flows along from the flues and is led into the feed-water heater, where it loses 8·6 per cent. of its original charge of heat across the heating surface to the cold feed water on its way to

the boiler. Finally it flows away up the chimney, carrying 11 per cent. of the heat of combustion with it.

If a particular mass of water be followed round the motive-power circuit it will be observed that it receives in the feed-water heater 8·6 per cent. of the heat produced by combustion, and further on 71·7 per cent. across the boiler heating surface, making altogether, with the small additions brought in by the jacket circulation, 85 per cent. as it flows into the engine. In the engine 14·9 per cent. is converted into work and 64 per cent. passes into the condenser, of which 60 per cent. is transferred to the cooling circuit.

The cooling circuit receives 60 per cent. of the heat produced by combustion, and carries it to a cooling pond, where the heat is gradually dissipated, in consequence of which the circulating water

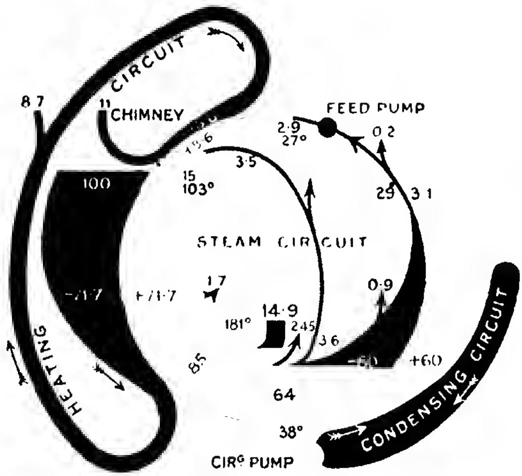


FIG. 17.—Interaction of circuits.

is cooled, and is able to pass through the condenser again and perform its duty of taking up more heat.

The quantities of heat-carrying materials flowing in the different circuits, together with other data, are brought together in Table 3 for the sake of comparison.

The numbers in the first horizontal line give the several quantities in terms of heat production at the rate of 100 lb.-cals. per minute. For this rate of flow to be established through the plant, 0·012 lb. of coal, requiring 0·24 lb. of air, must be burnt per minute; 0·128 lb. of water must be pumped into the boiler per minute; 3·6 lbs. of water must pass through the condenser per minute, and the result is a rate of heat transformation in the cylinder given by 0·63 H.P., equivalent to 20,790 ft.-lbs. per minute.

In line 2 all the quantities are increased in the proportion 0·63

to 1, in order to show them in relation to a rate of working of 1 H.P. In lines 3 and 4 the quantities are shown respectively in relation to a boiler feed of 1 lb. per minute and to a rate of combustion of 1 lb. of coal per minute. In line 5 the quantities actually flowing in the engine during the trial from which the data were measured are shown.

TABLE 3.—COMPARISON OF THE QUANTITIES OF THE HEAT-CARRYING MEDIA FLOWING IN THE THREE CIRCUITS IN TERMS OF DIFFERENT UNITS. THE UNIT OF TIME IS ONE MINUTE.

Line.	Heating circuit. Furnace gas flowing per minute.		Motive-power circuit. Boiler feed per minute.	Cooling circuit. Condenser feed per minute.	Horse-power.	Foot-lbs. per minute.
1.	Coal .	0·012 lbs.	0·128 lbs.	3·6 lbs.	0·63	20,790
	Air .	0·240 ,,				
	Gas .	0·252 ,,				
2.	Coal .	0·019 ,,	0·202 ,,	5·7 ,,	1·0	33,000
	Air .	0·380 ,,				
	Gas .	0·399 ,,				
3.	Coal .	0·094 ,,	1·0 ,,	28·1 ,,	4·91	162,000
	Air .	1·880 ,,				
	Gas .	1·974 ,,				
4.	Coal .	1·0 ,,	10·7 ,,	300·0 ,,	52·5	1,732,500
	Air .	20·0 ,,				
	Gas .	21·0 ,,				
5.	Coal .	12·25 ,,	131·0 ,,	3674·0 ,,	643	21,219,000
	Air .	245·0 ,,				
	Gas .	257·25 ,,				

The flow of heat in the plant is everywhere caused by a difference of temperature. Heat can only pass from one medium to another if there is a temperature fall between them. Fig. 18 has been drawn to emphasize this point, and the diagram shows the temperature gradient of the main flow of heat through the plant corresponding to the diagram Fig. 17. The diagram is not drawn to scale.

This diagram should, however, be considered in connection with Fig. 19, which shows the respective temperature gradients along the three circuits. The circuit paths are supposed to be pulled out straight, so that the profiles of the gradients all lie in a plane, and they are placed in relation to one another to show the "temperature falls" in the boiler and in the condenser. These falls determine the direction of flow of the main heat stream through the plant.

The course of the heat flow determined by these diagrams, both along the circuit paths and in the main stream, can be followed without difficulty.

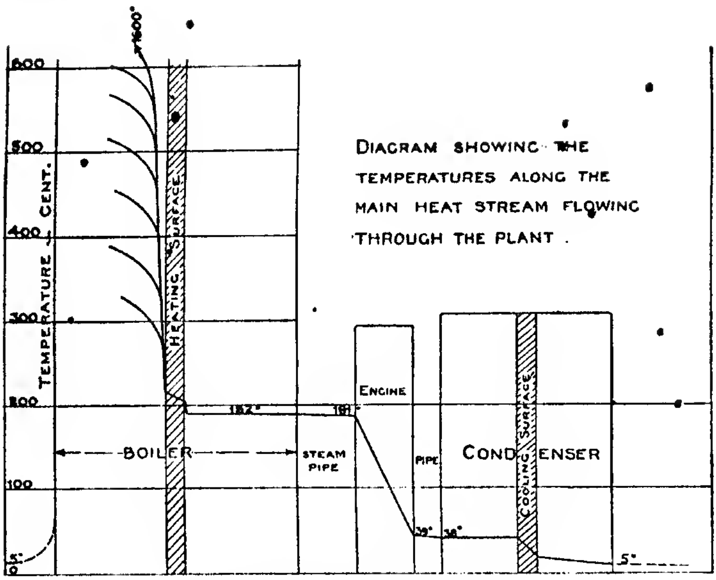


FIG. 18.—Main temperature gradient.

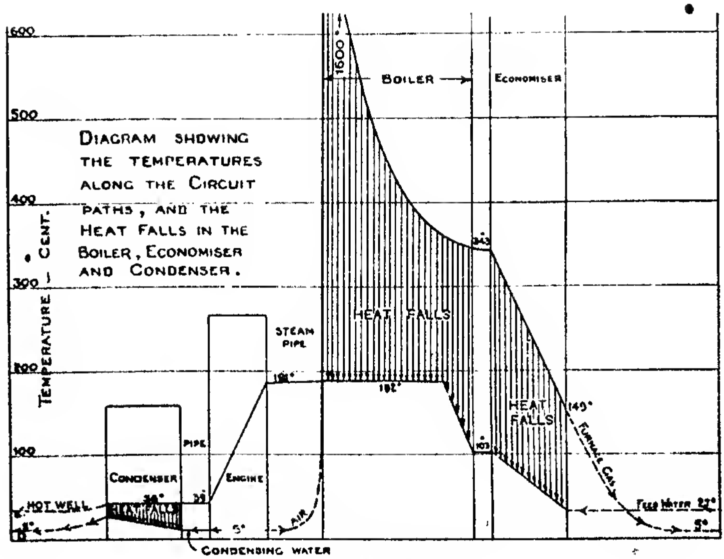


FIG. 19.—Temperature gradient across circuits.

To illustrate further the flow of energy through a steam plant an energy stream for a locomotive is sketched diagrammatically and not to scale in Fig. 20. The stream originated by the combustion of fuel at the grate is shown dividing into six branches.

Branch *a* shows the loss of heat energy by radiation.

Branch *b*, the energy carried away by the chimney gases.

Branch *c*, the energy expended in doing work against engine friction.

Branch *d*, the energy spent in overcoming the tractive resistances of the engine.

Branch *e*, the energy spent in overcoming the tractive resistances of the vehicles attached to the draw bar.

Branch *f*, the energy carried away by the exhaust steam.

Then, if there is no storage of energy anywhere along the flow, the sum of the energy flow in all the streams must be equal to the energy flow originated at the grate.

Notice that in the above statement the particular kind of energy is not specified. Whatever be its form, we know that the figures

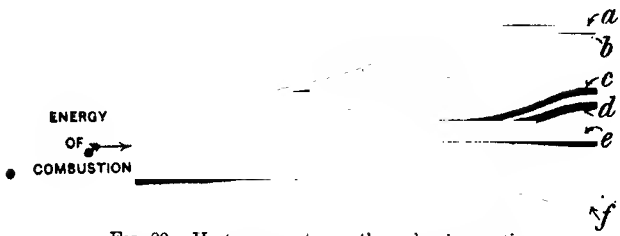


Fig. 20.—Heat energy stream through a locomotive.

representing its equivalent in any one form must add up to the same sum at every point of the flow.

Fig. 21 shows a heat stream drawn from data given in "High Steam Pressures in Locomotive Service," by Prof. Goss, and published by the Carnegie Institution of Washington in 1907. In this diagram the energy carried away by the exhaust steam is brought close to the stream representing the energy carried away by the chimney gases. The smaller branches of the heat flow have been neglected. From this diagram it will be seen that 95 per cent. of the heat produced by combustion passes away at the chimney top, and only 4 per cent. is available at the driving wheels for pulling the train. The results given on the diagram correspond to a test made on a testing plant, where the engine was driven at a speed corresponding to 50 miles per hour with a cut-off of 20 per cent. in the cylinders, and developing 464 I.H.P. Boiler pressure, 200 lbs. per square inch by gauge. From the figures given the thermal efficiency of the engine as a whole is 4 per cent.

The heat supply to the cylinder, reckoned from a feed temperature equal to the temperature of the exhaust steam, was in this

trial 111,000 lb.-cals. per minute. Therefore the thermal efficiency is $\frac{464 \times 33000}{1400 \times 111000} = 0{\cdot}099$. Or, expressed in another way, 238 lb.-cals. per minute must be supplied to the cylinders per I.H.P. minute.

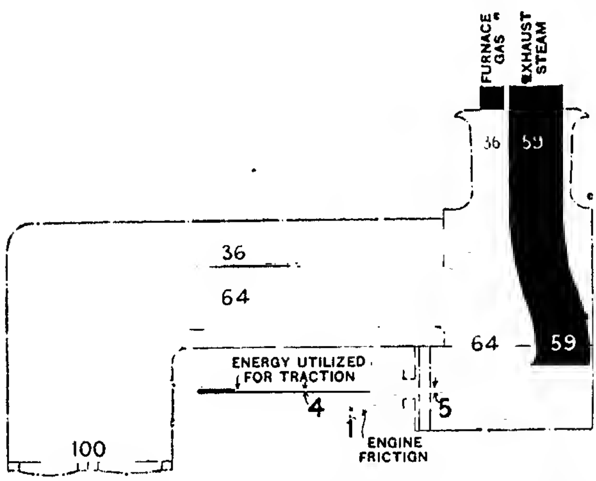


FIG. 21.—Heat energy stream through a locomotive.

11. Thermal Efficiency Possible.—In the last section figures were given showing that the thermal efficiency of a compound condensing engine is about 19 per cent. Such an efficiency seems at first sight to be very low, and if the efficiency of the whole plant is considered it is lower still, namely, 14·9 per cent. In the case of the locomotive matters are even worse, only 4 per cent. of the heat produced by combustion appearing *finally* as *mechanical* energy. These performances are, however, quite good for the respective types of engine.

The Principle of the Conservation of Energy places no limitation on the amount of energy which can be transformed from one kind to another; it merely implies that when any transformation takes place a quantity of one kind of energy is transformed into an equal quantity of another kind.

Experience shows, however, that there is a limitation, imposed by natural conditions, to the quantity of heat energy which can be transformed into mechanical energy. Thus, although by friction 1400 units of mechanical energy can be transformed into 1 unit of heat energy, there is no way of completely reversing the transformation so that 1400 units of mechanical energy can be obtained by the expenditure of 1 unit of heat energy.

The transformation of heat into mechanical energy is effected by a heat engine. A heat engine receives heat energy at the temperature

T degrees from a source or heating circuit, and can only work if it can reject heat to a cooling circuit at a lower temperature t.

If there is no temperature fall through the engine there can be no transformation of any of the heat energy into mechanical energy. If a perfect engine, perfect in the sense that there are no losses by radiation or conduction or unresisted expansion, receives a quantity of heat at an absolute temperature T, and rejects heat at the lower absolute temperature t, it cannot convert into mechanical energy a greater proportion of the heat received than the fraction given by

$$\frac{T - t}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Or, if U stands for the maximum theoretical quantity of heat which can be converted into mechanical energy by a perfect heat engine which receives a quantity of heat energy Q at the absolute temperature T, and rejects heat energy at a temperature t, then the relation between the quantities is

$$U = \frac{Q(T - t)}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Put in another way, the quantity of heat energy required by a perfect heat engine working between the temperature limits T and t, to perform the amount of work U, is

$$Q = \frac{TU}{T - t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When working at the rate of 1 H.P. a heat engine converts $\frac{33000}{1400} = 23{\cdot}56$ lb.-cals. into mechanical energy per minute. If the whole of the heat received by the working agent is received at the constant temperature $T = (195 + 273) = 468°$ C. absolute, and the whole of the heat rejected is rejected at the constant temperature $(40 + 273) = 313°$ C. absolute, the quantity of heat which must be supplied to the engine is by equation (3)

$$H = \frac{468 \times 23{\cdot}56}{468 - 313} = 71{\cdot}2 \text{ lb.-cals. per minute.}$$

Thus for a heat supply of 71·2 lb.-cals. per minute a perfect heat engine, working between the temperature limits stated, would convert 23·56 of them into mechanical energy.

The highest possible thermal efficiency for the given temperature limits is therefore

$$\frac{23{\cdot}56}{71{\cdot}2} = 0{\cdot}33$$

Any actual heat engine working between these limits of temperature would be found less efficient than this; but as it could never have a greater efficiency, the thermal efficiency actually realized by a heat engine should be compared with the thermal efficiency it could realize if it were perfect in the sense defined above.

The relations in equations (1) to (3) are general in their application,

and are independent of the physical properties of the working agent. No matter what kind of working agent be used, providing that the temperature limits remain the same as those given in the example, the thermal efficiency of the engine could not exceed 0·33.

When this law is applied to an actual engine, the first difficulty met with is that the heat supplied to the working agent is not in general supplied only at one temperature. The mechanical energy which can be obtained from a perfect engine receiving heat at different temperatures is found by calculating the work which can be done by the quantity of heat received at each temperature and then adding the several quantities of work together.

For example, suppose an engine to receive heat as follows:—

20 lb.-cals. per minute at 470° C. absolute,
10 lb.-cals. per minute at 350° C. absolute,
5 lb.-cals. per minute at 330° C. absolute,

and suppose that the temperature of rejection is constant at 300° C. absolute.

Applying the law expressed by equation (2),

Work obtained per minute from 20 lb.-cals. received at 470°,

$$\frac{20(470 - 300)}{470} = 7{\cdot}23 \text{ lb.-cals.}$$

Work obtained per minute from 10 lb.-cals. received at 350°,

$$\frac{10(350 - 300)}{350} = 1{\cdot}43 \text{ lb.-cals.}$$

Work obtained per minute from 5 lb.-cals. received at 330°,

$$\frac{5(330 - 300)}{330} = 0{\cdot}45 \text{ lb.-cals.}$$

Total work obtained per minute from the 35 lb.-cals. received, 9·11 lb.-cals.

That is to say a perfect heat engine could in these circumstances transform 9·11 lb.-cals. per minute into mechanical energy, but it could transform no more than this. Any actual engine would in similar circumstances not transform so much, and it would be doing well if it could transform two-thirds of this, that is 6·7 lb.-cals.

Assuming that this figure represents the performance of an actual engine receiving and rejecting heat as stated above, two efficiencies stand out, namely,

(1) the efficiency of the perfect engine $= \dfrac{9{\cdot}11}{35} = 0{\cdot}26$;

(2) the efficiency of the actual engine $= \dfrac{6{\cdot}7}{35} = 0{\cdot}19$.

The efficiency of the actual engine relatively to the perfect engine, sometimes called the "Efficiency Ratio," is 0·66.

Referring to similar calculations made for the condensing engine whose heat stream is drawn above, although the actual efficiency of

the engine appears low, namely 0·19, its efficiency relative to what is the possible limit of efficiency is 0·66, a much better performance.

The natural limitation to the transformation of heat into mechanical energy was perceived by Carnot, who, in 1824, published a remarkable paper entitled "Reflexions sur la Puissance Motrice du Feu," in which he developed the reasoning leading up to the relation given in equation (2). The ideas of Carnot given in this paper have been developed into the modern Science of Thermodynamics. What is called the Carnot engine is a perfect heat engine, receiving the whole of its heat at a temperature T absolute, and rejecting heat at the temperature t absolute, the quantity rejected being the difference between the heat received and the work done.

The perfect engine corresponding to a steam engine is called the Rankine engine of comparison, because Rankine stated the conditions of heat reception and rejection which should be regarded as corresponding in a perfect engine to the conditions of heat reception and rejection in an actual steam engine. Expressions for calculating the quantity of heat U, called the **available energy**, which can be converted into work per pound of steam received by a Rankine engine of comparison, are given in Section 54, page 194.

12. The Laws of Thermodynamics.—The first law is a statement founded on experimental evidence relating to the transformation of heat energy into mechanical energy, and may be regarded as a particular case of the general principle of the conservation of energy.

Law 1.—Heat and mechanical energy are mutually convertible, and when mechanical energy is produced from heat energy a definite quantity of heat disappears for every unit of work done; and when heat is produced from mechanical energy the same definite quantity of heat energy appears for every unit of work spent.

The relation between heat and work is, 1400 ft.-lbs. = 1 lb.-cal.

This law places no limitation on the amount of heat which can be transformed into mechanical energy, but the second law does. The second law is stated in many different ways, but for practical purposes it is conveniently stated in terms of the efficiency of a perfect engine as follows :—

Law 2.—If a Carnot engine receives a quantity of heat Q at an absolute temperature T, and rejects heat at an absolute temperature t, the greatest quantity of mechanical energy which can be obtained is

$$U = \frac{Q(T - t)}{T}$$

No engine working between the same limits of temperature can be more efficient than this.

This is a law based upon experience. It can be deduced in the form given from the more familiar fact that heat cannot of itself flow from one body to another of higher temperature.

CHAPTER II

STEAM BOILERS. THE HEATING CIRCUIT AND THE HEAT TRANSFERRED TO THE MOTIVE-POWER CIRCUIT

13. Steam Boilers.—A steam boiler is designed with the object of bringing near together the gases flowing along the heating circuit, and the water circulating in the motive-power circuit in conditions where the temperature difference between them is sufficient to cause a rapid transfer of heat energy from the gas to the water.

Four kinds of steam boiler are illustrated in Figs. 22 to 29, namely, a Lancashire boiler of the kind used largely for factory installations; a marine boiler; a locomotive boiler, and a water-tube boiler.

In each boiler, that part of the surface exposed to the action of the hot gases, and over which the gases sweep on their way from the grate to the chimney, is the heating surface. In no circumstance must any plate or tube exposed to the action of the furnace gas become dry on the water side when the furnace is in action, otherwise the heat which should be transmitted to the water accumulates in the plate, the plate becomes overheated, softens, and disaster may follow.

Referring to the Lancashire boiler shown in Figs. 22 and 23, the boiler barrel or shell is formed of steel plates riveted together. Two flue tubes are enclosed in the shell and are secured at each end to the end plates. A grate G is formed at the front end of each flue by means of fire-bars resting on carriers secured to the inside of the flue tubes. A cross-section through the flue tube is shown in Fig. 23. A bridge of fireclay, A, is shown at the end of the grate, the object of which is to protect the transverse plate and fire-bar carriers from the intense local heat there. The ashes fall into the part of the flue under the grate, from which they can be raked out. This type of boiler is set in brickwork arranged to form flues outside the boiler, so that the hot gases are brought into contact with the external surface of the parts of the boiler shell which are exposed in the brick flues. In the illustrations shown, the furnace gas, after leaving the flue F, passes into the chamber C, from which it flows along the flue U, under the boiler, when, arriving at the front end of the shell, the gas stream divides into two streams, and flows back along the side flues, S, S (Fig. 23), to the common flue leading to the chimney. The heating circuit in this class of boiler is thus folded back on itself twice.

The air supply is drawn into the ashpits and through the interstices between the fire-bars, and then through the fuel bed by the action of the chimney, or some equivalent apparatus for producing the flow. A certain quantity of air is drawn into the furnace through openings in the fire door, this additional and direct supply being required to burn the volatile products driven off from freshly fired coal. The heat from the fuel bed acts on the freshly fired coal to distil off these volatile products, and unless air is present in sufficient quantity they pass away in the flue gas unburned. Smoke from a chimney is a sure sign of unburnt products of some sort, and smoke prevention is largely a question of a properly arranged air supply to the volatile products when they are at a sufficiently high temperature to burn.

The water level should never fall nearer to the tops of the furnace crowns than 4 inches.

The distinguishing feature of the Lancashire boiler is the two straight flue tubes running from end to end of the shell.

The Galloway boiler is of the Lancashire type with a modified arrangement of flue tubes. At a short distance from the ends of the fire grates the two separate flue tubes in a Galloway boiler are brought together, and are united to form one arched and oval-shaped single flue

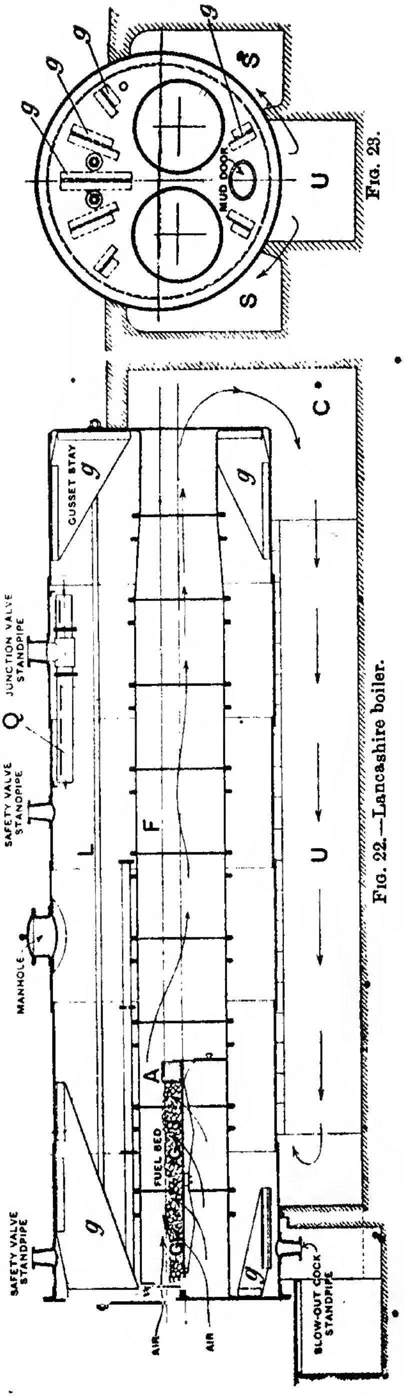
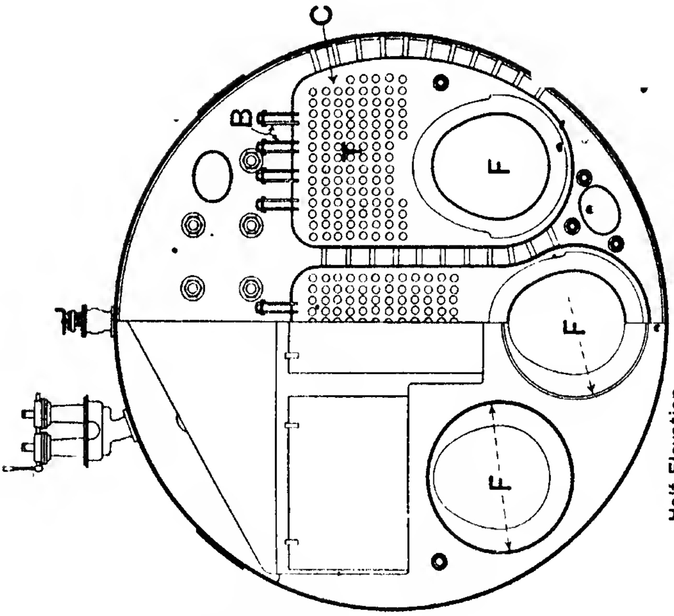


FIG. 23.

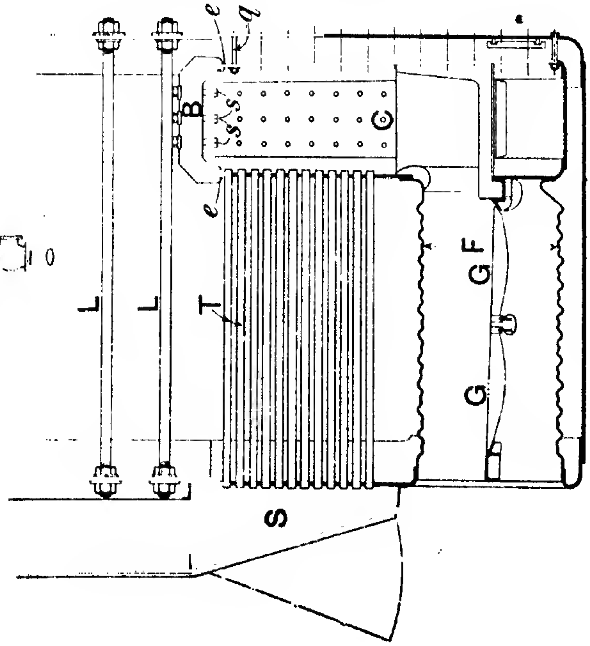
FIG. 22.—Lancashire boiler.



Half Section.

Half Elevation of Boiler Front.

FIG. 25.



Longitudinal Section.

FIG. 24.—Marine or tank boiler.

continued to the back end of the boiler. This single flue is crossed at intervals by small water tubes which serve the double purpose of strengthening the single flue against collapse, and of promoting the circulation of the water around the flue.

A Cornish boiler is of the same general construction as a Lancashire boiler, but it has only one flue tube instead of two.

A tank boiler of the marine type is shown in Figs. 24 and 25. Fig. 24 is a longitudinal section through the centre, and Fig. 25 is a half elevation of the front end and a half cross-section.

There may be one, two, three, or even four furnace tubes in a boiler of this type, depending upon the diameter of the shell. A three-furnace boiler is shown in the illustrations. A grate is formed in each furnace by fire-bars supported on carriers riveted across the furnace tube as indicated in Fig. 24 at GG. Each flue tube F leads into the lower part of a combustion chamber C, as it is called, and small flue tubes T run from the upper part of the combustion chamber back through the mass of water in the boiler to the front plate of the boiler. A smoke-box S is formed on the end plate to receive the furnace gas as it pours out from the tubes, and flues lead from the smoke-box to the chimney. Doors are formed in the front plates of the smoke-box to give access to the interior, so that the small tubes may be cleaned. No part of the external surface is utilized as heating surface in boilers of this kind, and there is no brickwork setting. The boiler is merely supported on suitable frames. The furnaces, the combustion chambers, and the flue tubes, are wholly immersed in the water in the boiler shell, and the heating surface is therefore the area of the inside surfaces of these elements. The water level should never fall nearer to the tops of the combustion chambers than 6 inches.

This type of boiler is also known as the Scotch boiler. Until the introduction of water-tube boilers, it was the type usually used for marine purposes.

A modification of this boiler, called a dry-back boiler, is sometimes used for land installations. The furnace tube and the small return tubes are carried through from end to end of the shell, and a combustion chamber is formed of fire-brick or of plating lined with brick or fireclay at the back end in the way illustrated diagrammatically in Fig. 1, Chapter I.

A locomotive boiler is shown in Figs. 26 and 27. The feature which distinguishes a locomotive boiler from all other types is the cubical fire-box, open on the lower side, and secured round the lower edges to an outer casing of similar shape. The boiler barrel is riveted to this outer casing, and small flue tubes connect the fire-box tube plate with the front tube plate of the boiler barrel. It may be noticed in passing that the front plate of a marine boiler is the plate immediately facing the fireman and containing the furnace openings. The front plate of a locomotive boiler is the end plate remote from the furnace end, and the nomenclature is derived from the association of the idea of motion with the locomotive boiler.

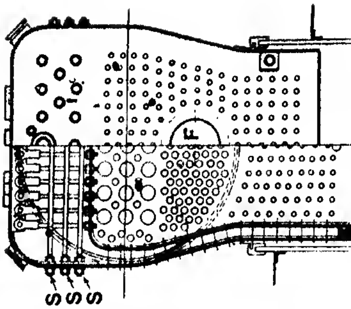


Fig. 27.

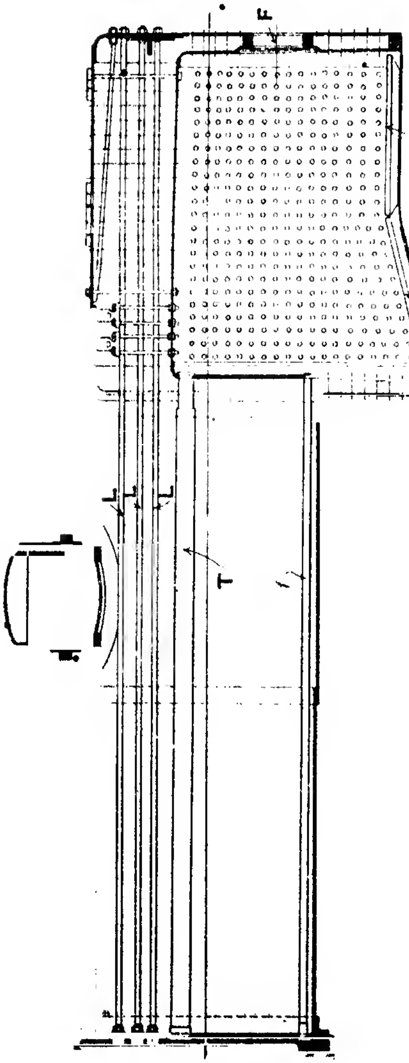


Fig. 26.—The locomotive boiler.

The term cubical as applied to the shape of the fire-box and its outer casing must be interpreted with wide generality. The actual shape of the fire-box is governed by the fact that the lower part, in most cases, must be narrow enough to go down between the engine frames, and its width is therefore limited by the gauge between the rails.

The upper part, which is clear of the frames, may be wider, and the cross-section, Fig. 27, shows a box which has been widened out in the part above the frames. The outer casing follows the same general shape, and usually there is about a 4-inch water space between the vertical sides of the fire-box and the outer casing, the top, however, of the outer casing is arranged at as great a distance as practicable above the fire-box crown to give steam room. The grate G is formed by fire-bars supported on carriers resting in supports secured to the sides of the fire-box. The ashpan is separately constructed of thin plate and angle iron, and is bolted or cottered up to the lower part of the outer casing. Dampers are placed in the front, or in the back, or sometimes in both the front and the back of the ashpan, for regulating the air supply to the furnace. A fire hole is shown at F. A smoke-box, formed of plate and angle iron, is secured to the front

end of the boiler, and the chimney is placed centrally on this. The end of the blast pipe is brought centrally under the chimney, and the steam from the engine cylinders, as it passes through the blast pipe and into the chimney, causes a draught powerful enough to supply air at the rate necessary to burn coal on the grate at the rate of 100 lbs. per square foot of grate area per hour.

The sides and top of the fire-box together with all the tubes are immersed in water, and in no circumstance must the level of water in the boiler fall nearer to the top of the fire-box than 4 inches. In this figure the upper tube T is a superheater tube, and the lower tube *t* is a normal flue tube. There are twenty-one of the larger superheater tubes. The construction of a superheater is described in detail in Section 75.

A common feature of the boilers just described is that the furnace gas flows through the flues and tubes, which are all immersed in the water carried in the shell. In a water-tube boiler the water is carried in a number of small tubes stacked in a furnace, so that the water-carrying tubes are immersed in the flames and hot gas. The heating surface is the external surface of the water-carrying tubes.

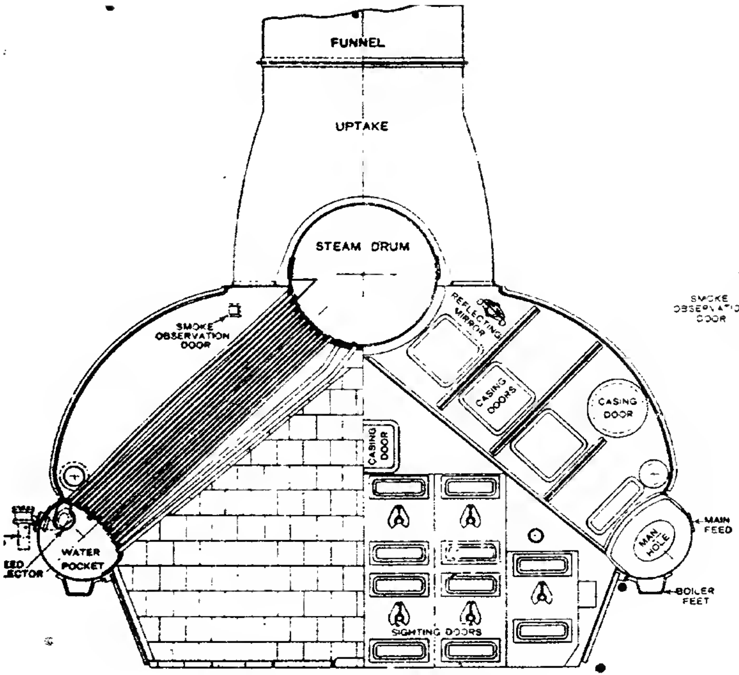
A water-tube boiler of the Yarrow type is shown in Figs. 28 and 29. In this type there are three drums or receptacles arranged with their axes parallel. The intersections of these axes with a vertical plane are the three corners of a triangle. Stacks of small tubes connect each lower receptacle or water pocket, as it is called, with the steam drum at the apex. The grate lies horizontally between the water pockets at the base of the inclined stacks of tubes. The action of the draught draws the flames right and left across the stacks of tubes and then into a common flue leading to the chimney.

The boiler shown is arranged for burning oil fuel. The grate is replaced by a fire-brick floor and the ends of the furnace space are also bricked. The oil burners are placed along the front and each burner is connected to a common oil main in which the oil is forced by a pump. The air necessary for combustion passes into the furnace through the spaces provided round the burners.

There is a continuous circulation of water between each of the water pockets and the common steam drum at the apex, and the circulation is started and maintained by the action of the flames on the stacks of water tubes immersed in them. The water flows from a water pocket upwards through some of the tubes to the common steam drum, being partially evaporated during the journey, whilst a corresponding quantity of water flows down from the drum to the water pocket through the remainder of the tubes.

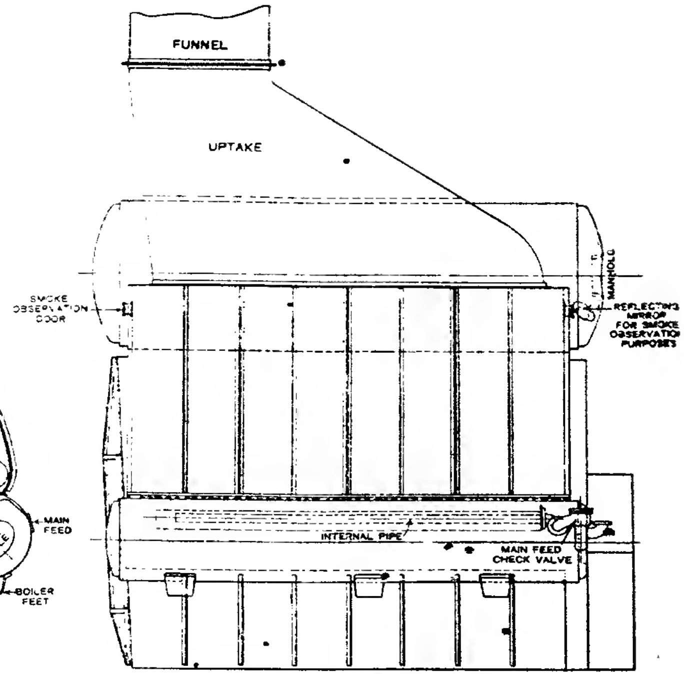
All the tubes in a stack are exactly alike and their division into two sets, namely, those carrying the upward flow and those carrying the downward flow, takes place automatically to meet the conditions of the moment.

The weight of dead water carried by a water-tube boiler in relation to the rate at which steam is produced is small compared with boilers of the tank type. This has the advantage that steam



Half Section. Half Elevation of Boiler Front.

Fig. 28.—Yarrow water-tube boiler.



Elevation

Fig. 29.

can be raised rapidly, an advantage which cannot be over-rated in connection with warships. Also the weight of a water-tube boiler per unit of power is considerably less than that of a tank boiler, and this is another feature of special advantage when the boilers are to be used in a warship. It is not surprising, therefore, that the water-tube boiler has practically displaced the tank boiler in warships.

Although reduction of weight is always an advantage, the reduction of dead water in a boiler is a disadvantage for some conditions of working. Working a water-tube boiler is like running a single-cylinder engine without a flywheel. There is no reserve to draw upon to steady the fluctuations between demand and supply. The maintenance of constant pressure and a proper water level in a water-tube boiler, in conditions where the demand for steam is variable, require closer attention than in the case of tank boilers.

There are many variations of type in boilers of the water-tube class, and there have been many workers in this field of development. The water-tube boiler fitted in H.M.S. *Speedy* in 1885 by Sir John I. Thorneycroft marked an epoch in the development of boilers of this class. The heating surface in the Thorneycroft boiler is made up of stacks of curved tubes, and in the more recent Thorneycroft-Schultz boiler there are three receptacles at the base of the boiler; the third is placed midway between the outer ones and is connected in a vertical direction with the drum at the apex by a stack of curved tubes.

In the Babcock and Wilcox boiler a stack of water tubes is placed in an inclined position in a furnace, and the axis of the stack lies in the same plane as the axis of the steam drum. Each end of the stack is separately connected to the drum, and the flow of the water is down from the drum through downcomers to the lowest end of the stack, along the stack towards the upper end, and then back into the drum.

In the Belleville boiler a stack of tubes is placed in the furnace, but the tubes are not all inclined the same way. The tubes, vertically over one another, incline in opposite direction, though the inclination to the horizontal is small. The connections at the ends of the tubes are made so that the tubes vertically over one another are in series. The water flowing into the bottom tube from the common water pocket is therefore forced to circulate through all the tubes vertically above it before it discharges into the drum at the top.

The Stirling boiler, the Niclausse boiler, the Dürr boiler are other water-tube boilers in use. The history of the development of these boilers, together with illustrations of the details of the different types, can be followed in the *Transactions of the Institution of Naval Architects*.[1]

It will be understood from these illustrations, that the constructional elements of a boiler are primarily, a shell or barrel of,

[1] Papers by J. I. Thorneycroft, vols. 30, 35, 36. Papers by A. F. Yarrow, vols. 40, 41. Prof. W. H. Watkinson, "Circulation in Water-tube Boilers," vol. 37. "The Niclausse Water-tube Boiler," Mark Robinson, vol. 37.

circular form, circular flue tubes, and flat plates arranged in different ways according to the type of boiler. In the water-tube boiler the tubes and drums are relatively small.

Wherever a flat surface is used in a boiler it must be stayed against the internal pressure. Flat surfaces in general will be found to occur in pairs, so that a stay between them is put in tension by the pressure acting between them. Examples of pairs of surfaces stayed to one another are exhibited in the tank boiler, Figs. 24 and 25, where the sides of the combustion chambers are connected by stays, of which *q* is one, pitched at approximately 8 ins.; and also in the locomotive boiler, Figs. 26 and 27, where the vertical sides of the fire-box are stayed to the sides of the outer casing by stays pitched at about 4 ins. In Figs. 26 and 27, the front plate of the barrel is shown stayed to the back plate of the outer casing of the fire-box by the longitudinal stays L, L, and the flat sides of the outer casing of the fire-box are held by the cross stays S, S. Similarly, the flat ends of the tank boiler shown in Figs. 24 and 25 are stayed together by the longitudinal stays L, L. In both the locomotive boiler and in the tank boiler the tubes act as stays. In some cases the ends of a few of the tubes are secured to the tube plates by nuts and washers, so that they may act as stay tubes with certainty, the usual way of securing a tube in the tube plate being to expand it by means of the tube expander. When the distance between the flat end plates is inconveniently great for longitudinal stays, each end plate is connected to the boiler shell by gusset stays, examples of which are shown in Figs. 22 and 23, in which the gusset stays are each marked by the letter *g*. A gusset stay is usually formed of a piece of plate running diagonally from the end plate to the barrel and secured at each end by angle irons riveted to the end plate and to the barrel respectively. Gusset stays are largely used in boilers of the Lancashire and Cornish type to stay the part of the end plate unstayed by the flue tubes. Local staying of the end plate is sometimes avoided by dishing the ends.

When a flat surface occurs unpaired with another it is stayed to girders carried across it. Thus the top of the combustion chambers of a tank boiler are stayed in this way, as shown in Figs. 24 and 25, where B is one of the girder stays supported at *ee* by the edges of the vertical side plates. The flat crown plate of the combustion chamber is held up to the girder by the stays *s*, *s*. The top of a locomotive fire-box is usually stayed in this way, though frequently it is stayed directly to the part of the outer casing above it. In the Belpaire boiler the top of the outer casing is flat and parallel to the top of the fire-box so that the surfaces form a pair and are stayed directly to one another, as shown in Figs. 26 and 27.

The circumferential stress s produced in a thin tube of diameter d and thickness t by a pressure p acting within it is easily shown to be

$$s = \frac{pd}{2t} \quad \dots \dots \dots \quad (1)$$

whilst the longitudinal stress f in the tube produced by the pressure on the end plates is

$$f = \frac{pd}{4t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus the stress produced in a tube 30 ins. diameter and ¼ in. thick by a fluid pressure of 200 lbs. per square inch is by (1) 12,000 lbs. per square inch, whilst from (2) the longitudinal stress is 6000 lbs. per square inch.

These expressions may be used to determine the stresses in a thin tube, or in a thin drum made without riveted joints, but in the case of a boiler shell constructed of plates riveted together, the thickness of the plates is determined, not by the above expressions, but from the strength of the riveted joints. The strength of riveted joints has been thoroughly investigated experimentally by Sir Alexander B. W. Kennedy in connection with a Research Committee of the Institution of Mechanical Engineers. The records of these experiments are published in the *Minutes of the Proceedings of the Institution of Mechanical Engineers* for 1881, 1882, 1885, 1888.

The compressive stress produced by a fluid pressure acting externally on a tube may be calculated by the expressions (1), (2) above. A tube subjected to external fluid pressure behaves very differently to a tube subjected to internal fluid pressure. With the internal pressure the tube tends to become circular in form, so that any initial deviation from the circular form is corrected by the action of the internal pressure; but the opposite is the case with an external fluid pressure. The slightest initial deviation from the circular form tends to increase, and the tube will collapse, generally with a relatively small pressure. Hence the expressions used for determining the collapsing pressure of flue tubes are generally derived from experimental data, and in this way allowance is made for initial imperfections in the circular form, which, however small, are serious.

However well a boiler is constructed, the continually changing form due to the expansion and contraction resulting from unequal heating, together with the action of the impurities contained in the water, combined with electric action in the nature of electrolysis, produce pitting, corrosion, and grooving of the plates; and systematic examination must always be made by experienced inspectors to see that these causes do not imperil the safety of the boiler.

14. Mountings Required for the Safe Working of a Boiler.— These are :—

1. Safety valves.
2. Main stop valve.
3. Water gauges.
4. Pressure gauge.
5. Clack-box, non-return valve, or feed check valve.
6. Blow-off cock.
7. Mud plugs and wash-out doors.

And sometimes

8. Low-water valve.
9. Fusible plug.
10. Air valve.
11. Scum valve.

The mountings applied to the Yarrow boiler shown in Fig. 28 are :—

Double safety valves.
1 main stop valve.
1 auxiliary stop valve.
1 auxiliary feed check valve.
1 scum valve.
2 water gauges.
1 pressure gauge and cock.
1 main feed check valve on each water pocket.
1 boiler blow-out valve on each water pocket.

The mountings applied to a locomotive include those numbered 1 to 7 above, with sometimes a fusible plug and an air valve, and in addition various fittings connected with the working of the locomotive, such as a steam whistle, injector, steam cocks, brake valves, warming cocks.

On land boilers a low-water valve is sometimes fitted.

Safety valves.—The valve is usually of the mushroom type, held down against the steam pressure acting to lift it off its seat by dead weights applied directly to it, or by springs applied directly to it, or either by dead weights or springs applied to it through a lever.

Fig. 30 shows a pair of direct-loaded safety valves of the kind which would be used on a marine boiler where dead-weight direct-loading is inadmissible.

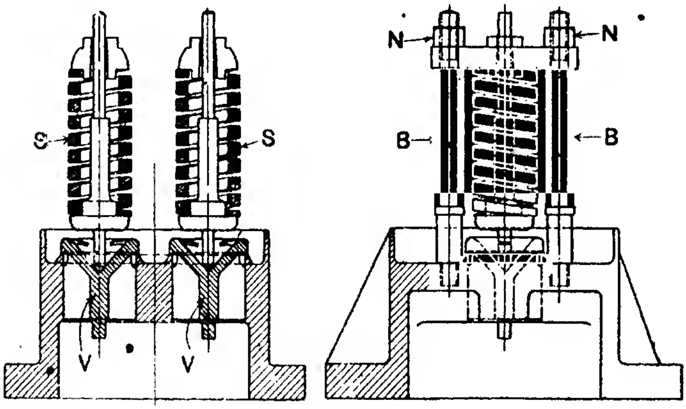
Each valve V, V, is held down by the action of a spring S, S, applied directly to it. The load on each valve is regulated by turning nuts, as N, N, and so increasing or diminishing the compression of the spring. The adjustment of the spring is made against a standard pressure gauge attached to the boiler. For example, if the boiler is to blow off at 200 lbs. per square inch, steam is raised, and the nuts are adjusted until the valves are just lifting when the pressure gauge indicates 200 lbs. per square inch. Special precautions must be taken with all kinds of spring-loaded safety valves to prevent tampering with the regulating nut after it has been set. In the example it will be seen that the nuts screw down hard on to the tubes B, B, so that their length, once determined, prevents any further compression of the spring by the nuts N, N.

The load on a spring-loaded valve increases as the valve lifts, because the lifting of the valve alters the length of the spring slightly. In order that the increase of load may be negligibly small the springs should be relatively long, so that the slight lifting of the valve necessary to give a free passage for the steam alters the length of the spring by only a small fraction of its original length.

If p is the steam pressure acting on the valve in pounds per square inch, and if A is the area of the valve in square inches, the total pressure acting to lift the valve is pA pounds. If the spring is of such strength that s pounds compress it 1 inch, and if x is the compression of the spring in inches reckoned from its free length to the actual length between ends when in place, then

$$pA = sx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

gives the relation between the boiler pressure, the spring strength, and its compression when the valve is just lifting.



When the boiler is blowing off freely, the spring is compressed to a new length, x_1, and the pressure then increases to p_1. When equilibrium is established

$$p_1A = sx_1 \quad . \quad (2)$$

Subtracting (1) from (2)

$$(p_1 - p)A = s(x_1 - x)$$

or the increase of pressure for a given lift, $(x_1 - x) = \delta x$, will vary directly as s, that is

$$\delta p = \frac{s}{A}\delta x \quad . \quad (3)$$

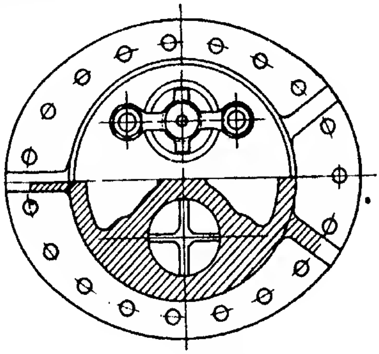


FIG. 30.—Spring safety valve.

For example, suppose a particular spring requires to be compressed 2 ins. in order to produce the load on a valve 5 ins. diameter necessary for a blow-off pressure of 200 lbs. per square inch. From equation (1), s, the strength of the spring, is 1960 lbs. per inch of compression.

If the valve lifts $\frac{1}{10}$ of an inch when blowing off freely, the increased pressure is from (3) 10 lbs. per square inch. If the spring had been of different design so that it was necessary to compress it 4 ins. to produce the required loading, the increase of pressure for $\frac{1}{10}$ in. lift would be reduced to 5 lbs. per square inch. There is another cause operating, however, to modify these relations, namely, the friction between the steam and the boundaries of the narrow annulus through which it flows away when the valve is lifted.

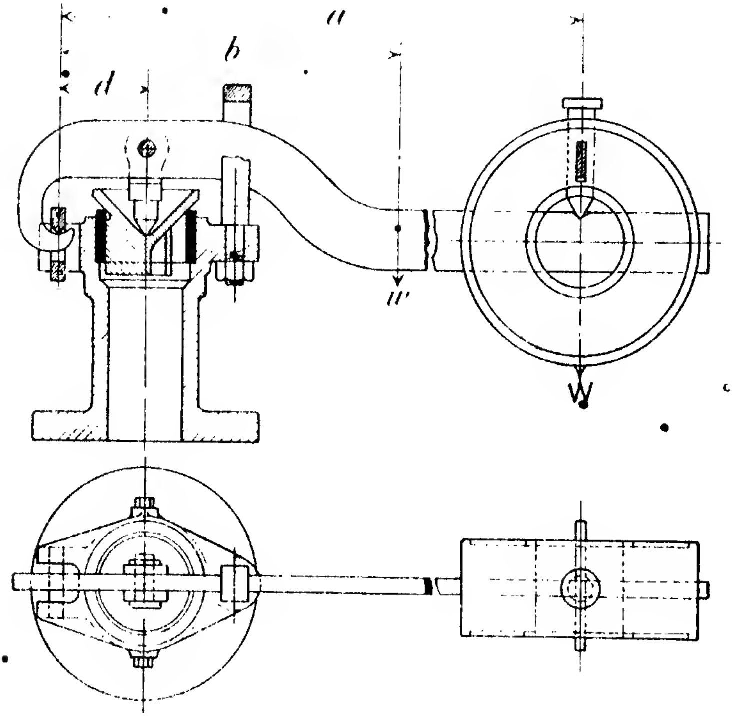


FIG. 31.—Lever safety valve.

In the dead-weight safety valve, weights are placed directly on the valve. The load on the valve is then accurately known without need of adjustment against a pressure gauge, and the load remains constant as the valve lifts. The relation between the boiler pressure p and the load W on the valve whose area is A is

$$pA = W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The valve and its load are suitably guided in the vertical direction to ensure that the valve can lift without danger of sticking.

In the lever safety valve, the load is applied to the valve by means of a weight or a spring balance acting at the end of a lever pivoted on the valve column and pressing on the valve at a point near the fulcrum.

Fig. 31 shows a lever safety valve loaded with a dead weight W. Let W be the dead weight on the lever, and let a be the horizontal distance from the fulcrum of a vertical through its centre of gravity. Further, let w be the weight of the lever and let b be the perpendicular distance from the fulcrum to a vertical through the centre of gravity of the lever. Finally, let d be the perpendicular distance from the fulcrum to a vertical through the centre of the valve. Then if p is the boiler pressure and A the area of the valve

$$pAd = Wa + wb \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

shows the conditions of equilibrium and the relation between the various quantities concerned. In valves of this type the weights should always be secured at the very end of the lever so that their leverage cannot be increased accidentally.

If a spring balance is used instead of a weight the adjustment of the length of the spring is made finally against a pressure gauge by means of the adjusting nut. When the proper position of the nut has been found, a brass tube or ferrule is placed beneath the nut to prevent any further screwing down of the nut. As in the direct-loaded valve, the loading on the valve increases slightly as it lifts against the spring.

The Ramsbottom or Duplex safety valve, Fig. 32, has been largely used on locomotives. It has the advantages that it cannot be accidentally overloaded, and that there are no small pins or joints to get corroded. There are two valves, one of which, V, is shown in position. The point on the lever which is applied to the second valve is shown at V_1. The valves are held down by a lever L which is loaded by spring S, placed between the valve columns. The length of the spring, and therefore the load which it applies to the valves through the lever L, is adjusted by means of the nuts N, N, until steam blows off at the assigned pressure and a standard pressure gauge is attached to the boiler in order to show when this pressure is reached.

When the adjustment of the length of the spring has been made, a packing piece P is prepared, equal in thickness to the gap between the piece K, holding the end of the spring, and the main casting. After the steam pressure has been taken off, the spring is slacked off, the packing piece is inserted in place, and the nuts N, N, are then screwed down hard, pulling the spring out again and permanently fixing it at the length corresponding to the blowing-off pressure, and fixing it in a way which prevents any further extension and consequent overloading of the safety valves either by design or by accident.

If the spring should break, safety links MM prevent the lever from being blown away. The lever is extended to form a handle so that the attendant can, by an alternate pull down and a push up, relieve first one and then the other valve of part of the spring load

and so, by the escape of steam which should take place from each valve in turn, ascertain if they are working freely.

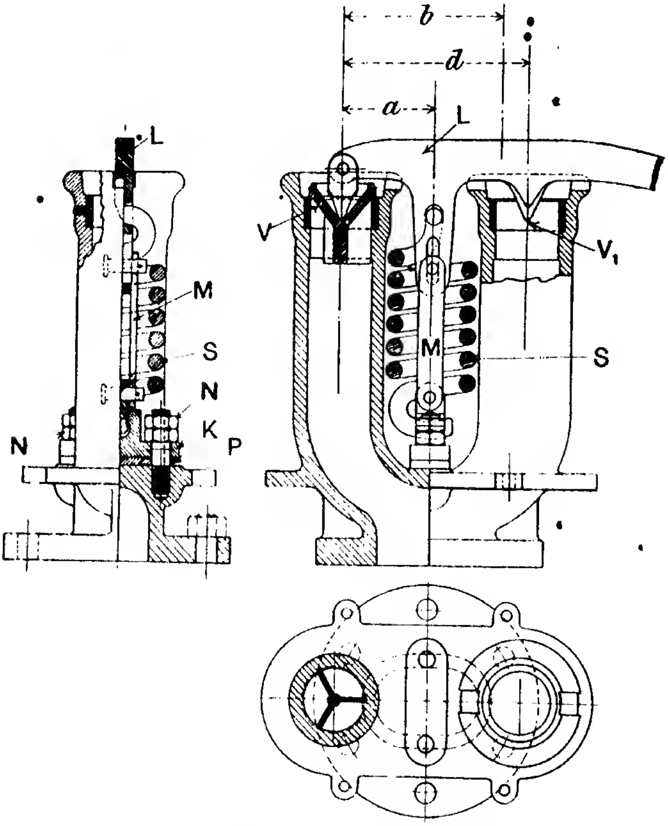


FIG. 32.—Ramsbottom or Duplex safety valve.

Let w be the weight of the lever, safety links, and springs;
- p, the blowing off pressure in pounds per square inch, and A the area of the valve in square inches;
- b, the perpendicular distance from the centre of the valve V to a perpendicular through the centre of gravity of the lever L;
- d, the distance between the centre of the valves;
- a, the distance from V at which the spring is applied to the lever;
- s, the force exerted by the spring on the lever.

Then taking moments about V,

$$wb + sa = pAd$$

and $$w + s = 2pA$$

Eliminating s,

$$\therefore \quad a = \frac{pAd - wb}{2pA - w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and $$s = 2pA - w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

from which a and the pull of the spring can be calculated.

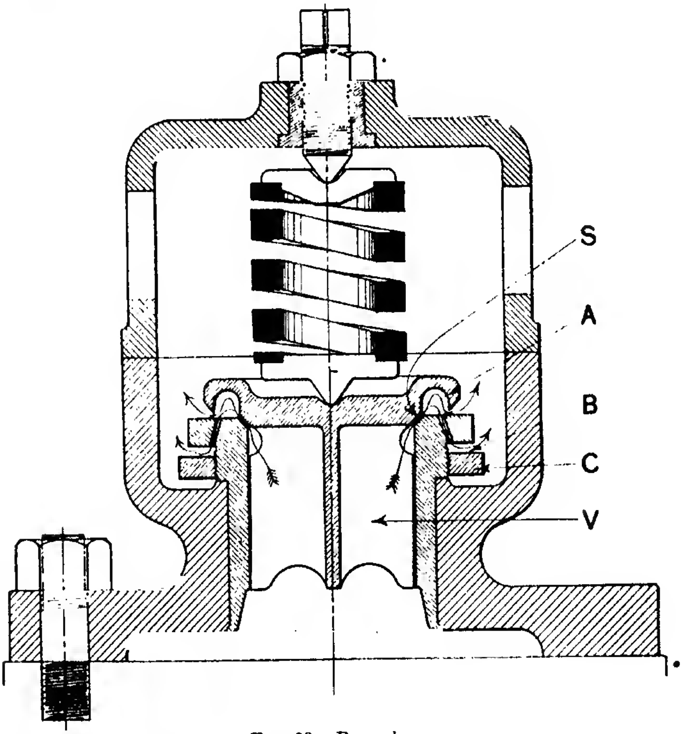


FIG. 33.—Pop valve.

A valve in which the disadvantage of using a short spring is evaded is shown in Fig 33. A valve V of the ordinary type is provided with a second seat A, which is practically closed when the valve is down on its main seating S. When the total steam pressure acting on the valve is just equal to the pressure applied by the spring, the valve lifts, and the steam escapes into the atmosphere, not freely, but with difficulty, through the space between the second seating A and the bush and through a ring of holes drilled in the metal of the bush, of which two are shown in the figure, and finally through the

narrow space B formed between the nut C and the underside of the flange of the bush. This throttling and change in direction of the flow of the steam downwards, produces an upward component of the pressure of the steam in the space between the two seatings which helps to lift the valve. The total pressure lifting the valve is therefore increased by the action of this upward component acting on the annulus between the two seatings, in consequence of which the valve lifts until equilibrium is established between this increased upward push of the steam and the downward push of the spring. The magnitude of this upward component is regulated by adjusting the space B by the nut C. Immediately the steam pressure in the boiler reaches the blowing-off pressure the valve lifts rapidly and blowing off begins with vigour. The name Pop Valve is generally associated with this type of valve, because of the peculiar noise made when blow-off begins. The figure is not a scale drawing but a diagram only, in which some small details are left out and in which the valve seats are exaggerated in size.

The *main stop valve* is placed on the top of the boiler as far as possible to ensure that dry steam only is admitted to the steam pipe. In certain conditions water may be carried mechanically along with the steam through the stop valve and into the steam pipe, that is to say, the boiler may prime. A horizontal "anti-priming" pipe attached to the extension of the steam pipe through the stop valve is added to prevent priming. It is perforated with numerous small holes or slots so that the steam can make its way through them without obstruction but the free passage of water is prevented. The term "regulator" is specifically applied to the main stop valve of a locomotive. A dome is sometimes built on the top of the boiler barrel and the regulator is placed in this so that the entry to the steam pipe through the regulator valve is as far as possible from the water surface. A dome is in fact an anti-priming device. Where domes are not provided on locomotive boilers a horizontal perforated pipe is generally fitted as the steam collector, and the regulator is then placed outside the boiler.

A *water gauge* consists of a glass tube, held at each end by suitable fittings, in which the level of the water in the boiler can be seen. The glass tube is connected by the top fitting to the steam space and by the bottom fitting to the water space. When the water level is normal the glass appears to be half full of water. The lower fitting should be so placed that when the water level falls to the bottom of the glass, the furnace crowns, or combustion chambers, or fire-box tops are well covered with water. Cocks in the fittings allow communication between the gauge glass and the boiler to be cut off at any moment, so that a new glass can be put in whilst the boiler is in steam, or the escape of steam and water can be stopped in case the glass breaks. A gauge glass should be surrounded by a plate glass protector to prevent broken fragments flying in case of fracture. Various devices are used to throw the water level into relief, such as a black cross hatching on a white ground placed behind

the glass, or a broad white line formed on the back of the glass. The refraction of the water causes a break in the cross hatching or in the width of the line at the water level. A cock is placed on the lower fitting to enable the cocks and the glass to be blown through in order to prevent the accumulation of dirt and scale in the comparatively small passages connecting the glass with the boiler.

The essential element of a *pressure gauge* is usually either a Bourdon tube or a corrugated diaphragm, connected by light mechanism with a pointer whose angular position indicates the pressure. A Bourdon tube is curved into the shape of a crook and is oval in section, the shorter diameter of any section lying in the plane of the crook. One end of the tube is fixed and the other end is connected to the pointer. The tube tends to straighten as the pressure inside it increases, and it is this property which enables it to be used as a pressure gauge. A sufficient number of points on the pressure scale over which the pointer moves are marked off whilst the tube is subject to known pressures. The scale for a pressure gauge fitted with a diaphragm is constructed experimentally in a similar way.

The pressure gauge is connected to the boiler by a copper tube bent into the form of a **U**, or a syphon as it is called. The object of the syphon is to condense water in the **U**. The water condensed transmits the steam pressure to the diaphragm or to the Bourdon tube, and at the same time protects the delicate mechanism of the pressure gauge from the hot steam. The connection between the boiler and the syphon is always made through a small cock.

The *clack-box* is one of the most important fittings on a boiler. A typical design is shown in Fig. 34. The stop valve V is normally open. The valve is formed of a ball B. The pressure of steam in the boiler forces the ball down on its seat. Water is forced through the valve by the feed pump, but immediately the pressure in the feed pipe falls below the pressure in the boiler the valve closes automatically, and prevents the return of any of the water in the boiler along the feed pipe. Should the valve be prevented from closing by the lodgment of dirt on the seating, or should it start to leak badly, it is isolated by closing the stop valve V, the cover C is unscrewed, and then the valve is withdrawn for examination.

The stop valve V is often omitted, in which case the valve can only be examined when the boiler contains no steam. Two clack-boxes should be fitted to each boiler, each with an isolating valve V, so that one can be used if the other fails.

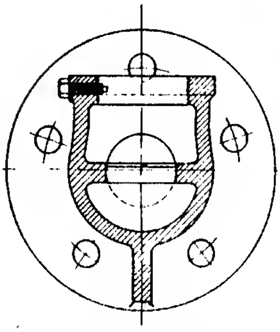
A *blow-off cock* of simple design is shown in Fig. 35. Its purpose is to empty the boiler of water, and it is attached to the lowest part by means of a branch piece, so that all the water can be drained from the boiler.

A *mud plug* is a screwed taper plug used to close an orifice in the boiler plate, through which the nose of the nozzle attached to a water hose is inserted when the boiler is being washed out. Several of these orifices are provided, arranged so that a stream of water may

be directed from the hose to places where mud and deposit tend to accumulate.

Wash-out or *mud-hole doors* are the covers to larger orifices of rectangular or oval shape through which the mud brought down by washing can be removed.

A *manhole door* covers the opening in a boiler through which a man can get inside. A typical manhole door is shown in Fig. 36.



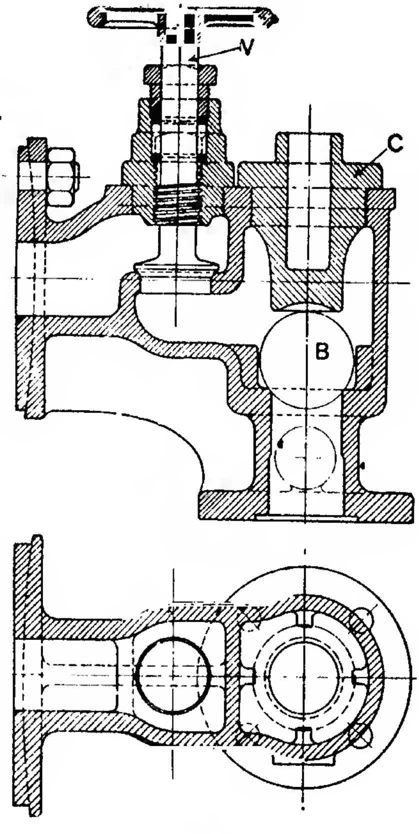


Fig. 34.—Clack-box.

A *low-water valve* is an automatic device for opening a safety valve when the surface of the water in the boiler falls below a prescribed level. It is really a safety valve held down by the joint action of two weights inside the boiler placed at opposite ends of a lever carrying, near the fulcrum, a rod connecting it to the valve. One weight is in the steam space and tends to close the valve, whilst the other weight is immersed in the water and tends to open the valve. As the water surface falls the immersed weight gradually loses the support of the water and a level is finally reached at which its weight is sufficient to open the valve. Low-water valves are generally fitted to stationary boilers of the Cornish and Lancashire type.

A *fusible plug* is another device to prevent serious results follow-

ing upon shortness of water in the boiler. It is a hollow metal plug screwed into the highest part of the furnace, or combustion chamber, or fire-box top. The hollow part of the plug is filled with a fusible metal, so that if the plate gets hotter than it ought to be, the fusible metal core melts out, leaving a hole through which steam and water flow into the furnace, thus damping the fire and relieving the pressure.

A *scum valve* is a blow-out valve connected with a pipe inside the boiler. The end of the pipe is formed into a shallow pan, which is fixed just below the normal level of the water. When the valve

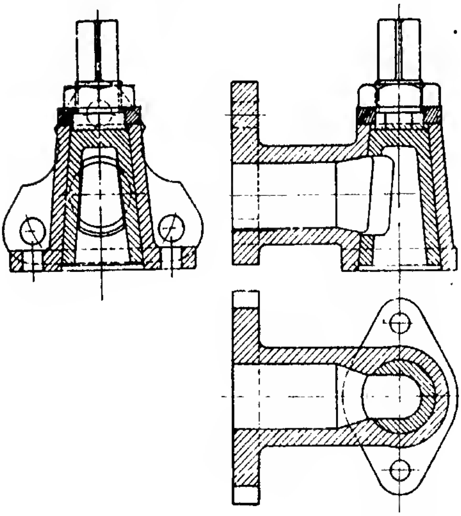


Fig. 35.—Blow-off cock.

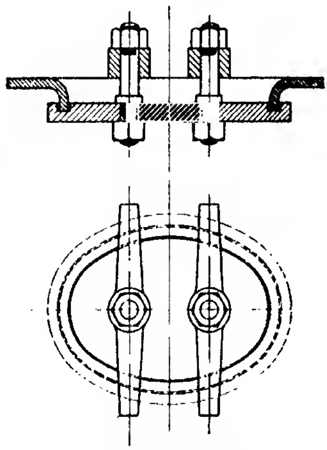


Fig. 36.—Manhole door.

is opened the surface water is therefore drawn off, carrying the floating grease, dirt, and scum with it.

15. The General Action of a Steam Boiler.—The power of a boiler depends primarily upon the rate at which fuel is burnt on the grate.

The **rate of combustion** is usually measured by the number of pounds of fuel burnt per square foot of grate area per hour.

The power of the boiler is measured therefore by the product of the rate of combustion and the grate area.

The rate of combustion is controlled by the rate at which air is supplied to the furnace.

Every pound of fuel requires a definite weight of oxygen to burn it, and this oxygen is supplied by feeding the furnace with air. The weight of air which must be fed to the furnace per pound of fuel burnt depends upon the chemical constitution of the fuel. In order

to ensure complete combustion, air is usually supplied in excess of the quantity theoretically necessary. If too little air is supplied the fuel is not properly burnt, and combustible gas passes away through the chimney; if too much is supplied, the temperature of the furnace gas is unnecessarily reduced.

One pound of good steam coal requires theoretically about 12 lbs. of air to burn it, but in practice from 15 to as many as 24 lbs. would be supplied, the actual quantity depending upon the kind of fuel, the rate of combustion, and the skill of the stoker.

For the present assume that the furnace is supplied with 16 lbs. of air for every pound of coal burnt. Assume also that the rate of combustion is 16 lbs. of coal per square foot of grate area per hour, a rate usual in stationary boilers with chimney draught. Then about 256 lbs. of air per hour must be supplied to the furnace for each square foot of grate area.

Now 256 lbs. of air at atmospheric pressure and at a temperature of 800° C., an assumed temperature for the air passing through the fuel bed, occupies about 12,000 cub. ft., and hence the velocity through the fuel bed would be of the order 10 ft. per second, if it is assumed that the area of the interstices through which the air can flow is one-third of a square foot per square foot of grate.

If the rate of combustion is 100 lbs. of coal per square foot of grate per hour, a rate often exceeded in express locomotives, the corresponding weight of air which must be drawn through the furnace is 1600 lbs. per square foot of grate per hour, and, with the assumptions above, the volume is about 76,000 cub. ft., and the velocity through the fuel bed is of the order 60 ft. per second, that is, 40 miles per hour.

These calculations are of course very approximate, but they serve to show the order, both of the volume of air required and its velocity through the fuel bed, and to raise the important practical question: what is the limiting velocity at which air can be drawn through the fuel bed? No very definite answer can be given to this question because there are so many factors concerned, but the practical limit is reached when the air velocity through the fuel bed is sufficient to carry away the lighter parts of the fuel in a partially burnt condition, and ultimately to carry them away through the chimney.

The air is set in motion through the furnace by the action of a **chimney**, which is said to produce a **natural draught**. The action of the chimney is sometimes supplemented by fans, in which case the draught is said to be **forced**, that is to say, a forced draught is understood to be a rate of air supply greater than that which can be produced by a chimney acting alone. The **steam blast**, as applied to a locomotive, produces a draught compared with which the action of the engine chimney is negligible.

When the draught is established, it is controlled by means of dampers, which act by varying the size of the openings through which the air or the furnace gas passes. Forced draught is controlled by regulating the speed of the fans.

The actual amount of heat produced by combustion depends upon the calorific value of the fuel.

The **calorific value** of a fuel is the number of heat units produced by the burning of one pound of the fuel. A strict definition of the Calorific Power is given below in section 22, and the meaning of "Higher" and "Lower" values is there explained.

The calorific value of a fuel may be determined experimentally in a calorimeter, or it may be calculated from a chemical analysis.

One pound of good steam coal gives about 8000 lb.-cals. when properly burnt. One pound of oil of the kind burnt in boiler furnaces in place of coal gives about 10,000 lb.-cals., whilst one pound of dry wood furnishes only about 4000 lb.-cals.

These figures are subject to large variations, but they serve to give some idea of the relative calorific values of the fuels mentioned.

These preliminary remarks indicate that prominent amongst the subjects which require detailed consideration in connection with a boiler plant are, the means for producing and regulating the draught, the heat energy produced by the combustion of different fuels together with the air supply necessary to produce complete combustion, and also methods of measuring the efficiency of the heating circuit and the efficiency of heat transmission from it to the motive-power circuit.

16. Draught.—Air flows through the furnace because the pressure at the base of the chimney is lower than the pressure in the ash-pit. A comparatively small difference of pressure determines the flow necessary. For example, a difference of pressure of 0·3 lb. per square inch is sufficient to cause a flow fast enough to carry away the lighter parts of the fuel. In practice the difference of pressure is measured by means of liquid contained in a U-tube, one limb of which is put into communication with the gas stream at the base of the chimney, whilst the other limb is open to the boiler room. A U-tube is shown in Fig. 37. The pressure in the chimney is less than the pressure in the boiler-room, therefore the liquid rises in the limb connected to the chimney until it reaches a level sufficiently high to balance the difference between the pressure in the chimney and the pressure in the limb open to the boiler-room.

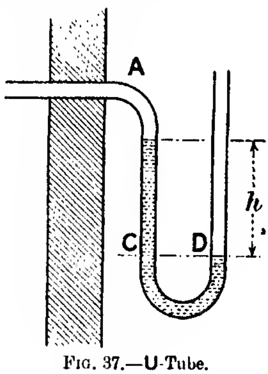


FIG. 37.—U-Tube.

Let P be the pressure of the air in the boiler-room in pounds per square inch, p the pressure of the flue gas at the place where the U-tube is inserted in the flue, D the density of the liquid in the

tube, a the sectional area of the tube. Then when equilibrium is established the pressure in the limb A at the level CD is

$$ap + haD \text{ lbs.}$$

and the pressure at D is Pa lbs.

When there is equilibrium,

$$P - p = \delta p = hD.$$

When the units are in inches and the liquid is water, $D = 0{\cdot}0362$, so that

$$\delta p = 0{\cdot}0362h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

That is to say, a difference of level of 1 in. of water corresponds to a difference of pressure between the ash-pit and the chimney base of 0·0362, or approximately $\frac{1}{28}$ lb. per square inch. Or if δp is measured in lbs. per square foot, and h is the difference of level in inches,

$$\delta p = 5{\cdot}204h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If A lbs. of air flow through a furnace per pound of fuel burnt there will be formed approximately A + 1 lbs. of furnace gas. In equal conditions of temperature and pressure, however, the volume of the air supply per pound of fuel burnt is practically equal to the volume of furnace gas produced. The volume of the furnace gas corresponding to a given air supply can therefore be calculated from the characteristic equation for air, namely,

$$PV = 96\ AT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where P is the pressure in pounds per square foot, V is the volume in cubic feet, T is the temperature in degrees Cent. absolute, and A stands for the number of pounds of air supplied per pound of fuel burnt.

Again, combustion takes place at atmospheric pressure, and this may be taken constant and equal to 2120 lbs. per square foot. Inserting this value in (3)

$$V = \frac{AT}{22} \text{ cub. ft. per lb. of fuel burnt} \quad . \quad . \quad . \quad (4)$$

The density of the air supply and, with only small error, the density of the furnace gas is

$$D = \frac{22}{T} \text{ lbs. per cub. ft.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let Z be the sectional area of the flue at any point in the gas stream where the temperature is T, then the mean velocity of the gas flow across the section is

$$u = \frac{ATw}{22Z} \text{ ft. per second} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where w is the weight of fuel burnt per second.

Consider the application of these equations to the case where air

is supplied at the rate of 16 lbs. per pound of fuel burnt, and the temperature at the base of the chimney is 300° C., corresponding to 573° C. absolute.

From (4) the volume of furnace gas flowing into the base of the chimney per pound of coal burned is

$$V = \frac{16 \times 573}{22} = 417 \text{ cub. ft.}$$

and the density of the gas at the base of the chimney is

$$D = \frac{22}{573} = 0{\cdot}0384 \text{ of a lb. per cub. ft.}$$

Also if coal is burned at the rate of $\frac{1}{3}$ lb. per second, and if the area of the chimney at the base is 16 sq. ft., from (6)

$$u = \frac{16 \times 573 \times 0{\cdot}33}{22 \times 16} = 8{\cdot}6 \text{ ft. per second.}$$

The volume of the furnace gas flowing through any section of the flue or chimney, and also the mean velocity of flow through the section can be calculated in a similar way for given values of A and w when the temperature at the section is measured and the area of the section is known.

Or if w, T, u, and Z are measured, A, the air supply in pounds per pound of fuel burnt, can be determined.

17. Chimney Draught.—A draught is produced by a chimney because the pressure at its base is less than the pressure in the ash-pit. Let H be the height of the chimney in feet, and let D be the mean density of the hot gas which it encloses. Also let P_1 be the pressure of the air on a plane, level with the top of the chimney. Then the pressure inside the chimney on a plane at the base of the chimney, level with the ash-pit, is $P_1 + HD$ lbs. per square foot; and the pressure on this plane at the entrance to the ash-pit is $P_1 + HD_1$, where D_1 is the density of the outside air. The difference of pressure tending to set the air in motion through the grate, the fuel bed, and the flues, is thus

$$(P_1 + HD_1) - (P_1 + HD) = H(D_1 - D) \text{ lbs. per sq. ft.} \quad (1)$$

This is the pressure difference between the gas at the base of the chimney and the outside air when the dampers are all shut, and the chimney is completely filled with gas at the mean density D.

Let $\mathfrak{H}$ be the height in feet of a column of gas, whose density is equal to D, the mean density of the chimney gas, which will produce a pressure equal to this pressure difference. Then

$$\mathfrak{H}D = H(D_1 - D)$$

from which

$$\mathfrak{H} = H\left(\frac{D_1}{D} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This may be regarded as the difference of head between the ash-pit

and the base of the chimney which produces the flow of gas into the chimney from the outside air.

This head is used partly in producing the velocity of flow, and partly in overcoming the frictional resistance to flow through the fuel bed and the flues. Let u be the velocity of flow at the base of the chimney. Then the head corresponding to the flow is $\frac{u^2}{2g}$. Let J be the head required to maintain the flow against the frictional resistances. Then

$$\mathfrak{H} = \frac{u^2}{2g} + J \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The velocity head is small relatively to J and may be neglected. With this simplification

$$\mathfrak{H} = J$$

The head J is measured directly by means of a U-tube in the way explained in section 16. The U-tube is inserted in the base of the chimney, and the difference of level of the liquid in the two limbs of the tube is observed. Let h be this difference in inches, and let D_2 be the density of the liquid in the U-tube. Then h inches of liquid is equal to $\frac{hD_2}{12D}$ ft. of hot gas, and this quantity is therefore equal to J.

Therefore
$$\frac{hD_2}{12D} = \mathfrak{H} = H\left(\frac{D_1}{D} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the liquid used is water, $D_2 = 62{\cdot}5$ lbs. per cubic ft. And with the approximation explained in section 16, $D = \frac{22}{T}$ lbs. per cubic ft. where D and T are corresponding values of the density and the absolute temperature. Introducing these values into equation (4)

$$h = 4{\cdot}2H\left(\frac{1}{T_1} - \frac{1}{T}\right) \text{ ins. of water} \quad . \quad . \quad . \quad . \quad (5)$$

and
$$\mathfrak{H} = H\left(\frac{T - T_1}{T_1}\right) \text{ ft. of hot gas} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Equation (5) gives a relation between the draught h measured in inches of water at the base of the chimney, the temperature T_1 of the outside air, and the mean temperature T of the gas enclosed by the chimney stack.

Returning now to a consideration of the velocity of flow, let f be the fraction of the head which is used to produce the velocity.

Then
$$\frac{u^2}{2g} = f\mathfrak{H}$$

Solving this for u, and putting c for the constant outside the root sign

$$u = c\sqrt{\mathfrak{H}} = c\sqrt{H\left(\frac{T - T_1}{T_1}\right)} \text{ ft. per second} \quad . \quad . \quad (7)$$

This relation connects the velocity of flow with the head created by the chimney. The constant c has the value 2 when $\frac{1}{16}$ of the head is used to produce the velocity.

From an examination of the data recorded in the *Proceedings of the Institution of Mechanical Engineers* in connection with the Marine Engine Trials, the author has found that the value of c for the chimneys of the ships tested ranged from 1·6 to 2·7.

The weight of gas discharged into the chimney through the section at the base is equal to the product of the velocity of flow u; the density D_b at the base of the chimney; and the area Z of the section at the base.

Let F be the discharge in pounds per second. Then

$$F = D_b u Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

D_b is here the actual density of the gas as it crosses the section, and is equal to $\frac{22}{T_b}$, where T_b is the absolute temperature of the gas at the section. Using this value of the density, and the velocity of flow from (7), equation (8) becomes

$$F = 22c\sqrt{H}\sqrt{\frac{1}{T_1}}\left\{\frac{\sqrt{T - T_1}}{T_b}\right\}Z \quad . \quad . \quad . \quad . \quad (9)$$

This discharge is a maximum when $\frac{\sqrt{T - T_1}}{T_b}$ is a maximum.

Assume that the temperature of the gas in the chimney is uniform and equal to the temperature at the base. Then $T = T_b$. Inserting this value of T_b in the above expression, differentiating with regard to T and equating to zero, it will be found that the expression is a maximum when $T = 2T_1$. Substituting this value of T in (9), it will be found that

$$F_{max} = \frac{11}{T_1}c\sqrt{H}Z \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It is more convenient to state the discharge in pounds per hour, F_h say. Further, assuming an outside temperature of 15° C., corresponding to $T_1 = 288$ degrees absolute, and taking 2 as a suitable value for c, the expression becomes

$$\left.\begin{array}{c}\text{Maximum discharge of furnace}\\ \text{gas in pounds per hour}\end{array}\right\} = F_h = 275\sqrt{H}Z \quad (11)$$

These results are illustrated by the following example. A round chimney is 43 ft. high and 16 sq. ft. area. The temperature at the base of the chimney is 576° C. absolute. The mean temperature in the chimney is 510° C. absolute. The temperature of the outside air is 286° C. absolute. Furnace gas produced per pound of fuel burnt 22·8 lbs. Find the draught in inches of water; the velocity with which the gas enters the base of the chimney; the total weight of gas discharged into the chimney per hour; the weight of fuel which can be burnt per hour.

From equation (5)

$$h = 4 \cdot 2 \times 43 \times \left(\frac{1}{286} - \frac{1}{510}\right) = 0 \cdot 28 \text{ in. of water.}$$

From equation (6) the head producing the flow is

$$\mathrm{H} = 43\left\{\frac{510 - 286}{286}\right\} = 33 \cdot 7 \text{ ft. of hot gas.}$$

Take $c = 1 \cdot 75$. Then from (7)

$$u = 1 \cdot 75 \times \sqrt{33 \cdot 7} = 10 \cdot 1 \text{ feet per second.}$$

The density at the chimney base is $\frac{39}{510} = 0 \cdot 0382$ lb. per cubic foot. The discharge in pounds per second is from (8).

$$F = 0 \cdot 0382 \times 10 \cdot 1 \times 16 = 6 \cdot 2$$

Discharge in pounds per hour $= 3600 \times 6 \cdot 2 = 22{,}300$.

$$\text{Weight of fuel which can be burnt per hour} = \frac{22300}{22 \cdot 8} = 980 \text{ lbs.}$$

F can also be calculated directly from equation (9).

In order to determine the size of a chimney for a plant of stated horse-power an estimate must be made of the weight of furnace gas produced per hour.

Let W be the weight of fuel burnt per horse-power-hour in a plant of HP total horse-power; and let A be the weight of air supplied per pound of fuel burnt. Then the total weight of gas flowing into the chimney per hour is $W(A + 1) \times HP$ pounds.

If the chimney is worked at a temperature to give the maximum flow, and also if the mean temperature of the chimney approximates to the temperature at the base, the discharge per hour is given approximately by equation (11). Equating these two equations, and solving for Z the cross-section of the chimney at the base is

$$Z = \frac{W(A + 1) \times HP}{275\sqrt{H}} \quad \ldots \ldots \quad (12)$$

Or solving for the horse-power, a chimney of height H and cross-section Z will draw air to provide for working at the rate of HP horse-power, thus

$$HP = \frac{275Z\sqrt{H}}{W(A + 1)} \quad \ldots \ldots \quad (13)$$

The expressions (10), (11), (12), and (13) are based on the assumption that the temperature in the chimney is twice the temperature in the outside air. The chimney may, however, be worked at a lower temperature without serious loss of power. For instance, with $c = 2$ and $T_1 = 288$, equation (9) becomes

$$F = 2 \cdot 6\sqrt{H}\left\{\frac{\sqrt{T - 288}}{T}\right\} \quad \ldots \ldots \quad (14)$$

The factor in the brackets reduces to $0 \cdot 0295$, if $T = 576$, the temperature for the maximum discharge, but it only falls to $0 \cdot 0288$ if the temperature T is reduced by 100 degrees.

The expressions above are based on the assumption that the resistance to flow is produced by the fuel bed and the flues, and the resistance to flow through the chimney itself has not been included. The resistance to flow in the chimney itself reduces the discharge for a given height, and as the height increases the resistance to the flow may so reduce the discharge that nothing is gained by increasing the height.

To allow for the loss due to this cause the actual area of the chimney may be made slightly larger than the area Z calculated from the equation (12). In some chimneys an area equal to the perimeter of Z multiplied by 1/6 is added. This is equivalent to allowing a thickness of two inches of inert gas over the whole of the inside surface.

The difference of pressure established by a chimney working with natural draught enables enough air to be drawn through the fuel bed to burn properly from 15 to 20 lbs. of coal per square foot of grate per hour. This rate of combustion may be increased by increasing the draught, and a limit is reached when the velocity of the gas is great enough to carry away incompletely burned fuel along with it.

The general method of forcing draught is by means of fans, applied to increase the velocity of flow either of the furnace gas or the air supply to the furnace. A method largely used in warships and fast passenger boats is to make the stokehole air-tight; and then to drive air into it by fans, so that the pressure inside is slightly greater than the pressure outside. The only exit for the air from the stokehole is through the ash-pits and fuel beds. The draught produced by the combined action of the fan and the chimney varies usually from 1 to 2¼ ins. of water, an intensity sufficient to burn from 30 to 50 lbs. of coal per square foot of grate per hour. A fan is sometimes applied to draw the air through the furnace and to cause what is called an induced draught. In this case it is placed at the base of the chimney, and all the hot gases pass through it. The fan in fact draws furnace gas from the flues and discharges it into the chimney. A bye-pass flue is arranged so that, by means of dampers, the furnace gas may be guided direct into the chimney without passing through the fan.

Another way of forcing the draught is to close the ash-pits, as in the Howden system, where the fan drives air into the ash-pit through a chamber heated by tubes through which some of the furnace gas flows. The furnaces are thus supplied with warm air under pressure.

The advantages of mechanical draught are, that the draught may be regulated to a nicety by regulating the speed of the fans, and that the furnace gas may be reduced to a lower temperature than is desirable when working with chimney draught, and thus the furnace efficiency may be increased. Also a rate of combustion may be maintained at the grate, greater than is possible with natural draught, thus reducing the weight of the boiler per horse-power. Against

these advantages, however, must be set the disadvantage of maintaining the running machinery necessary with mechanical draught, and the greater wear and tear on the boiler which is inseparable from higher rates of combustion. A chimney once built requires further expenditure only for occasional pointing; and the rate of combustion which it determines is so moderate that the cost of boiler maintenance is small.

18. The Steam Blast.—In a locomotive the draught is produced and maintained by the action of the exhaust steam on the gases in the smoke-box as the steam passes from the blast pipe to the chimney. The immediate effect of the action is to reduce the pressure in the smoke-box below that of the surrounding atmosphere, and thus to establish a difference between the pressure in the smoke-box and the pressure in the ash-pan, under the action of which air flows in through the ash-pan dampers, through the grate, tubes, and away up the chimney. The draught caused in this way is so powerful that sufficient air can be drawn through the furnace to burn considerably over 100 lbs. of coal per square foot of grate per hour, though this rate of combustion cannot be much exceeded without serious loss of furnace efficiency. Some idea of the powerful draught caused by the steam blast can be obtained from the data recorded in the experiments of Adams and Pettigrew[1] in connection with the trials of an express locomotive.

The maximum draught and the mean draught at various points in the path of the flow observed during a journey from Exeter to Waterloo at a mean speed of 44·45 miles per hour, are shown in the following table:—

TABLE 4.—Draught Observed during a Locomotive Trial.

	Maximum draught observed during the journey. (Inches of water.)	Mean draught during the journey. (Inches of water.)
At base of chimney	15	7·33
At blast pipe orifice	12·8	6·27
In smoke-box, level with middle row of tubes	9	4·77

There was a slight pressure in the ash-pan, caused by the motion of the engine through the air, which had a maximum value of 0·8 in. of water, and a mean value of 0·19 in.

The average rate of combustion was 80 lbs. of coal per square foot of grate per hour.

Comparing these figures with the results obtained by natural chimney draught, it will be seen that the steam blast is much more powerful.

[1] Adams and Pettigrew, "Trials of an Express Locomotive," *Proc. Inst. C.E.*, vol. 125, 291.

The dimensions of the smoke-box, chimney, and blast pipe require careful adjustment in relation to one another in order to secure a draught suitable for the boiler. The actual proportions can only be found experimentally.

An exhaustive series of experiments were carried out by a Committee of the Railway Master Mechanics Association on Exhaust Pipes and Passages[1] as a result of which the Committee recommended that the dimensions indicated in Fig. 38 should have the

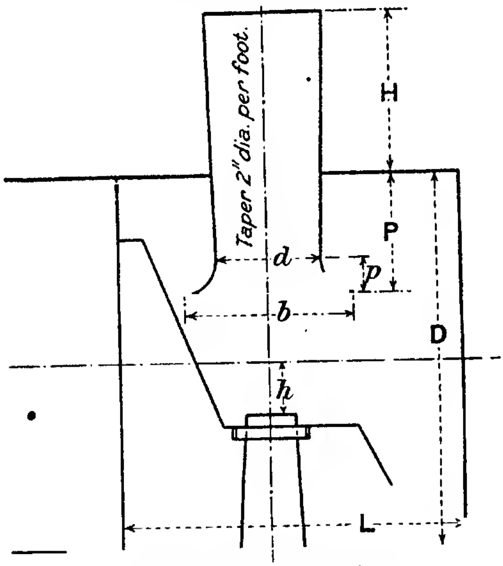


Fig. 38.—Smoke-box proportions.

following relations among themselves after making H and h as great as practicable:—

$d = 0{\cdot}21D + 0{\cdot}16h$
$p = 0{\cdot}22D$
$b = 2d$ or $0{\cdot}5D$
$P = 0{\cdot}32D$
$L = 0{\cdot}6D$ or $0{\cdot}9D$, but not of intermediate values.

This last relation is, however, not well established.

Fig. 39 shows the smoke-box of a London and North-Western Locomotive of the Precedent Class. The dimensions agree fairly well with those which would be found from the above expressions. The top of the blast pipe is encircled by a **ring-blower** B, perforated with holes. Steam is supplied to the ring when the engine is standing or when steam is shut off from the cylinders, and the jets from the holes cause a draught sufficient to keep the fire going and to prevent a back draught.

[1] *Proc. Amer. Railway Master Mechanics Assoc.*, 1906.

The powerful draught caused by the blast entrains a considerable weight of sparks. These sparks are either all thrown out of the chimney top or are in part arrested and caught in the smoke-box. The arrangements for arresting sparks are somewhat elaborate in American and in Continental locomotives. In English practice a spark arrester, when one is fitted, usually takes the form of a wire netting placed below the level of the blast orifice, and above the top

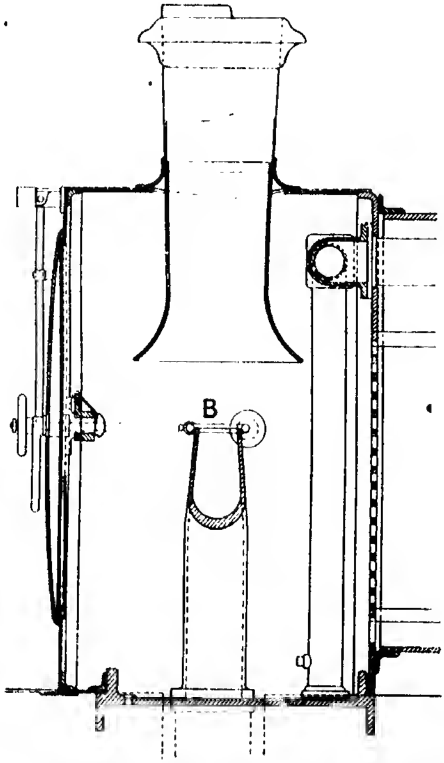


FIG. 39.—Smoke-box, London and North-Western Railway.

row of tubes. The heavier sparks and cinders issue from the tubes with such a high velocity that they continue in their straight paths until they impinge on the smoke-box door, and it is on the rebound that they are caught up in the current of hot gas and projected from the chimney top. Taking advantage of this Mr. James Holden fitted to some engines of the Great Eastern Railway, at suitable places in the smoke-box, louvres, which deflect the sparks downwards so that they collect in a heap on the floor of the smoke-box round a tube

which is essentially an ejector in communication with the blast pipe. At every blast a small quantity of steam passes through the ejector and expels the sparks into receptacles at the side of the engine. Detailed information with regard to smoke-box practice in the United States will be found in "Locomotive Sparks," W. F. M. Goss. London, 1902.

The action of the steam jet on the gas in the smoke-box is intermittent, but it was shown by the Committee of the Railway Master Mechanics Association that a jet of steam flowing steadily from the exhaust pipe, the engine being at rest, produces draught conditions which are in every respect similar to those obtained when the engine is running, providing that equal weights of steam per unit of time are discharged through the chimney in each experiment.

Consider, therefore, a jet of steam flowing steadily from the blast-pipe orifice. It flows with a velocity due to the difference between the back pressure in the cylinders and the atmosphere, and as it flows it spreads laterally into a conical form, the increasing diameter of which becomes equal to the diameter of the chimney near the top, thus leaving a conical annulus in the lower part of the chimney.

As the jet issues from the orifice of the blast pipe, its surface sets the layer of gas in contact with it in motion, and owing to the viscosity of the gas, this motion is transmitted in varying degree to all parts of the gas in the smoke-box. As the jet flows towards the chimney, the gas in contact with it penetrates across the boundary of the steam jet and mingles with it, this penetration going deeper and deeper into the steam jet as the flow approaches the upper part of the chimney. Meanwhile the conical annulus in the lower part of the chimney is filled with furnace gas drawn upwards by the increasing draught towards the throat of the chimney, until near the top this gas cone mingles with the jet also, so that finally a jet of mixed steam and gas is discharged from the chimney top. The flow from the blast-pipe orifice up to a height in the chimney a little way below the top may be analysed into a cone of pure steam, round which is formed a cone of mixed steam and gas, and round this again a cone of pure gas.

The velocity of the central core of pure steam diminishes towards the top as the mixed cone entrained by it enlarges, and at the same time its temperature increases, though this does not occur at the centre until near the top when the mixture is more complete.

The velocity of the outer cone of pure gas increases as it flows upwards, and its temperature falls until finally towards the top the boundaries of the separate cones disappear, and the stream issuing from the chimney top is a mixture of gas and steam of sensibly uniform temperature and velocity.

The boundaries of these separate cones are well marked in the throat of the chimney, that is the narrowest part of a conical chimney, or the section where the area becomes uniform in a cylindrical chimney. These facts have been established by means of an elaborate research on the action of the chimney and blast pipe made by F. C.

Huygen,[1] of Delft, on a Belgian locomotive built by Beyer Peacock & Co. in 1891, having the following principal dimensions :—

Grate area 23·7 sq. ft. Total heating surface 1100 sq. ft.

Cylinders 18 ins. diameter 26 ins. stroke. Area of exhaust orifice 17 sq. ins.

Holes were drilled at vertical intervals in the chimney through which instruments could be inserted for the measurement of the temperature, the velocity, and the pressure of the flow, at any point on a horizontal diameter of the chimney level with each hole. Consider the observations of temperature at one horizontal level. Immediately inside the chimney the temperature shown is that of the outer annulus of pure furnace gas, and as the thermometer is pushed towards the centre of the chimney this temperature remains constant until it reaches the boundary of the mixed cone. Here it begins to fall and the temperature goes on falling as the thermometer passes through the cone until it reaches the boundary of the cone of pure steam. After this the temperature remains constant up to the centre at 100° C. Fig. 40 shows a section of the chimney on which several experiments were made. The horizontal lines correspond to the different levels of the observation holes. Consider level number 5. The letter *a* marks the point where the temperature was observed to fall, and the letter *b*, the point where the temperature fell to 100°. The lines through these points connect the points found in a similar way at different levels and give the boundaries of the cone as determined from the observed temperature changes. Curve No. 1 in Fig. 40, p. 88, shows the temperature of the outer cone of gas, and curve No. 2 that of the centre of the cone of pure steam. At level 11 the temperature of the core begins to rise, showing almost complete penetration of the gas to the centre, and finally discharge takes place at the common temperature 130° C. The section of this temperature cone on the right side is cross-hatched.

The velocity of the furnace gas across a horizontal section is approximately constant up to the boundary of the mixed cone. Here the velocity begins to increase and the increase continues through the mixed cone until in the core of pure steam it becomes constant again. The letter *c* shows the point where the velocity of the gas begins to increase, and the letter *d* the point where it reaches the velocity of the inner core of steam, and these points, therefore, mark the boundaries of the cones at level 6. The dotted lines drawn through them pass through points found in a similar way at different levels. The section of the mixed velocity cone on the left side is cross-hatched. Curve No. 3, Fig. 40, shows the velocity of the gas in the outer cone and indicates the way the velocity increases as the chimney top is approached, and curve No. 4 shows the velocity of the inner jet.

In a similar way points *e f* mark the observed boundaries of the pressure changes at level 7, and the thick lines through them connect

[1] "Over de Exhaust-werking bij Locomotiven," Huygen, F. C. "A Thesis for a Doctorate at the Technical High School," Delft, 1907.

all points found in the same way at different levels. Curve No. 5, Fig. 40, shows the draught in inches of water. It will be observed how rapidly the draught increases towards the throat of the chimney.

The research included experiments on long and short chimneys, both cylindrical and conical in form, and the conclusions arrived at were that the forms of the steam and mixed cones were independent of the form of the chimney, and that at the top the cones merged one into another unless the chimney were too short, and that the cones kept their individuality to a higher level in a conical chimney than in a cylindrical chimney.

The form of chimney recommended as a result of these experiments is cylindrical, of the same general design as that shown in Fig. 40, the distance from the exhaust orifice to the throat of the chimney being 31 ins. At this level the diameter of the steam jet itself enlarges to about 12·7 ins. from an orifice $5\frac{1}{4}$ ins. diameter.

Some account of the experiments made at Purdue University by Prof. Goss is given in "Locomotive Sparks," mentioned above.

The size of the blast-pipe orifice may be found on the assumption that the continuous flow of steam at a uniform rate, equal to the mean rate of the actual intermittent flow, produces blast conditions which are practically the same as the actual conditions.

Let A be the area of the blast-pipe orifice in square feet;
- D, the density of the steam as it flows through the orifice;
- w, the velocity of flow through the orifice in feet per second;
- F, the weight of steam discharged through the orifice in pounds per second;
- Q, the weight of steam produced by the boiler per second.

Then the weight of steam flowing through the orifice per second is

$$F = DAw \text{ lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But this must be equal to the weight of steam produced by the boiler per second, neglecting small losses, so that

$$Q = DAw = \frac{\pi d^2}{4 \times 144} Dw$$

where d is the diameter of the orifice in inches. Solving for d,

$$d = 13{\cdot}5\sqrt{\frac{Q}{Dw}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or solving for the product Dw, which is the weight of steam in pounds discharged per second per square foot of the orifice,

$$Dw = \frac{182Q}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The quantity Q may be reckoned in various ways. One of the most convenient is to compute the indicated horse-power which the engine

is to exert continuously, and then to allow 25 lbs. of steam per indicated horse-power-hour. With this assumption

$$Q = \frac{\text{I.H.P.}}{144} \text{ lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Or Q may be found from the heating surface by allowing an average of $1\frac{1}{4}$ lbs. of water evaporated per square foot of heating surface per hour. This gives

$$Q = \frac{H.S.}{327} \text{ lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and corresponds to about 2·3 sq. ft. of heating surface per indicated horse-power.

It is shown below, Section 180, equation (5), that the product Dw can be calculated when the inital and the final pressures producing the discharge are given, providing that the flow is assumed to be frictionless and adiabatic. The pressure against which the discharge

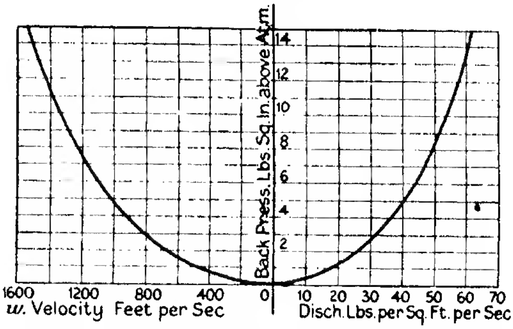


Fig. 41.—Flow through blast pipe.

from a blast pipe is made differs so little from the atmospheric pressure, that it may be taken as constant and equal to 14·7 lbs. per square inch. The initial pressure producing the discharge is the time average of the back pressure in the cylinder multiplied by a factor to allow for the pressure required to overcome the frictional resistance to the flow of the steam from the cylinder to the orifice. It is impossible to make even an approximate estimate of this average pressure and the factor to use with it, by any direct process. But an indirect evaluation of the time average of the back pressure required to produce the flow through the orifice can be made in the following way.

The curves in Fig. 41 have been plotted by methods explained below in Section 181. The pressures marked on the vertical axis are the continuous back pressures which must be maintained in the cylinder to produce the velocity and the discharge determined by a horizontal drawn through any particular back pressure to cut the velocity curve on the left, and the Dw curve on the right.

For a given value of Q and of d the value of the product Dw can be calculated from equation (3). This value of Dw is then found on the horizontal axis to the right of the diagram, a vertical is drawn to cut the curve, and through the point on the curve so determined a horizontal is drawn to cut the vertical axis and the velocity curve. The point of intersection on the pressure axis fixes the continuous back pressure in the cylinder which is required to maintain the discharge Dw; the intersection on the velocity curve projected downwards to the velocity scale shows the corresponding actual mean velocity of flow through the orifice. The pressure is relatively small; the greater part of the back pressure in the cylinder is required to overcome the frictional resistance of the exhaust ports and passages.

For example, suppose an engine is required to develop continuously 1000 I.H.P., and that the diameter of the blast nozzle is 5·0 ins. From (4), Q = 6·95. From (3), Dw = 50·4.

From the curves it will be found that the corresponding back pressure is 8 lbs. per square inch, and that the velocity of discharge is about 1200 ft. per second.

The author has found from an analysis of the data collected from a number of engines that the velocity of discharge is generally in the neighbourhood of 1000 ft. per second, with an equivalent back pressure in the cylinder of 5 lbs. per square inch. The corresponding rate of discharge per square foot of orifice is 40. This figure, therefore, may be substituted for Dw in equation (2), and the diameter of the orifice can then be found when Q is known. With this substitution

$$d = 2·14\sqrt{Q} \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{If } Dw = 30, \text{ then } d = 2·5\sqrt{Q} \quad . \quad . \quad . \quad . \quad (7)$$

19. **Fuels.**—The natural fuels available are wood, peat, coal, oil, and gas. Of these coal is by far the most generally used for firing steam boilers, though oil fuel is used in countries where it is cheap and plentiful, and also in special cases where it is even more expensive than coal because of its greater convenience. Natural gas is in some parts of the United States used as a fuel, and occasionally gas obtained as a bye-product is used locally as a fuel, as, for instance, where the blast furnace gases are utilized instead of allowing them to burn away at the furnace tops. In countries where other natural fuels are scarce, such vegetable refuse as cotton stalks and megasse or sugar cane refuse are used in the factories where they are produced.

Coal is a composite fuel consisting mainly of carbon, but containing in general hydrogen, oxygen, nitrogen, sulphur, with a certain amount of moisture and a quantity of mineral matter, which is left behind as ash when the coal is burned.

"Dry coal" is coal from which the moisture has been driven off artificially, and "pure dry coal" is a term used to denote the combustible matter left when the water has been driven off and the ash has been subtracted. Thus, if a sample of coal contains 5 per cent.

of moisture and 5 per cent. of ash, it contains only 90 per cent. of combustible matter.

When dry coal is heated out of contact with air, as in a retort or in a closed crucible, volatile gas is driven off and the solid substance remaining is described as the combustible residue or as fixed carbon, though it contains other elements than carbon, or more briefly as coke. Both the volatile gas and the coke are fuels. The act of heating has merely divided the coal into two fuels of different kinds, the volatile gas, consisting chiefly of hydrocarbons, and the coke consisting chiefly of carbon. The volatile gas burns with a long flame and leaves no residue. Coke burns with hardly any flame and leaves the ash behind.

Industrially, coals are distinguished into Anthracite coals, Bituminous coals, and Lignites.

Anthracite contains about 90 per cent. of carbon, and when heated gives only a small percentage of volatile gas. It burns with no smoke, and gives intense local heat.

Bituminous coal contains from 80 to 90 per cent. of carbon, and from $4\frac{1}{2}$ to 6 per cent. of hydrogen, and from 3 to 14 per cent. of oxygen. It softens when heated, and produces from 10 to 35 per cent. of volatile matter.

A bituminous coal high in carbon is called a semi-anthracite, or steam coal, because it burns more easily than anthracite, evolving almost as much heat per pound burnt, and is practically smokeless. Bituminous coals low in carbon approach the lignites or brown coals in character.

Lignite as a fuel stands between wood and the true coals. There are many varieties, generally low in carbon, of which there is rarely more than 70 per cent. present. They contain larger proportions of water and oxygen than the true coals.

Cannel coal is used chiefly for gas-making, and contains a relatively large proportion of volatile matter, usually over 35 per cent.

Coals are also distinguished into the caking and the non-caking varieties. Caking coals soften when heated, and ultimately yield a coherent coke. Non-caking coals burn freely, and in fact are often described as free-burning coals. Anthracite and semi-anthracite coals are free from any tendency to cake.

The composition of coal may be given either by a proximate analysis or by an ultimate analysis. A proximate analysis is more easily made than an ultimate analysis. It shows five constituents of the fuel, namely

1. Moisture.
2. Volatile combustible matter.
3. Combustible residue or coke.
4. Sulphur.
5. Incombustible matter, that is, the ash.

The volatile gas consists of the elements carbon, hydrogen, oxygen, and nitrogen, combined in unknown ways to form in general hydro-

carbons. The combustible residue, consisting chiefly of carbon, may contain these elements in small proportions also.

Although a proximate analysis gives valuable information regarding the character of the coal, it does not give sufficient data from which to calculate the calorific value, or the needful air supply. For this purpose an ultimate analysis is required. An ultimate analysis shows the percentage of

1. Moisture.	5. Sulphur.
2. Carbon.	6. Nitrogen.
3. Hydrogen.	7. Ash.
4. Oxygen	

The two most variable constituents are the moisture and the ash. Moisture may be absorbed after the coal is mined. Ash is mineral earthy matter mixed with the coal, and may vary considerably with the same essential coal according to the place in the seam from which the coal is mined. It is usual, therefore, to state the composition of coal in terms of the carbon, hydrogen, oxygen, and nitrogen present, excluding the water, sulphur, and the ash from the account, in this way obtaining analyses which are more comparable with one another.

The difficulty of making an ultimate analysis of coal has led Professor L. S. Marks, of Harvard University, to suggest a method whereby the ultimate analysis can in part be derived from the proximate analysis. The United States Geological Survey has published both the proximate and ultimate analyses of two hundred and forty kinds of fuel collected from twenty-eight different States, the samples representing every kind of fuel from anthracite to peat, and having a range of volatile matter from 6 to 70 per cent. From these analyses Professor Marks has found that the percentage of hydrogen and nitrogen in pure dry coal can be inferred from a curve, within about a half per cent. maximum error, for coals which contain from 18 to 48 per cent. of volatile matter. The curves are shown in Fig. 42. Thus, given a coal containing 36·5 per cent. of volatile matter, the hydrogen present in the dry combustible will be, reading from the curve, 5·3 per cent., and the nitrogen 1·7 per cent. It is interesting to compare the results obtained in this way with the results of the actual ultimate analysis of the coal, Nixon's Navigation, whose analysis is given in Section 24, below. The volatile matter shown in the proximate analysis is 11·72 per cent. Referring to the curves, the hydrogen and nitrogen in the ultimate analysis would be 4 per cent. and 1·15 per cent. respectively. The actual amounts given in the analysis are 4·12 per cent. of hydrogen and 0·98 per cent. of nitrogen. The carbon in the volatile matter cannot be so easily determined. Professor Marks suggests, however, that the total carbon present can be found from an analysis of the products of combustion remaining in a bomb calorimeter used for the actual determination of the calorific value of the fuel. The sulphur can be determined in a similar way, and finally the oxygen can then be found by difference. The method of carrying out the determination is fully described in

Professor Marks' paper, which is published in the Harvard Engineering Journal, April, 1909.

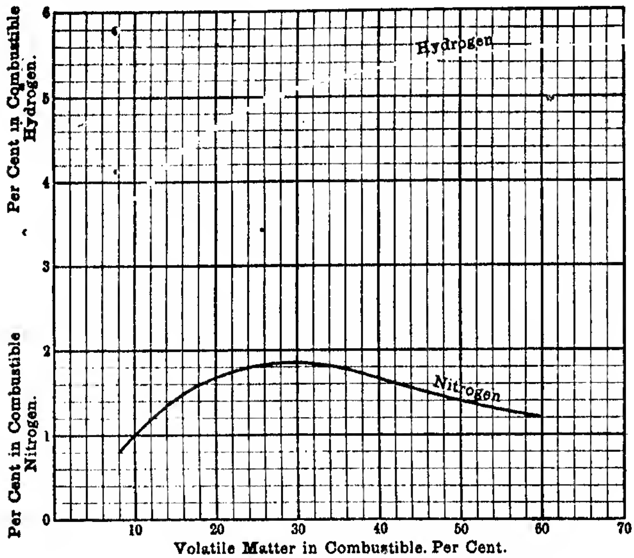


FIG. 42.—Professor Marks' diagram.

20. Combustion of Coal.—The combustion of coal involves the simultaneous burning of the two fuels into which it is separated when it is heated, namely, the volatile hydrocarbons and the combustible residue.

Air must be drawn through the fuel bed to burn the combustible residue, and air must be admitted over the top of the fuel bed through regulated openings in the fire-hole door, to burn the volatile hydrocarbons. When the volatile products form a large proportion of the fuel it may be necessary to admit even a further supply of air to the flue of boilers of the Lancashire type, and a hollow fire-bridge through which air can flow from the ash-pit directly into the flue is sometimes added, along with a damper to regulate the supply.

In some stationary and marine boilers the fuel is sometimes fired on to a dead plate, just inside the furnace door. A dead plate is really part of the grate area, but it is unpierced with any openings into the ash-pit, and in fact it terminates in a support for carrying the furnace door end of the fire-bars. The volatile gases are driven off as the fuel lies on the dead plate, and at the next firing the fixed residue is pushed along on to the fire-bars, and the fresh supply of fuel is placed on the dead plate.

When combustion is complete the carbon present in the fuel is burnt to carbon dioxide and the hydrogen is burnt to water, any

sulphur present being burnt to sulphur dioxide. If the air supply is *insufficient, the carbon is only partly burnt to carbon dioxide*, and carbon monoxide appears in the furnace gas. If the air supply through the furnace door is insufficient, the volatile hydrocarbons are not properly consumed, and smoke is formed. The object of the brick arch in a locomotive fire-box is to guide the volatile gases backwards across the fire, where meeting the air admitted through the fire-hole door they are subsequently burnt to carbon dioxide and water. Skilful firing consists in so adjusting the amount of air admitted through the ash-pit, grate, and fuel bed, in relation to the amount admitted through the furnace door, that no carbon monoxide appears finally in the furnace gas, and that no smoke is produced, and at the same time so regulating the total quantity of air that it is just sufficient for the purpose. If too much air is admitted the temperature of the furnace will be reduced and the general efficiency of the boiler will be diminished because an unnecessarily large weight of furnace gas has been produced per pound of coal burned, and this entails a consequent reduction of the temperature head available for transferring heat from the heating circuit to the water in the motive-power circuit.

21. Rate of Combustion.—Excess Air.—The rate at which fuel can be consumed on a grate is determined by the rate at which air is supplied to the furnace. For every pound of fuel fired there is a definite minimum theoretical quantity of air required, and this can be calculated when the chemical constitution of the fuel is known in the way illustrated in Section 24.

It is always found that even when regulated by a skilful fireman the supply of air admitted to the furnace is in excess of this theoretical quantity.

For example, in the experiments of Donkin and Kennedy on stationary boilers, the coal used, Nixon's Navigation, required theoretically 10·37 lbs. of air for the complete combustion of a pound of the coal. In the range of the twenty-one experiments the air actually admitted varied from 16·2 lbs. per pound of coal in a locomotive boiler tested as a stationary boiler, to 44 lbs. per pound of coal in the experimental boiler at University College, an intermediate figure being furnished by an experiment on a Lancashire boiler where 37·35 lbs. were used per pound of coal.

In general the amount of excess air depends upon the rate at which the combustion is effected, and is generally considerably less in locomotive practice than in land and marine practice. In Prof. Goss' experiments with an engine on a testing plant, recorded in "High Steam Pressure in Locomotive Service," the excess air varied from 1·7 to 55 per cent. of the theoretical quantity. The general average of the excess recorded was 15 per cent.

The results of an interesting and instructive series of experiments regarding the completeness of combustion in locomotive practice, with which question is bound up that of the air supply, were

communicated to the Institution of Mechanical Engineers in a paper entitled "Combustion in Locomotive Fire-boxes," by Dr. F. J. Brislee, in March, 1908. The experiments were made on the London and North-Western Railway. *Engines of the Precursor and the Experiment class were fitted with apparatus for collecting samples of the furnace gases* from the smoke-box during their ordinary service. Numerous samples were taken whilst the engines were working in their usual services on the main line, and the subsequent analyses of the samples indicated that, notwithstanding the high rate at which the fuel was consumed, the combustion was efficient. One set of analyses may be quoted relating to a journey from Preston to Carlisle and back, with an engine of the Experiment class. The air drawn through the furnace per pound of carbon in the furnace gases, calculated from equation (6), Section 26, is added in the last column of the table.

TABLE 5.

ANALYSES OF GAS SAMPLES collected from the smoke-box of an engine of the Experiment class during a journey from Preston to Carlisle and back, taken from a paper communicated to the Institution of Mechanical Engineers in 1908 by Dr. F. J. Brislee, to which is added the corresponding air supply to the furnace per pound of carbon in the furnace gas.

Experiment number.	Number of coaches.	Gradient.	Speed. Miles per hour.	Percentage by volume.				Vacuum in smoke-box. Inches of water.	Air supply per pound of carbon in the gases from $\left\{\frac{250}{CO_2+CO}-1\right\}$
				CO_2	O_2	CO	N_2, etc., by difference.		
1.	18	1 in 1199 up	61·0	16·3	0·2	1·6	81·9	7·5	13·0
2.	9	1 in 75 up	64·3	16·1	1·1	0·7	82·1	7·5	13·9
3.	9	1 in 75 up	42·8	13·6	3·0	1·6	81·8	8·0	15·4
4.	9	1 in 125 down	75·0	15·0	1·0	1·0	83·0	4·1	14·6
5.	12½	1 in 184 up	42·8	13·5	0·0	2·2	84·3	6·0	14·9
6.	12½	1 in 136 up	51·4	15·0	1·6	2·2	81·2	5·0	13·1
7.	12½	1 in 125 up	42·0	15·8	0·0	1·0	83·2	5·9	13·9
8.	12½	1 in 104 down	69·2	15·9	2·8	0·2	81·1	6·4	14·5
9.[1]	12½	1 in 90 up	22·0	14·6	3·6	0·0	81·8	4·9	16·1

The general conclusions drawn from these experiments were that the combustion was efficient, and that a thin fire, as used in the Experiment class, tends to promote efficient combustion, the thicker fire of the Precursor class appearing to determine more carbon monoxide in the furnace gases.

It must not, however, be overlooked that a thin fire is not easy to manage when an engine is steaming hard. The blast is more easily able to produce holes in the fuel bed and thus let air through at a few points, which of course reduces the efficiency of combustion greatly. The fire should be thick enough to prevent this happening. The actual thickness in any case depends upon the kind of coal to be burnt.

[1] Starting.

The limiting velocity of air through the fuel bed, that is, the velocity at which the lighter parts of the fuel are caught up in the current of air and carried away, is never produced by the natural *draught of a chimney, but it can, of course, be reached by any method of forced draught working.*

In locomotive practice the steam blast from an engine working at full load generally produces a draught in the neighbourhood of the limiting velocity, and sometimes this velocity is exceeded: the fuel is not properly burned, the permanent way is littered with cinders, and the furnace efficiency falls.

Dean Goss has devoted considerable time to the investigation of this question, and the results of many experiments are given in his book, "Locomotive Sparks," Chapman & Hall, London, 1902. The following figures, quoted from this book, are instructive. They relate to an experiment with Brazilian block coal. As would be expected, the sparks increase in calorific value as the rate of combustion increases.

It will be observed that a loss equivalent to 14·2 lbs. of coal per square foot of grate per hour was incurred when firing at a rate of 121 lbs. of coal per square foot per hour with a draught of about 5 ins. of water.

TABLE 6.—SPARK LOSSES DEDUCED FROM EXPERIMENTS MADE BY DEAN GOSS.

Speed in miles per hour.	Coal fired per square foot of grate per hour.	Sparks produced per square foot of grate per hour.	Coal equivalent to 1 lb. of sparks.	Per cent. of energy lost.	Draught.
	lbs.	lbs.	lbs.		Inches of water.
15	45·23	1·95	0·665	2·9	1·93
35	83·7	5·96	0·8	5·7	2·98
35	86·5	5·43	0·8	5·1	3·02
35	88·98	6·20	0·81	5·7	3·00
55	107·68	16·25	0·84	12·7	3·57
35	115·29	10·1	0·85	7·5	4·88
15	121·2	16·74	0·85	11·9	4·99

In the experiments made by Adams and Pettigrew[1] with an express locomotive on the London and South-Western Railway, the efficiency of working was high and the respective rates of combustion during five runs were, 62, 70, 72, 80, and 72 lbs. of coal per square foot of grate per hour of running time, with corresponding consumptions of 2·31, 2·61, 2·34, 2·52, and 2·45 lbs. of coal per I.H.P. per hour. This is a fine performance, and shows that a locomotive is an economical plant when the rate of combustion is moderate. As the rate of combustion increases the completeness of combustion and therefore the plant efficiency become less, since a larger proportion of the fuel is carried away unconsumed, and, as indicated in the above table, when Brazilian block coal is burnt at the rate of 107 lbs. per square foot of grate per hour, 12 per cent. of the energy

[1] *Proc. Inst. C.E.*, vol. 125.

may be lost in the form of sparks, and this is quite apart from *the increased loss due to incomplete combustion.*

The full significance of the effect of the rate of combustion on the economy will, however, be more clearly brought out in connection with the diagrams of performance in Chapter VI.

The general facts regarding combustion and boiler performance are exhibited in table 10, page 122.

22. Calorific Value.—The Combustion of certain Definite Chemical Compounds.—Chemical reactions between substances are in general accompanied by the production or the absorption of heat energy. For instance, the combination of lead with sulphur to form lead sulphide is accompanied by the production of 89 lb.-cals. of heat energy for every pound of lead entering into the reaction. And the combination of carbon with carbon dioxide to form carbon monoxide absorbs 1625 lb.-cals. for every pound of carbon present in the gas formed.

Oxygen combines readily with most substances, and in many cases the combination is accompanied with the evolution of large quantities of heat energy. Any substance which by its combination with oxygen furnishes heat energy rapidly in sufficient quantity and at sufficiently low cost to enable the energy produced to be used for industrial purposes is called a fuel. The calorific value of a fuel may be defined as follows:—

The calorific value of a fuel is the number of thermal units which must be abstracted from the products produced by the complete combustion of unit weight of the fuel in order to reduce them to the temperature at which the oxygen was supplied to the fuel.

The importance of considering the temperature to which the products are reduced is indicated by the prevailing practice of assigning two values to the calorific value of a substance in cases where hydrogen is one of the constituent elements. What is called the "Higher Calorific Value" is calculated on the supposition that the products of combustion are cooled down to something below 100° C., so that the steam produced by the combustion of the hydrogen is reduced to water in a liquid state and so yields up its latent heat as an addition to the heat produced. The "Lower Value" does not include the latent heat of the steam produced, since the steam is supposed to pass away in a gaseous condition. In most cases in practice the steam does pass away in a gaseous condition, and hence the lower calorific value approximates nearer than the higher calorific value to the heat which unit weight of the fuel can develop in the conditions in which the fuel is actually burned in practice.

Table 7 shows in the third column the calorific values of some of the chemical elements and compounds which enter into the composition of fuels.

The oxygen required for the combustion of the various substances is shown in column 5 of the table, and the products formed are given in column 6.

TABLE 7.—CALORIFIC VALUES OF CERTAIN SUBSTANCES TOGETHER WITH THE PRODUCTS OF COMBUSTION AND THE THEORETICAL AIR SUPPLY.

1	2	3	4	5	6		7			
1 lb. of the substance	Burned to —	produces heat energy measured in pound-centigrade-thermal-units per pound of substance (lb. calories).		Weight of oxygen required per pound of substance.	Products formed when burned with oxygen.		Theoretical weight of air required per lb. of substance (oxygen supply × 4·33).	Products formed when burned with air.		
		Higher val.	Lower val.		Carbon dioxide.	Steam.		Carbon dioxide.	Steam.	Nitrogen.
				lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Hydrogen, H_2	Steam	—	29,000	8	—	9	34·6	—	9	26·6
"	Steam and cooled to water	34,500	—	8	—	9	34·6	—	9	26·6
Marsh Gas, CH_4	Steam and carbon dioxide	—	11,650	4	$2\frac{3}{4}$	$2\frac{1}{4}$	17·3	$2\frac{3}{4}$	$2\frac{1}{4}$	13·3
" "	Steam and carbon dioxide and steam cooled to water	13,100	—	4	$2\frac{3}{4}$	$2\frac{1}{4}$	17·3	$2\frac{3}{4}$	$2\frac{1}{4}$	13·3
Ethylene, C_2H_4	Steam and carbon dioxide	—	11,060	$3\frac{3}{7}$	$3\frac{1}{7}$	$1\frac{2}{7}$	14·9	$3\frac{1}{7}$	$1\frac{2}{7}$	11·4
" "	Steam and carbon dioxide and steam cooled to water	11,900	—	$3\frac{3}{7}$	$3\frac{1}{7}$	$1\frac{2}{7}$	14·9	$3\frac{1}{7}$	$1\frac{2}{7}$	11·4
Carbon, C	Carbon dioxide	8080		$2\frac{2}{3}$	$3\frac{2}{3}$	—	11·54	$3\frac{2}{3}$	—	8·88
Carbon monoxide CO	" "	2420		$\frac{4}{7}$	$1\frac{4}{7}$	—	2·5	$1\frac{4}{7}$	—	1·9
Carbon, C	Carbon monoxide	2420		$1\frac{1}{3}$	$2\frac{1}{3}$ CO	—	5·7	$\frac{1}{3}$CO	—	4·4
Sulphur, S	Sulphur dioxide	2220		1	2 SO_2	—	1·33	$2SO_2$	—	3·41

These weights may be calculated from the chemical equations which represent the initial and the final stages of the process of combustion. For example, consider the case of the burning of carbon to carbon dioxide. The chemical equation is

$$C + O_2 = CO_2$$

which indicates that solid carbon combines with a molecule of oxygen to form, after complete combustion, a molecule of carbon dioxide gas. Knowing that the atomic weights of carbon and oxygen are 12 and 16 respectively, the equation becomes

$$\begin{aligned} C + O_2 &= CO_2 \\ 12 + 32 &= 44 \end{aligned}$$

showing that 1 molecule of carbon dioxide contains 32 parts by weight of oxygen and 12 parts by weight of carbon. Reducing these numbers proportionally it follows that 1 lb. of carbon combines with $2\frac{2}{3}$ lbs. of oxygen to form $3\frac{2}{3}$ lbs. of carbon dioxide.

In a similar way it is found that 1 lb. of hydrogen combines with 8 lbs. of oxygen and produces 9 lbs. of steam.

And 1 lb. of marsh gas combines with 4 lbs. of oxygen to form $2\frac{3}{4}$ lbs. of carbon dioxide and $2\frac{1}{4}$ lbs. of steam.

The oxygen necessary for combustion of fuel in steam boilers is supplied by feeding the furnace with air.

Column 7 of the table shows the weight of air required per pound of substance in column 1 in order to provide the oxygen given in column 5. Air consists of 23·1 parts by weight of oxygen and 76·9 parts by weight of nitrogen with a small and in general unknown quantity of water vapour held invisibly by the air. So that 1 lb. of oxygen is contained in 4·33 lbs. of air. The theoretical air supply per pound of fuel is found, therefore, by multiplying the necessary weight of oxygen by 4·33.

The nitrogen of the air passes through the furnace unchanged, so that it is found mixed with the products of combustion. Referring to the table, it will be seen that 1 lb. of carbon combines with 11·54 lbs. of dry air, and the gaseous products contain 3·66 lbs. of carbon dioxide, 8·88 lbs. of nitrogen.

When the air contains moisture some of the heat produced by combustion is used to evaporate the moisture into steam. The curves, Fig. 89 (page 314), show the heat required for this purpose for various degrees of moisture. The effect of moisture is to reduce the temperature attained by combustion in any particular set of conditions.

The actual process of combustion is a complicated phenomenon. The chemical equations given above represent the initial and the final stages of a process in which the intermediate steps are very complicated. It was shown by Professor H. B. Dixon in 1883 that carbon monoxide will not combine with oxygen unless a trace of moisture be present. The moisture, though taking no part in the combination produced by combustion, is necessary to determine the

combination. In 1885 Professor Baker showed that charcoal, and even the inflammable substances phosphorus and sulphur, would not burn unless a trace of water vapour were present; and in 1902 Professor Baker further showed that the water vapour present should contain some impurity, as even a mixture of oxygen and hydrogen in presence of water would not explode unless there were some trace of impurity in the water. The combustion of marsh gas and ethylene has been specially studied by Professor Bone, and reference should be made to his papers to obtain an idea of the most recent work on the subject.

23. Determination of the Calorific Value of a Fuel Experimentally.—This is done by means of a fuel calorimeter. There are only a few types, but there are several forms of each type, differing in details. The general principle governing the design of them all is that a weighed sample of fuel is placed in a small receptacle which is completely immersed in a known mass of water. The oxygen necessary for complete combustion is supplied to the receptacle, and combustion is started by chemical or by electrical means. The heat produced by combustion flows from the receptacle to the surrounding water. The rise in temperature of the water gives the remaining data from which to calculate the quantity of heat produced from the sample. The term, "water," means the actual weight of water present in the calorimeter plus the weight of water equivalent in heat absorbing capacity to the solid parts of the calorimeter.

In the calorimeter originated by Berthelot the fuel is placed in a small steel bomb, and oxygen is forced into the bomb until the pressure is about 375 lbs. per square inch. Combustion is started by raising a fine platinum wire embedded in the fuel to incandescence by means of an electric current. The weight of fuel placed in the bomb is 1 gram, and the bomb apparatus is immersed in 2200 grams of water. Combustion here takes place at constant volume, and the final temperature of the water is not many degrees above the original temperature.

In the Thomson calorimeter and similar forms, the fuel is burnt in a stream of oxygen supplied from an oxygen bottle. Here, combustion takes place at constant pressure, and the gas produced, bubbles up through the water in the calorimeter.

In still another type the fuel is mixed with a substance which provides the necessary oxygen, like potassium chlorate, sodium peroxide, or potassium nitrate.

Scientifically the different forms may be classified into two types, namely, those in which combustion takes place at constant volume, and those in which it takes place at constant pressure.

Detailed descriptions of the different varieties of fuel calorimeters would hardly fill any useful purpose here, since a real knowledge of their peculiarities and the way to use them can only be learnt in a laboratory. Much useful information with examples of actual

experiments will be found in "Heat for Engineers," by C. R. Darling, published by E. & F. N. Spon, London.

When considering the results obtained from fuel calorimeters it must not be forgotten that they are obtained from an exceedingly small quantity of fuel, microscopic, in fact, in relation to the bulk from which the sample was selected. When it is considered what large quantities of coal are purchased by shipping companies and railway companies, even a quantity equivalent to a truck load is not too large to burn as a sample in order to obtain reliable data relating to the calorific value of the fuel. Burnt in the furnace of a boiler, in as nearly as possible the conditions of service, the rate of evaporation of the water in the boiler furnishes data from which an average value of the calorific value of the fuel can be found, and for practical purposes a test of this kind would be preferable to a test in a laboratory calorimeter.

24. Calculation from the Ultimate Analysis of a Fuel of the Theoretical Air Supply; the Weight of the Products of Combustion; and the Calorific Value. The Higher and the Lower Calorific Values and the Available Calorific Value.—The weight of air theoretically necessary for complete combustion and also the weight of the furnace gas produced by the combustion of a given weight of a fuel, can be calculated when the ultimate analysis of the fuel is given.

Theoretical Air Supply.

It has been shown in Section 22 that 1 lb. of carbon requires $2\frac{2}{3}$ lbs. of oxygen to burn it completely to carbon dioxide, and that when so burnt the weight of carbon dioxide formed is $3\frac{2}{3}$ lbs.

Similarly, 1 lb. of hydrogen combines with 8 lbs. of oxygen to form 9 lbs. of water.

And 1 lb. of sulphur requires 1 lb. of oxygen and forms 2 lbs. of sulphur dioxide.

If 1 lb. of coal contains 0·86 lb. of carbon, 0·04 lb. of hydrogen, 0·01 lb. of sulphur, and 0·02 lb. of oxygen, the remainder being ash, the calculation for the theoretical air supply is made as follows:—

0·86 lb. of carbon requires	$0·86 \times 2\frac{2}{3}$ lbs. of oxygen	=	2·29 lbs.
0·04 lb. of hydrogen ,,	$0·04 \times 8$,, ,,	=	0·32 lb.
0·01 lb. of sulphur ,,	$0·01 \times 1$ lb. ,,	=	0·01 lb.
1 lb. of this coal therefore requires of oxygen		=	2·62 lbs.

But of this the coal itself supplies 0·02 lb., leaving 2·60 lbs. to be supplied by the air. Air consists of 23·1 per cent. by weight of oxygen. The weight of air required to supply 2·60 lbs. of oxygen is therefore $\frac{2·60}{0·231} = 11·25$ lbs.

Weight of the Products of Combustion.

The combustible material together with the oxygen in the fuel passes into the gaseous products of combustion.

The total weight of the products per pound of fuel burnt is therefore the weight of the oxygen shown in the fuel analysis together with the combustible material, added to the weight of air required to burn it.

In the numerical example taken above the weight of the products is therefore 0·93 + 11·25 = 12·18 lbs. per pound of coal burnt.

Alternatively the weight of the products may be calculated in this way.

Weight of carbon dioxide produced from the carbon	$= 0{\cdot}86 \times 3\frac{2}{3}$	$= 3{\cdot}15$ lbs.
Weight of steam produced from the hydrogen	$= 0{\cdot}04 \times 9$	$= 0{\cdot}36$ lb.
Weight of sulphur dioxide produced from the sulphur	$= 0{\cdot}01 \times 2$	$= 0{\cdot}02$ lb.
Weight of nitrogen in the air supply		$= 8{\cdot}65$ lbs.
Total weights of gaseous products per pound of coal		$= 12{\cdot}18$ lbs.

Calculation of the Calorific Value.

Although coal, oil, and fuels generally, consist of complex molecular arrangements of the carbon, hydrogen, oxygen, nitrogen, and other elements which form their substance, yet a fair approximation to the calorific value of a fuel can be obtained by calculation from a chemical analysis which shows the proportion by weight of each element present.

The calculation is made on the assumption that each element found in the analysis contributes to the total heat produced the amount it would produce if burnt separately.

The total heat calculated in this way, and the heat actually obtained by burning in a calorimeter, differ, because, although the chemical analysis shows the weight of each element present, it gives no clue to the way in which these elements are built up into molecules. And the calorific value of a molecule is different from the sum of the calorific values of its constituent elements.

For example, marsh gas, CH_4, consists of 12 parts by weight of carbon and 4 parts by weight of hydrogen, so that 1 lb. of the gas contains 0·75 lb. of carbon and 0·25 lb. of hydrogen. The heat produced by the burning of these weights of each element separately is therefore

$$0{\cdot}75 \times 8080 + 0{\cdot}25 \times 34{,}500 = 14{,}685 \text{ lb.-cals.}$$

But the value determined in a calorimeter is 13,100 lb.-cals. per pound. The inference is that 1585 lb.-cals. per pound are required to break up the molecule into its constituent elements.

Again, acetylene, C_2H_2, which consists of 24 parts by weight of

carbon, and 2 parts by weight of hydrogen, contains per pound 0·923 lb. of carbon and 0·077 lb. of hydrogen. The heat produced by burning separately these weights of each element is 10,100 lb.-cals. The heat produced by burning a pound of acetylene in a calorimeter is, however, 11,900; 1800 lb.-cals. more than that which is produced by burning each element separately. The inference is that this quantity of heat was absorbed in the making of the molecule, and it is restored when the molecule is broken up again into its constituent elements.

In general the breaking down of the molecular construction of coal appears to absorb heat.

The ultimate analysis of a fuel shows the amount of each element present expressed as a fraction of the weight of the dry fuel. This is an analysis of a sample of the Nixon Navigation coal from Glamorganshire, which was used in Donkin and Kennedy's experiments on steam boilers.

Carbon	0·8877
Hydrogen	0·0412
Oxygen	0·0238
Nitrogen	0·0098
Sulphur	0·0068
Ash	0·0307
	1·0000

The proximate analysis, though not required for our present purpose, is added.

Moisture	0·0112
Volatile gas	0·1172
Combustible residue	0·8405
Ash	0·0311
	1·0000

The combustible elements present in this coal are, therefore, carbon, hydrogen, and sulphur. From the table given in Section 22, it will be seen that the calorific values of these elements when burnt separately are respectively, 8080, 34500, and 2220 lb.-cals. per pound. The heat produced by burning separately the weights of the combustible elements in the ultimate analysis above is therefore

$$0{\cdot}8877 \times 8080 + 0{\cdot}0412 \times 34500 + 0{\cdot}0068 \times 2220 = 8607 \text{ lb.-cals.}$$

Of this, part is used to break down the molecular structure of the fuel substance, but since this molecular structure is unknown the amount cannot be calculated.

It has long been the practice to regard the oxygen in the fuel as neutralizing a corresponding weight of the hydrogen in the fuel. It is assumed, therefore, that for every pound of oxygen present 0·125 lb.

of hydrogen is neutralized, and that only the remainder of the hydrogen is available for combustion.

Applying this correction to the above calculation, the 0·0238 lb. of oxygen in the analysis neutralize $\frac{0 \cdot 0238}{8}$ lbs. of hydrogen, thus depriving the furnace gas of $34500 \times \frac{0 \cdot 0238}{8} = 102$ lb.-cals. per pound. The calorific value found above is therefore reduced to 8505 lb.-cals. per pound.

This process of calculation is expressed by the following formula :—

Calorific value in lb.-cals. per pound of coal

$$8080\ C + 34500 \left\{H - \frac{O}{8}\right\} + 2220\ S$$

where C, H, O, S, are the weights of these respective elements present in 1 lb. of dry coal.

This formula, originally given by Rankine, has been much used in practice.

From what has been said above it is clear that the factor $\left(H - \frac{O}{8}\right)$ represents the weight of hydrogen left after subtracting the quantity assumed to be neutralized by the oxygen present. It is better, however, to regard the term $\frac{O}{8}$ as an arbitrary correction, applied to allow for the heat absorbed in breaking down the molecular structure of the fuel. From this point of view the 102 lb.-cals. found above are looked upon as the heat absorbed for this purpose.

The justification of the formula is that it gives values approximating fairly closely to the actual calorific values of coals ordinarily used in steam boilers when these values are determined in a calorimeter. The correcting term is, however, unsatisfactory in form from the scientific point of view, because it depends upon the quantity of oxygen in the analysis, and this again bears little if any relation to the heat required for molecular dissociation.

The above formula may be applied to calculate the calorific value of oil fuel. The following is an analysis of a sample of liquid fuel known as "Astatki," which was at one time used for firing some of the express locomotives on the Great Eastern Railway :—

Carbon	0·8610
Hydrogen	0·1270
Oxygen, sulphur, nitrogen by difference	0·0120
	1·0000

Specific gravity of the oil, 0·906.

Flash point, 101° C., Close Test.

Assuming that the whole of the difference is oxygen, the formula gives—

Calorific value per pound

$$= 8080 \times 0{\cdot}8610 + 34500\left(0{\cdot}1270 - \frac{0{\cdot}0120}{8}\right)$$

$$= 11183 \text{ lb.-cals.}$$

Higher and Lower Calorific Value.

The calorific value calculated by the above formula is the calorific value defined in Section 22. The temperature at which the oxygen is supplied, and to which the products of combustion are reduced, is the atmospheric temperature at which the experiment is made. When the calorific value is found experimentally in a calorimeter, the products are also cooled down to atmospheric temperature. The value calculated from the formula is therefore comparable with the value determined in a calorimeter. A mean value of 18° C. may be taken as the atmospheric temperature to which the products are reduced.

The **higher calorific value,** sometimes called the **gross calorific value,** is the value found either by calculation or by experiment when the products of combustion are reduced to the temperature at which the oxygen is supplied, usually about 18° C.

In practice furnace gas cannot be rejected from chimneys at such a low temperature as this. And when gas is burned in the cylinder itself, as in all internal combustion engines, the products of combustion leave the engine at high temperatures. The calorific value found as above is therefore greater than it is possible to realize when the fuel is burnt in any practical way.

The **lower or net calorific value** is the value obtained when the products are cooled down to 100° C., it being assumed that any steam produced by combustion escapes as steam, and is not condensed to water at 100° C.

It will be seen that the lower calorific value differs from the higher calorific value by the number of heat units which must be abstracted from the products in order to cool them from 100° C. to 18° C.

Consider first the higher and the lower calorific values of hydrogen. Hydrogen burns to steam. To cool 1 lb. of steam from 100° to 18° C. there must be abstracted from it the latent heat, and then a quantity of heat sufficient to cool the water formed from 100° to 18°. The latent heat per pound of steam at 100° C. is 539, and the heat which must be abstracted to cool the water to 18° is 82 lb.-cals. The heat which must be abstracted per pound of steam is thus 621 lb.-cals. But every pound of hydrogen burnt produces 9 lbs. of water, and hence the difference between the higher and the lower calorific values of hydrogen is $621 \times 9 = 5589$ lb.-cals. The lower calorific value of hydrogen is therefore $34500 - 5589 = 28911$, and this is sometimes taken at 28900 and sometimes 29000.

Again, the specific heat of carbon dioxide is 0·217; consequently the heat which must be abstracted from 1 lb. of carbon dioxide to

reduce its temperature from 100° to 18° is $0{\cdot}217 \times 82 = 17{\cdot}8$ lb.-cals. And 1 lb. of carbon produces $3\frac{2}{3}$ lbs. of carbon dioxide. Therefore, the heat abstracted per pound of carbon is $3\frac{2}{3} \times 17{\cdot}8 = 65$ lb.-cals. The lower calorific value of carbon is thus $8080 - 65 = 8015$.

The correction for carbon is small compared with the correction for hydrogen, and is, in fact, generally neglected. Neglecting this correction, and also the still smaller correction for sulphur, the formula becomes

$$\left.\begin{array}{l}\text{Lower calorific value in pound-}\\ \quad\text{calories per pound of fuel}\end{array}\right\} = 8080\,C + 29000\left(H - \frac{O}{8}\right) + 2220S.$$

This formula gives for the typical Welsh coal, whose analysis is given above, a calorific value of 8300 lb.-cals. per pound.

Available Calorific Value.

All the heat corresponding to the lower calorific value cannot be transferred even theoretically from the furnace gas to the water in a steam boiler, because the furnace gas must always be hotter than the water, in order to determine a flow of heat from the gas to the water. A limiting condition may be imagined in which the transfer of heat from the furnace gas to the water goes on until the gas is reduced to the temperature of the water, but this would necessitate an infinitely extended heating surface, and is quite outside the range of practical possibilities. This condition may, however, be taken as an ideal standard with which to compare the actual transference of heat achieved by the boiler, just as the Rankine Engine of Comparison is taken as an ideal standard with which to compare the actual efficiency achieved by a steam engine. The efficiency ratio of a steam engine is its efficiency measured against the ideal efficiency of the corresponding Ideal Rankine Engine. Similarly, the actual efficiency of a boiler may be compared with the efficiency of an Ideal Boiler of Comparison, in which the furnace gas is reduced to the temperature of the water in the boiler before it leaves the heating surface and passes into the chimney.

The quantity of heat which the ideal boiler can transfer to the water per pound of fuel burnt on its grate may appropriately be called the **available calorific value** of the fuel. No boiler will, in the same conditions, transfer more heat, and any actual boiler will transfer less per pound of coal burnt.

The available calorific value of a fuel is determined by two conditions:—

1. The products of combustion cannot be cooled below the temperature of the water in the boiler, and this temperature is fixed by the pressure at which the steam is produced.

2. The fuel is burnt in air which contains not only oxygen, the essential element of combustion, but a large proportion of nitrogen, which passes through the furnace with the products of combustion, and deprives the hot gas of the amount of heat necessary to raise

its temperature from the temperature of the incoming air to the temperature of the furnace gas at exit, which in the ideal case is the temperature of the water in the boiler.

An example will make the matter clear.

Suppose steam is being produced at 150 lbs. per square inch with Nixon's Navigation coal having the ultimate analysis given above.

The temperature of the water in the boiler at this pressure is 181° C. The furnace gas cannot therefore be cooled below this temperature.

One pound of this dry coal produces on burning 3·25 lbs. of carbon dioxide, 0·37 lb. of steam, and 0·0136 lb. of sulphur dioxide. If all of these gases are cooled to 18° C., the heat given out by the burning of 1 lb. of coal is the higher calorific value of the coal, and is equal to 8505 lb.-cals. as calculated above. If the products are cooled only to 181° C., the heat not abstracted from the gases is calculated as follows :—

Carbon dioxide, specific heat 0·217 : $3{\cdot}25 \times 0{\cdot}217 \times (181 - 18) = 115$

Steam, specific heat 0·48 :

$$0{\cdot}37\{(100 - 18) + 539 + 0{\cdot}48(181 - 100)\} = 245$$

Sulphur dioxide, negligible

Total . . 360

Again, the quantity of oxygen required to burn 1 lb. of this coal is 2·69 lbs., corresponding to 8·97 lbs. of nitrogen, and this deprives the furnace gas of

$$8{\cdot}97 \times 0{\cdot}244 \times (181 - 18) = 356 \text{ lb.-cals.}$$

per pound of coal to raise its temperature from 18° to the temperature of the furnace gas at exit from the heating surface.

The number of heat units which the ideal boiler could transfer to the water per pound of coal burned on its grate is thus

$$8505 - 360 - 356 = 7789 \text{ lb.-cals.},$$

and this is the available calorific value of the fuel when steam is produced at 150 lbs. per square inch.

Compare now the boiler efficiencies determined by these three calorific values, assuming that in given conditions of working 10 lbs. of steam are produced per pound of coal burned, and that the total energy per pound of the steam produced is 662 lb.-cals.

The efficiency calculated from the higher calorific value is $\dfrac{662 \times 10}{8505} = 0{\cdot}78$

The efficiency calculated from the lower calorific value is $\dfrac{6620}{8300} = 0{\cdot}80$

The efficiency calculated from the available calorific value is $\dfrac{6620}{7789} = 0{\cdot}85$

The last efficiency expresses the actual efficiency in terms of the efficiency of the ideal boiler of comparison, and is analogous to the efficiency ratio of a steam engine.

In practice it is impossible to cool the furnace gas down to the temperature of the water in the boiler, because an economical limit to the extent of the heating surface is reached when the temperature is considerably higher than the temperature of the water. As shown in Chapter I, the heat-fall determines the rate of flow of heat, and there is little advantage gained when the fall is less than 100° C., because the quantity of heat transferred per square foot of heating surface per second is too small to make it worth while providing heating surface when this limit is reached. Again, a high temperature of flue gas is required for the maintenance of the draught. The flue gases can therefore seldom be reduced in the boiler flue to a lower temperature than 300° C. with any advantage. Further, it is not possible to burn coal properly with only the amount of air chemically necessary. As shown above, excess air must be admitted to ensure good combustion, and this excess air must be warmed from the atmospheric temperature to the temperature of the furnace gas at exit. But, on the other hand, the quantity of heat abstracted from the flue gas for this purpose is far less than the loss which would be incurred by incomplete combustion due to a restricted air supply.

No actual boiler, therefore, can exceed the efficiency of the ideal boiler of comparison.

Carbon Value of a Fuel.

The heating qualities of fuels are sometimes compared with the heating value of carbon. By "carbon value" is meant the number of pounds of pure carbon which produce, when burnt to carbon dioxide, as many heat units as are produced by the combustion of 1 lb. of fuel. Thus the carbon value of a fuel having a calorific value of 8480 is $\frac{8480}{8080} = 1{\cdot}05$.

25. Temperature in the Furnace.—The heat produced by the combustion of fuel is carried initially by the products of combustion. From these products it is transmitted to the furnace boundaries in all directions where there is a temperature fall. The temperature produced in the furnace by combustion is not calculable with any certainty. Direct observations show that it may reach and even exceed 1000° C.

Some interesting records of the temperatures observed in the furnace and at points in the path of the furnace gas amongst the nest of tubes in a water-tube boiler fired with oil fuel are shown in Fig. 43, which is taken from a paper by Mr. Harold Yarrow.[1]

The tubes are shown in plan and the points at which the temperatures were measured are indicated. Curve No. 1 shows the temperatures corresponding to a rate of evaporation of 16 lbs. of water per square foot of heating surface per hour. The temperature

[1] "Results of Experiments with a Water-tube Boiler with Special Reference to Superheating," *Trans. Inst. Naval Arch.*, 1912.

of the furnace gas just before passing the first row of tubes is 2250° F. = 1232° C., and this is reduced by the first row of tubes by 450° F. = 250° C. The rate of fall of temperature gets less and less as the gas flows towards the outer rows of tubes, but the rate suddenly increases when the gas flows by the tubes through which the cold feed water is passing.

The line BC shows the constant temperature of the water in the tubes corresponding to the pressure in the boiler, namely, 388° F. = 198° C. and the line EF shows the temperature corresponding to the

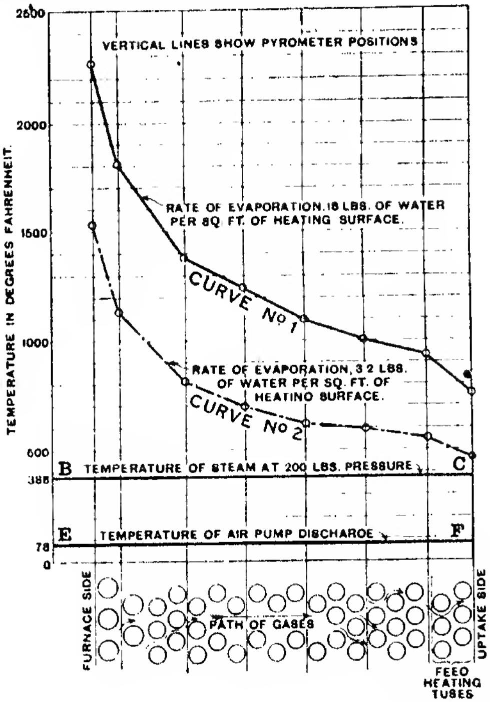


FIG. 43.—Curves showing drop of temperature through flues of Yarrow boiler.

air pump discharge, namely, 78° F. = 25·6° C. Vertical ordinates between BC and curve No. 1 show the temperature differences along the path of the flow.

When the rate of evaporation was reduced to 3·2 lbs. of water per square foot of heating surface the temperatures fell to those shown by curve No. 2.

When evaporating at the higher rate of 16 lbs. per square foot of heating surface per hour, the consumption of oil fuel was at the rate of 1 lb. for 14·8 lbs. of water evaporated from and at 100° C., whilst

at the lower rate of evaporation oil fuel was burned at the rate of 1 lb. for 16 lbs. of water evaporated from and at 100° C.

Fire-box temperatures were measured in the trials of locomotives at the St. Louis Exhibition. The highest recorded temperature is 1280° C., which was obtained in the fire-box of a passenger locomotive fitted with a brick arch. The corresponding smoke-box temperature was 395° C. The rate of combustion was 134 lbs. of coal per square foot of grate per hour.

The temperature reached by the gases depends largely upon the skilfulness of the firing. The quantity of heat which can be produced by the perfect combustion of a pound of coal of known quality does not vary within very wide limits, but the temperature produced by its combustion will vary with the amount of excess air admitted to the furnace, and therefore it is a difficult matter to obtain comparable figures regarding temperature from the records of experiments on boilers, because of the uncertainty of the quantity of excess air admitted to the furnace.

26. Determination of the Air Supply from an Analysis of the Flue Gas and an Analysis of the Fuel.—The quantity of air drawn through a furnace is so large that a direct measurement of the amount is impracticable.

The amount is determined indirectly from an analysis of the flue gas and an analysis of the fuel.

A sample of the flue gas is collected in a gas bottle over brine or mercury and then by means of an Orsat or a similar apparatus the gas is analysed into its constituents In the process of collecting the sample the steam formed by the combustion of the hydrogen in the fuel condenses to water and thus disappears from the gas, leaving what is called "dry flue gas".

The constituents of dry flue gas are carbon dioxide, sometimes carbon monoxide, oxygen, and nitrogen, with negligibly small quantities of other gases produced from any sulphur or phosphorus in the fuel. The analysis is made by bringing a known volume of the gas, generally 100 c.c., into successive contact with solutions which absorb one or other of the constituent gases. Carbon dioxide is absorbed by caustic potash; carbon monoxide by an acid solution of cuprous chloride; oxygen by an alkaline solution of pyrogallic acid, and the unabsorbed residue is assumed to be nitrogen.

The results of a volumetric analysis made in this way are usually expressed as a percentage as illustrated by the following analysis:—

Carbon dioxide, CO_2	13·6	per cent.
Carbon monoxide, CO	1·6	,,
Oxygen, O_2	3·0	,,
Nitrogen, N_2, by difference . . .	81·8	,,
	100·0	

The fuel which produced this gas had the following composition:—

Carbon	0·886
Hydrogen	0·045
Ash	0·069
	1·000

With these data the weight of air per pound of fuel burnt can be calculated to a fairly close approximation.

The first step is to calculate the weights of the several constituents of the gas and the total weight of gas produced per pound of carbon appearing in the furnace gas itself.

Equal volumes of gases at the same temperature and pressure have weights proportional to their molecular weights. The molecular weights of the constituent gases are:—

Carbon dioxide	44
Carbon monoxide	28
Oxygen	32
Nitrogen	28

A molecule of carbon dioxide contains 12 parts by weight of carbon and 32 parts by weight of oxygen, making a total of 44.

A molecule of carbon monoxide contains 12 parts by weight of carbon and 16 parts by weight of oxygen, making a total of 28.

If each of the volumetric percentages be multiplied by the respective molecular weights figures will be obtained which are proportional to the weights of the respective substances present in the furnace gas.

Thus the above volumetric analysis of the furnace gas gives that there are 13·6 × 44 = 598·4 lbs. of carbon dioxide present, and this consists of 13·6 × 12 = 163·2 lbs. of carbon and 13·6 × 32 = 435·2 lbs. of oxygen.

Also there are 1·6 × 28 = 44·8 lbs. of carbon monoxide present, consisting of 1·6 × 12 = 19·2 lbs. of carbon and 1·6 × 16 = 25·6 lbs. of oxygen.

Of oxygen there are 3·0 × 32 = 96 lbs., and of nitrogen 81·8 × 28 = 2290 lbs.

Adding these results together, 3029·2 lbs. of dry flue gas contain 182·4 lbs. of carbon; 556·8 lbs. of oxygen, and 2290 lbs. of nitrogen.

Dividing the total weight of gas by the weight of carbon, it gives that 1 lb. of carbon is contained in 16·6 lbs. of flue gas.

These processes of calculation may be tabulated.

TABLE 8.—SHOWING DETAILS OF CALCULATIONS TO DETERMINE THE WEIGHT OF THE SUBSTANCES IN THE FLUE GAS PER POUND OF CARBON.

Analysis of flue gas by volume.		Multipliers for			Proportional weight of			
		Carbon.	Oxygen.	Nitrogen.	Carbon.	Oxygen.	Nitrogen.	Total.
Carbon dioxide,	13·6	12	32	—	163·2	435·2	—	598·4
Carbon monoxide,	1·6	12	16	—	19·2	25·6	—	44·8
Oxygen,	3·0	—	32	—	—	96·0	—	96·0
Nitrogen,	81·8	—	—	28	—	—	2290	2290·0
Total					182·4	556·8	2290	3029·2
Per pound of carbon . . .					1	3·05	12·55	16·6

This table shows at a glance the weight of each constituent substance present in 3029·2 lbs. of dry furnace gas, and also, by proportionate reduction, the weights of each substance per pound of carbon in the dry furnace gas.

The quantity of air drawn through the furnace per pound of carbon in the dry furnace gas can now be estimated in three ways:—

(*a*) with fair accuracy from the total weight of gas formed by the burning of a known weight of fuel;
(*b*) with less accuracy from the nitrogen found in the flue gas;
(*c*) with still less accuracy, except in the case of coals containing very little hydrogen, from the oxygen found in the flue gas.

(*a*) This method requires an ultimate analysis of the fuel in addition to the analysis of the dry flue gas, in order to estimate from the hydrogen in the fuel analysis the weight of steam formed.

Referring to the table it will be seen that the dry furnace gas produced for each pound of carbon burnt is 16·60 lbs. From the fuel analysis it is clear that $\frac{0{\cdot}045}{0{\cdot}886}$ lb. of hydrogen are burnt along with each pound of carbon, producing $9 \times \frac{0{\cdot}045}{0{\cdot}886} = 0{\cdot}457$ lb. of steam. The total weight of flue gas per pound of carbon burnt is thus $16{\cdot}6 + 0{\cdot}457 = 17{\cdot}06$ lbs. The weight of fuel burnt to produce this is 1 lb. of carbon and 0·051 lb. of hydrogen, making 1·051 lbs. in all. Therefore the air drawn through the furnace per pound of carbon in the flue gas is $17{\cdot}06 - 1{\cdot}051 = 16{\cdot}01$ lbs. But the analysis of the fuel shows that the carbon content is 0·886 lb. per pound. Consequently the weight of air per pound of coal burnt is

$$16{\cdot}01 \times 0{\cdot}886 = 14{\cdot}2 \text{ lbs.}$$

This calculation may be thrown into a formula. Thus, from the table it will be clear that

Weight of dry furnace gas per pound of carbon in the gas

$$= \frac{11CO_2 + 7CO + 8O_2 + 7N_2}{3(CO_2 + CO)} \text{ lbs.} \quad . \quad . \quad . \quad (1)$$

Weight of steam formed per pound of carbon in dry gas

$$= \frac{9H}{C} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Total weight of flue gas formed per pound of carbon in the gas = equation (1) + equation (2).

Weight of combustible fuel burnt to produce this total weight of flue gas

$$= 1 + \frac{H}{C} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Therefore weight of air per pound of carbon in the flue gas = A

$$= \frac{11CO_2 + 7CO + 8O_2 + 7N_2}{3(CO_2 + CO)} + \frac{8H}{C} - 1 = A \quad (4)$$

The value of the numerator in equation (1), in all practical cases, falls between 740 and 760. Taking a mean value therefore of 750, the expression reduces to

$$A = \frac{250}{CO_2 + CO} + \frac{8H}{C} - 1 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If no fuel analysis is available, then the term $\frac{8H}{C}$ must be omitted and the expression below gives the air supply with fair accuracy, namely,

Weight of air per pound of carbon in the flue gas

$$= \frac{250}{CO_2 + CO} - 1 = A \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(*b*) Assuming that the nitrogen shown in the analysis is all derived from the air drawn through the furnace, the air supply itself is found by multiplying the percentage of nitrogen present by 1·3, since by weight 77 per cent. of air consists of nitrogen.

Thus from the table it will be seen that there are 12·55 lbs. of nitrogen in the flue gas per pound of carbon. Therefore the corresponding weight of air is 12·55 × 1·3 = 16·3.

The calculation can be thrown into a formula, as follows:—

Nitrogen present in the dry furnace gas . $= 28N_2$

Carbon present in dry furnace gas . . . $= 12(CO_2 + CO)$

Nitrogen in dry furnace gas per pound of carbon . . . . $= \frac{28N_2}{12(CO_2 + CO)}$.

Air per pound of carbon in dry furnace gas $= \frac{1{\cdot}3 \times 28N_2}{12(CO_2 + CO)} = A$

Which with sufficient accuracy is

$$A = \frac{3N_2}{(CO_2 + CO)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The objection to this direct use of nitrogen is that in the process of analysis its quantity is found by difference, and it thus includes the errors of analysis, and also there may be small quantities present due to the reduction of the ammonia formed from the fuel.

(c) The oxygen shown in the analysis is less than the total amount used by the quantity required to burn the hydrogen, and if there is much hydrogen in the fuel, methods (a) or (b) are preferable. It will be seen from the table, and also by analogy with the construction of the nitrogen expression, that the oxygen in the flue gas per pound of carbon is

$$\frac{32(CO_2 + O_2) + 16CO}{12(CO_2 + CO)}$$

The corresponding quantity of air is found by multiplying this by 4·33, since air contains 23 per cent. of oxygen by weight. This gives

$$A = 5{\cdot}77\,\frac{2(CO_2 + O_2) + CO}{(CO_2 + CO)} \quad . \quad . \quad . \quad . \quad (8)$$

If no fuel analysis is available a good approximation to the value of A, the weight of air drawn through the furnace in pounds per pound of carbon in the dry flue gas, can be obtained by expression (6).

To compute the air supply per pound of fuel supplied to the furnace it is necessary to know the proportion of carbon in the fuel. Also when fuel is burnt at a great rate, as in the case of a locomotive working at its maximum power, a certain proportion is thrown away at the chimney top containing combustible carbon, and this proportion must of course be ascertained before the air supply can be expressed in terms of a pound of fuel actually supplied to the furnace.

For example, if coal contains 90 per cent. of carbon, and 10 per cent. of the quantity fired is lost by cinders and sparks, the amount of carbon appearing in the furnace gas is 81 per cent. of the actual weight of coal fired. The weight of air per pound of coal fired would then be 0·81A, where A is the weight computed above. This point is considered again below in Section 27.

27. Construction of a Heat Balance Sheet for the Heating Circuit.—The principles developed in the preceding sections may be used to construct a balance-sheet showing the energy received into the heating circuit and the way it is accounted for, when sufficient data are available.

In the cases of boilers in which the velocity of the air supply is well within the limiting velocity, like stationary boilers working with natural draught, all the cinders formed can be collected from the ash-pit, and the sparks are usually of negligibly small calorific

value, and the quantity is negligibly small also. When a boiler is worked at a high rate, however, the heat energy carried away through the chimney by the cinders and sparks is considerable. As much as *one-third of the total energy of the fuel fired into the fire-box of an express locomotive can be lost in this way.*

The general case where the cinder and spark loss is appreciable includes the case where it is negligible, and therefore the construction of a balance sheet for the first case will be explained in detail.

The left side of the balance sheet should show the heat actually produced by combustion, and the right side the quantity of heat in the various channels along which heat energy flows away, each item being expressed on a basis of 1 lb. of dry coal.

The estimation of the heat actually produced per pound of coal supplied to the furnace requires the determination of:—

1 the calorific value of the coal as determined in a calorimeter;
2 the heat energy unproduced in the cinders and sparks;
3 the heat energy unproduced because of the imperfect combustion of part of the carbon to carbon monoxide instead of to carbon dioxide.

The heat produced is accounted for under:—

4 heat taken up by the water in the boiler;
5 heat flowing away by radiation;
6 heat used in warming and evaporating the water in the fuel and the water produced by combustion, and then superheating the steam formed to the temperature of the chimney gas;
7 heat carried away by the dry chimney gas.

Heat produced per Pound of Coal supplied to the Furnace.

In order to compute items (1), (2), (3), from which this quantity is found, it is necessary to have either a calorimeter with which the calorific value of the fuel may be measured, or an analysis of the fuel from which it may be calculated, together with an analysis of the chimney gas to find the proportion of carbon monoxide present. There must also be given the weight of cinders and sparks, together with their respective calorific values.

Let K be the calorific value of the fuel as determined in a calorimeter; and let x pounds of cinders and sparks be formed per pound of coal supplied to the furnace, having an average calorific value k. Then assuming combustion to be perfect, the heat produced per pound of coal supplied to the furnace is $K - kx$ lb.-cals.

Suppose, however, that the combustion is imperfect and that there is an appreciable quantity of carbon monoxide appearing in the furnace gas. If M represents the proportion of carbon monoxide shown in the volumetric analysis of the furnace gas, 12M represents the corresponding weight of carbon; and if D represents the

proportion of carbon dioxide in the volumetric analysis, 12D is the corresponding weight of carbon.

The total weight of carbon is 12(M + D), and the fraction of this which is burnt to carbon monoxide is

$$\frac{12M}{12(M + D)} = \frac{1}{1 + \frac{D}{M}}$$

This fraction gives the weight in pounds of the carbon burnt to carbon monoxide per pound of carbon in the dry furnace gas when the ratio $\frac{D}{M}$ is known.

Referring to Table 7, page 97, it will be seen that

1 lb. of carbon burnt to carbon dioxide gives .	8080 lb.-cals.
1 lb. of carbon burnt to carbon monoxide gives	2420 lb.-cals.
Energy unproduced per pound of carbon burnt to carbon monoxide is	5660 lb.-cals.

The heat energy unproduced in the carbon monoxide per pound of carbon appearing in the dry furnace gas is then

$$5660\left\{\frac{1}{1 + \frac{D}{M}}\right\} \text{ lb.-cals.} = U \text{ say} \quad . \quad . \quad . \quad . \quad (1)$$

Assuming that each pound of coal contains the fraction c lbs. of carbon, and that x pounds of cinders and sparks are formed containing the same proportion of carbon as the coal, then the energy unproduced per pound of coal fired to the furnace is

$c(1 - x)$U lb.-cals. per pound of coal supplied to the furnace . (2)

Thus, if coal containing 0·8 lb. of carbon is burnt to furnace gas containing 12 per cent. of carbon dioxide by volume and 3 per cent. of carbon monoxide by volume, and if further 0·2 lb. of cinders and sparks are thrown away per pound of coal supplied to the furnace, the energy unproduced through imperfect combustion is approximately

$$0{\cdot}8(1 - 0{\cdot}2)0{\cdot}2 \times 5660 = 725 \text{ lb.-cals.}$$

per pound of coal supplied to the furnace.

Collecting together these expressions, the heat actually produced per pound of coal supplied to the furnace is given by

$$K - kx - c(1 - x)U \text{ lb.-cals.} = Z$$

and this quantity of heat energy should be placed on the debit side of the balance sheet.

Heat accounted for.

To determine the several items on the credit side of the balance sheet, the weight of water evaporated from and at 100° C. per pound of coal supplied to the furnace must be measured. Let this weight be V, then the heat taken up by the motive-power circuit from the

heating circuit per pound of coal supplied to the furnace is LV lb.-cals., where L is the latent heat at 100° C.

An estimate of the heat lost by radiation can be made by finding how much coal must be supplied to the furnace in order to keep the steam pressure constant over several hours, during which no steam is taken from the boiler and no feed water is supplied to it. In the Donkin and Kennedy trials the radiation loss, measured in this way, varied from 3 to 8 per cent. In the particular case of a locomotive boiler the loss was 3 per cent. when tested as a stationary boiler, and nearly 4 per cent. when tested on the road whilst it was working a train. Failing any direct experimental evidence, the radiation loss may be estimated at 5 per cent.

The heat required to evaporate and superheat the moisture in the coal, as fired, can be estimated with fair accuracy, but the matter is complicated by the fact that the air brings with it into the furnace an uncertain quantity of water vapour according to the hygrometric state of the atmosphere. To find this quantity a reading of the wet and dry bulb thermometer must be taken, and then the curve, Fig. 89, Chapter V, may be used to find the weight of steam brought in by the air. It is seldom, however, that experiments on boilers furnish data accurate enough to require this refinement in the reduction of the results.

Roughly, 1 lb. of air completely saturated at 15° C. contains $\frac{1}{80}$ of a pound of vapour, so that for every 20 lbs. of air supplied to the furnace, a weight corresponding approximately to 1 lb. of coal supplied, the weight of vapour taken in would not at this temperature exceed $\frac{1}{4}$ of a pound, and would generally be less.

The water produced by the combustion of hydrogen is found by multiplying the hydrogen shown in the analysis of the fuel by 9. Thus 4 per cent. of hydrogen in the fuel produces 0·36 lb. of water per pound of fuel supplied.

Let w be the whole weight of steam in the furnace gas per pound of fuel supplied. Then, assuming that this originated from liquid water at the air temperature, this weight has to be raised from the air temperature to 100° C.; evaporated at this temperature and afterwards superheated to the temperature of the chimney gas, a process represented by the formula—

$$w\{(100 - t) + 540 + (T - 100)0{\cdot}48\} \text{ lb.-cals.} \quad . \quad (3)$$

The figure 0·48 is taken to be the specific heat of superheated steam at constant pressure.

To compute item 7, namely, the heat carried away by the dry chimney gas, first from the analysis of the chimney gas calculate A, the air supply per pound of carbon appearing in the chimney gas, from expression (5), (6), (7), or (8), pages 112, 113. Then the weight of furnace gas per pound of carbon in the gas is (A + 1) lbs., and the heat carried away per pound of carbon in the furnace gas is

$$(A + 1)(T - t)\,0{\cdot}24 \text{ lb.-cals.} = F \text{ say} \quad . \quad . \quad . \quad (4)$$

T and t are respectively the chimney and the air temperatures and

0·24 is the specific heat of the furnace gas at constant pressure. Then the heat carried away by the dry chimney gas per pound of coal supplied to the furnace is

$$c(1 - x)\mathrm{F} \text{ lb.-cals.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

since, as stated above, it is assumed that each pound of coal contains the fraction c of carbon and that only $1 - x$ lbs. of coal are actually burnt, the remainder being carried away as cinders and sparks.

Finally, there will always be some heat unaccounted for due to slightly different initial and final conditions; the escape of hydrocarbons in the gases, the escape of gas unconsumed, and the heat in the ashes.

Collecting these results together and putting B for the heat taken up by the water in the motive-power circuit, or, as it is usually put, by the boiler feed; R for the heat escaping by radiation; W for the heat absorbed in warming, evaporating, and superheating the water brought in by the fuel and produced by combustion, and the vapour brought in by the air, the heat accounted for is

$$\mathrm{B} + \mathrm{R} + \mathrm{W} + c(1 - x)\mathrm{F} \text{ lb.-cals.}$$

per pound of coal supplied to the furnace.

And finally, if there is an exact balance between the heat produced and the heat accounted for, this expression should be equal to expression Z above. Therefore

$$\mathrm{K} - kx - c(1 - x)\mathrm{U} = \mathrm{B} + \mathrm{R} + \mathrm{W} + c(1 - x)\mathrm{F} \quad . \quad (6)$$

In this expression it is assumed that the quantity of sulphur in the fuel is small, so that the gas produced by its combustion is small enough to be neglected in the gaseous and air calculations; K is here the higher calorific value. There will be no difficulty in modifying the terms of the equation to meet the circumstances of any particular case. As the equation stands it presents a general relation for the case where there is an appreciable loss of heat energy in the cinders and sparks. In cases where the rate of combustion is low, x may be put equal to zero.

28. Example. Heat Balance Sheet for the Heating Circuit of a Compound Locomotive.—The data for this example are selected from the records of 10 trials made at the St. Louis Exhibition, in 1904, on a De Glehn compound Atlantic-type locomotive, No. 2512, belonging to the Pennsylvania Railroad Co. The trial number is 508. The trial lasted 3 hours and the general conditions were:—

Speed, 160 revolutions per minute, equivalent to 38 miles per hour.

Cut off, high-pressure cylinder, 50 per cent.

Cut off, low pressure, 70 per cent.

Regulator full open.

Indicated horse-power, 944·6.

Coal fired per square foot of grate per hour, 91 lbs.

TABLE 9.—DATA FROM TRIAL 508, ST. LOUIS EXHIBITION.

Dry coal analysis.		Smoke-box gas analysis. By volume.		Calorific values by calorimeter. (Higher value.)
Carbon	84·20	Carbon dioxide	12·47	
Hydrogen	4·28			
Oxygen	2·94	Carbon monoxide	1·03	1 lb. dry coal 8260 lb.-cals.
Nitrogen	1·44			
Sulphur	0·80	Oxygen	5·50	1 lb. cinders 6990 ,,
Ash	6·34			
		Nitrogen	81·00	1 lb. sparks 6400 ,,
	100·00			
Moisture as fired, 0·94 p. ct.			100·00	

Dry coal fired, 9116 lbs.

Cinders collected in smoke-box, 2135 lbs.

Sparks discharged from chimney, 345 lbs.

Evaporation from and at 100° C. per pound of dry coal fired, 7·9 lbs. of water.

Barometric pressure, 14·48 lbs. per sq. in.

Temperature of laboratory, 22·7° C.

Temperature in smoke-box, 310° C.

Cinders and Sparks.

From these data the unproduced heat energy in the cinders per pound of dry coal fired is

$$\frac{2135 \times 6990}{9116} = 1640 \text{ lb.-cals.}$$

and in the sparks

$$\frac{345 \times 6400}{9116} = 242 \quad ,,$$

$$\text{Total} \quad . \quad . \quad . \quad . \quad 1882 \quad ,,$$

Weight of cinders and sparks produced per pound of dry coal fired 0·27 lb. $= x$ in the general equation (6).

Heat Energy unproduced because of Imperfect Combustion.

The gas analysis shows 12·47 per cent. by volume of carbon dioxide and 1·03 per cent. by volume of carbon monoxide. The ratio of these, namely, $\frac{D}{M}$ in equation (1), page 115, is 12·1.

The coal analysis shows that c, the proportion of carbon present, is 0·84; and x, the weight of cinders and sparks formed per pound of dry coal fired, is 0·27.

Therefore by using expression (2), page 115, it will be found that the heat unproduced per pound of dry coal fired is

$$c(1 - x)U = 266 \text{ lb.-cals.}$$

The value of U in this expression is, from equation (1), page 115, 432 lb.-cals.

Heat Energy transferred to the Motive-power Circuit.

The heat energy transferred across the heating surface to the water in the boiler per pound of dry coal fired is

$$540V = 540 \times 7{\cdot}9 = 4265 \text{ lb.-cals.}$$

since from the data the observed evaporation from and at 100° C. per pound of dry coal fired is 7·9 lbs.

Heat Energy carried away by the Dry Chimney Gases.

To compute this, the first step is to calculate the weight of air drawn through the furnace per pound of carbon in the dry furnace gas. Using expressions (4), (5), (7), and (8) in order, page 112, it will be found that A = 18·13 lbs. from (4), 17·93 from (5), 18·00 from (7), and 15·8 from (8). Taking A equal to 18 lbs., the weight of furnace gas per pound of carbon in the gas, which is (A + 1) lbs., is 19 lbs.

The heat carried away can then be computed by equation (4) of Section 27, page 116, that is

$$19(310 - 24)0{\cdot}24 = 1303 \text{ lb.-cals.} = F.$$

And by equation (5), Section 27, page 117, the heat carried away by the furnace gas per pound of coal fired reduces to

$$0{\cdot}842 \times 0{\cdot}73 \times 1303 = 802 \text{ lb.-cals.}$$

Heat Energy corresponding to the Steam in the Furnace Gas.

Water produced by the combustion of hydrogen = 0·043 × 9 =	0·39 lb.
Water introduced with the coal	0·01 „
Total	0·40 „

Add to this the water equivalent to the vapour introduced by the air. Assume this to bring the total amount up to half a pound. Then apply equation (3), page 116, the heat absorbed in warming, evaporating, and superheating this quantity is 357 lb.-cals. per pound of coal fired.

The heat carried away by radiation will in this example have to be found by difference, because no experiments were made to determine it directly.

Collecting these results together the balance sheet is formed *thus:—*

Balance Sheet *for the Heating Circuit of a De Glehn Compound Atlantic-type* **Locomotive, constructed from data obtained from Experiment No. 508 in the Report of the Trials at the St. Louis Exhibition.**

Heat energy produced per pound of dry coal fired.					Heat energy accounted for per pound of dry coal fired.		
			lb.-cals.	per cent.		lb.-cals.	per cent.
Calorific value per pound of dry coal as measured in a calorimeter			8260	100	Transferred to the motive-power circuit	4265	51·7
	lb.-cals.	per cent.			Carried away by the dry chimney gases .	802	9·7
Less— heat unproduced because of imperfect combustion .	266	3·2			Absorbed and carried away in heating, evaporating, and superheating water in furnace gas . .	357	4·3
heat unproduced in cinders . . .	1640	19·9			Radiation, heat in ashes, and generally unaccounted for .	688	8·3
heat unproduced in sparks thrown out of the chimney	242	2·9					
	2148	26·0					
Less			2148	26			
			6112	74		6112	74

The figures in the per cent. column express the various quantities in terms of the gross calorific value of the pound of dry coal.

29. Boiler Efficiency. Furnace Efficiency.—It will be seen from the balance sheet just constructed that 51 per cent. of the heat in the coal is transferred to the water in the boiler. This is the measure of the efficiency of the boiler as a whole. But the heat actually produced in the furnace is only 74 per cent. of the heat in the coal, and hence if all the heat produced by the combustion of the coal had been transmitted, the boiler efficiency could not have been greater than 74 per cent. Without entering into the refinement of available heat discussed in Section 24, it may be said that the efficiency of transmission is $\frac{51}{74} = 70$ per cent. That is to say 70 per cent. of the heat actually produced in the furnace is transferred across the heating surface from the hot furnace gas to the water in the boiler.

It is clearly necessary to distinguish between the efficiency of the

furnace and the efficiency of transmission in order to obtain a clear insight into the working of a boiler.

Three efficiencies may be defined as follows :—

$$\text{Boiler efficiency} = \frac{\text{Heat transferred to the motive-power circuit per pound of dry coal fired}}{\text{Calorific value per pound of dry coal}}$$

$$\text{Furnace efficiency} = \frac{\text{Heat produced in the furnace per pound of dry coal fired}}{\text{Calorific value per pound of dry coal}}$$

$$\left.\begin{array}{l}\text{Efficiency of trans-}\\ \quad\text{mission}\end{array}\right\} = \frac{\text{Heat transferred to the motive-power circuit per pound of dry coal fired}}{\text{Heat produced in the furnace per pound of dry coal fired}}$$

The product of the furnace efficiency and the efficiency of transmission is equal to the boiler efficiency.

The "boiler efficiencies" obtained in certain authoritative trials, together with details relating to the heating circuit, are given in Table 10 (p. 122).

The efficiency varies greatly with the rate of combustion. In order to show the performance of a boiler through its whole range of action it is necessary to construct a heat balance sheet for several different rates of combustion. It is seldom that data sufficiently complete can be obtained for this purpose.

The data furnished by the trials of locomotives at the St. Louis Exhibition, and the data published by Prof. Goss are, however, so complete, that so far as the locomotive boiler is concerned it is possible to follow its working through wide ranges of action.

As the rate of combustion increases, the cinder and spark loss increases, and this loss is difficult to measure. In fact, it is the one item amongst the data which is open to most doubt. For this reason, the weight of cinders and sparks formed per pound of coal supplied to the boiler, and which is denoted by x in the general equation (6), page 117, is assumed to be unknown. The other items are then filled in and the equation solved for x. When x is found it is used in the way already illustrated in the construction of the balance sheet.

The trials of a Simple Consolidation Engine No. 734 have been selected for illustrating the influence of the rate of combustion on the boiler efficiency, because it was worked through a range varying from 32 to 139 lbs. of coal per square foot of grate per hour. The coal used was the same as that for which the analysis is given in the example of Section 28, page 118.

Solving equation (6), page 117, for x, we have

$$x = \frac{K - B - R - W - c(U + F)}{k - c(U + F)} \quad . \quad . \quad . \quad (1)$$

TABLE 10.—Showing Results Obtained with Boilers of Various Types.

Line No.	Authority.	Admiralty Committee on Naval Boilers.					Donkin & Kennedy.	Locomotive Tests and Exhibits, St. Louis Exhibition Trials.			
1.		Tank. Single-ended. Cylindrical. H.M.S. *Minerva* (Report 1902).		Tank. Single-ended. Cylindrical. R.M.S. *Saxonia* (Rep. 1902).	Water tube. H.M.S. *Hyacinth* (Rep. 1902).	Water tube. H.M.S. *Hermes* (Report 1904).	Galloway boiler at works of Bryan Donkin & Co. (Exp. No. 2).	Loco. freight. Lake Shore and M. & S. Railway. (Test No. 201.)	Loco. freight. Lake Shore and M. & S. Railway. (Test No. 212.)	Loco. passenger. New York Central & H. R. Railway. (Test No. 801.)	Loco. passenger. New York Central & H. R. Railway. (Test No. 812.)
2.	Area of firegrate. Sq. ft.	81		63·3	44	62	15	33·76	—	49·9	—
3.	Area of heating surface. Sq. ft.	2308		2867	1357	2210	582	2541	—	3006	—
4.	Ratio heating surface to grate area	28·5		45·3	30·3	35·7	38·8	75	—	60	—
5.	I.H.P. developed by the engines per boiler	644	1016	1010	437	672	—	299	1054	567	1641
6.	Heating surface per I.H.P.	3·58	2·27	2·83	2·79	3·29	—	8·5	2·4	5·3	1·84
7.	Description of coal used	Hand-picked Welsh.		Welsh.	Hand-picked Welsh.	Hand-picked Welsh.	Nixon's Navigation Welsh.	Friable coal, 75% residue, 16% volatile matter.			
8.	Carbon value of coal as fired	1·01	1·0	1·0	1·0	0·97	1·07	1·02	1·04	1·03	1·04
9.	Weight of coal fired per sq. ft. of grate per hour. Lbs.	13·9	30·3	20·6	19·8	20·5	10·2	32·8	139	25·8	134·2
10.	Weight of air drawn through flues per lb. of coal fired. Lbs.	23·3	21·8	23·4	17·0	—	24·4	14·4	6·2	14·5	6·88
11.	Flow of furnace gas, lbs. per min. per sq. ft. of grate	5·6	11·42	8·43	5·46	—	4·3	8·4	16·7	6·7	17·6
12.	Air temperature outside, Cent.	8·9	6·1	10·0	15·6	12·7	1·7	20	27	11	6·2
13.	Average temperature of flue gas, Cent.	338·0	429	202·0	372·0	331	204	267	353	266	395
14.	Difference between temperature of flue gas and air	329	423	192	356	318	202	247	326	255	389
15.	Mean air pressure above atmosphere in closed stock hole. Inches of water	0	0·81	0	0	0	—	—	—	—	—
16.	Ditto, in ash-pits. Inches of water	—	—	1·53	—	—	—	− 0·25	− 0·15	− 0·11	− 0·79
17.	Mean vacuum at base of funnel, or chimney, or in smoke-box of locomotives. Inches of water	0·5	0·39	0·58	0·43	0·42	0·35	1·33	5·9	1·4	8·86
18.	Pressure difference causing flow of furnace gas, draught. Inches of water	0·5	1·2	2·1	0·43	0·42	0·35	1·13	5·75	1·29	8·07
19.	Equivalent evaporation per lb. of coal fired, from and at 100° C.	10·33	9·27	12·33	11·03	11·04	9·32	10	6·52	11·68	7·32
20.	Heat energy in lb.-calories transferred across each sq. ft. of heating surface per hour, 540 × line 19 × line 9 ÷ line 4	2720	5320	3005	3360	3401	1405	2360	6520	2710	8850
21.	Boiler efficiency. Per cent.	68	61·4	82·3	73·3	75·8	70·4	65·0	42	75·3	46·9
22.	CO_2 in furnace gas, by weight. Per cent.	12·56	13·45	17·10	11·94	10·44	13·0	By vol. 10·6	By vol. 13·0	By vol. 12·03	By vol. 13·9
23.	Funnel area per sq. ft. of grate. Sq. ft.	0·116		0·23	0·156	0·166	—	—	—	—	—
24.	Height of funnel above dead plates. Feet	75		129·8	75	75	—	—	—	—	—
25.	Weight of boiler with all accessories. Tons	69·6		100	25·2	40·0	—	—	—	—	—
26.	Weight of boiler per I.H.P. Tons	0·068		0·1	0·052	0·059	—	—	—	—	—
27.	Steam pressure, by gauge. Lbs. per sq. in.	144	148	199	240	—	50	200	200	209	212
28.	Area of blast pipe orifice. Sq. in.	—	—	—	—	—	—	21·6	21·6	24·24	24·24

It is generally more convenient to estimate the various items as a percentage of the calorific value of the coal. Multiplying all through by $\frac{100}{K}$, the expression becomes

$$x = \frac{100 - B\% - R\% - W\% - \dfrac{100c(U+F)}{K}}{\dfrac{100k}{K} - \dfrac{100c(U+F)}{K}} \quad . \quad . \quad . \quad (2).$$

The results of the calculations are shown by the curves, Fig. 44,

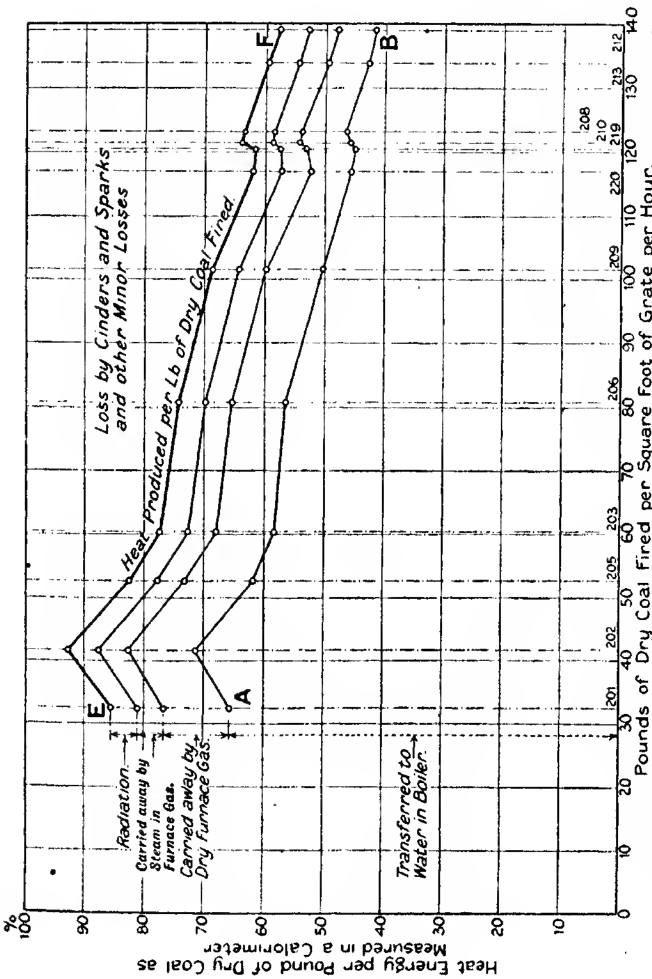


FIG. 44.—Diagram showing the influence of the rate of combustion on the boiler efficiency and on the furnace efficiency. St. Louis Trials, Locomotive No. 734.

where AB shows the boiler efficiency; EF the heat actually produced in the furnace per pound of dry coal fired.

The peculiarity brought out by the diagram is that as the rate of combustion increases the furnace efficiency decreases, mainly due to the increasing loss by cinders and sparks due to the increasing velocity of the air through the fuel bed; but the efficiency of transmission remains nearly constant and equal to about 70 per cent. through the whole range of the experiments.

The author has worked out corresponding figures for the whole of the trials at St. Louis Exhibition, and for the published trials of Prof. Goss given in "High Steam Pressures in Locomotive Service," and these peculiarities occur in each case. The general conclusion to be drawn is, that the efficiency of transmission is only slightly affected by variations in the rate of combustion, but the furnace efficiency falls, as the rate of combustion increases, and the fall is rapid after a certain limiting velocity of the draught is reached, a velocity which depends upon the nature of the fuel. With small coal the limiting velocity is considerably lower than with large coal.

With the relatively low rates of combustion used in boilers with natural draught the furnace efficiency is higher, but the transmission efficiency is not generally so high.

For example, in the case of a Cornish boiler at the City and Guilds Engineering College, included in the trials of Donkin and Kennedy, the furnace efficiency was nearly 100, that is to say, the heat in the coal was actually produced by combustion there being no carbon monoxide found in the chimney gas and no appreciable loss by cinders and sparks, so that the boiler efficiency, namely, 62 per cent., was practically the efficiency of transmission.

There is no doubt that a high rate of combustion seems to increase the efficiency of transmission, due probably to the fact that the temperature of the furnace gas both in the fire-box and in the flues is higher than with low rates. The best results, therefore, would seem to be obtained by arranging for a rate of combustion as high as possible, but not sufficiently high to require a draught powerful enough to entrain any appreciable quantity of cinders and sparks.

30. The Efficiency of Transmission.—The efficiency of transmission as defined above is the average efficiency taken over the whole of the heating surface. The efficiency at any one point depends mainly upon the local difference of temperature between the furnace gas sweeping over the one side of the plate and the water on the other side. This difference of temperature falls rapidly as the gas passes along the flue, until at last it becomes so reduced that the rate of transmission of heat from the gas across the plate is not sufficiently great to make it worth while to extend the heating surface any further, the heat in the furnace gas then passes away into the chimney, unless it is in part used to heat the feed water.

In general heat is transmitted from the furnace gas to the

boundaries of the furnace and flues in two fundamentally different ways:—

(1) By radiation from the incandescent fuel, the flames, and the hot gases;

(2) By the agency of the matter in the path of flow, that is the hot gas, the metal boundary, and the water on the other side of the boundary.

In the first case, transmission is effected by the vibrations of the ether in the same way that heat is transmitted to the earth from the sun. In the second case the matter in the path of flow transmits heat in two mechanically different ways, namely, by convection and by conduction.

Thus, in a furnace or flue there are in general three methods of heat transmission in operation simultaneously, namely, transmission by radiation, by convection, and by conduction. It is practically impossible to analyse the results of experiments so that the quantity of heat transmitted by each of these three methods may be separately, stated. It is instructive, however, to consider each method separately, in order to obtain an estimate of the relative importance of each method of transmission.

Radiation.—Physicists accept Stefan's Law for the total radiation from a black body. This law asserts that the total radiation from a black body into the space surrounding it is proportional to the difference between the fourth powers of the absolute temperatures of the body and of the space.

Let R be the lb.-calories radiated from each square foot of the surface of a black body per hour; and let T be the absolute temperature of the black body; and t be the absolute temperature of the space into which the radiation takes place. Then

$$R = 9{\cdot}33 \times 10^{-9}(T^4 - t^4) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The term black body may be held to apply to carbon at any temperature, and includes carbon lumps or carbon particles in a state of incandescence. The flame in the fire-box of a boiler is usually full of particles of glowing carbon, which can be seen by looking into the furnace through a blue glass. The radiant heat which reaches the boundaries of a furnace has its origin partly in the surfaces of the incandescent carbon at the fire level, partly in the surfaces of the incandescent carbon caught up in the flame and the gas, and partly in the flame itself which radiates heat from all points in its volume.[1]

An upper limit to the quantity of heat which can be transmitted by radiation to the boundaries of a furnace or fire-box may be found by making the assumption that the fire-box is filled with flame, and that the surface of the flame is composed of a continuous surface of incandescent carbon. The radiant heat received per square foot

[1] Callendar, "On the Radiation from Flame". Third Report of a Committee of the British Association presented at the Sheffield meeting of 1910.

of heating surface in contact with this flame can then be calculated from Stefan's formula.

Thus, suppose the flame temperature to be 1600° C. absolute, and the temperature of the metal boundary or absorbing surface to be 400° C. absolute; then, applying Stefan's formula, the absorption of heat per square foot per hour by the fire-box surface is 60,000 lb.-cals. This corresponds to an evaporation of approximately 1·27 lbs. of water per hour at a pressure of 200 lbs. per square inch, and allowing 30 lbs. of water per boiler horse-power gives nearly 4¼ H.P. per square foot of furnace boundary. This is, of course, an upper limit, and represents an unattainable condition, but it gives some idea of the order of magnitude towards which the radiation tends in the case of a fire-box filled with flame.

Transmission by Conduction and Convection.—Transmission by conduction and convection is generally held to be conditioned by the fact that at the boundary of the hot gases in the fire-box and flues there is clinging to the plates a thin film of gas, in which convection currents do not apparently take place and which offers a high resistance to the passage of the heat other than radiant heat, since the heat can pass across the film by conduction only. There is a similar film of water on the water side of the plate, though the resistance to the passage of heat through it is considerably smaller.

Starting from a point in the interior of the mass of gas in the fire-box, the path of the heat-flow to a point in the interior of the mass of water may be considered as made up of the following parts:—

1. The gaseous part of the path from the starting point to the gas film clinging to the plates.
2. The gas film itself.
3. The surface of contact between the plate and the gas.
4. The part of the path through the metallic plate.
5. The surface of contact between the plate and the water.
6. The water film.
7. The water from the film to the point in the mass of water.

If the plate is dirty, there must be added a layer of sooty deposit on the gas side and a layer of scale on the water side, on which there may be in some cases a deposit of oily matter. The method of transmission along this path differs in the different elements. Starting from the assumed point in the mass of the hot gas, the heat is carried to the boundary by a movement of the hot particles of the gas, caused partly by convection currents and partly by currents caused by the draught. Conduction plays a negligible part in transferring the heat along this element of the path. Transference across the bounding gas film, through the metal of the heating surface and across the water film, takes place by conduction, after which the heat is carried to the interior of the water mass mainly by convection.

So far as transmission is concerned, the part of the path which requires special consideration is the metallic plate with its film coatings. Transmission of heat along this path may be likened to

a flow of electricity through a circuit made up of the elements of different resistances. The potential difference, which causes the current to flow along against the resistance of the elements of an electric circuit, may be compared to the differences of temperature along the elements of a heat circuit. Just as it requires a large potential difference to force a current of electricity through a small element of a circuit of high resistance, so it requires a large temperature difference to force heat to flow along a small element of the path offering a great resistance. Difference of temperature may be regarded also as a "temperature head," by analogy with the flow of water; with known data the temperature head could be plotted in a diagram. Such a diagram is sketched in Fig. 45, not, however,

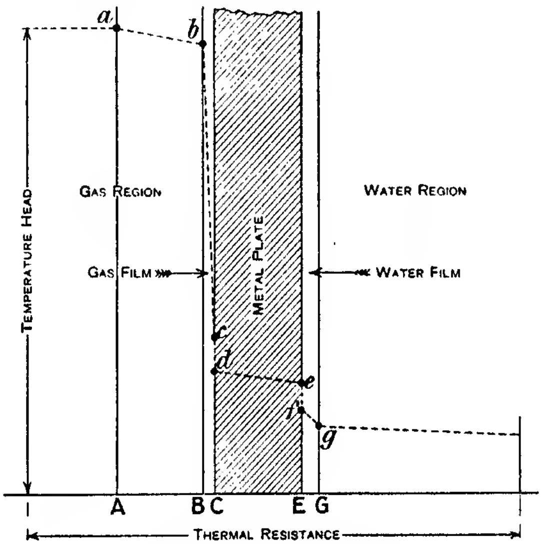


Fig. 45—Temperature head.

from any data, but merely to show the main characteristics of the flow.

Starting from a point a in the interior of the hot gas, Aa represents the temperature, and this is shown as gradually falling to the value Bb at the surface of the gas film. The resistance of the film is extremely great, consequently the temperature falls to Cc in forcing heat through it, the transfer of heat is thus determined by the temperature head Bb—Cc. There is probably a further drop of temperature head represented by cd which is required to force the flow across the surface where the gas film is in contact with the metal, corresponding with the potential difference at a joint in an electric circuit. The temperature head required to cause the flow through the metal itself is Cd—Ee with a further fall of head ef at the surface of contact

between the metal and the water film. The fall of temperature head through the water film is Ef—Gg, until finally the temperature has fallen to that of the water, and the heat transmission through the mass of the water is continued by movement of the water particles.

To give some idea of the order of magnitude of these temperature differences, some figures from a paper by Mr. Hudson[1] may be cited. In this paper it is deduced from experimental evidence that 98 per cent. of the total available temperature head is required to force the heat from the gas into the plate, the remaining 2 per cent. being all that is required to transfer the heat from the plate to the water in the boiler. Or, referring to the diagram, the total available temperature head is Aa—Gg. The above figures indicate that

$$(Aa - Cd) = 0{\cdot}98\ (Aa - Gg)$$
$$\text{and that } (Cd - Gg) = 0{\cdot}02\ (Aa - Gg)$$

Mr. Hudson also arrives at the conclusion that the gas furnace side of the plate of the heating surface is never more than 20° C. hotter than the water side. The experiments of Sir John Durston[2] substantially confirm these conclusions. They show that, when the temperature of the water was 100° C. and the plate clean, the temperature on the hot side of the plate was 38° C. higher than that of the water. Although the actual figures, namely, 20° and 38°, appear to be discrepant, yet, when the temperature difference is expressed as a percentage of the whole temperature head, there is sufficient agreement to show what a small temperature head is concerned in causing the flow of heat through the plate, compared with the total temperature head used to produce the flow from the interior of the hot gas.

Sir John Durston in the paper quoted above gave experiments to show the effect of a coating of grease on the water side of the heating surface. Briefly, when 5 per cent. of mineral oil was added to the water, the temperature difference increased from 38° C. to 54° C.; with $2\frac{1}{2}$ per cent. of paraffin, the difference increased to 65° C.; and with a greasy deposit $\frac{1}{10}$ inch thick, the temperature rose to 188° C. above the temperature of the water.

The flame temperature in these experiments varied from 1150° C. to 1370° C.

On the Existence of a Gas Film Coating the Plate.—The idea of a film coating the plate is a useful mental conception, though actually it is probable that there is a thin layer of gas in contact with the plate which is nearly at the temperature of the plate forming the first of a succession of thin layers, gradually increasing in temperature to the temperature of the flame. The total thickness of temperature stratification is probably of the order $\frac{1}{40}$ in. Justification for this statement is found in an experiment by Mr. Fletcher.[3] If a piece of

[1] "Formulæ for the transmission of heat, taking into account the velocity of the gases," *Engineer*, **70**, 523.

[2] "Transmission of heat through tube plates; also measurements of fine temperature," *Trans. Inst. Naval Arch.*, 1893, **34**, 130.

[3] *Engineering*, vol. **60**, 1895, p. 50.

paper be stuck to the bottom of a metal pot, water can be boiled in the pot by means of a strong gas flame underneath without even charring the paper. This shows that the temperature of the paper cannot be anywhere near the temperature of the flame; in fact, the paper must be immersed in a comparatively cool film of gas at a temperature not much higher than the temperature of the boiling water. In this experiment, whatever the flame temperature may be, it is clear that a large proportion of the head corresponding to it is used up in forcing the heat across the thin film of cool gas in which the paper is immersed; and that the remaining head is used up in forcing the heat through the paper, the metallic bottom of the pot and through the water film.

If the eye is brought to the level of the paper, the existence of this film can be seen. Mr. Fletcher, who found it to be about $\frac{1}{40}$ in. thick, measured it by placing strips of copper on the paper next the gas flame. When the copper exceeded $\frac{1}{40}$ in. thick the paper charred.

On the Existence of a Water Film Coating the Plate.—There is considerable evidence to show that a water film clings to the plate on the water side separating the bulk of the water from the plate. This film is the cause of anomalous results in such a simple case as the measurement of the heat required to raise the temperature of water in an open vessel with a gas flame underneath. If in one case the water is maintained absolutely at rest, so that nothing is done to break up the film, then the heat transmitted to the water will be considerably less for a given temperature head than it would be if means were taken to break up the film. This is because heat must pass by conduction without convection currents, consequently it is the conductivity of water which is called into play, and this is known to be exceedingly small. The water film is further considered below. In a boiler the question is more complicated, because of the formation of steam bubbles.

31. Conductivity of the Elements in the Path of the Flow.—*Conductivity of the Gas.*—The transmission of heat from one point to another through a gas is accomplished partly by radiation, partly by convection, and partly by conduction. Transmission by radiation and convection takes place so readily that it is difficult to devise experiments in which heat transferred by conduction can be separated from that transmitted by other methods. Clerk Maxwell predicted from the kinetic theory of gases that the conductivity of a gas is proportional to the product of the coefficient of viscosity of the gas and its specific heat at constant volume. For practical purposes it may be assumed that the specific heat of the gas at constant volume is constant, and that the coefficient of viscosity varies as the $\frac{3}{4}$ power of the absolute temperature of the gas. Then, knowing the value of the conduction of the gas at any one temperature the value at any other can be calculated. Thus, let K be the number of lb.-calories flowing per second across 1 sq. ft. when the

temperature gradient falls at the rate of 1° C. per foot; T the absolute temperature of the gas. Also let the experimental value of K at 0° C. be A.

Then the value of K at the temperature T is

$$K = A\left\{\frac{T}{273}\right\}^{\frac{3}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Some experimental values of A are given in Table 11.

TABLE 11.—VALUES OF THE CONSTANT A FOR VARIOUS SUBSTANCES.

	Experimental values of K at 0° C. = A.
Air	$3{\cdot}7 \times 10^{-6}$
Carbon dioxide	$2{\cdot}1 \times 10^{-6}$
Nitrogen	$3{\cdot}5 \times 10^{-6}$
Oxygen	$3{\cdot}8 \times 10^{-6}$
Carbon monoxide	$3{\cdot}4 \times 10^{-6}$
Hydrogen	$22{\cdot}2 \times 10^{-6}$

These figures are reduced from those given in the C.G.S. system in the Physikalisch-Chemische Tabellen of Landolt-Bornstein published by J. Springer, Berlin.

The use of the expression and the tables may be illustrated by calculating the value of K for air at 500° C.

Choosing the value of A for air from the table,

$$K = 3{\cdot}7 \times 10^{-6} \times \left\{\frac{773}{273}\right\}^{\frac{3}{4}} = 8{\cdot}1 \times 10^{-6} \quad . \quad . \quad (2)$$

The heat flowing per square foot per second is expressed by

$$K(T - t)$$

when the temperature gradient is such that $T - t$ is the fall of temperature in degrees C. per foot.

Conductivity of Water.—No experiments exist which give the conductivity of water as a function of the temperature up to the temperatures used in a steam boiler. The experiments of C. H. Lees show that the conductivity appears to decrease as the temperature increases. Quoting two values, reduced to lb.-C. units per square foot per second per degree C. difference of temperature per foot,

$$K \text{ at } 25^\circ \text{ C. is } 119 \times 10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$K \text{ at } 47^\circ \text{ C. is } 105 \times 10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

When ebullition takes place in a boiler the presence of steam bubbles on the heating surface adds a new element to the problem of heat transmission since they offer a large resistance to the passage of heat.

Conductivity of the Metal Plate.—In measuring the conductivity of a metal plate experiments must be devised which do not require the assumption that the faces of the metal are at the same temperature as the medium in contact with it. There is a definite joint

resistance to the flow of heat at places where two dissimilar metals, or a metal and a gas, or a metal and a liquid are in contact. When a hole is drilled in a plate and filled with fusible metal, the temperature at which the metal melts is not necessarily the temperature of the plate with which it is in contact, because of the temperature head required to force the heat across the surface of contact. This joint resistance is termed the "Penetrability". When the heat-flow has crossed the surface of the plate into the interior the property of conductivity is sometimes termed the "Permeability of the metal". For a complete account of the work which has been done on the conductivity of metals for heat, the reader should consult works on physics. It is sufficient for our purpose to give probable expressions for iron, copper, and brass.

Ångstrom found that

$$K = 0{\cdot}0135(1 - 0{\cdot}0029\ T) \text{ for iron} \quad . \quad . \quad . \quad . \quad (5)$$
$$K = 0{\cdot}0692(1 - 0{\cdot}00214\ T) \text{ for copper} \quad . \quad . \quad . \quad (6)$$

Lees[1] found

$$K = 0{\cdot}0177 \text{ between } 25^\circ \text{ and } 35^\circ \text{ C. for brass.}$$

A good idea of the general nature of the problem can be obtained by applying the expressions given for calculating K to a hypothetical case of a copper plate 1 in. thick, coated with a film of water on one side $\frac{1}{100}$ in. thick, and with a film of gas on the furnace side $\frac{1}{50}$ in. thick. Assume the average temperature of the plate to be 200° C. Using expression (6), it will be found that at this temperature K, the conductivity of the copper plate is 40250×10^{-6}. Assuming also that the average temperature of the gas film is 500° C., its conductivity K may be regarded as equal to the conductivity of an air film, and this is, from expression (2), 8×10^{-6}. Estimating the conductivity of the water film to be 100×10^{-6}, the ratio of the conductivities of the copper plate to the water film is 400 to 1, and the ratio of the conductivities of the copper plate to the gas film is 5000 to 1. Calling therefore the resistance of the copper plate to the passage of heat unity, the resistance of the air film will be expressed by $\frac{5000}{50} = 100$, and the water film by $\frac{400}{100}$, that is 4. The total resistance from a point just outside the air film to a point just outside the water film is therefore $100 + 1 + 4 = 105$. Assume that the temperature head required to determine the flow between these two points is unity. Then the head will be distributed between the three elements of the path as follows:—

Drop of temperature across air film $= \frac{100}{105} = 0{\cdot}95$
" " " " the plate $= \frac{1}{105} = 0{\cdot}01$ app.
" " " " water film $= \frac{4}{105} = 0{\cdot}04$ app.

If the gas temperature at the point just outside the air film is say 1500° C., and the water temperature just outside the water film is 200° C., the total drop of temperature along the path of flow is

[1] *Phil. Trans. A.*, 183, p. 506, 1892.

1300° C. Of this 1235° drop is required to force the heat across the gas film, 12½° drop is all that is necessary to cause the flow across the copper plate, and 53° is the drop across the water film.

Assuming all the conditions to remain the same with the single exception that an iron plate is used instead of a copper plate, it will be found that the distribution of the temperature head is only slightly changed. The air film would absorb 90 per cent. of the head, the iron plate about 6 per cent., and the water film about 4 per cent.

A comparison of these results indicates the reason that so far as heat transmission is concerned a steel fire-box is practically as good as a copper one, and that the metal for the tubes may be chosen without specially considering its heat-conducting properties, since the existence of the gas film on the furnace side reduces the question of relative conductivity of the metal forming the tubes to insignificant proportions.

Many experiments have been made which indicated that the distribution of temperature head is of the order brought out in the above hypothetical examples, some of which have been mentioned above.

32. On the Influence of the Velocity of Flow of the Furnace Gases.—If the gas film and the water film could be completely or even partially destroyed, the difference of temperature required to determine a given flow of heat across the plate would be considerably reduced. Or put in another way, a given difference of temperature would produce an increased flow. The movement of the furnace gases over the heating surfaces tends to destroy, or break up, the gas film clinging to the plates, and the action is greater the greater the velocity of the gases. Most of the expressions which have been suggested in recent times for the calculation of the quantity of heat-flow across the heating surface contain a velocity term; and no expression could have any generality without a velocity term. Attention was first called to the matter by Professor Osborne Reynolds,[1] in 1874, in a remarkable paper in which the suggestion was made that the transmission of heat depended upon the diffusibility of gases, and from the properties of diffusion deduced that

$$H = At + BDvt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where H is the rate of heat-flow across the heating surface per unit area of surface;
D is the density of the gas;
t is the difference of temperature between the surface and the gas;
v is the velocity of the gas;
A and B are constants.

Many other expressions have been proposed, but the work of

[1] *Proc. Manchester Lit. and Phil. Soc.*, 1874, p. 9.

Stanton[1] and Nicholson show that the above expression interprets the results of experiments in a remarkably consistent manner.

33. On the Breaking Up of the Water Film.— Although the water film does not require a large temperature drop to cause the heat to flow across it, yet it is very important to keep the water moving over the heating surface not only to break up the film, and thus facilitate the transfer of heat from the plate to the water, but to sweep off the bubbles of steam which form on the plate. At the point where a steam bubble forms the plate is practically covered with a gas film, and if the bubble is not swept off quickly the resistance opposed to the transfer of heat from the plate to the water will cause the plate to get hotter. It is not difficult to imagine that a group of bubbles in a part of the boiler where circulation of the water is difficult may coat the plate with a film which may lead to serious overheating of the plates.

The overheating may not be dangerous, but the plate may be locally so much hotter than the metallic parts surrounding it, that the consequent differential expansion may produce considerable wear and tear at that part of the boiler. Some interesting experiments on the circulation of water in a locomotive boiler, are described in a paper by Mr. G. J. Churchward.[2] Mr. Churchward states that the observations indicated that the circulation was generally along the bottom barrel from the front end down the fire-box front, and up the sides and back of the box. A little alteration in the firing had the effect of changing the direction of the currents, and in some cases of completely reversing them.

It will be gathered from the brief account given in the last few sections that heat transmission is an extremely complicated thing, and that the conditions which determine the heat-flow across the heating surface of a steam boiler vary from point to point along the gas stream. In the furnace, radiation is a predominant factor: at the end of the tubes or flues, conduction and convection have the greater influence. Everywhere the velocity is an operating cause, and everywhere the velocity is changing, and the density of the furnace gas is changing.

A more detailed account of the problem, together with the bibliography of 406 papers or articles on the subject, will be found in a report entitled "Heat Transmission" made by the author, with the assistance of Mr. Thieme, for the Institution of Mechanical Engineers, and published in the *Proceedings* of the Institution for October, 1909.

34. Liquid Fuel.—Fuel in the liquid form is easier to store and easier to handle than coal. When burnt liquid fuel leaves no ash, and the calorific value of some varieties is nearly 50 per cent.

[1] "Passage of heat through metal surfaces in contact with liquids; effect of speed," *Phil. Trans. Roy. Soc.*, vol. 190, p. 67.

[2] "Large Locomotive Boilers," *Proc. Inst. Mech. Eng.*, 1906.

higher than that of good steam coal. For example, on page 103 it was shown that a sample of Astatki gave a calorific value by calculation from its chemical analysis of 11,183 lb.-cals. per pound as against 8505 lb.-cals. for a sample of Nixon's Navigation coal.

Thus, weight for weight, 40 per cent. more energy can be carried in the form of liquid fuel than in the form of coal. The advantage of this cannot be overestimated in the case of warships, where every ton saved can be used to increase the offensive or defensive power of the ship.

The following table is given by Mr. James Holden in a paper communicated to the Institution of Civil Engineers.[1]

The last column to the right has been added by the author, and the several items are calculated by multiplying the theoretical evaporation given in column 4 by 540.

TABLE 12.—SHOWING THE CALORIFIC VALUES OF VARIOUS KINDS OF LIQUID FUEL.

Variety of liquid hydro-carbon.	Specific gravity.	Flash point.	Water evaporated from and at 100° C. Theoretical.	Water evaporated from and at 100° C. Actual in G.E.R. practice.	Calorific value.
		Deg. C.	lbs.	lbs.	lb.-cals.
Creosote	1·075	82	17·4	12·5	9,400
Coal-gas tar	1·220	66	15·6	11·6	8,420
Russian astatki . .	0·900	93	21·0	14·0	11,340
Texas oil	0·935	66	20·3	13·5	11,000
Borneo oil	0·960	76	20·3	13·5	11,000
Green oil	1·115	104	17·3	12·7	9,350
Oil-gas tar	1·070	49	17·8	12·8	9,600

Liquid fuel cannot be properly burnt until it is reduced to a state of fine mechanical subdivision so that the air necessary for its combustion can mix intimately with it. The liquid must, in fact, be broken up into a fog as it enters the furnace. One way of producing this fog is to project a stream of fuel against a mass of hot brickwork in the furnace,[2] and this was the method used by Mr. Thomas Urquhart in the design of the liquid fuel burning locomotives running on the Russian railways. Another way is to break up the liquid jet by the action of small steam jets directed towards it, a method successfully developed by Mr. James Holden. The Holden burner is shown in Fig. 46 fitted to the fire-box of a locomotive, and the burner itself is shown in greater detail in Fig. 47.

Referring to Fig. 46, the burner A is fastened to the casting forming the engine footplate. It projects into a ferrule or circular channel B, passing through the water space C of the fire-box. The burner is connected to the fuel supply pipe D by the flexible pipe E. Steam is supplied to the burner through the pipe F, and also through

[1] "Note on the Application of Liquid Fuel to the Engines of the Great Eastern Railway Co.," *Proc. Inst. C.E.*, 1910-11, vol. 185, Part 3.

[2] "Petroleum Fuel in Locomotives," Thomas Urquhart, *Proc. Inst. Mech. E.*, 1884 and 1889.

the pipe H, to a ring G encircling the nose of the burner. I is the spindle of the valve by which the driver regulates the supply of oil to the burner. The steam through F induces oil to flow through the burner into the furnace, and as the liquid stream emerges, inclined jets of steam fall upon it from small holes drilled round the ring G. These jets effectively atomize the oil. The necessary quantity of air for combustion is drawn in partly through the opening round the nose of the burner and partly through the centre of the burner itself. In fact, the jet issuing from the burner is composed of three jets, namely, an inner conical jet of air, a surrounding jet of steam, and a conical jet of oil fuel surrounding the steam. This arrangement of jets will be understood from Fig. 47. O is the casing of the

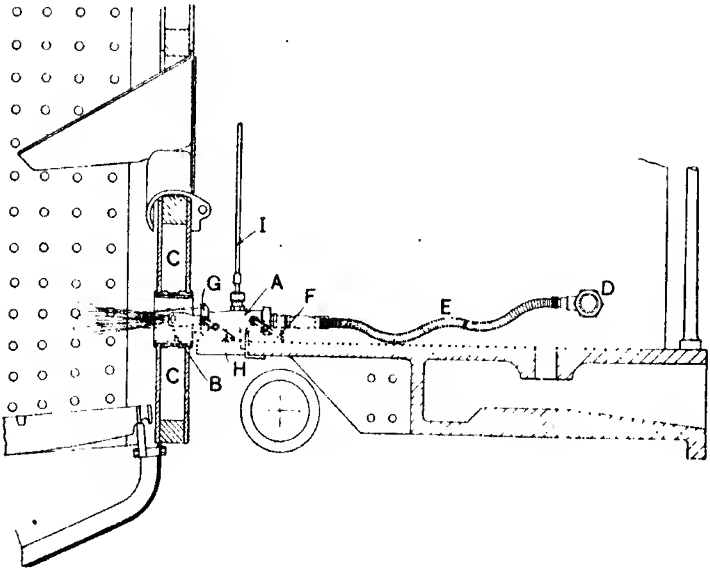


Fig. 46.—Holden oil fuel burner.

burner, and there are three cones K, L, and M, arranged to produce the composite flow when steam is blown through it. The burner is in fact an injector, injecting both oil and air into the furnace. The ring blower is shown at *e*. Oil orifices to the inner cone are shown at A and B.

Two of these burners project sufficient oil in the fire-box to run an express locomotive. Oil fuel may be used exclusively, in which case the fire-bars are covered with a layer of broken fire-bricks, or it may be used to supplement coal fuel. The liquid fuel is filtered through a wire gauze strainer as it enters the storage tank carried by the engine tender, and a steam-heated warming coil placed in the tank serves to keep the oil thin in cold weather. Arrangements are made for

blowing steam through the pipes in case a clot of thickened fuel finds its way into the supply pipes and obstructs the flow, and the interior of the injector itself can be withdrawn easily during a run for cleaning purposes by slackening off the nut N, Fig. 47. Ventilating coils are provided in the storage tanks for the purpose of allowing air to replace the fuel as it is drawn off, and to allow any gas formed to escape into the air. Oil is a more convenient fuel with which to fire a locomotive than coal. Its adoption depends upon the relative price of oil and coal.

An oil burner of another kind is shown in Fig. 48. The spindle A of the burner terminates in a pointed piston B. The conical point on the piston together with the corresponding seating in the casing of the burner form a regulating valve. The end of the piston is

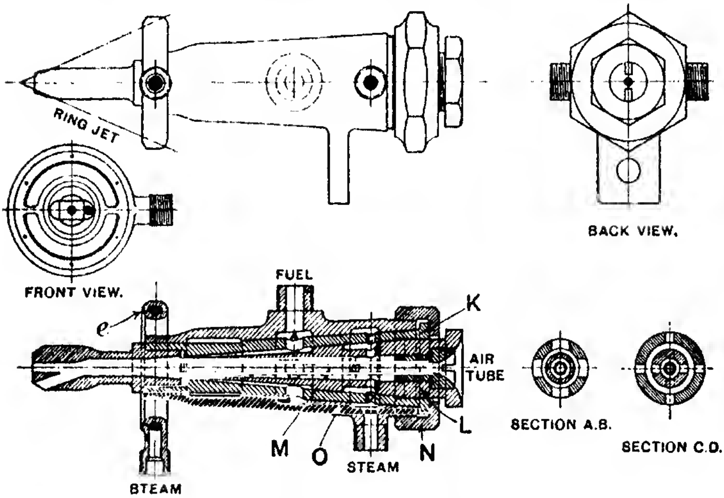


FIG. 47.—Holden oil fuel burner. Details.

shown at C. Small holes are drilled through the piston at an angle with the face. Oil is forced into the burner at D at a considerable pressure, and passing through the inclined holes in the piston emerges into the furnace in a whirling vortex of such energy that the liquid is thrown into a fog. In fact, the energy of flow due to the pressure drop between the entrance to the burner at D and the exit at E appears as kinetic energy in the vortex, and the angular velocity produced is so great that the liquid is fogged by the immediate action of centrifugal force through the liquid. At the same time air flows into the furnace through the central channel into which the burner projects, and also through an annular channel, surrounding the central channel, in which vanes are fixed. These vanes throw into a vortex the air passing through the annular channel. Just at the threshold of the furnace, fuel vortex and air

vortex coalesce into an intimate mixture, and the oil is rapidly and completely burned. The general view of a burner attached to the boiler front is shown in Fig. 49. Z is the plate fronting the boiler-room, and Y is a plate in the furnace against which a fire-brick lining is built. Air under pressure from a fan is driven into the space between the plates Z and Y, and from there flows through the annular channel X, in which the vanes V, V, V, are fixed, and also through the central channel W immediately surrounding the burner. The boiler to which the burner is attached is fitted with 10 burners supplied from a common oil main in which the oil pressure is maintained by a pump at from 50 to 200 lbs. per square inch according to circumstances.

The oil from the pump passes through a steam-heated calorifer

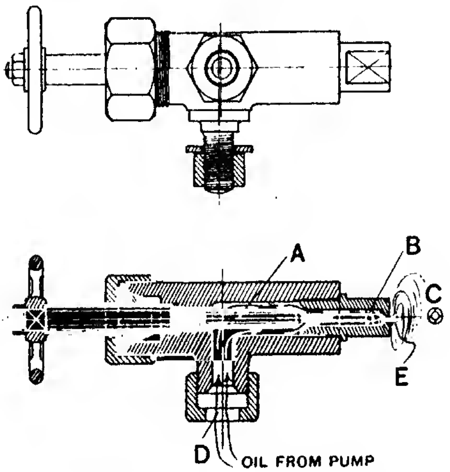


Fig. 48.—Oil fuel burner. Marine boilers.

and a filter on its way to the burner. Its viscosity can be reduced by regulating the steam supply to the heater and thus facilitating the fogging in the vortex. Excess air is used to the extent of about 50 per cent.

As illustrating the convenience of this method of firing, an approximate calculation of the size of the oil pump required to deliver oil necessary to produce 10,000 H.P. may be made. Assume that 1 lb. of oil evaporates 16 lbs. of water, and that the turbines require 12 lbs. of steam per I.H.P.-hour. Then the quantity of oil required per I.H.P.-hour is $\frac{3}{4}$ lb. Oil must therefore be delivered to the furnace at the rate of 7500 lbs. per hour. If the oil weighs 58 lbs. per cubic foot this corresponds to a displacement of 129 cub. ft. per hour. A ram $2\frac{1}{2}$ ins. diameter and 1 ft. stroke driven at 63 strokes per minute will supply the required quantity. The

horse-power developed depends upon the speed of the oil pump, and this can be regulated to a nicety.

Regulation of the speed of the pump is necessarily accompanied by regulation of the speed of the fan driving air into the furnace, because it must not be forgotten that 1 lb. of oil must be supplied with about 20 to 24 lbs. of air to properly burn it.

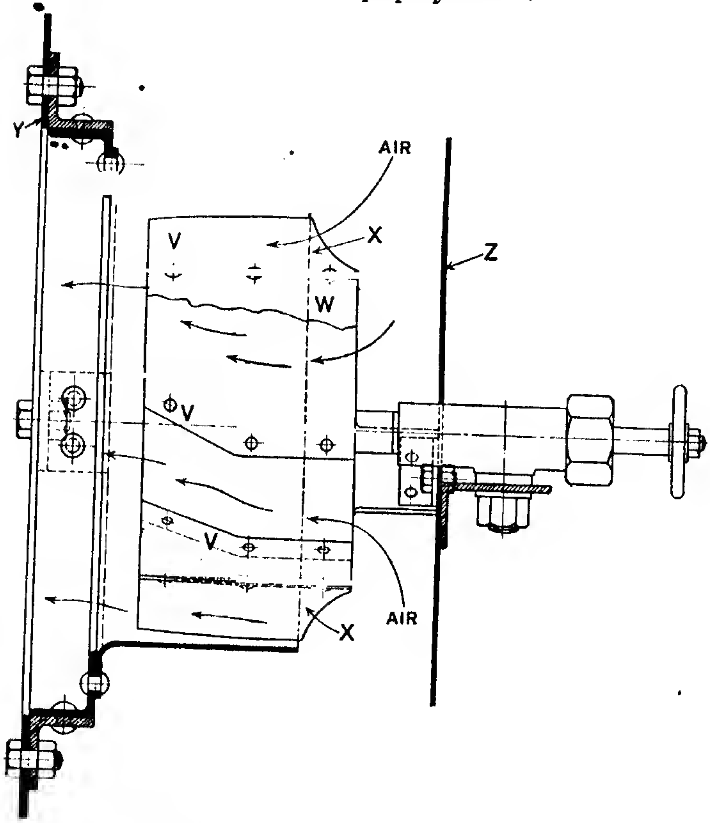


FIG. 49.—General arrangement of oil fuel burner on front of a water-tube boiler.

The results of a series of trials made on a water-tube boiler fired with liquid fuel are shown in Table 13. This table is taken from a paper by Mr. Harold Yarrow read at the spring meeting of the Institution of Naval Architects in 1912. The paper is valuable as showing also the results which may be obtained when a water-tube boiler is fitted with a superheater.

A general view of the boiler is shown in Fig. 50, from which it will be seen that the uptake is divided into two parts by a partition

TABLE 13.—RESULTS OF TRIALS OF A YARROW WATER-TUBE BOILER FITTED WITH A SUPERHEATER AND FIRED WITH OIL FUEL.

Trials with Damper Open.

Heating surface: Large Nest of Generator Tubes = 3247 sq. ft.; Small Nest of Generator Tubes = 2188 sq. ft.; Superheater = 1265 sq. ft.; 6700 sq. ft. total.

On these trials the heating surface is taken as the total heating surface of 6700 sq. ft.

Steam pressure lbs. per square inch.	Superheat in degrees C.	Air pressure, inches of water.	Lbs. of water evaporated per hour.	Lbs. of oil fuel burnt per hour.	From and at 100° C. Lbs. of water evaporated per lb. of oil.	From and at 100° C. Lbs. of water evaporated per square foot of heating surface per hour.	Lbs. of oil fuel burnt per square foot of heating surface per hour.	Temperature of feed water, degrees C.	Temperature between small nest of generator tubes and superheater, degrees C.	Temperature of uptake, degrees C. Above superheater.	Temperature of uptake, degrees C. Above large nest of generator tubes.
242	52	5·0	94,659	8286	14·6	18·0	1·237	14	605	442	475
243	51·5	3·16	76,021	6454	15·0	14·4	0·963	17·5	496	370	386
243·7	46	2·44	68,387	5695	15·2	12·9	0·850	17·5	484	362	364
242·8	34	1·7	46,041	3630	15·9	8·6	0·542	17·8	341	280	288
241·8	17	0·998	20,059	1540	16·1	3·7	0·230	17	250	222	231
242·2	11	0·625	8,478	649	16·1	1·55	0·096	17	240	210	213

Trials with Damper Shut.

Heating surface: Large Nest of Generator Tubes = 3247 sq. ft.; Small Nest of Generator Tubes = 2188 sq. ft.; Superheater = 1265 sq. ft.; 6700 sq. ft. total.

On these trials the heating surface of boiler is taken as heating surface of Large Nest of Generator Tubes = 3247 sq. ft.

Steam pressure, lbs. per square inch.	Air pressure, inches of water.	Lbs. of water evaporated per hour.	Lbs. of oil fuel burnt per hour.	From and at 100° C. Lbs. of water evaporated per lb. of oil.	From and at 100° C. Lbs. of water evaporated per square foot of heating surface per hour.	Lbs. of oil fuel burnt per square foot of heating surface per hour.	Temperature of feed water, degrees C.	Temperature of uptake, degrees C. above large nest of generator tubes.
242·0	4·85	68,648	6287	13·25	25·66	1·936	16·1	490
242·25	3·97	57,693	5065	13·84	21·6	1·56	15·6	450
242·4	2·491	44,050	3504	15·3	16·5	1·09	15·1	356
242·5	1·46	31,481	2473	15·4	11·75	0·76	17·5	317

plate, the part to the left of the centre line being fitted with an air-cooled damper. When steam is shut off from the engines the damper is closed to stop the passage of hot gas across the superheater tubes. Then all the hot gas passes away to the right. When working with the superheater in action the hot gas divides, part flowing to the right across the wide nest of the generator tubes, and part to the left across the narrow set of generator tubes, and then across the superheater tubes, and away to the chimney. It will be seen from the results given in the table that when working with the moderate air pressure of 0·62 in. of water 16 lbs. of water were

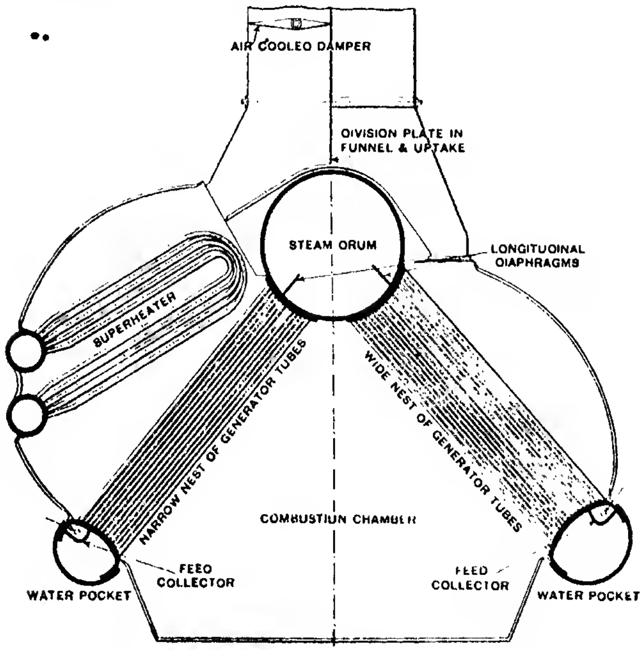


FIG. 50.—General arrangement of Yarrow boiler with superheater.

evaporated per pound of oil burned per hour, and the steam was superheated 11° C. With the air pressure increased to 5 ins. the evaporation per pound of oil fell to 14·6, but the evaporation per square foot of heating surface was increased more than eleven times, namely, from 1·55 lbs. of water per square foot per hour to 18 lbs.

The lower table shows the results obtained with the damper shut so that only the large nest of generating tubes was in action. These results indicate how perfectly oil fuel can be applied to marine purposes, and indicate further that the only bar to the general adoption of oil fuel is the question of a steady supply at a reasonable cost.

CHAPTER III

The Motive-Power Circuit: Thermodynamics

35. Carnot's Reflections on the Motive Power of Heat.—A detailed examination of the heat transferences and transformations occurring in an engine developing 643 horse-power is made in Section 10. From this it will be seen that the working agent transforms into mechanical energy only 19 per cent. of the heat which is received from the heating circuit, the remainder being rejected to the cooling circuit. This engine is an economical one as engines go. Why is it, then, that such a large proportion of the heat received by the working agent should be rejected in the condenser? Can any better results be obtained by using a working agent more easily vaporized than steam, such as alcohol or ether?

The answers to these questions are found in a remarkable essay by Sadi Carnot, entitled "Reflexions on the Motive Power of Heat," and published in Paris in 1824, an essay containing the fundamental principles from which the modern science of Thermodynamics has been developed, by Clausius, Rankine, Kelvin, Maxwell, and other workers.

Reverting again to the steam circuit, Section 10, a particular mass of the working agent may be imagined, as it circulates, to pass through a series of changes of pressure and volume and state of aggregation, changes which are repeated each time the mass makes the circuit, that is to say, the mass passes through a cyclical variation of state.

Carnot begins his theory by stating as an axiom that "when a body has undergone any changes, and after a certain number of transformations is brought back identically to its original state considered relatively to density, temperature, and mode of aggregation it must contain the same quantity of heat as it originally contained".

The next idea which Carnot emphasizes is, that motive power cannot be obtained from heat by the alternate expansion and contraction of a working substance without the combination of a source and a sink of heat at different temperatures to act as boiler and condenser respectively. That is to say, there must be a heating circuit to supply heat to the working agent, and a condensing circuit to receive heat from it, otherwise the pressure-volume cycle in the cylinder by means of which heat energy is transformed into motive power cannot be established. Generalized, this means, that whenever a difference of temperature exists, it is possible to use this

difference to obtain motive power from heat. In order to realize to the full the motive power obtainable from a quantity of heat energy when a difference of temperature or a "temperature fall" exists, Carnot laid down the condition that "there should not be any direct interchange of heat between bodies at sensibly different temperatures". If such a direct interchange takes place, a temperature fall is wasted, because if such a fall exists, its presence makes it possible to obtain motive power from heat, in other words, to transform heat energy into mechanical energy.

The transfer of heat from one body to another at the same temperature, that is, without any temperature fall between them, is an ideal process, which may be regarded as the transfer produced by an infinitely small temperature fall operating for an infinitely long time. A working agent expanding slowly in a cylinder may be theoretically maintained at a constant temperature during expansion by ideally regulating the heat supply from the walls; and if this is imagined done, heat energy is transferred from the walls to the working agent at constant temperature, the temperature of the agent. Conversely, during the process of compression the heat produced may be removed by the ideal regulation of the action of the walls, so that the agent is maintained at a certain temperature, and heat is transferred from it to the walls at constant temperature. The term **Isothermal expansion** is applied to the mode of expansion during which the temperature of the working agent does not change. **Isothermal compression** is the compression of the working agent at constant temperature.

During isothermal expansion the heat energy received from the walls is transformed into mechanical energy, and the working agent by its expansion does work on the piston. If the mechanical process is reversed so that work is done on the piston from without, the mechanical energy expended in compressing the gas is transformed into heat energy.

If the working agent expands in a cylinder with walls assumed to be made of a perfectly non-conducting material, work is done at the expense of the heat energy in the working agent itself, because, by hypothesis, no heat can be transferred from the walls to the working agent during the process. In these circumstances, as the stock of heat energy possessed by the working agent is gradually changed into mechanical energy, the temperature falls. If the working agent is compressed, the mechanical energy used to compress it is transformed into heat energy, and since the walls are non-conducting, this heat is added to the stock of energy possessed by the working agent, and its temperature rises.

Adiabatic expansion or **compression** are terms applied to the particular modes of expansion or compression in which no heat energy is received from or given to the walls.

Thus heat energy may be received or rejected by a working agent at constant temperature by the process of isothermal expansion or isothermal compression, and the temperature of the working agent

may be lowered or raised by the process of adiabatic expansion or compression.

Any process which when reversed results in corresponding exact reversals of the work and heat transformation made through the *physical properties of the working agent*, is defined by Carnot as a **reversible process.**

For example, suppose that during isothermal expansion from a given initial to a given final state the working agent receives Q units of heat energy from the walls and does W units of work on the piston; then, if during compression from the final to the initial state W units of work are done on the piston from without, and Q units of heat energy are returned to the walls of the cylinder, the process is a reversible one, since the reversal of the process of expansion to compression results in the exact reversal of the heat and work transformation.

Similarly, during a reversible adiabatic process of expansion and compression between stated initial and final conditions of the working agent, the quantity of heat energy abstracted from the stock possessed by the agent and transformed into mechanical energy during expansion is exactly equal to the quantity of heat restored to the working agent as the result of a transformation of the work done during compression, and the drop of temperature during expansion is equal to the rise of temperature during compression.

In order that a process may be reversible there must be no transfer of heat between the working agent and the walls at different temperatures, and the pressure exerted on the piston as the agent changes its volume must be exactly balanced by the resistance against which the piston is moved.

Also there must be no loss due to mechanical friction of the piston in the working cylinder, and no losses in the working agent itself due to molecular friction or viscosity.

No working agent exists in which this condition is exactly fulfilled, but air and the permanent gases fulfil them very nearly over wide ranges of temperature.

A perfect gas is an ideal working agent, in which there are no internal losses during a change of state. Within the ordinary ranges of temperature dry air may be regarded as a perfect gas.

The next step in Carnot's theory is the definition of a cycle of reversible operations by means of which the maximum quantity of motive power obtainable from a given temperature difference may be calculated.

In order to make the calculation a working agent must be selected whose characteristic properties are known. Carnot selected air, and described his cycle with air as the working agent.

The air is imagined to pass through a cycle of reversible operations beginning and ending with the air at the same pressure, volume and temperature. Heat is received by the reversible process of isothermal expansion; the air is cooled by the reversible process of adiabatic expansion; heat is rejected by the reversible process

of isothermal compression; and the temperature is raised to the initial temperature finally by the reversible process of adiabatic compression. These four processes constitute Carnot's Cycle.

During the cycle a definite quantity of heat energy is received from the source at the higher temperature T, a definite quantity is rejected in the condenser at the lower temperature t, and the difference is the quantity transformed into mechanical energy in the cylinder. A continuous repetition of the cycle enables heat energy to be transformed into mechanical energy continuously. If all the processes in the steam circuit of an actual engine were reversible, the work done by the piston per minute would be exactly equal to the difference between the heat received by the steam circuit from the heating circuit and that rejected to the condensing circuit per minute. Before continuing Carnot's argument it will be convenient to consider some points in the thermodynamics of a perfect gas.

36. Some Points in the Thermodynamics of a Perfect Gas.—The relations between the pressure P, the volume V, and the absolute temperature T of a perfect gas are defined by the equation

$$PV = cwT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which c is a constant, and w is the mass of gas undergoing change.

The equation may, in fact, be regarded as a convenient summary of the results which have been obtained from experiments made on the so-called permanent gases.

If the temperature of a mass of gas is kept constant during a change of pressure and volume, the equation becomes PV = a constant, which is Boyle and Marriott's law that the pressure of a gas varies inversely as the volume at constant temperature.

If again the pressure is kept constant during a change of volume and temperature, the equation becomes $\frac{V}{T}$ = a constant, which is Charles' law that the volume of a gas at constant pressure varies directly as the absolute temperature.

The results of experiments on the permanent gases show that a volume V of any one gas or a mixture of gases increases by $\frac{1}{273}$rd part for an increase of 1° in temperature measured on the centigrade scale. If, therefore, a gas at zero temperature on the centigrade scale has the volume V, it will increase at a temperature t to the volume $V_t = V\left(1 + \frac{t}{273}\right)$. If the temperature is reduced by 273° below the zero of the centigrade scale, the second term of the quantity in the brackets becomes minus 1, and the actual volume of the gas has shrunk to nothing. Before this temperature would be reached, however, the gas would have first liquefied and then afterwards solidified, so that it is reasonable to suppose that at a temperature 273° below the melting point of ice the volume to which any gas would be reduced would be the volume of the solid into which it had been changed.

The fact that all the gases which are liquefied only at low temperatures follow very nearly the same law of contraction points to the existence of a zero of temperature below which it is impossible to go. Thus the existence of an absolute zero of temperature and its position below the temperature of melting ice are inferred from a comparison of the behaviour of different gases when subject to change of temperature. The temperature itself is also a state of the gas or solid inferred from its volume, since with the ordinary thermometers the temperature scale marked on the instruments is really a scale showing the changes of volume of the substance. It will be shown later that a truly absolute scale of temperature may be defined by means of the principles of thermodynamics alone in a way which makes the scale independent of the properties of any substance.

For the present the temperature T in the characteristic equation is to be found by adding 273 to the temperature measured on a Centigrade thermometer.

When heat is received or rejected by any of the permanent gases or by mixtures of them, equal amounts of heat energy cause very nearly equal changes of temperature in a particular mass of gas when the addition of heat is made whilst the pressure of the gas is kept constant.

The quantity of heat energy in lb.-calories required to raise the temperature of 1 lb. of gas by 1° when the pressure is kept constant, is defined as the "specific heat of the gas at constant pressure," and may be represented by the symbol K_p. This quantity increases slightly as the temperature increases, the increase being, however, negligible except in the case of high temperatures.

The specific heat of a perfect gas at constant pressure may, by inference from the behaviour of the permanent gases, be regarded as constant.

Again, when heat is received or rejected by a gas, equal amounts of heat energy cause nearly equal changes of temperature when the volume of the gas is kept constant whilst the heat is added, and the quantity of heat in lb.-calories required to raise the temperature of 1 lb. of the gas 1° is called the "specific heat of the gas at constant volume," and is represented by the symbol K_v.

It will be shown immediately (Section 38) that the difference between the specific heat of a perfect gas at constant pressure and the specific heat at constant volume is a constant, and equal in value to the gas constant c in the characteristic equation.

37. Application of the First Law of Thermodynamics to a Perfect Gas.—The first law of thermodynamics may be stated thus—

$$\left\{\begin{array}{l}\text{The rate at which}\\ \text{a working agent}\\ \text{receives heat}\\ \text{energy}\end{array}\right\} = \left\{\begin{array}{l}\text{Rate at which the}\\ \text{internal energy}\\ \text{possessed by the}\\ \text{agent is changed}\end{array}\right\} + \left\{\begin{array}{l}\text{The rate at which}\\ \text{work is done}\\ \text{by the gas ex-}\\ \text{ternally}\end{array}\right\} \quad (1)$$

or symbolically over a definite period

$$Q = E + W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This equation is perfectly general in its application and may be applied to find the relation between the heat received by the working agent and the work done, providing that the characteristic equation of the working agent is known.

The **internal energy** of a working agent is the stock of energy which it possesses by virtue of its state of pressure, volume, temperature, and state of aggregation. It is, in fact, the whole energy which has been put into it to bring it into that state. There is no way of measuring how much this is, because so little is known of the molecular energy of substances.

Referring to the first law above, it will be understood that when a working substance receives heat energy it is in general employed partly in changing the internal energy and partly in doing work. If the heat energy supplied and the work done during a change can be measured, the consequent change in the internal energy can be found, and hence the internal energy of a working agent can be measured from some arbitrary state.

Joule demonstrated that the internal energy of a quantity of air was practically independent of its pressure and volume, and depended only on its temperature. It was shown that the common temperature of two masses of gas at different pressures remained unchanged after mixing them together without changing their total volume. One mass of gas was compressed into a vessel A; a second vessel B was exhausted so that the mass of gas in it was small and at a low pressure. The two vessels were connected by a closed stop valve and were placed in water. After the apparatus had come to a common temperature the stop valve between the vessels was opened and gas from the vessel A rushed into B until the pressures in the two vessels were equal. After this experiment the common temperature was found to be unchanged. No external work had been done, and therefore the internal energy had not been diminished, and as the temperature at the end of the operation was the same as it was at the beginning, it was concluded that the internal energy of a gas depends only upon its temperature.

Kelvin and Joule, using subsequently more delicate experimental methods, found that there is a small drop of temperature when air expands without doing external work, and this is due to the fact that some work is actually done in what may be called molecular friction. Joule's law applies, therefore, only to the ideally perfect gas. It is obvious that if the volume of a gas remains constant, all the heat which it receives goes to increase its stock of internal energy. If, therefore, a perfect gas changes its temperature from T_1 to T_2, the inference is that the internal energy has changed by the amount $K_v(T_2 - T_1)$ per unit mass of the gas, and this is true whatever be the changes in pressure and volume which may be going on at the same time.

This expression may therefore be substituted in equation (2) above for the first term on the right side when it is assumed that a perfect gas is used as the working agent.

Again, when heat is transformed into mechanical energy by the expansion of a perfect gas, all the work done is due to the expansion of the gas, that is to say, there is no mechanical energy used to overcome the friction or the viscosity of the gas molecules during expansion. All the work is shown on the corresponding indicator diagram.

Let the state of a mass w of gas be represented on the pressure-volume diagram, Fig. 51, by the point A. Let the gas change to a new state represented by the point B. And let the pressure pass through the intermediate values defined by the curve joining A to B. Then the work done during the change is represented by the area included between the volume ordinates through the points A and B, the volume axis, and the curve AB.

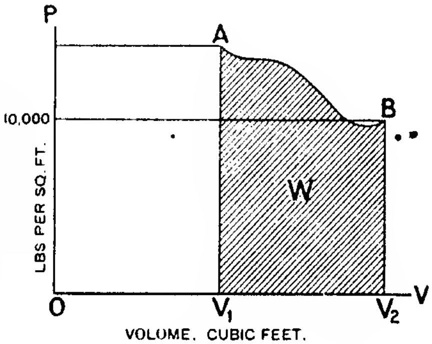


FIG. 51.—Work area during change of state.

This area may be computed graphically, or mechanically by a planimeter, or analytically if the equation to the curve AB is known.

In general the work done may be represented analytically by the well-known expression for the area—

$$W = \int_{V_1}^{V_2} P dV$$

The work W is measured in ft.-lbs. when P is given in pounds per square foot, and V in cubic feet. The number of thermal units corresponding to the quantity of work W is found by dividing W by J, the mechanical equivalent of heat.

Introducing this, together with the expression for the change of internal energy into the general energy equation (2) (p. 145), it becomes

$$Q = wK_v(T_2 - T_1) + A\int_{V_1}^{V_2} P dV \quad . \quad . \quad . \quad . \quad (3)$$

where A is written for $\frac{1}{J}$.

This together with the characteristic equation

$$PV = cwT$$

enables the relations between heat received and work done to be calculated when the working agent is a perfect gas.

In these expressions it is to be remembered that T is the absolute temperature found by adding 273 to the temperature measured on the ordinary centigrade scale;

P is the pressure in pounds per square foot;
V is the volume in cubic feet;
J, the mechanical equivalent of heat, is 1400;
Q, the heat energy, is measured in lb.-calories;
w is the weight of gas in pounds; and
c is 96 foot-lbs. units when the gas is dry air.

EXAMPLE.—Let the pressure and volume of a mass of half a pound of dry air be 14,400 lbs. per square foot and 2 cub. ft. respectively, and let A be the point on the P.V. diagram, Fig. 51, representing this state. Let the air change along the arbitrarily drawn curve AB to the new state represented by the point B, namely, a pressure of 10,000 lbs. per square foot and a volume of 4 cub. ft., $K_v = 0{\cdot}17$.

Find the heat received or rejected by the air during the change of state.

The first step is to calculate the temperatures in the initial and final states. This is done by means of the characteristic equation thus—

$$\text{Temperature at A} = \frac{PV}{cw} = \frac{14400 \times 2}{96 \times 0{\cdot}5} = 600^\circ \text{ C. abs.}$$

$$\text{Temperature at B} = \frac{PV}{cw} = \frac{10000 \times 4}{96 \times 0{\cdot}5} = 833^\circ \text{ C. abs.}$$

$$\text{Change of temperature} = 233^\circ \text{ C.}$$

Since the increase of internal energy is $wK_v(T_2 - T_1)$ and K_v is 0·17, therefore the numerical value of the increase is

$$0{\cdot}5 \times 0{\cdot}17 \times 233 = 19{\cdot}8 \text{ lb.-cals.}$$

Again, the area of the curve, graphically measured, represents 24,000 ft.-lbs., equivalent to 17·1 thermal units. The heat received by the working agent during the change is then—

$$Q = 19{\cdot}8 + 17{\cdot}1 = 36{\cdot}9 \text{ thermal units.}$$

Before the work area can be measured, the form of the expansion curve must be known. Certain relations may be established by assuming a form, or the equation of the curve may be found for certain conditions of heat-flow.

38. On the Difference of the Specific Heats.—The difference between the specific heat at constant pressure and the specific heat at constant volume is equal to the constant c in the characteristic equation, divided by J.

Let the change of state from A to B be made at constant pressure, therefore AB is a straight line parallel to the axis of volume as shown by the dotted path, Fig. 52, and the work done is $P(V_2 - V_1)$ ft.-lbs. This is equivalent to the expression $cw(T_2 - T_1)$, since the characteristic equation gives that $PV = cwT$. Also the heat received as the gas expands at constant pressure from

T_1 to T_2 is $wK_p(T_2 - T_1)$ lb.-cals. And the change of internal energy is $wK_v(T_2 - T_1)$.

So that—

$$Q = E + W$$

becomes

$$wK_p(T_2 - T_1) = wK_v(T_2 - T_1) + \frac{cw(T_2 - T_1)}{J}$$

That is

$$K_p - K_v = \frac{c}{J}$$

A gas may change from a particular state, represented by the point A on a pressure-volume diagram, Fig. 52, to a volume V_2 along

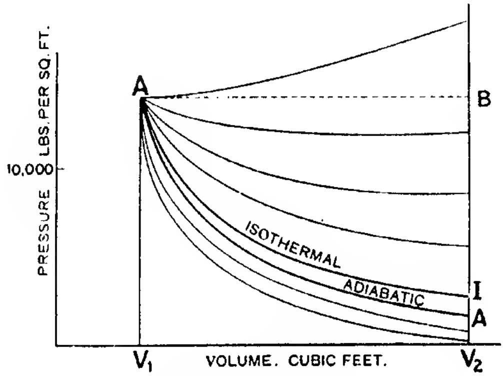


Fig. 52.—Expansion curves.

any one of an infinitely great number of possible paths as indicated, according to the rate at which heat energy is received or rejected during the change. There are two cases of particular interest, namely, the case where the change of state is made at constant temperature, that is isothermally, and the case where a change of state is made adiabatically, that is to say, without receiving any heat energy during the change. The particular curves corresponding to these conditions are shown thick on the diagram, assuming the initial conditions, $P = 14{,}400$ lbs. sq. ft., $V = 1$ cub. ft., $w = 1$ lb.

If the expansion curve through A lie above the adiabatic curve, the gas is receiving heat during expansion; if it lie below the adiabatic the gas is losing heat during expansion. Also, if the expansion curve lie above the isothermal, the gas increases in temperature during expansion. On curves between the adiabatic and isothermal the gas is receiving heat from outside but is falling in temperature.

39. Form of the Expansion Curve and the Heat received during the Isothermal Expansion of a Perfect Gas.—Since the

change is made at constant temperature the form of the curve is obtained directly from the characteristic equation. Putting T constant in this, it is at once apparent that the change of state must be made so that the product of the pressure and volume is constant at all points of the path, that is

$$P = \frac{cwT}{V} = \frac{P_1V_1}{V}$$

where P_1V_1 represents a particular pair of values of the pressure and volume anywhere along the expansion line.

The area under the P.V. curve is given by—

$$\int_{V_1}^{V_2} PdV = P_1V_1\int_{V_1}^{V_2}\frac{dV}{V} = P_1V_1 \log_e \frac{V_2}{V_1} = P_1V_1 \log_e r$$

where r in the last form is the **ratio of expansion.**

There is no change in the internal energy since there is no change of temperature, therefore the equation

$$Q = E + W$$

becomes

$$Q = 0 + P_1V_1 \log_e r$$

which shows that when a perfect gas expands isothermally the whole of the heat received is transformed into mechanical energy during the change of state.

40. Form of the Expansion Curve during the Adiabatic Expansion of a Perfect Gas. Expressions for the Work Area.—When the gas expands adiabatically no heat is received during a change of state, and the energy equation becomes

$$0 = E + W$$

that is, for an infinitesimal change

$$0 = K_v dT + PdV$$

per unit mass.

Now by differentiating the characteristic equation $PV = cT$ we obtain

$$dT = \frac{(PdV + VdP)}{c}$$

Substituting this in place of dT above, and simplifying with the aid of the relation $K_p - K_v = c$, the equation reduces to

$$0 = \frac{K_p}{K_v} PdV + VdP$$

Let γ stand for $\frac{K_p}{K_v}$, the ratio between the specific heat at constant pressure and the specific heat at constant volume; then, separating the variables,

$$\gamma\frac{dV}{V} = -\frac{dP}{P}$$

from which by integration

$$\gamma \log_e V = -\log_e P + \log_e \text{ of a constant.}$$

That is

$$PV^\gamma = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The work area during an adiabatic change of state can thus be found by substituting for P in the general integral $\int PdV$, the expression

$$P = \frac{P_1V_1^\gamma}{V^\gamma}$$

$P_1V_1^\gamma$ represents the constant, and its value may be found from any pair of values of P_1 and V_1 taken along the adiabatic curve. The result of the integration shows that the work area may be expressed in either of the following three forms during an adiabatic change of state from the initial condition P_1V_1 to the final condition P_2V_2:—

$$W = \frac{P_1V_1(1 - r^{1-\gamma})}{\gamma - 1} \text{ where } r = \frac{V_2}{V_1} \quad . \quad . \quad . \quad (2)$$

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$W = \frac{cw(T_1 - T_2)}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The temperatures T_1 and T_2 may be calculated for the initial and final conditions from the characteristic equation.

The quantity W, when divided by J, represents in each case the amount by which the internal energy of the gas is reduced during the expansion from the state P_1V_1 to the state P_2V_2.

41. Change of Temperature during an Adiabatic Change of State of a Perfect Gas.—The state points A and B, which in the pressure-volume diagram define the initial and final conditions of pressure and volume, lie on an adiabatic curve, consequently—

$$P_1V_1^\gamma = P_2V_2^\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also the characteristic equation $PV = cwT$ must be satisfied at each point, so that

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Divide (1) by (2) to eliminate P, and

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

from which the ratio of expansion can be found which is required to produce a given change in the temperature ratio.

Thus, suppose that a gas is initially at 600° temperature, and it is desired to change it to a temperature of 400° by the

process of adiabatic expansion, how much must its volume be increased to effect the change?

Taking $\gamma = 1{\cdot}4$

$$\frac{V_2}{V_1} = (1{\cdot}5)^{\frac{1}{\gamma-1}} = 1{\cdot}5^{2{\cdot}5}$$

That is
$$\log\frac{V_2}{V_1} = 2{\cdot}5 \log 1{\cdot}5$$

from which
$$\frac{V_2}{V_1} = 2{\cdot}75$$

Therefore, to reduce the temperature from 600° to 400° the gas must be expanded to $2\frac{3}{4}$ times its original volume. Conversely, it will be seen from the equations that to increase the temperature from 400° to 600° requires that the gas be compressed to a final volume $\frac{1}{2{\cdot}75}$ of the original volume.

That is to say, the ratio of expansion required to produce a fall of temperature, is equal to the ratio of compression required to produce an equal rise of temperature over the same range.

Again, raising equation (2) to the γ power and dividing (1) by (2)

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

from which the ratio of the pressure change required to produce a given ratio of temperature change may be calculated.

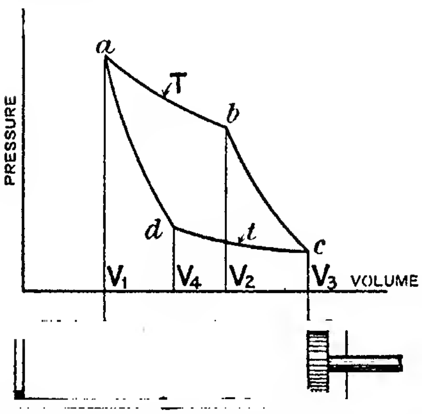


FIG. 53.—Carnot cycle with air as the working agent.

42. The Carnot Cycle with Air as the Working Agent.—With the aid of the propositions just established the heat transferences and transformations in the Carnot Cycle, with air as the working substance, can easily be followed in detail. The air is assumed to follow the law of the perfect gas, namely, $PV = cwT$. Imagine a cylinder fitted with a frictionless piston to be made of perfectly non-conducting material, but with a cover such that it can be replaced at will by another which is perfectly conducting.

Let the state of the air as regards pressure and volume be represented by the point a on the pressure-volume diagram, Fig. 53, and let T be its absolute temperature. Then imagine the diagram $abcda$ to be described in consequence of the four regulated processes detailed in the table below, that is isothermal expansion from a to b with reception of heat at constant temperature T; adiabatic expansion from b to c cooling the gas from T to t without loss of heat; isothermal compression from c to d with rejection of heat at constant temperature t; adiabatic compression from d back to a without loss of heat. In order that the adiabatic curve may pass through the initial point a, the third process must be stopped at d, when the ratio $\frac{V_4}{V_1} = \frac{V_3}{V_2}$, since the temperature is to be raised by compression through a step equal to the fall by expansion, and it is shown in Section 41 that to do this the ratio of compression must be equal to the ratio of expansion, r.

During adiabatic expansion and compression the non-conducting cylinder cover is supposed to be in place; during isothermal expansion and compression the perfectly conducting cylinder cover is supposed to be in place, so that heat may be freely received or rejected through it. Then during each process—

Heat received or rejected = Change of internal energy ± work done.

The following table gives the value of each term in this relation corresponding to each process:—

TABLE 14.—Table showing Heat Transference and Transformation in a Carnot Cycle with Air as the Working Agent. R is written for the Product cw.

Process.	Heat received or rejected.	Change of internal energy.	External work done.
1. Isothermal expansion at temperature T	$RT \log_e r$	0	$+ RT \log_e r$
2. Adiabatic expansion from T to t	0	$-\frac{R(T-t)}{\gamma - 1}$	$+\frac{R(T-t)}{\gamma - 1}$
3. Isothermal compression at temperature t	$- Rt \log_e r$	0	$- Rt \log_e r$
4. Adiabatic compression from t to T	0	$+\frac{R(T-t)}{\gamma - 1}$	$-\frac{R(T-t)}{\gamma - 1}$

The heat received during the cycle is $RT \log_e r$; and adding together the items in each of the last two columns, we have

$$\text{Change of internal energy} = 0$$
$$\text{Work done externally} = R(T - t) \log_e r.$$

The thermal efficiency of the cycle is therefore

$$\frac{\text{Work done}}{\text{Heat received}} = \frac{R(T - t) \log_e r}{RT \log_e r} = \frac{T - t}{T}.$$

With air as the working agent it is quite clear, therefore, that the efficiency of the cycle depends only upon the temperature limits at which air receives and rejects heat. And the motive power which can be obtained from one unit of heat energy received at the temperature T with air as the working agent is the fraction $\frac{(T - t)}{T}$ of a thermal unit, where t is the temperature of rejection.

But would the employment of any other working agent enable a greater fraction of the total to be obtained within the same limits of temperature? The answer is No, because the processes set out in detail above are all reversible, as will easily be seen if the cycle is described in the reverse way by starting with adiabatic expansion from a to d and continuing from d to c; c to b; and from b to a. In the reversed cycle—

$$\left.\begin{array}{r}\text{Heat rejected at the higher tem-}\\ \text{perature T during the cycle}\end{array}\right\} = \mathrm{RT}\log_e r$$

$$\text{Change of internal energy} = 0$$

$$\text{Work spent upon the gas} = \mathrm{R}(\mathrm{T} - t)\log_e r$$

So that in the reverse process work is spent upon the air exactly equal in amount to the work which the air does on the piston in the direct process, and a quantity of heat is transferred from the gas to the hot body exactly equal in amount to the quantity received from the hot body in the direct process.

The ideal engine is therefore reversible.

Carnot's proof (slightly modified in form by Callendar) that it is impossible to produce by the agency of heat alone a quantity of motive power greater than that which is obtained from a reversible engine, runs as follows:—

If it were possible to produce from a given quantity of heat supplied a greater quantity of motive power than that obtained from a reversible engine, it would suffice to divert a portion of this power to return to the source (the heating circuit) by means of a reversible engine the quantity of heat taken from it. We should thus obtain at each repetition of the cycle a balance of motive power without taking any heat from the source (the heating circuit), that is to say, without any consumption of fuel.

The extreme improbability of such a result is sufficient *reductio ad absurdum* to satisfy any reasonable intelligence.

Thus, having proved that, with air as the working agent, the efficiency of the Carnot engine depends only upon the temperature limits, and having shown that the process is reversible, and then having shown that if an engine is more efficient than this reversible Carnot engine with air as a working agent, it would be possible to obtain motive power without burning fuel, what is known as Carnot's principle follows, namely, using Carnot's own words:—

"The motive power of heat is independent of the agents set to work to realize it; its quantity is fixed solely by the temperatures of

the bodies between which in the limit the transfer of heat is effected."

Therefore the efficiency calculated for the Carnot engine using air as the working agent is the maximum efficiency of any heat engine, whatever be the physical properties of the working agent employed.

Therefore, from the practical point of view, the second law of thermodynamics may be stated as follows:—

The maximum quantity of motive power which can be obtained from a quantity of heat, Q, received by a working agent at absolute temperature T, and rejected at absolute temperature t, is

$$\frac{JQ(T - t)}{T} \text{ ft.-lbs.}$$

43. The Carnot Cycle with Vapour as the Working Agent.—When the working agent is a vapour assumed to be always in contact with the liquid from which it is formed the isothermal curves in the pressure-volume diagram of the cycle become straight lines, as shown in the Fig. 54, where ab represents the first process of

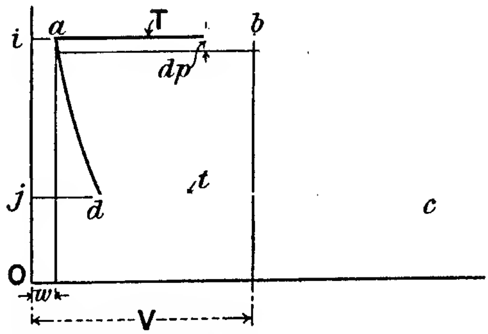


FIG. 54.—Carnot cycle with vapour as the working agent.

isothermal expansion with reception of heat; bc the second process of adiabatic expansion from the temperature T to the temperature t; cd the third process of isothermal compression with rejection of heat; and da the fourth process of adiabatic compression from temperature t to temperature T.

The temperature at which a liquid begins to boil depends upon the pressure, and boiling goes on as heat is received until the liquid is all turned into vapour, the volume increasing during the process. The heat received during the boiling is the latent heat of the vapour.

In the diagram the point a represents unit weight of liquid at temperature T; the pressure Oi on the piston is that at which the liquid boils at the temperature T.

Having made these adjustments heat is received into the liquid from the source and boiling goes on until when the volume of the

vapour has reached ib the liquid has just disappeared. This is the end of the first process.

Adiabatic expansion bc to the condenser temperature corresponding with the pressure Oj is the second process.

Isothermal compression takes place at the constant pressure Oj, during which heat is removed by the condenser, and the vapour gradually liquefies. The process is stopped at a point of the condensation such that the fourth process of adiabatic compression completes the condensation and at the same time raises the temperature to the upper temperature T.

In order to measure the work done or the motive power obtained during the cycle the form of the adiabatic curves must be known. There is no simple relation between pressure and volume from which an adiabatic curve can be drawn, and in practice adiabatic expansion curves of vapours are either set out from empirical equations or are plotted from calculations which ultimately depend upon tables of the properties of vapours. This point will be considered in detail in connection with steam later on. Meanwhile the difficulty can be avoided by considering a cycle in which the lower temperature is brought very near to the higher temperature so that the small portions of the adiabatics required in the diagram reduce to straight lines.

Let dt represent the difference between the upper and the lower temperature, that is $T - t$; and let dp represent the corresponding difference of pressure. Then if w is the volume of the liquid at the commencement of the cycle, and V the volume of the vapour formed from it by the reception of the quantity of latent heat L, the motive power corresponding to the cycle is in the limit

$$dp(V - w)$$

The heat received is the quantity L. But by the second law of thermodynamics the work which can be obtained from this is $\frac{JLdt}{T}$ ft.-lbs.

Therefore
$$dp(V - w) = \frac{JLdt}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or
$$\frac{dp}{dt}(V - w) = \frac{JL}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

an equation usually attributed to Clapeyron. The rate with regard to the temperature at which heat is transformed into work, or the work which can be done, per unit of heat supplied is

$$\frac{\frac{dp}{dt}(V - w)}{JL} = \frac{1}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The left side of the equation involves the properties of the working agent, but the right-hand side shows that whatever the properties may be the rate at which heat is transformed per unit of heat supplied

at the temperature T can be calculated by simply taking the reciprocal of the absolute temperature at which the heat is supplied.

Suppose, for example, that heat is supplied at the temperature 30° C. = 303° C. absolute. The equation shows that the rate at which heat is transformed as the temperature falls per unit mass per unit of heat supplied at this temperature is 0·0033 lb.-cal. whatever the working agent may be.

Tables giving the properties of vapours are usually prepared from experimental determinations of corresponding values of the pressure p, the boiling temperature t, the latent heat L; but the specific volume V is generally calculated from expression (2), after substituting therein the values of $\frac{dp}{dt}$, L, and w deduced from experimental data, and then solving for V. Experimental values of V agree with the calculated values as nearly as experimental errors permit; and therefore it is instructive for a student to assume that all the values given in the vapour tables are determined experimentally and then to use these values to convince himself of the truth of equation (3).

The following table, calculated for me by Mr. Hewson, shows the details of the calculations to find the motive power obtainable per unit of mass per unit of heat received at 30° C., for a fall of temperature from 30° to 29° C. For the purpose of this calculation equation (2) may be written

$$\frac{\Delta p \,.\, 144 \,.\, (V - w)}{JL} = \frac{\Delta t}{T}$$
$$= \frac{1}{T} \text{ when } \Delta t = \text{unity.}$$

Δp is then computed from the tables, and L, V, w are taken from the tables for the temperature 30° C.

TABLE 15.—MOTIVE POWER OBTAINABLE PER POUND OF A WORKING SUBSTANCE PER UNIT OF HEAT RECEIVED AT THE TEMPERATURE 30° C. (303° ABSOLUTE $\cdot\frac{1}{T} = 0{\cdot}0033$) FOR A FALL OF TEMPERATURE FROM 30° TO 29° C.

Col. 1. Working Agent.	Col. 2. Δp.	Col. 3. $(V - w.)$	Col. 4. $\Delta p.144.(V - w)$.	Col. 5. L.	Col. 6. $\frac{\Delta p.144.(V - w)}{JL}$
		cub. ft. per lb.	ft.-lbs.	lb.-cals.	lb.-cal.
Steam	0·03507	528·0	2666·0	579·6	0·003286
SO_2	2·1276	1·346	412·4	89·77	0·003281
NH_4	4·995	1·845	1327·0	276·2	0·003432
Ether	0·4539	6·406	418·7	90·86	0·003291
Alcohol	0·08355	92·14	1108·6	240·5	0·003292
Chloroform	0·1990	10·30	295·2	64·10	0·003289
Carbon bisulphide	0·3053	9·093	400·4	86·88	0·003292
Carbon tetrachloride	0·1180	13·62	231·5	50·21	0·003293
Aceton	0·2282	18·97	623·4	135·5	0·003286

The data for this table were taken from "Tables of the Properties of Saturated Steam and other Vapours," by Cecil H. Peabody. J. Wiley & Sons, New York. 1909.

The various working agents are tabulated in the first column. The second column gives the change of pressure corresponding to a change of temperature from 30° to 29°, and is found from the tables by interpolation where necessary. The third column shows the change of volume corresponding to the evaporation of one pound of the substance from the liquid to the dry saturated state. The products of the figures in the second and third column, multiplied by 144, are shown in the fourth column, and give in each case the external work done during the process. The fifth column shows the heat taken in during the process of evaporation (the latent heat of the substance), whilst the sixth column gives the final value of the quantity of motive power obtainable per unit of heat for a fall of temperature from 30° to 29°.

The close agreement of the figures in column 6 amongst themselves is apparent and should convince the student of the truth of Carnot's Principle, that the motive power obtained is independent of the physical properties of the working agent; and the near approximation to the result obtained by merely taking the reciprocal of the absolute temperature shows to what a simple conclusion Carnot was able to lead the way by the truly extraordinary insight his genius enabled him to get of the relations involved in the thermal efficiency of a heat engine.

Carnot endeavoured to verify his conclusions experimentally in the way illustrated above, but the inaccurate data of the time prevented him from getting any further than the result that the motive power per degree per unit of heat supplied at temperature T was a function of the temperature fall only. It was Clausius who first stated that Carnot's Function is the reciprocal of the absolute temperature at which the heat is received.

44. Absolute Temperature.—The general relations between mechanical energy and thermal energy are thus conditioned by two laws; the one established by Joule relating to the equivalence of heat and work leading to the formulation of the first law of thermodynamics; the second established by the reasoning of Carnot leading to the formulation of the second law of thermodynamics, a law which enables the work which can be done to be calculated in terms of the temperature difference between the boiler and the condenser only, no reference to the physical properties of the agent employed being necessary.

Kelvin[1] pointed out that these laws enabled an absolute scale of temperature to be defined which is independent of the physical properties of any substance.

Consider the pressure-volume diagram, Fig. 55. Let any working agent in the state A, receive at constant temperature T a quantity of heat Q in consequence of which its state changes to B, along the isothermal curve AB. Through A and B draw two adiabatic curves.

[1] Lord Kelvin's collected papers, vol. 1, p. 333.

Let ab be another isothermal cutting them at any lower temperature t. Then if the working agent passes through the cycle of changes ABba and back to its initial state A, the area represented by the indicator diagram ABba is the difference between the heat received at the temperature T along the isothermal AB and the heat rejected at the lower temperature t along the isothermal ba. This area increases as the lower temperature is reduced until a limiting value of the temperature is reached where the work area is equal to the heat received at the upper temperature T. This limiting temperature is defined as the absolute zero.

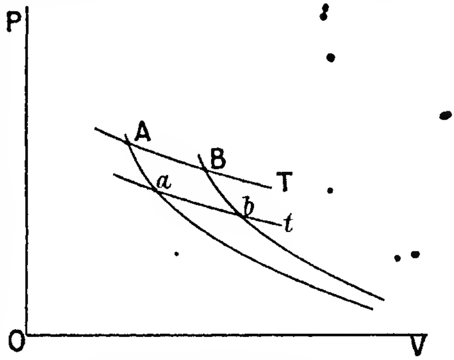


FIG. 55.—Absolute temperature.

The idea of the absolute temperature can be obtained directly from the expression of the second law, namely—

$$W = \frac{Q(T - t)}{T}$$

in which Q is the heat received by the working agent at the temperature T, W is the work done in thermal units, and t is the temperature of rejection.

It is clear that if the lower temperature t is reduced to zero all the heat received is then converted into work and no further reduction of temperature is possible. This fixes a limiting value for the absolute zero.

Again, if q is the heat rejected at the lower temperature, then by the first law—

$$q = Q - W = Q\left(1 - \frac{T - t}{T}\right) = \frac{Qt}{T}$$

That is, $\frac{q}{t} = \frac{Q}{T}$. Therefore the ratio $\frac{Q}{T}$ is a constant, C say. So that $W = C(T - t)$, where T and t may have any values, and therefore the work done for a given temperature drop is the same at any part of the scale.

Therefore a degree on the scale may be defined in terms of the work corresponding to the temperature difference it represents.

The absolute scales obtained by the use of actual gases do not quite correspond with the absolute thermodynamic scale, because the specific heat of none of the permanent gases is accurately

constant. Professor Callendar[1] has calculated the correction which must be applied to the readings of certain of the permanent gas thermometers in order to reduce them to the absolute thermodynamic scale.

For all practical purposes the readings of the air thermometer may be taken as corresponding with the thermodynamic scale. In the ideally perfect gas equal quantities of heat produce equal changes of temperature and equal changes of volume at constant pressure, and the perfect gas scale and the absolute thermodynamic scale are therefore identical.

45. General Thermodynamic Relations.—It follows from the two laws of thermodynamics that there are certain abstract relations between the changes produced in the pressure, temperature, and volume of a working agent by the gain or loss of heat which apply generally to all working agents. The principal relations can be readily obtained by the aid of the pressure-volume diagram.

Let the point A represent the pressure and volume of unit mass of a gas. The term gas is used here in a general sense, and for the moment it is convenient to assume that the gas can be reduced to the absolute zero without liquefying.

As the pressure and the volume change the point A describes a curve, and if the change takes place without loss or gain of heat the curve is, by definition, called an adiabatic curve. If it gains or loses heat at the rate required to keep the temperature constant the curve described by A as it moves in the diagram is defined as an isothermal curve.

If the change takes place at constant pressure the point moves in a straight line parallel to the horizontal axis. If the change is made at constant volume the point moves in a vertical line. In whatever path the point moves, the area enclosed by the curve, the horizontal axis, and the two lines of constant volume drawn through the state points marking the beginning and end of the path, represents the external work done during the change of state. If the point moves in a closed curve the area of the curve represents the work done every time the point makes the circuit of the curve. If the point moves in the direction of the hands of a clock round the path the gas does work on its environment. If the point moves round the path in a direction opposite to the hands of a clock work is done on the gas. In the first case heat is converted into work, in the second case work is converted into heat.

The **internal energy** of a gas in the state A, Fig. 56, is represented by the area enclosed by the indefinitely prolonged adiabatic curve drawn through A to cut the volume axis at infinity, the line of constant volume A*a* through A, and the volume axis OV. Because, if the gas be imagined to expand until its pressure falls to zero, it can do no more work and hence the energy it contained is exhausted.

[1] "On the Thermodynamical Correction of the Gas Thermometer," Callendar, *Phil. Mag.*, Jan., 1903.

Thus the area $ZaACfZ$ represents the internal energy E_A of the gas in the state A, Z being a point in OV at infinity. Let the state point now move to B along the path AxB. Draw through B the adiabatic Be. Then the internal energy of the gas in the state B is represented by the area $ZbBDeZ$.

Again, the change of internal energy due to the change of state along the path AxB is the difference between the area $ZbBDeZ$ and

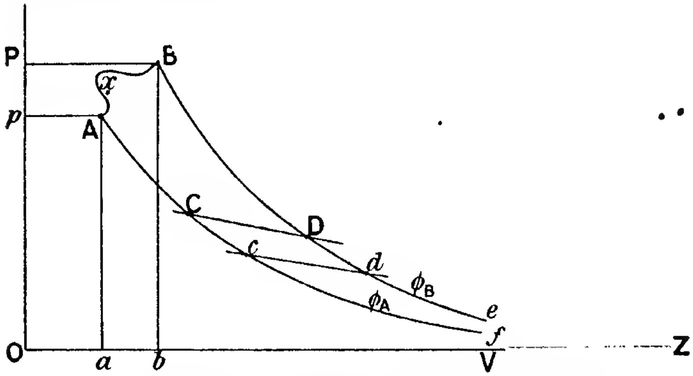


FIG. 56.—General thermodynamic relations and the P.V. diagram.

$ZaACfZ$, and this is clearly independent of the shape of the path AxB. Therefore the change in the internal energy depends upon the initial and the final position of the state point only.

This illustrates Carnot's axiom, p. 141. The value of the internal energy corresponding to a point in any path is therefore represented by the area formed like $ZaACfZ$ for the particular point in question.

Again, during the change of state from A to B, by the first law the heat absorbed is the difference between the internal energies at B and A plus the work done. The work done is represented by the area $aABb$. Therefore, Q, the heat absorbed, is represented by the area $ZbBDeZ - ZaACfZ + aABb =$ the indefinitely prolonged area $ZfABeZ$. Therefore the heat received during a change of state from A to B is represented by the area contained between the actual path of the state point and two indefinitely prolonged adiabatic curves drawn through the initial and final points of the path.

This theorem demonstrated by Rankine is in the *Phil. Trans.* for 1854. It enables the heat received during a change of state to be followed on the pressure-volume diagram, though it cannot actually be measured because the area representing it is unclosed.

Total Energy.—The **total energy** of a substance in any state is defined as the sum of the internal energy and the product of the pressure and the volume, expressed in thermal units, corresponding to the state.

Let the state of a gas as regards its pressure and volume be denoted by A, Fig. 56. Through A draw the line of constant pressure pA, and the adiabatic curve Af. Then the internal energy of the gas

in the state A is represented by the indefinitely prolonged area under the adiabatic curve Af; and the product of the pressure and the volume corresponding to the state is represented by the rectangle OpAa. The total energy in the state A is thus represented by the area ZOpAfZ.

Representing the total energy by the symbol I—

$$I = E + PV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly, the total energy of the gas in the state B is represented by the area ZOPBZ.

The difference between the total energy at B and the total energy at A is the work area ZfApPBeZ, and this area is independent of the path connecting A and B. If A moves so that the area ZOpAZ is constant, A describes a curve of constant total energy.

In practical applications of these functions of total energy and internal energy it is the difference between two states that is generally in question, so that any temperature may be chosen from which to reckon the internal energy of a substance.

For example, when the substance is a perfect gas, the temperature is conveniently chosen to correspond with the absolute zero. But in the case of the working substance used in internal combustion engines, it is found convenient to choose 0° C., and sometimes even 100° C. In the case of vapours 0° C. is usually chosen, and this is the temperature from which internal energy is reckoned in the steam tables at the end of the book.

In general the total energy does not reduce to zero at the temperature from which internal energy is reckoned. For example, referring to Table 2 of the steam tables, it will be seen that at 0° C. the substance, steam, reduced to the state of water, has a vapour pressure of 0·09 lb. per square inch, and a volume of 0·016 cub. ft. per pound. Therefore, although by convention the internal energy is reckoned zero at this temperature, the product of the pressure and the volume has a small value. The value in thermal units is

$$\frac{144 \times 0{\cdot}09 \times 0{\cdot}016}{1400} = 0{\cdot}00015 \text{ lb.-cals., a negligible quantity.}$$

Entropy.—Consider two adiabatic curves Af and Be, Fig. 56, and let CD and cd be any two isothermals cutting them, the upper one corresponding to a temperature T, and the lower to a temperature t. Suppose the gas to receive Q units of heat at the upper temperature T, and that in consequence the state point moves from C along the isothermal to D. That is, in consequence of receiving Q units of heat, it moves by an isothermal path from one adiabatic to a second adiabatic. Let the point now move down the adiabatic to the temperature t, and then move back to the first adiabatic along the isothermal path dc, rejecting heat Q′, and, finally, suppose it to pass into its initial position along the adiabatic. It describes a Carnot cycle; that is to say, it receives the heat Q, does the work W, and rejects the heat Q′. The relations between these quantities are fixed by the second law.

Work done $= \frac{Q(T - t)}{T}$

And by the first law,

Heat rejected $Q' = Q - W = Q - Q\frac{(T - t)}{T} = \frac{Qt}{T}$

So that $\frac{Q}{T} = \frac{Q'}{t}$

Thus the reception of heat Q at temperature T moves the state point from one adiabatic to the other, and the rejection of heat Q' at temperature t moves the state point back again to the first adiabatic. But the process is reversible, so that the reception of the quantity of heat Q' at temperature t would move the state point from c to d, just as the rejection of Q' at temperature t moves it from d to c.

Therefore, if Q is the heat required to change the state from any point A on the adiabatic ϕ to B on the adiabatic ϕ_B at constant temperature T, the ratio $\frac{Q}{T}$ is a constant quantity. This constant increment is called the increase of entropy from the adiabatic ϕ_A to the adiabatic ϕ_B. The particular adiabatic from which these increments are measured is a matter of choice, and when chosen it becomes the adiabatic of zero entropy. A scale of entropy may be imagined, in which equal intervals corresponded to equal values of the quotient $\frac{Q}{T}$.

There is no change of entropy when the state point moves reversibly along an adiabatic, since in that case $Q = 0$; it therefore follows that whether the path from A to B is isothermal, or is any arbitrary curve, the change of entropy is just the same, so that the change of entropy does not depend upon the path AB, but the heat required to make the change does, since, as shown above, it is the indefinitely prolonged area contained between the adiabatics and the path AB.

The general expression connecting heat received with change of entropy is, for an arbitrary path,

$$dQ = Td\phi$$

To calculate the change of entropy produced by the addition of the heat Q along an arbitrary path, it is necessary to evaluate the integral—

$$\phi = \int\frac{dQ}{T}$$

The entropy temperature diagram may be constructed when the properties of the working agent are known, as will be illustrated below for steam, in which case the integral can be evaluated as an area on a $t\phi$ diagram.

Summarising, the work done during a change of state AB is shown by a definite area on a pressure-volume diagram, and the heat received during the change of state is shown by an indefinite area contained

by the path AB and the adiabatics through A and B. But the curves, although not finite, show that the internal energy of a substance depends upon the position of the state point only; that the total energy depends upon the position of the state point only; and that the value of the entropy depends upon the position of the adiabatic curve only. Therefore, the expressions for these quantities are perfect differentials, which means that any change in the value of either the internal energy, the total heat, or the entropy is independent of the shape of the path, and depends upon the initial and final states A and B only.

On the other hand, the work done during the change from A to B, and also the heat received during the change, depend upon the shape of the path, so that neither value can be computed from the initial and the final conditions only.

46. General Expressions for the Principal Thermodynamic Relations.—Draw two isothermal lines, Fig. 57, at a distance apart δT, the upper one corresponding to the temperature T. Let A be any point on the upper isothermal. Through A draw Aa, a line of constant volume, and AP, a line of constant pressure. Through B, the intersection of Aa with the lower isothermal, draw a line of constant pressure BD, and through D draw a line of constant volume Dd. Then δp and δv are respectively the changes of pressure and volume corresponding to the change of temperature δT. Finally, draw adiabatics Aϕ and Dϕ_D through the points A and D.

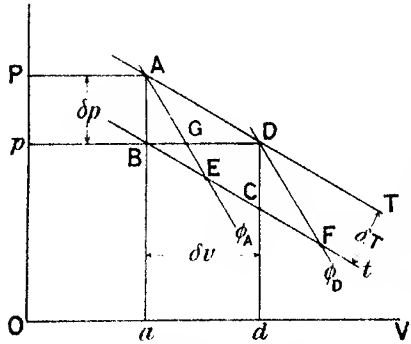


FIG. 57.—Thermodynamic relations.

The change of temperature is small, so that the parts of the isothermal, and the adiabatic curves in the diagram, may be considered as intersecting pairs of parallel straight lines. The figure ADFE may then be regarded as a parallelogram.

Let a working agent pass through the Carnot cycle defined by the indicator diagram ADFE. The heat, Q, required to do the work represented by this area is received by the working agent at the higher temperature T during the isothermal change of state from A to D. By the second law of thermodynamics, the relation between the heat received and the work done during the cycle is

$$W = \frac{JQ(T - t)}{T}$$

that is, using the notation for small increments,

$$\delta Q = \frac{T}{J} \cdot \frac{\delta W}{\delta T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But by the geometry of the figure the area ADFE = the area ADCB, since both are on the same base and between the same parallels, and this area is therefore equal to the product AB × BD, that is to $\delta p . \delta v$. Substituting this product for δW in (1) it becomes

$$\delta Q = \frac{T}{J} \cdot \frac{\delta p . \delta v}{\delta T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

This may be expressed in two ways. Dividing by δp and proceeding to the limit,

$$\left(\frac{dQ}{dp}\right)_T = -\frac{T}{J}\left(\frac{dv}{dT}\right)_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

That is to say, the rate of heat addition with change of pressure at a given temperature T is equal to *minus* $\frac{T}{J}$ into the rate of change of volume with change of temperature at the corresponding pressure p. The suffixes T and p in the formula denote corresponding temperatures and pressures. A minus sign is prefixed because absorption of heat during isothermal expansion corresponds to diminution of pressure.

Dividing by δv and proceeding to the limit,

$$\left(\frac{dQ}{dv}\right)_T = \frac{T}{J}\left(\frac{dp}{dT}\right)_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this case absorption of heat corresponds to an increase of volume, so that the sign is positive.

The heat required to change the temperature by 1° at constant pressure is $\left(\frac{dQ}{dT}\right)_p$, and this is the specific heat at constant pressure. Let this be represented by the symbol K_p.

Similarly, the heat required to change the temperature 1° at constant volume is $\left(\frac{dQ}{dT}\right)_v$, and this is the specific heat at constant volume. Let this be represented by K_v.

That the difference between these specific heats is given by

$$K_p - K_v = \frac{T}{J}\left(\frac{dp}{dT}\right)_v\left(\frac{dv}{dT}\right)_p \quad . \quad . \quad . \quad . \quad . \quad (4)$$

is proved as follows (Callendar): Imagine unit mass of the working agent in the state A to be cooled at constant volume to B through the temperature δT. The heat abstracted is $K_v \delta T$. Imagine now that it is heated at constant pressure until its state is changed at constant pressure from B to the point G on the adiabatic AE. Then it has received as much heat in the process as it lost in the cooling from A to B because the points A and G are both on an adiabatic, and hence the gas has neither gained nor lost heat up to the point G. Therefore, the heat added from B to G is equal to $K_v \delta T$. The heat which must be added in addition to complete the change of temperature at constant pressure is the heat from G to D, and which

is $(K_p - K_v)\delta T$. Also by the theorem of Rankine, page 161, this heat is represented by the indefinitely prolonged area between the path GD and the adiabatics through G and D, and this area is in the limit equal to the area between AD and the adiabatics, and thus becomes the quantity of heat dQ which must be added in order to do the work ADCB, and this is seen from *e.g.* (1a) page 165 to be $\frac{T}{J}\cdot\frac{dp\,.\,dv}{dT}$.

Therefore, $$dQ = (K_p - K_v)dT = \frac{T}{J}\cdot\frac{dp\,.\,dv}{dT}$$

From which $$K_p - K_v = \frac{T}{J}\cdot\left(\frac{dp}{dT}\right)_v\cdot\left(\frac{dv}{dT}\right)_p \quad . \quad . \quad . \quad . \quad (4)$$

This is a general relation, and applied to the characteristic equation of a perfect gas, $PV = JRT$, shows at once that the difference between the specific heats is constant and equal to R, because

$$\frac{dP}{dT} = \frac{JR}{V} \text{ and } \frac{dV}{dT} = \frac{JR}{P}$$

so that, from (4) $K_p - K_v = \frac{TJR^2}{PV} = R$.

The quantity of heat required to produce any change of state may be estimated in two steps, namely, a temperature step and an expansion step at constant temperature.

Let a gas change its state in consequence of the reception of the small quantity of heat δQ. Then

$$\delta Q = \left(\frac{dQ}{dT}\right)_v \delta T + \left(\frac{dQ}{dv}\right)_T \delta v$$

But the first term is $K_v dT$.

And from equation (3) $$\left(\frac{dQ}{dv}\right)_T = \frac{T}{J}\left(\frac{dp}{dT}\right)_v$$

Therefore $$dQ = K_v dT + \frac{T}{J}\left(\frac{dp}{dT}\right)_v dv \quad . \quad . \quad . \quad . \quad (5)$$

On the diagram, Fig. 58, let the state change from A to B. Through B draw an isothermal corresponding to the temperature at B, and through A draw a line of constant volume to meet it in a. Then the heat absorbed by the change from A to B is the heat required to change the state at constant volume from A to a, the quantity represented by the first term; and the second addition of heat required to expand down the isothermal to the state B, a quantity represented by the second term in the expression.

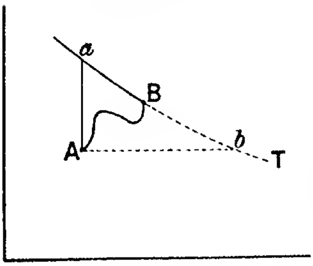


FIG. 58.—Absorption of heat from A to B in two steps.

An alternative way of making the change is shown by dotted

lines in Fig. 58. Heat may first be absorbed at constant pressure from A to b, taking in the quantity $K_p dT$, after which a quantity is rejected during compression along the isothermal to B. Analytically this process is represented by taking T and p as the variables.

Thus $$\delta Q = \left(\frac{dQ}{dT}\right)_p \delta T + \left(\frac{dQ}{dp}\right)_T \delta p$$

But from (2) the second term is equal to $-\frac{T}{J}\left(\frac{dv}{dT}\right)_p$

Therefore $$dQ = K_p dT - \frac{T}{J}\left(\frac{dv}{dT}\right)_p dp \quad . \quad . \quad . \quad . \quad . \quad (6)$$

General Expression for the Change of Internal Energy.

The change of internal energy as the state point moves from A to B along any path is equal to the difference between the heat received and the work done. That is

$$dE = dQ - \frac{p}{J}dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

On the diagram, Fig. 58, it is the difference between the indefinitely prolonged area between the adiabatics drawn through A and B and the path itself, and the work area under the path AB.

The infinitesimal change may be made in two equivalent steps, namely, Aa, a change at constant volume, and aB, a change at constant temperature. Or the two equivalent steps may be taken, namely, Ab at constant pressure, and bB at constant temperature. Expressions for dQ in both cases have just been found. Choosing first the steps of constant volume, and of constant temperature, and substituting for dQ its value from (5) above,

$$dE = K_v dT + \frac{1}{J}\left\{T\left(\frac{dp}{dT}\right)_v - p\right\}dv \quad . \quad . \quad . \quad . \quad (8)$$

Since this expression is a perfect differential,

$$\left(\frac{dK_v}{dv}\right)_T = \frac{d}{dT}\left\{T\left(\frac{dp}{dT}\right)_v - p\right\}_v\frac{1}{J} = \frac{T}{J}\left(\frac{d^2p}{dT^2}\right)_v \quad . \quad . \quad . \quad (9)$$

an equation giving the variation of the specific heat at constant volume with volume, in terms of the pressure and temperature, when the temperature is kept constant.

Applying these results to the perfect gas, PV = JRT;

from equation (8), $T\left(\frac{dP}{dT}\right)_v = \frac{JRT}{V} = P$; therefore $dE = K_v dT$

and from equation (9) $\frac{d^2P}{dT^2} = 0$; therefore K_v is constant.

It was shown above that $K_p - K_v = R$ = a constant for the perfect gas. Therefore, since K_v is constant, K_p is also constant.

Again, assuming that the change from A to B is made by two steps,

the first Ab at constant pressure and the second bB at constant temperature, and substituting for dQ its value from (6) above, and also substituting for $p \,.\, dv$ its equal value with p as the independent variable, namely, $d(pv) - v \,.\, dp$,

$$dE = K_p dT - \frac{T}{J}\left(\frac{dv}{dT}\right)_p dp - \frac{1}{J}\left\{d(pv) - v \,.\, dp\right\}$$

$$= K_p dT - d\frac{pv}{J} - \frac{1}{J}\left\{T\left(\frac{dv}{dT}\right)_p - v\right\}dp \quad . \quad . \quad . \quad (10)$$

Since this is a perfect differential,

$$\left(\frac{dK_p}{dp}\right)_T = - \frac{T}{J}\left(\frac{d^2v}{dT^2}\right)_p \quad . \quad . \quad . \quad . \quad . \quad (11)$$

an expression giving the variation of specific heat at constant pressure with the pressure when the temperature is constant.

Applying these expressions to the perfect gas,

$$T\left(\frac{dv}{dT}\right)_p = \frac{JRT}{P} = V, \text{ and hence } dE = K_p dt - d\frac{pv}{J}.$$

General Expression for the Change in Total Heat Energy.

Total heat energy is defined by Callendar to be the internal energy E added to the product of the pressure and volume. That is,

$$I = E + \frac{pv}{J}$$

A change of total heat energy is thus

$$dI = dE + d\frac{pv}{J} = dQ - \frac{p}{J}dv + d\frac{pv}{J}$$

since from (7) dE is $dQ - \frac{p}{J}dv$.

But $$d\frac{pv}{J} - \frac{p}{J}dv = \frac{v}{J}dp$$

Therefore $$dI = dQ + \frac{v}{J}dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Substituting the value of dQ from equation (6),

$$dI = K_p dT - \frac{1}{J}\left\{T\left(\frac{dv}{dT}\right)_p - v\right\}dp \quad . \quad . \quad . \quad . \quad (13)$$

Since this expression is a perfect differential

$$\left(\frac{dK_p}{dp}\right)_T = - \frac{T}{J}\left(\frac{d^2v}{dT^2}\right)_p \text{ as already found in (11)}$$

Expression (13) for dI could have been found directly from the definition, $dE + d\,.\,\frac{pv}{J}$, by adding $d\frac{pv}{J}$ to the expression for dE in terms of the temperature and pressure given in equation (10). It is, however, more convenient to consider dI as stated in (12).

From (13) it will be seen that the total variation of I at constant pressure is given by $K_p dT$, so that increase of I from a given temperature is the heat energy which must be added at constant pressure to change its temperature from the initial to the final temperature.

When a change of state is made at constant volume it will be seen from (8), that the change of internal energy is

$$dE = K_v dT$$

or the heat energy which must be added to change the temperature from an initial to a final temperature measures the change of internal energy.

But when the change of state is made at constant pressure, the heat energy which must be added measures the change in total heat energy, that is

$$dI = K_p dT$$

General Expression for the Change of Entropy.

Change of entropy is defined as

$$\frac{dQ}{T} = d\phi$$

Therefore the change of entropy produced by the addition of heat dQ is found by dividing the expressions for dQ by T.

Thus, in terms of temperature and volume we derive from (5)

$$d\phi = K_v \frac{dT}{T} + \frac{1}{J}\left(\frac{dp}{dT}\right)_v dv \quad . \quad . \quad . \quad . \quad . \quad (14)$$

or, in terms of the temperature and pressure using (6)—

$$d\phi = K_p \frac{dT}{T} - \frac{1}{J}\left(\frac{dv}{dT}\right)_p dp \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and both expressions are perfect differentials.

Applying these expressions to a perfect gas, for which $PV = JRT$, and therefore $\frac{dP}{dT} = \frac{JR}{V}$, equation (14) becomes

$$d\phi = K_v \frac{dT}{T} + \frac{R}{V} dv,$$

that is
$$\phi - \phi_0 = K_v \log_e \frac{T}{T_0} + R \log_e \frac{V}{V_0} \quad . \quad . \quad . \quad (16)$$

and similarly
$$\phi - \phi_0 = K_p \log_e \frac{T}{T_0} - R \log_e \frac{P}{P_0} \quad . \quad . \quad . \quad (17)$$

Relation between the Saturation Pressure and the Temperature.

An expression connecting the pressure and the temperature has been obtained in Section 43. It connects the latent heat with the rate

of change of the saturation pressure with the temperature, and the volume of 1 lb. of dry saturated steam. As mentioned on page 157, the expression has generally been used to calculate the volume of unit mass of dry saturated vapour from observed values of the latent heat and the corresponding changes of pressure and temperature, because the experimental determination of the volume of dry saturated vapour is an extremely difficult matter owing to the difficulty of avoiding, on the one hand, condensation, and on the other hand, slight superheating of the vapour.

Quoting the equation from page 156, it is

$$\frac{dp}{dt}(v - w) = \frac{\mathrm{JL}}{\mathrm{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or

$$v = \frac{\mathrm{JL}}{\mathrm{T}} \cdot \frac{dt}{dp} + w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

It was this relation which enabled Professor J. Thomson to predict that the melting point of ice is lowered by increasing the pressure on it. It was done in this way.

Solving the equation for $\frac{dp}{dt}$,

$$\frac{dp}{dt} = \frac{\mathrm{JL}}{(v - w)\mathrm{T}}$$

It is an experimental fact that the volume of 1 lb. of ice w is greater than the volume of water from which it is formed. Therefore the term $(v - w)$ is negative. Therefore $\frac{dp}{dt}$ is negative, and hence, as the pressure p increases the temperature t diminishes.

Large pressures are required to produce a perceptible diminution of the temperature of freezing. Thus 1 lb. of water in freezing increases its volume from 0·016 to 0·0174 cub. ft., and gives out 80 lb.-cals.

So that—

$$dp = \frac{1400 \times 80}{-0{\cdot}0014 \times 273{\cdot}1}dt$$

$$dp = -29300\,dt.$$

That is, to reduce the freezing point by 1° requires a pressure of 29,300 lbs. per square foot = 203 lbs. per square inch.

The equation will be used immediately to obtain an analytical relation between the saturation pressure p and the temperature t of steam by means of which the steam pressure corresponding to any given temperature can be calculated in the way used by Callendar.

47. The Characteristic Equation for a Vapour and Callendar's Equation for Steam.—All gases and vapours, including steam, are imperfect in the sense that they do not quite follow the relations expressed by the equation PV = JRT, the equation of a perfect gas.

Let A, Fig 59, be a point on a P.V. diagram representing the volume of 1 lb. of a perfect gas at a pressure P and temperature T. The ideal volume corresponding to this is $\frac{JRT}{P}$ cub. ft.

An equal weight of an actual gas with the same gas constant R and in similar conditions of temperature and pressure would have a smaller volume, as indicated by the state point B. This difference between the ideal volume and the actual volume is considered by Callendar to be due to the co-aggregation of a proportion of the molecules into pairs, the result of which is to change the volume but not the temperature or the pressure. It is as if a regiment of 1000 soldiers were ordered to form 10 lines of 100 men per line, each man being 10 ft. from his neighbour, and that, instead of extending as ordered, 300 of them linked arms and co-aggregated into 150 pairs so that the ground actually covered would be short of the full area by the ground required for 150 men.

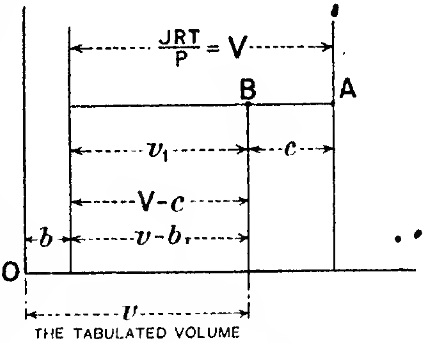


FIG. 59.—Volume relations.

The gaseous equation may be adjusted to allow for this co-aggregation by writing it—

$$(v_1 + c)P = JRT \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This expression tacitly neglects the volume of the molecules themselves and assumes that the volume can be reduced to zero.

Actually, a gas cannot be reduced to a volume smaller than the volume of the liquid from which it is formed. Therefore the actual volume of gas expanding is the ideal volume $\frac{JRT}{P}$, less the co-aggregation volume, plus a minimum volume b, say. Let v be the actual volume, and b the minimum volume to which the gas can be reduced by cooling, then the characteristic equation for a vapour may be written

$$(v - b) = \frac{JRT}{P} - c \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and this is the form of the characteristic equation of steam devised by Callendar.

Callendar shows [1] that for a vapour the "co-aggregation" volume

[1] Callendar, H. L., "On the Thermodynamical properties of gases and vapours as deduced from a modified form of the Joule-Thomson equation with special reference to the properties of steam," *Proc. Roy. Soc.*, vol. 67. Callendar, H. L., "On the Thermodynamical correction of the gas Thermometer," *Proc Phys. Soc. London*, vol. 18; also *Phil. Mag.*, Jan. 1903.

c may be regarded over a wide range of pressure as a function of the temperature only, of the form

$$c = C\left(\frac{T_o}{T}\right)^n$$

For steam, C = 1·192 cub. ft. per lb. at the absolute temperature 273·1° Cent., that is 0° Cent.

$n = \frac{10}{3}$.

b = 0·016 cub. ft. per lb.

R = 0·11012 mean calories; RJ = 154·168 ft.-lbs.

v is the volume in cub. ft. of 1 lb. of steam, either saturated or superheated.

P is the pressure in pounds per sq. ft.

T is the absolute temperature reckoned equal to 273·1 + t.

Inserting these values in equation (2), page 171, the characteristic equation for steam reduces to—

$$(v - 0{\cdot}016) = \frac{0{\cdot}11012\,\mathrm{JT}}{\mathrm{P}} - 1{\cdot}192\left(\frac{273{\cdot}1}{\mathrm{T}}\right)^{\frac{10}{3}} \quad . \quad . \quad (3)$$

If the saturation pressure P is inserted in this equation along with the corresponding temperature T, the calculated volume v is the volume of 1 lb. of dry saturated steam at that temperature. Also if P is the pressure of steam superheated to a temperature T, then v is the volume of 1 lb. of the superheated steam.

The equation applies generally to dry saturated and to superheated steam.

As an example, calculate the volume of 1 lb. of dry saturated steam at a temperature of 200° C. The corresponding saturation pressure is 225·2 lbs. per square inch.

The first step is to calculate the magnitude of the co-aggregation volume c. Thus—

$$c = 1{\cdot}192\left(\frac{273{\cdot}1}{473{\cdot}1}\right)^{\frac{10}{3}} = 0{\cdot}1909 \text{ cub. ft. per lb.}$$

$$\text{Therefore } v = \frac{0{\cdot}11012 \times 1400 \times 473{\cdot}1}{225{\cdot}2 \times 144} - 0{\cdot}1909 + 0{\cdot}016$$

$$= 2{\cdot}249 - 0{\cdot}191 + 0{\cdot}016 = 2{\cdot}074 \text{ cub. ft.}$$

To facilitate calculations, values of c are given in Table 2, p. 742. As a further illustration calculate the volume when the steam is superheated 300° at this pressure, that is its actual temperature is 500° C. and the absolute temperature is 773·1°.

The co-aggregation volume is—

$$c = 1{\cdot}192\left(\frac{273{\cdot}1}{773{\cdot}1}\right)^{\frac{10}{3}} = 0{\cdot}0371 \text{ cub. ft. per lb.}$$

$$\text{Therefore } v = \frac{0{\cdot}11012 \times 1400 \times 773{\cdot}1}{225{\cdot}2 \times 144} - 0{\cdot}037 + 0{\cdot}016$$

$$= 3{\cdot}654 \text{ cub. ft.}$$

It should be noticed that at this high pressure the co-aggregation volume, though small, is a considerable fraction of the actual volume and is therefore an important term in the characteristic equation. For instance, in the example just worked the co-aggregation volume is 0·19 cub. ft. per pound for dry saturated steam at 225·2 lbs. per square inch, a magnitude which is over 9 per cent. of the actual volume 2·074 cub. ft.

Providing the temperature remains the same, the co-aggregation volume does not change, so that if the condition of the steam is so altered that its pressure is reduced to atmospheric pressure whilst the temperature, namely, 200° C., remains unaltered, the value of c remains unaltered. In this new condition the steam would be superheated 100° and the volume per pound from the characteristic equation would be 34·33 cub. ft. so that the co-aggregation volume is only about 0·5 per cent. of the actual volume. The co-aggregation volume at 100° C. is 0·421 cub. ft. The volume of 1 lb. of dry saturated steam is, from the characteristic equation, 26·79 cub. ft. per lb. at 100° C.; the co-aggregation volume is therefore about 1·6 per cent. of the actual volume. As the pressure is reduced towards zero and the volume increases the co-aggregation term becomes negligible and the vapour tends to behave as an ideally perfect gas.

Values of the co-aggregation volume are given in the third column of Steam Table 2.

The volume per pound, both for dry saturated and for superheated steam, can be rapidly calculated from the simpler expression (8), page 176, when tables are available in which corresponding values of the total energy I and the pressure p are given, as in Tables 1, 2 and 3, pages 738 to 744. This expression, though approximate, gives values of the volume which differ from the values found from the characteristic equation only by negligibly small quantities.

48. Expressions for the Specific Heats, the Total Energy, the Internal Energy, and the Entropy of Steam; for the Total Energy and the Entropy of Water; and for the Latent Heat and Pressure of Steam.—The general expressions of Section 46 may now be applied to the characteristic equation of steam to establish the values of these functions belonging to steam.

The Specific Heat at Constant Pressure K_p, *and the Specific Heat at Constant Volume* K_v.—The general expression for the specific heat at constant pressure is, quoting equation (11), Section 46, page 168,

$$\left(\frac{dK_p}{dP}\right)_T = -\frac{T}{J}\left(\frac{d^2v}{dT^2}\right)_P$$

The solution is

$$K^p = -\frac{T}{J}\int\left(\frac{d^2v}{dT^2}\right)dP + \text{a term which is a function of T} \quad (1)$$

The characteristic equation for steam is, from equation (2), page 171,

$$v = \frac{JRT}{P} - C\left(\frac{T_o}{T}\right)^n + b$$

therefore
$$\frac{dv}{dT} = \frac{JR}{P} + nC\left(\frac{T_o^n}{T^{n+1}}\right)$$

and,
$$\frac{d^2v}{dT^2} = -n(n+1)C\left(\frac{T_o^n}{T^{n+2}}\right)$$

that is, writing c for the co-aggregation volume $C\left(\frac{T_o}{T}\right)^n$

$$\frac{d^2v}{dT^2} = -\frac{n(n+1)c}{T^2}$$

Substituting in (1) and integrating with regard to P

$$K_p = \frac{n(n+1)Pc}{JT} + \text{a constant.}$$

Callendar gives reasons in his papers above quoted for assuming that as the pressure approaches zero the vapour behaves as a perfect gas and that the specific heat tends to a limiting value K_p^o, which is independent of the temperature. Therefore the constant in the equation is K_p^o.

The specific heat of steam at constant pressure is thus—

$$K_p = K_p^o + \frac{n(n+1)Pc}{JT} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

According to Callendar, $K_p^o = (n+1)R$. Its numerical value is therefore $\frac{13}{3} \times 0{\cdot}11012 = 0{\cdot}4772$. This is the value which has been used in the calculation of the Steam Tables in the Appendix.

In a similar way from the general relation in (9), page 167, and the characteristic equation, it can be shown that

$$K_v = K_v^o + \frac{ncP}{JT}\left(n - 1 - \frac{nc}{V}\right) \quad . \quad . \quad . \quad . \quad (3)$$

In this expression V is the ideal volume $\frac{JRT}{P}$

And the numerical value of K_v^o is $nR = \frac{10}{3} \times 0{\cdot}11012 = 0{\cdot}3671$

The Total Energy of Dry-Saturated or Superheated Steam, I.—From equation (13), Section 46, page 168,

$$dI = K_p dT - \frac{1}{J}\left\{T\left(\frac{dv}{dT}\right)_p - v\right\}dp.$$

From the characteristic equation for steam (2), page 171,

$$v = \frac{JRT}{P} - C\left(\frac{T_o}{T}\right)^n + b$$

and by differentiating this with regard to T, and then multiplying by T, we obtain

$$T\frac{dv}{dT} = \frac{JRT}{P} + nc.$$

Inserting this and the value of v in the equation for dI together with the value of K_p given in (2), it becomes

$$dI = \left\{K_p^o + \frac{n(n+1)Pc}{JT}\right\}dT - \left\{\frac{(n+1)c - b}{J}\right\}dP$$

The integral of this equation is

$$I = K_p^o T - \frac{(n+1)Pc}{J} + \frac{bP}{J} + \text{a constant} \quad . \quad . \quad . \quad (4)$$

The value of the constant is found by substituting for I, T and P, a set of values found from experiment and then solving the equation for the constant.

Many observations have been made on the value of I at 100° C, and at atmospheric pressure. Its value reckoned from zero on the Centigrade scale (273·1 abs.) is 639·3. The pressure corresponding to 1 atmosphere reduced, from 760 mm. of mercury in latitude 45°, to lbs. per square inch in the latitude of Greenwich is 14·689.

Defining the heat unit as the $\frac{1}{100}$ part of the heat required to raise 1 lb. of water from 0° to 100° C., the value of the mechanical equivalent of heat is 1400.

This unit of heat is the mean calorie as distinguished from the 20°-calorie and the 15°-calorie, in which the unit is defined as the quantity of heat required to raise the temperature 1° at 20° or 15° respectively. In each case J is slightly different.

Substituting in (4)

$$639{\cdot}3 = 0{\cdot}4772 \times 373{\cdot}1 - \frac{14{\cdot}689 \times 144}{1400}\left\{\frac{13}{3} \times 1{\cdot}192\left(\frac{273{\cdot}1}{373{\cdot}1}\right)^{\frac{10}{3}} - 0{\cdot}016\right\} + B$$

from which $B = 463{\cdot}994 = 464.$

Inserting the numerical values in equation (4)

$$I = 0{\cdot}4772T - 0{\cdot}10286p\left\{\frac{13}{3}c - 0{\cdot}016\right\} + 464 \qquad (5)$$

In this equation p is in pounds per square inch, and T is the absolute temperature. I stands for lb.-calories per pound reckoned above 0° C.

The total energy per pound, I, may be expressed in terms of the product Pv by the elimination of the temperature T from equation (4). From the characteristic equation—

$$T = \frac{P(v - b + c)}{JR}$$

Substituting this value of T in (4), and remembering that $K_p^o = (n+1)R$, it reduces to

$$I = \frac{(n+1)}{J}Pv - \frac{nbP}{J} + 464 \quad . \quad . \quad . \quad . \quad (6)$$

Inserting the numerical values, $J = 1400$, $n = \frac{10}{3}$, and $P = 144p$, the equation reduces to

$$I = 0{\cdot}4464pv - 0{\cdot}3435pb + 464 \quad . \quad . \quad . \quad . \quad (7)$$

Solving this for v,

$$v = \frac{2{\cdot}243(I - 464)}{p} + 0{\cdot}0123 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The constant term 0·0123 is negligible at low pressures, and is only about $\frac{1}{2}$ of 1 per cent. of the whole volume at 200 lbs. per square inch, and even at 450 lbs. per square inch it is only just over 1 per cent. of the volume. For calculations relating to practical problems, the constant may therefore be neglected, giving an exceedingly simple relation wherewith to calculate the volume of a pound of steam in either the dry and saturated or in the superheated condition when steam tables giving corresponding values of I and p are available.

The Internal Energy of Dry Saturated or Superheated Steam, E. —The expression for E can be found from equation 8, page 167, and the characteristic equation in a similar way to that in which the expression for I has just been found. There is a more expeditious method, however. Since by definition

$$I = E + \frac{Pv}{J}$$

$$E = I - \frac{Pv}{J}.$$

Substituting for I its value from (4), page 175, remembering that K_p^o in (4) is equal to $K_v^o + R$, and substituting for v its value $\left\{\frac{JRT}{P} - c + b\right\}$ from the characteristic equation for steam (2), page 171,

$$E = K_v^o T - \frac{ncP}{J} + \text{a constant} \quad . \quad . \quad . \quad . \quad (9)$$

The constant term is the same as in the equation for I. So that numerically

$$E = 0{\cdot}3671T - 0{\cdot}3427cp + 464 \quad . \quad . \quad . \quad (10)$$

p is in pounds per square inch, and T is the absolute temperature.

Substitute the value of I from equation (6) into the general equation,

$$E = I - \frac{Pv}{J}$$

and it reduces to

$$E = \frac{nPv}{J} - \frac{nbP}{J} + 464 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which with $n = \frac{10}{3}$ becomes

$$E = 0{\cdot}3435p(v - b) + 464 \quad . \quad . \quad . \quad . \quad (12)$$

p is here in pounds per square inch, and v in cubic feet.

The Total Energy of Water, I_w.—The total energy of water may be expressed by the relation

$$I_w = sT + dh + \text{a constant},$$

where I_w is the total energy in lb.-calories, s is the mean specific heat assumed to be constant, dh is the small quantity of heat which added to sT brings I_w to the correct experimental value; and the constant is added to adjust I_w to the zero from which energy is measured, namely, 0° C.

Callendar[1] points out that water at any temperature may be regarded as a mixture of liquid with its vapour distributed through it. In this mixture s is the specific heat of the liquid, and is constant, whilst dh, the "correction," is the latent heat of a mass of the vapour equal in volume to the volume of the liquid. With this assumption

$$dh = \frac{Lw}{v - w}$$

where L is the latent heat of the vapour; w is the volume of a pound of the liquid, and v is the volume of 1 lb. of the dry saturated vapour. Therefore

$$I_w = sT + \frac{wL}{v - w} + C \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The constants C and s are now to be found, so that the energy is 0 at 0° C., and 100 lb.-cals at 100° C., since the unit of heat has been defined as one-hundredth part of the heat required to warm a pound of water from 0° to 100° C. The value of dh must be found before the pair of simultaneous equations required can be formed. Thus

At 0° C. $\qquad dh = \dfrac{594{\cdot}27 \times 0{\cdot}016}{3275 - 0{\cdot}016} = 0{\cdot}0029 = 0{\cdot}003$ say.

At 100° C. $\qquad dh = \dfrac{539{\cdot}3 \times 0{\cdot}0167}{26{\cdot}78 - 0{\cdot}0167} = 0{\cdot}337.$

Therefore, introducing these values together with the corresponding values of I_w into the equation for I_w

$$0 = s\,273{\cdot}1 + 0{\cdot}003 + C.$$
$$100 = s\,373{\cdot}1 + 0{\cdot}337 + C.$$

From which

$$s = 0{\cdot}99666$$
$$C = -\,272{\cdot}187$$

Therefore, when temperature is measured on the absolute scale,

$$I_w = 0{\cdot}9966T + \frac{wL}{v - w} - 272{\cdot}187 \quad . \quad . \quad . \quad (14)$$

And if the temperature is measured on the ordinary scale,

$$I_w = 0{\cdot}9966t + \frac{wL}{v - w} - 0{\cdot}003 \quad . \quad . \quad . \quad . \quad (15)$$

[1] "Continuous Electrical Calorimetry," *Phil. Trans. A.*, 1902, vol. 199.

The most uncertain element in this equation is w, the volume of a pound of water in terms of the temperature. The values of this given in Table 2, page 742, represent the best available data. The values of I_w tabulated in the tables have been derived from this equation. As an example, calculate the total energy of water at 250° C. Inserting the values from the tables in expression (15) above,

$$I_w = 0{\cdot}9966 \times 250 + \frac{0{\cdot}02016 \times 420{\cdot}96}{0{\cdot}869 - 0{\cdot}02016} - 0{\cdot}003 = 259{\cdot}16 \text{ lb.-cals.}$$

Entropy of Water.—To find an expression for the entropy, equation (14) must first be differentiated with regard to the temperature T to get an expression for dI_w, and then, after dividing through by T, the result must be integrated. But, first, the term $\dfrac{wL}{(v - w)}$ must be replaced by a function of the temperature.

It has been shown above that for vapours

$$\frac{dp(v - w)}{J} = \frac{L \,.\, dT}{T}$$

Multiplying both sides by w and rearranging the factors

$$w\frac{Tdp}{JdT} = \frac{wL}{(v - w)} = dh \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Therefore the equation for I_w (13), page 177, may be written

$$I_w = sT + \frac{wT}{J}\frac{dp}{dT} + C \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The assumption that dh can be expressed in this way was made by MacFarlane Gray,[1] in his investigation of the specific heat of water. Callendar[2] points out, as mentioned above, that the expression $\dfrac{wL}{(v - w)}$ is the latent heat of a volume of steam equal to the volume of the liquid. In other words, the liquid contains its own volume of vapour in solution.

Differentiating (17) with regard to T, and dividing by T, it becomes

$$d\phi = \frac{s}{T}dT + \frac{w}{JT}\frac{dp}{dT}dT + \frac{w}{J}\frac{d^2p}{dT^2}dT$$

The last term is small, and may usually be neglected.

The solution of this equation with 273·1 for the lower limit is then

$$\phi = s \log_e \frac{T}{273{\cdot}1} + \frac{w}{J}\left(\frac{dp}{dT}\right) - \frac{w}{J}\left(\frac{dp}{dT}\right)_0$$

And using the relation in (16), the expression may be written

$$\phi = s \log_e \frac{T}{273{\cdot}1} + \frac{wL}{T(v - w)} \quad . \quad . \quad . \quad . \quad (18)$$

[1] MacFarlane Gray, "The Variable and Absolute Specific Heats of Water," *Proc. Inst. C.E.*, 1901-02, Part I.

[2] Callendar, "Continuous Electrical Calorimetry," *Proc. Roy. Soc.*, 1902, 145.

This is the expression used for the calculation of the values of ϕ_w given in the Steam Tables in the Appendix.

As an example, calculate the entropy of a pound of water at 250° C.

$$\phi = 0{\cdot}9966 \log_e \frac{523{\cdot}1}{273{\cdot}1} + \frac{0{\cdot}02016 \times 420{\cdot}96}{523{\cdot}1(0{\cdot}869 - 0{\cdot}0201)}$$

$$\phi = 0{\cdot}9966 \times 0{\cdot}6482 + \frac{10}{523{\cdot}1}$$

$$\phi = 0{\cdot}666 \text{ as tabulated.}$$

It is to be observed that the numerical value of the second term undivided by T is calculated in the process of finding I_w.

The Latent Heat, L.—The latent heat of a vapour at a particular temperature is the difference between the total energy corresponding to the saturation pressure and the total energy of the water from which the steam is formed at that temperature. That is

$$L = I_s - I_w$$

And substituting the value of I_s from (4), page 175, with 463·994 for the value of the constant; and I_w from (13), page 177, with − 272·187 for the value of the constant, and combining the values of the constants,

$$L = K_p^0 T - \frac{1}{J}(n + 1)Pc + \frac{1}{J}bP - sT - \frac{wL}{v - w} + 736{\cdot}181 \qquad (19)$$

from which L can be found. In this equation P is the saturation pressure corresponding to the temperature T.

The Relation between the Saturation Pressure and the Temperature.—This relation is found by integrating the fundamental thermodynamical expression

$$dp(v - w) = \frac{JLdt}{T}$$

After substituting the expression for the latent heat in terms of the temperature and the pressure given in (19), also writing $\left\{\frac{JRT}{P} - c + b\right\}$ for v and dividing through by RT, the relation becomes

$$\frac{1}{J}\left\{\frac{J}{P} - \frac{(c - b)}{RT} - \frac{w}{RT}\right\}dp$$

$$= \frac{\left\{K_p^0 T - (n + 1)\frac{cP}{J} + \frac{bP}{J} - sT - \frac{wL}{v - w} + 736{\cdot}181\right\}}{RT^2}dt$$

Noting that $\frac{wdp}{dt} = \frac{wL}{(v - w)}$, the integration of this expression gives

$$\log_e P = \frac{(K_p^0 - s)}{R}\log_e T + \frac{(c - b)P}{JRT} - \frac{736{\cdot}181}{RT} + C$$

The constant C can be eliminated by evaluating the equation between limits, and moreover the equation found in this way is more convenient for calculation. Thus if P_o and T_o are a pair of observed values of the saturation pressure and temperature, and if c_o is the corresponding co-aggregation volume; and multiplying through by $m = 0{\cdot}43429$ to convert the natural to common logarithms,

$$\log_{10}\frac{p}{p_o} = \frac{(K_p'' - s)}{R}\log_{10}\frac{T}{T_o}$$
$$+ \frac{m}{JR}\left\{\frac{p(c - b)}{T} - \frac{p_o(c_o - b)}{T_o}\right\} + \frac{m736{\cdot}181}{RT_o}\left\{\frac{t - 100}{T}\right\} \quad . \quad (20)$$

In this expression the pressure p is in pounds per square inch, and t is the temperature on the ordinary centigrade thermometer. The temperature T is absolute. The lower limit is at 100° C. so that

$p_o = 14{\cdot}689$; $T_o = 373{\cdot}1$; $c_o = 0{\cdot}42134$; $b = 0{\cdot}016$; $K_p'' = 0{\cdot}4772$; $s = 0{\cdot}99666$; $R = 0{\cdot}11012$; $J = 1400$.

With these data the equation reduces to—

$$\log_{10}\frac{p}{p_o} = -4{\cdot}7172\log_{10}\frac{T}{T_o} + \frac{0{\cdot}40565p(c - b)}{T} - 0{\cdot}006474$$
$$+ 7{\cdot}7817\frac{(t - 100)}{T} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

This form is convenient for calculation.

As an example, calculate the saturation pressure corresponding to 200° C. = 473·1° absolute.

It is necessary to assume a value of p to begin with in order to calculate the value of the second term on the right side. Assume provisionally that $p = 225$.

The first step is the calculation of the co-aggregation volume c at the temperature 473·1 from

$$c = 1{\cdot}192\left(\frac{273{\cdot}1}{473{\cdot}1}\right)^{\frac{10}{3}}$$

With the data given this becomes 0·19091 cub. ft. per pound.

Then the middle term on the right side becomes—

$$\frac{0{\cdot}40565 \times 225(0{\cdot}19091 - 0{\cdot}016)}{473{\cdot}1} = 0{\cdot}03374$$

The first term, with $T = 473{\cdot}1$ reduces to 0·48647.

The last term becomes, remembering that $t = 200$—

$$\frac{7{\cdot}7817 \times 100}{473{\cdot}1} = 1{\cdot}64484$$

Then the right side reduces to 1·18564.

So that

$$\log p = 1{\cdot}18564 + \log 14{\cdot}689 = 2{\cdot}35263$$
$$= \log 225{\cdot}23$$

The initial assumption regarding p was therefore sufficiently accurate and the saturation pressure corresponding to 200° C. may therefore be recorded as 225·2 lbs. per square inch.

Expression for the Entropy of Steam ϕ_s.—Equation (15), Section 46, page 169, is

$$d\phi = K_p\frac{dT}{T} - \frac{1}{J}\left(\frac{dv}{dT}\right)_p dp$$

Substituting the value of K_p from (2), page 174, and differentiating the characteristic equation to get $\left(\frac{dv}{dT}\right)_p$ this becomes

$$d\phi = \left(K_p^o + \frac{n(n+1)Pc}{JT}\right)\frac{dT}{T} - \frac{1}{J}\left(\frac{JR}{P} + \frac{nc}{T}\right)dp$$

The solution of this is—

$$\phi = K_p^o \log_e T - R \log_e P - \frac{ncP}{JT} + \text{a constant}. \quad (22)$$

The constant is found by solving the equation after inserting a known value of the entropy along with the corresponding values of T, P, and c.

At 100° C. = 373·1° absolute, ϕ = 1·7573.

Also c = 0·4212 from Table 2 (page 742).

Therefore

$$1{\cdot}7573m = 0{\cdot}47720 \log_{10} 373{\cdot}1 - 0{\cdot}11012 \log_{10} 14{\cdot}689 \times 144 - \frac{\frac{10}{3} \times 0{\cdot}4212 \times 14{\cdot}689 \times 144 \times m}{1400 \times 373{\cdot}1} + C$$

where m is the modulus of the common system of logarithms and is equal to 0·43429.

This gives C = − 0·095.

And (22) becomes, after dividing through by m

$$\phi = 1{\cdot}0988 \log_{10} T - 0{\cdot}2536 \log_{10} P - \frac{ncP}{JT} - 0{\cdot}219 \quad (23)$$

T is the absolute temperature C.

P is the pressure in pounds per square foot

$n = \frac{10}{3}$.

The expressions are used in the following order to calculate a line in the steam table corresponding to a given temperature.

(1) Use expression (21) to find the saturation pressure corresponding to the given temperature in the way exemplified for the temperature 200° C.

(2) Insert T, the given temperature; and P, the calculated value of the saturation pressure, in the characteristic equation (3), page 172, and solve for v the volume of 1 lb. of dry saturated steam in the way exemplified when the temperature is 200° C. and the calculated value of the saturation pressure is 225·2 lbs. per square inch.

(3) Find the value of the total energy I_s of 1 lb. of dry saturated

steam from equation (5), page 175, using T and the calculated value of the saturation pressure p.

(4) Calculate the value of the latent heat L from equation (19), page 179.

(5) Calculate the value of the total heat of water I_w from equation (15), page 177.

(6) Calculate the value of the entropy of water ϕ_w from equation (18), page 178.

(7) Tabulate the increase of entropy due to the addition of the latent heat, namely, $\frac{L}{T}$.

(8) The entropy of 1 lb. of dry saturated steam is then $\phi_w + \frac{L}{T}$.

The total energy of superheated steam is calculated from equation (5), page 175, after inserting the pressure and the temperature, T, to which the steam is superheated, and the entropy is calculated from equation (23), page 181.

49. The Steam Tables and the Total Energy Temperature Diagram.—The physical properties of 1 lb. of dry saturated steam are tabulated on pages 738 to 744. The tables were calculated by Professor Callendar from the data given on page 743, from expressions like those given in the preceding section. Briefly, Table 2, on page 742, was calculated using the temperature as an independent variable, first in equation (21), page 180, to obtain the corresponding values of the saturation pressure, and then in the other expressions together with the saturation pressure, for calculating the other functions given in the table. Afterwards, by methods of interpolation, Table 1 was constructed.

The figures in the table are thermodynamically consistent, and the table is thus distinguished from those based simply on experimentally determined data.

The steam tables hitherto used are based mainly on the experimental researches of Regnault published in the *Memoirs of the Institute of France*, 1847, Vol. XXI.

These classical researches of Regnault, made with extreme care and with extraordinary skill, cover the whole range of the steam tables ordinarily used by engineers, and confirm in a remarkable way the accuracy of Callendar's characteristic equation and the results derived from it. Callendar's work has made possible the correction of the small inaccuracies and thermodynamical inconsistencies involved in the empirical equations devised to represent Regnault's results, and it further enables calculation to be made of the properties of steam at low pressures where experimental determinations are difficult and unreliable.

The physical properties of steam set out in the tables are reduced to a diagram in Fig. 62, page 200. The temperature is taken as an independent variable, and the total energy and other properties are

plotted against it. The diagram may be called the **total energy temperature diagram**.

The author has found this form of diagram very useful in teaching thermodynamics to engineering students. It was first brought into use for this purpose in connection with some special lectures given at the Technical College, Finsbury, in 1898.

The line OQ is the graph of the water energy I_w and is plotted from the figures in column 4 of the Steam Table 1, on page 738.

The curve through R is plotted from column 6 of Steam Table 1, and its ordinates give for any temperature the total energy of one pound of dry saturated steam. The curve may be called the saturation curve, because a point on it represents steam in the dry saturated state; or it may be called the boundary curve, because it divides the superheated region of the diagram from the wet region.

Curves of saturated pressure and saturated volume are added together with a curve showing the external work $P(v - w)$ expressed of course in lb.-cals.

The properties of the diagram will be described below.

50. The Formation of Dry Saturated Steam at Constant Pressure.—The energy additions to a pound of water necessary to change it into one pound of steam in the way it is produced in a steam boiler can now, by the aid of the steam tables and the diagram, Fig. 60, be followed in detail.

It is convenient to imagine that the steam is formed in a cylinder fitted with a piston which is loaded to the pressure of formation as illustrated in Fig. 60. The experiment begins with the piston in contact with the cylinder cover and the pound of water outside the cylinder at 0° C. and at the corresponding negligibly small vapour tension. The energy added to the water is reckoned from this initial condition. The water is first pumped into the cylinder against the pressure P, the load on the piston. The work done on the water and stored in the water in the cylinder is Pw ft.-lbs., where w is the volume of one pound of water in cubic feet. Values of w are given in column 4 of Table 2, page 742. The state point is now at Y. Heat is now added until the temperature of the water is raised from 0° C. to the temperature t corresponding to the saturation pressure P.

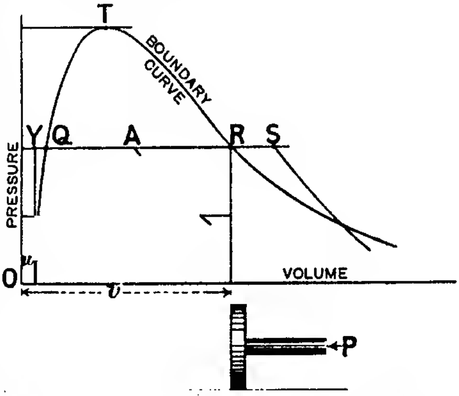


FIG. 60.—Formation of steam at constant pressure.

The liquid expands during this heat addition and work is done equal to P . dw, where dw is the small change in volume caused by the increase of temperature. The heat added during this process, sometimes called the "liquid heat," is denoted by the symbol h. The state point has now moved along a constant pressure line from Y to Q. The further addition of heat energy causes evaporation to begin, and evaporation goes on until, after the addition of the quantity of heat L, the latent heat, all the water is converted into steam and the cylinder contains one pound of "dry saturated" steam. The term "dry" is used because there is no water in the cylinder, and it is immediately qualified by the term "saturated" because the addition of heat stopped the instant the last drop of water disappeared. During the process the state point has moved along the line of constant pressure from Q to R. In the first position Q, there is no steam, but only water at the temperature of evaporation; in the second position R, there is no water but the slightest diminution in the heat energy stored in the steam produces water by condensation. If heat energy is continued after the state point has arrived at R, the steam is **superheated**, and if the constant pressure conditions are maintained and the state point moves to S, the steam has then been superheated at constant pressure, but the temperature begins to increase as soon as the state point moves to the right of R.

The total energy required to bring the pound of water from the conditions at 0° C. to the state of dry saturated steam at a stated temperature may thus be divided into three parts. Namely, the energy $\frac{Pw}{J}$, required to pump the water into the cylinder; the heat energy h, added to warm the water from 0° C. to the temperature of evaporation; and the latent heat energy L required to evaporate the heated water into steam. Calling the sum of these parts the **total energy** of 1 lb. of dry saturated steam, and representing this quantity by I_s, and writing A for $\frac{1}{J}$

$$I_s = APw + h + L \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This quantity is tabulated in column 6 of Table 1, page 738.

Alternatively it may be calculated from equation (5), page 175, after substituting the saturation pressure P and the corresponding temperature T, the values of which can either be taken from the first two columns of the tables or determined from equation (21), page 180.

During the addition of the energy I_s, external work is done as the piston moves from its initial position touching the cylinder cover to its final position corresponding to the state point R and the volume v. The **external work** is APv lb.-cals.

The increase of **internal energy** during the whole process is found by subtracting the external work from the total energy, thus

$$E = I_s - APv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or, substituting the value of I_s from (1),

$$E = h + L - AP(v - w) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The term $AP(v - w)$ gives the external work done during the addition of the heat energy $h + L$.

The value of E can be calculated from either expression by the aid of the tables.

Alternatively E can be calculated from the expression (12), page 176, after substituting the saturation pressure and the corresponding temperature, which can either be taken from the tables or determined from equation (21), page 180.

The entropy of 1 lb. of dry saturated steam is the entropy of the water plus the increase of entropy due to the addition of the latent heat L at the constant temperature of evaporation T, namely, $\frac{L}{T}$

Thus
$$\phi_s = \phi_w + \frac{L}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Alternatively, the entropy may be calculated from equation (23), page 181, after substituting the values of the saturation pressure and temperature.

An approximate expression for the entropy in terms of the temperature alone is obtained and has often been used by assuming that the specific heat of water is constant and equal to unity.

Then
$$d\phi = \int\frac{dh}{T} = \int\frac{dT}{T}$$

giving
$$\phi = \log_e \frac{T}{T_0}$$

taking the integral between limits. Then reckoning from 0° C.

$$\phi = \log_e \frac{T}{273 \cdot 1} + \frac{L}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It is, however, better to use the accurate and simpler expression (4) above, since both ϕ_w and $\frac{L}{T}$ are given separately in Table 1, page 738, so that ϕ_s can be found at once by a simple addition.

As the pressure of formation and therefore the temperature are increased, the state points Q and R, Fig. 60, move closer together until in the limit they coincide at T, and their two separate paths merge into one **boundary curve** separating the region of all steam or all water from the region of a mixture of steam and water. The temperature corresponding to the pressure line which is tangent to the boundary curve is called the **critical temperature.** At this temperature the latent heat vanishes and immediately the water has been heated to this temperature it turns into steam without an intermediate state corresponding to the addition of latent heat. For steam the critical temperature is about 365° C., corresponding to a pressure approaching 2,900 lbs. per square inch. The properties of the critical temperature have been investigated on vapours with lower critical temperatures,

notably carbon dioxide, where the critical temperature is 31° C. Callendar's characteristic equation for steam does not apply above temperatures of about 250° C. Consequently the expressions for the properties of steam obtained from it do not apply above this temperature.

In general, the expressions apply to all points on the boundary curve (which is the locus of R), up to 250 C., and to all points in the superheated region to the right of this. If the state point falls on the boundary curve, corresponding pairs of pressure and temperature values are determined by equation (21), page 180. And then substituting the values of P and T so obtained in the appropriate expressions given in Section 48, the values of the total heat I_s, the internal energy E, the latent heat L, the entropy ϕ can be calculated; the volume v can be found from the characteristic equation (3), page 172, or from the simpler expression (8), page 176.

If the state point falls in the superheated region, then the total energy, internal energy, volume, entropy, can be calculated for an assigned pair of values of P and T in this region by substitution in the same set of equations, Section 48, using the characteristic equation for the volume.

If, however, the state point falls within the boundary curve the temperature at which evaporation is taking place can be found from equation (21), page 180, as before, but other expressions must be devised for the total energy, the volume, and the internal energy: expressions which give the values of these functions in terms of the mixture of steam and water which exists in the cylinder during the process of evaporation.

The changes of state from the initial condition of 0° C. may now be traced on the total heat temperature diagram, Fig. 62, page 200.

Assume that a pound of dry saturated steam is to be formed at the pressure corresponding to a temperature of 200° C.

Starting with 1 lb. of water at 0° C. the state point is at the origin O. As the water is warmed by the addition of heat its energy increases gradually, and the state point moves along the water energy curve I_w until it arrives at Q. Then the ordinate 200-Q represents the energy which has been added, namely, APw to pump the water in the cylinder and h to warm it, to bring it from 0° C. to the condition of 1 lb. of water at the temperature and pressure of evaporation.

As the addition of heat energy is continued, the state point moves vertically from Q to R until, when the water is all converted into steam by the addition of the latent heat, the state point arrives at R. The saturation curve through R is the boundary curve between superheated steam and wet steam. The curve through Q is the boundary between hot water and a mixture of water and steam, that is wet steam. It will be observed that the two curves approach one another as the temperature is increased. They merge into one another at the critical temperature where the latent heat vanishes.

It will be observed from the figures on the path traced by the state point that the energy added to bring the water to 200° C. shown

by the ordinate 200–Q is 203·5 lb.-cals., that the latent heat is 467·4 lb.-cals., and that the total energy is 670·9 lb.-cals., and that the saturation pressure is 225·2 lbs. per square inch, whilst the volume of the steam is 2·07 cub. ft.

51. Wet Steam.—If the supply of heat is stopped during the process of evaporation, the water is only partly converted into steam and the cylinder is filled with a mixture of steam and water. In these circumstances the steam present is said to be "wet". If q is the fraction of the latent heat L which is supplied to the mixture in the cylinder, the heat energy supplied during the process of partial evaporation is qL lb.-cals. per pound. Then it will be clear that—

The **total energy** of 1 lb. of wet steam is $I_w + qL$ lb.-cals (1)

The corresponding **volume** is $qv + (1 - q)w$ cub. ft. per pound (2)

and this, unless the steam is very wet, is qv cub. ft. per pound (3)

The **external work** is $APqv$ lb.-cals. per pound . . . (4)

The **internal energy** is $I_w + qL - APqv$ . . . . . . (5)

The **entropy** is $\phi = \phi_w + \dfrac{qL}{T}$ . . . . (6)

All of these functions can be calculated by the aid of the steam tables when q is assigned.

The fraction q is generally called the "dryness fraction".

Referring to the total energy temperature diagram, Fig. 62, it will be seen that the vertical distance between the total energy line of saturated steam and the total energy line of water, that is the distance representing the latent heat, is divided by lines which cut every vertical intercept into ten equal divisions. These are lines of **constant dryness.** The value of q corresponding to each line is written against it.

The total energy of a pound of wet steam can therefore be read off the diagram when the value of q is assigned, so that the position of the state point can be marked down on the diagram.

52. Superheated Steam.—If the supply of heat is continued after evaporation is complete, the steam is said to be superheated. Assuming that the load on the piston is kept constant during the process, the steam is said to be superheated at constant pressure. This is, in fact, the condition in which steam is superheated in practice.

The total energy I′ of steam superheated at the constant pressure p to the temperature t can be calculated from the following equation, which is deduced from equation (4), page 175.

$$I' = K_p^o T + 464 - \frac{144}{1400}\{(n + 1)c - b\}p \quad . \quad . \quad (1)$$

In this equation T is the absolute temperature corresponding to t, p is the pressure of formation in pounds per square inch, n is $\frac{10}{3}$, b is 0·016,

and c is the co-aggregation volume. K_p^0 is the limiting specific heat 0·4772.

Values of I′ for 10° intervals of temperature and 50 lbs. per square inch intervals of pressure are given in Table 3, page 744. Values of the coefficient of p in the equation above are given in the last column of the table for intervals of 10 lbs. per square inch pressure in order to facilitate the calculation of values of the total heat for pressures intermediate to those given in the table. For example, find the total heat of 1 lb. of steam formed at 180 lbs. per square inch and then superheated to 300° C.

The nearest pressure in the table is 200 lbs. per square inch; and I′ for this pressure and 300° temperature is 728·8 lb.-cals.

This value is increased by 0·43 lb.-cal. for every 10 lbs. per square inch decrease of pressure. Therefore, since 180 lbs. correspond to 20 lbs. decrease of pressure, the value of I′ above must be increased by $0·43 \times 2 = 0·86$. So that

$$I' = 729·6 \text{ lb.-cals.}$$

The heat required to superheat the steam from a given temperature of formation to a temperature t is found by subtracting the total heat of the dry saturated steam from the total heat of the superheated steam. Thus

$$Q' = I' - I_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

To facilitate the use of Table 3 the values of I_s, together with the corresponding saturation temperatures, are given at the foot of each column in the table, the temperature being enclosed in brackets.

For pressures intermediate to those in Table 3, I′ must be calculated from the table and then I_s can be found from Table 1.

A glance at the figures in Table 3 will show how little the total heat of superheated steam is influenced by the pressure.

For example, the total heat of steam formed at 150 lbs. per square inch and then superheated to 400° is 781·5, and the total heat at a pressure of 200 lbs. per square inch and the same degree of superheat is 780·2; a difference, therefore, of 50 lbs. per square inch in the pressure of formation makes only a difference of 1·3 lb.-cals. at the superheat temperature of 400°.

The mean specific heat of the steam at constant pressure in any region of the table is found by merely taking the difference of the total heats of two adjacent temperatures and dividing by 10. Thus, the mean specific heat between the temperatures of 390° and 400° at 300 lbs. pressure per square inch is

$$K_p \doteq (777·8 - 772·6)0·1 = 0·52$$

Between the temperatures of 220° and 230° and at 300 lbs. per square inch

$$K_p = (683·8 - 677·5)0·1 = 0·63$$

The external work done during the whole process of evaporation and superheating from the initial condition of 0° C is APv', where v' is the

volume calculated from the characteristic equation for steam, equation (3), page 172, after substituting for P the pressure of formation in pounds per square foot, and for T the absolute temperature to which the steam is superheated.

The increase of internal energy from the zero conditions is

$$E = I' - APv'$$

The volume of 1 lb. of steam, superheated at constant pressure p until its total energy is I', can be calculated with negligibly small error from equation (8), page 176, which is quoted here,

$$v = \frac{2{\cdot}243(I' - 464)}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The volume is in cubic feet, and the pressure p is in pounds per square inch.

The value of I' is to be taken from Steam Table 3.

This equation applies also to dry saturated steam, corresponding values of I and p being found in Steam Table 1, page 738, or from Steam Table 2, page 742. Example: Find the volume of 1 lb. of steam superheated at constant pressure 200 lbs. per square inch to 350° C.

From Table 3, I' = 754·8.

Therefore from (4), $v = \dfrac{(754{\cdot}8 - 464)2{\cdot}243}{200} = 3{\cdot}26$ cub. ft.

The entropy of 1 lb. of superheated steam can be calculated directly from equation (23), page 181, after substituting for p the pressure of formation, and for T the absolute temperature to which the steam is superheated.

It is sometimes calculated approximately by adding to the entropy of the saturated steam the increase of entropy due to superheating, assuming that the specific heat of steam is constant during the process of superheating. With this assumption, the increase of entropy during superheating is

$$K_p \int \frac{dT}{T} = K_p \log_e \frac{T}{T_s}$$

where T is the temperature to which the steam is superheated, and T_s is the temperature of saturation at which the superheating begins. With this approximation the entropy of superheated steam is

$$\phi = \phi_w + \frac{L}{T_s} + K_p \log_e \frac{T}{T_s} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or with the approximation mentioned above, page 185, as sometimes used for water,

$$\phi = \log_e \frac{T_s}{273{\cdot}1} + \frac{L}{T_s} + K_p \log_e \frac{T}{T_s} \cdot \quad . \quad . \quad . \quad (6)$$

Referring to Fig. 62, page 200, a line of total heat corresponding to various degrees of superheat is drawn above the boundary line when

superheating is done at the constant pressure 14·7 lbs. per square inch, also a second line, RZ, for the pressure corresponding to 200° C.

53. The Adiabatic Expansion of Steam.—When, during a change of state, the working agent neither receives heat from nor loses heat to the walls of the vessel in which it is contained or through which it flows, the change of state is said to be adiabatic.

When the change of state is made by a reversible process, the entropy remains constant during the change. Advantage may be taken of this property to obtain an expression from which the state of steam may be computed after an adiabatic change by a reversible process from given initial conditions.

Referring to Fig. 60, page 183, it will be seen that the addition of heat to the stuff in the cylinder may be stopped at any stage of the process, leaving the steam wet, dry and saturated, or superheated. The steam may then be allowed to expand adiabatically it being assumed that the load on the piston is in some way automatically adjusted as the expansion proceeds, so that the external load is always in exact equilibrium with the pressure to which the steam falls. This assumption is necessary in order that the process may be reversible.

The entropy of the steam at the beginning of expansion from an initially wet condition is $\phi_{w_1} + \frac{q_1 L_1}{T_1}$, and the value can be calculated from the tables when q_1 is assigned. At the end of expansion $\phi_{w_2} + \frac{q_2 L_2}{T_2}$ gives the entropy.

But since the entropy does not change during reversible adiabatic expansion, these two expressions must be equal. Equating them and solving for q_2

$$q_2 = \frac{\phi_{w_1} - \phi_{w_2} + \frac{q_1 L_1}{T_1}}{\frac{L_2}{T_2}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The total energy of 1 lb. of wet steam is (equation (1), page 187),

$$I = I_w + qL$$

Therefore, the total energy of 1 lb. of steam, after expanding adiabatically and reversibly from T_1 to T_2, can be calculated directly from this expression after inserting for I_w the tabular value corresponding to T_2, and for q the value q_2 calculated from (1) above.

If the initial and final pressures are given, the corresponding temperatures can be found from the tables.

Also the volume of 1 lb. of wet steam after adiabatic expansion from T_1 to T_2 is qv cub. ft. per pound where q has the value q_2 found from (1), and v is the volume given in the tables corresponding to the final temperature T_2.

The total energy of steam at any point during a reversible

adiabatic change can be calculated by a more convenient process. Since the entropy is constant during a reversible adiabatic expansion, its value at a point where the dryness is q is equal to its value at every other point during the change. Therefore

$$\phi_w + \frac{qL}{T} = \text{a constant} = \phi_1 \text{ say.}$$

Also the total energy at the point where the dryness is q is

$$I_w + qL = I$$

Eliminating q from these two equations

$$I = T\phi_1 + I_w - T\phi_w \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$I = T\phi_1 - G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where G stands for the variable term $T\phi_w - I_w$.

The value of G can be calculated directly the final conditions are stated. Values of G are given in the last column of Table 2, page 742.

The value of the adiabatic constant ϕ_1 may be calculated from the initial conditions.

As an example, calculate the total energy of 1 lb. of steam after adiabatic expansion from the initially dry and saturated state at 200° C. to the final temperature of 100° C.

In this case, since the steam is initially dry and saturated, the adiabatic constant ϕ_1 may be calculated directly from the tables by merely adding the entropy of water at 200° to the entropy $\frac{L}{T}$ at 200°. Thus

$$\phi_w \text{ at } 200^\circ \text{ C.} = 0{\cdot}556$$

$$\frac{L}{T} \text{ at } 200^\circ \text{ C.} = 0{\cdot}987$$

$$\phi_1 = 1{\cdot}543 \text{ and is the adiabatic constant.}$$

Therefore, during the reversible adiabatic change from the initial conditions corresponding to dry saturated steam at 200° C., from equation (3)

$$I = 1{\cdot}543T - G$$

When the temperature falls to 100°, $T = 373$, and from Table 2, page 742, $G = 16{\cdot}37$.

Therefore, I, after adiabatic expansion to 100°,

$$= 1{\cdot}543 \times 373 - 16{\cdot}37 = 559 \text{ lb.-cals.}$$

In practice this method of calculating the total energy during a reversible adiabatic change will be found convenient.

The example may be continued by using the alternative method of calculating I by means of q_2.

Inserting the values required on the right side of equation (1), page 190, it becomes

$$q_2 = \frac{0{\cdot}556 - 0{\cdot}311 + 0{\cdot}987}{1{\cdot}445} = 0{\cdot}853$$

The total energy of 1 lb. of the expanded steam at the lower temperature 100° is then

$$I = I_w + 0{\cdot}853L = 100 + 0{\cdot}853 \times 539{\cdot}3 = 559$$

as before.

The tabular volume at the final temperature is 26·78 cub. ft. from Steam Table 2. Therefore the volume after adiabatic expansion from the given initial conditions is 0·853 × 26·78 = 22·83 cub. ft.

It is sometimes convenient to use the approximate relation, PV^m = a constant, for the adiabatic expansion of wet steam similar to the relation PV^γ = a constant for the adiabatic expansion of a perfect gas. An appropriate value of m for given initial and final conditions may be found as follows:—

Assume that PV^m = a constant. Then—

$$P_1V_1^m = P_2V_2^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so that the final volume V_2 is

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{m}} V_1$$

For the sake of generality, the steam may be assumed initially wet, so that the initial volume V_1 is equal to $q_1V_{1_{tab}}$ (the subscript "tab" means that $V_{1_{tab}}$ is the volume recorded in the Steam Tables), and the final volume is $q_2V_{2_{tab}}$.

Therefore
$$q_1V_{1_{tab}}\left(\frac{P_1}{P_2}\right)^{\frac{1}{m}} = q_2V_{2_{tab}}$$

And
$$m = \frac{\log \dfrac{P_1}{P_2}}{\log \dfrac{q_2V_{2_{tab}}}{q_1V_{1_{tab}}}} \quad . \quad . \quad . \quad . \quad (5)$$

This result shows that m cannot be constant.

Its value when initially dry steam expands abiabatically from 150 lbs. per square inch to 90 lbs. per square inch is 1·148; if the expansion is carried on to 20 lbs. per square inch the index falls to 1·135; and if the expansion is continued down to 1 lb. per square inch the index falls to 1·126.

Also if steam initially wet with a dryness fraction of 0·7 expands adiabatically from 150 to 20 lbs. per square inch, m has the value 1·115.

In using the approximate equation (4) it is therefore advisable to calculate a value of m suitable to the conditions of the problem.

When superheated steam expands adiabatically, so long as it

remains superheated, the relation between the pressure and the temperature is given by

$$\frac{P}{T^{(n+1)}} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

the relation between the pressure and the volume is given by

$$P(v - b)^{\frac{n+1}{n}} = P(v - b)^{1\cdot3} = \text{a constant} \quad . \quad . \qquad (7)$$

and the relation between the volume and the temperature by

$$(v - b)T^{n} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

These relations are proved as follows.

The expression for the entropy of superheated steam is, from equation (22), page 181,

$$\phi = K_p^o \log_e T - R \log_e P - \frac{ncP}{JT} + \text{a constant}$$

But, according to Callendar, $K_p^o = (n + 1)R$. And c is equal to $C\left(\frac{T_o}{T}\right)^n$, page 172. Therefore

$$\phi = R \log_e \left(\frac{T^{n+1}}{P}\right) - \frac{nCT_o^n}{J}\left(\frac{P}{T^{n+1}}\right) = \text{a constant during reversible adiabatic expansion.}$$

This relation can only be true if

$$\frac{P}{T^{(n+1)}} = \text{constant}$$

and therefore relation (6) is proved.

Also, it follows from (6) that $\frac{ncP}{T}$ is constant.

Again, from the characteristic equation for steam, equation (2), page 171, multiplying through by P and dividing by T,

$$\frac{P(v - b)}{T} = JR - CT_o^n\left(\frac{P}{T^{n+1}}\right)$$

Therefore, since the right side is a constant,

$$\frac{P(v - b)}{T} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

Eliminating T between (6) and (9), the relation given in (7) is obtained, and eliminating P the relation in (8) is obtained.

Supposing therefore that expansion starts from the initially superheated state marked by the state point S, Fig. 60, page 183, the expansion will follow the relation given in (7) until it comes to the boundary line, after which it will cross the boundary and follow the approximate law given by (4), page 192, or the accurate law expressed by relation (1), page 190; or by relation (3), page 191.

54. Work per pound of Steam. The Rankine Cycle.—The quantity of work which can be obtained from a pound of steam depends upon the cycle of operations through which it is made to pass in the engine.

Whatever be the cycle of operations established, the theoretical greatest quantity of work which can be obtained per pound of steam passing through the cycle can be computed by the aid of the Steam Tables and the second law of thermodynamics.

The particular cycle of operations which yields the maximum amount of work per pound of steam is the Carnot Cycle for a vapour, described in Section 43, page 155. No cycle can be devised which enables a greater quantity of work to be obtained, because in the Carnot Cycle the whole of the heat is taken in at the highest temperature, the temperature of evaporation, and all the heat rejected is rejected at the lowest temperature, the temperature of condensation. The heat taken in during evaporation is, in this cycle, simply the latent heat of the steam. Therefore, applying the second law directly, the greatest quantity of work which can be obtained from this quantity of heat is

$$\frac{L_1(T_1 - T_2)}{T_1} \text{ lb.-calories} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where L_1 is the latent heat corresponding to the temperature of reception T_1, and T_2 is the temperature of condensation.

This cycle involves, however, an operation which is practically impossible, namely, the fourth. In the fourth operation the working agent is raised from the lower to the higher temperature by the process of adiabatic compression. Practically applied, this means that the third process, the process of condensation, must be stopped at a point short of complete condensation, and that condensation must be completed and the temperature must be raised to the temperature of evaporation by adiabatic compression alone.

Rankine,[1] and independently Clausius,[2] devised a cycle of operations in which the fourth operation is replaced by one which has some relation to what is practically possible. The first two processes of the Carnot Cycle are retained, but in the third process condensation is carried out completely, and in the fourth process the temperature of the condensed water is raised to the higher temperature by adding heat instead of by adiabatic compression.

In order to realize Carnot's rule of maximum efficiency the heating circuit, and the water must be at the same temperature when heat is transferred, and therefore as the water increases in temperature the heating circuit or source must be imagined to increase in temperature also, so that there is no drop of temperature between them.

[1] Rankine, "On the geometrical representation of the expansive action of heat and the theory of thermodynamical engines," *Phil. Trans.*, 1855.

[2] Clausius describes the same cycle in a Memoir published in *Poggendorff's Annalen*, in March and April, 1856. See Appendix VIII. of the Report of the Committee on the Thermal Efficiency of Steam Engines, *Proc. Inst. C.E.*, vol. 134, p. 311.

In practice the increase of temperature from the temperature of condensation to the temperature of evaporation is made either in a feed water heater or in the boiler itself.

Imagine 1 lb. of water, at the temperature of condensation T_2, occupying the volume w in the cylinder Fig. 61. The stages of the Rankine Cycle may be enumerated as follows, it being assumed that the load P on the piston is automatically regulated to the pressure inside the cylinder, so that the pressure of the working agent on the piston is always in equilibrium with the pressure P applied by the piston :—

Stage a.—Reception of heat $h_1 - h_2$ until the temperature is increased from the temperature of condensation to the temperature of evaporation, the pressure being increased as the process goes on.

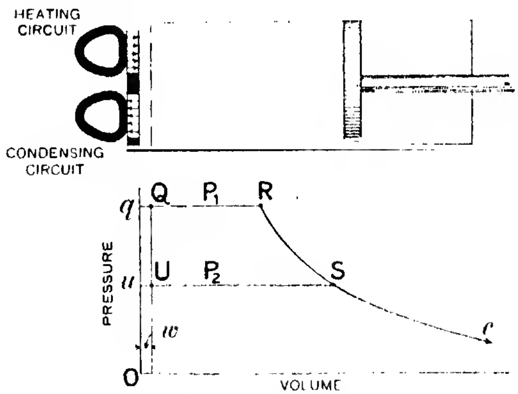


FIG. 61.—The Rankine Cycle.

Stage b.—Reception of heat L at constant pressure of evaporation, the volume increasing from Q to R.

Stage c.—Adiabatic expansion from the pressure of evaporation to the temperature and pressure of condensation, along the adiabatic curve RS.

Stage d.—Rejection of heat at constant pressure until the steam is reduced to 1 lb. of water at the temperature corresponding to the pressure of condensation; the volume decreasing along SU to U. The state points U, Q, R, S, U, mark off the beginning and end of the four stages in succession.

The figure QRSU is the indicator diagram for the process and represents the work done.

Let I_1 be the total energy of the steam at R. This is represented by the area bounded by Oq, qR, the indefinitely prolonged adiabatic curve Re, and the volume axis. Let I_2 be the total energy at S. This is represented by the area OuSeO.

The difference between these areas is the area uqRS, which is

the indicator diagram for the cycle increased by the small area $uqQU = A(P_1 - P_2)w$ lb.-cals., a negligible quantity in all practical calculations.

Therefore U, the work done in a Rankine Cycle, is

$$U = I_1 - I_2 \text{ lb.-cals. per pound of steam} \quad . \quad . \quad (2)$$

This is true whether the steam be initially wet, dry and saturated, or superheated.

If the steam is initially dry, $I_1 = I_s$ and its value is found directly from the Steam Tables.

If the steam is initially wet, then—

$$I_1 = I_{w_1} + q_1L_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the total energy cannot be found unless the initial dryness fraction q_1 is given.

If the steam is initially superheated, $I_1 = I'$, the value of which can be found directly from Table 3, page 744.

The value of I_2, the total heat corresponding to the lower temperature in the cycle, can be found from—

$$I_2 = T_2\phi_1 - G_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which ϕ_1 is the adiabatic constant (see page 191), T_2 is the lower absolute temperature in the cycle, and G_2 is tabulated in the last column of Table 2, page 742, and in the equation corresponds to the tower temperature T_2.

Alternatively, the value of I_2 may be calculated from—

$$I_2 = I_{w_2} + q_2L_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which the dryness fraction q_2 is calculated from equation (1), page 190.

The efficiency of the Rankine Cycle is the ratio between the work done and the heat received. Thus

$$\text{Thermal efficiency} = \frac{U}{Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Q has different values according as the steam is initially wet, dry and saturated, or superheated.

The following three equations give the form of the ratio in the three cases:—

Steam initially dry $\qquad \dfrac{U}{Q} = \dfrac{I_s - I_{w_2} - q_2L_2}{I_{w_1} - I_{w_2} + L_1} \quad . \quad . \quad . \quad . \quad (7)$

Steam initially wet $\qquad \dfrac{U}{Q} = \dfrac{I_{w_1} + q_1L_1 - I_{w_2} - q_2L_2}{I_{w_1} - I_{w_2} + q_1L_1} \quad . \quad (8)$

Steam initially superheated $\dfrac{U}{Q} = \dfrac{I' - I_{w_2} - q_2L_2}{I' - I_{w_2}} \quad . \quad . \quad . \quad . \quad (9)$

The general expression for the efficiency is

$$\frac{U}{Q} = \frac{I_1 - I_2}{I_1 - I_{w_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In this expression I_1 is the total energy of the steam at the higher temperature of the cycle. I_2 is the total energy of the steam after adiabatic expansion to the lower temperature of the cycle.

I_{w_2} is the energy of the water after condensation at the lower temperature. It is, in fact, the energy of the water when it arrives in the hot well, assuming that the condenser is perfect in the sense that it delivers water to the hot well at the temperature of condensation.

Consider again the indicator diagram QRSU, the area of which represents U, the work done in the Rankine Cycle between the pressure P_f and P_2. This area may be evaluated by using the approximate expression for the adiabatic curve PV^m = a constant $= P_1V_1^m$, from the integral

$$JU = \int_{P_2}^{P_1} v\,dp$$

This integral includes the negligibly small area $uqQU$.

From the adiabatic relation assumed

$$V = \frac{P_1^{\frac{1}{m}}V_1}{P^{\frac{1}{m}}}$$

Therefore $$JU = P_1^{\frac{1}{m}}V_1\int_{P_2}^{P_1}\frac{dp}{P^{\frac{1}{m}}} = \frac{P_1^{\frac{1}{m}}V_1\left(P_1^{1-\frac{1}{m}} - P_2^{1-\frac{1}{m}}\right)}{1 - \frac{1}{m}}$$

which reduces to

$$JU = \frac{m(P_1V_1 - P_2V_2)}{m - 1} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The value of m is to be calculated from equation (5), page 192.

The final volume V_2 is calculated from the initial conditions by aid of the assumed relation that is

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

As shown in equation (7), page 193, the pressure-volume relation in the superheated region is $P(V - b)^{1 \cdot 3}$ = a constant.

Therefore $$JU = 1\cdot 3\left\{\frac{P_1(V_1 - b) - P_2(V_2 - b)}{0\cdot 3}\right\} \quad . \quad . \quad . \quad (13)$$

This equation ceases to apply when the steam becomes wet.

55. Use of the Rankine Cycle as a Standard of Comparison for the Performance of Actual Steam Engines.—In the explanation of the cycle of changes constituting the Rankine Cycle given

in the preceding section, all the changes of state of the working agent are assumed to take place in the cylinder itself. In practice the different stages of the cycles take place in separate parts of the steam plant. The energy is, in fact, given to the water, and the steam is formed in the boiler, from which it is transferred through the steam pipe to the engine cylinder, in which a part of the energy is transformed into mechanical work or motive power, and the remainder of the energy is abstracted from the steam in the condenser. To reproduce the cycle in an actual steam plant consisting of boiler, engine, and condenser, it must be assumed that:—

(1) The steam is formed in the boiler at the constant pressure corresponding to the upper temperature of the cycle, and is then transferred to the cylinder without loss of heat energy, so that when 1 lb. of steam has been transferred the volume occupied in the cylinder is equal to the volume which it would have occupied if it had been evaporated in the cylinder.

(2) That adiabatic expansion takes place in the actual cylinder exactly as in the ideal cylinder.

(3) That the steam is pushed out through the exhaust pipe and appears in the condenser at the same pressure as the back pressure in the cylinder, and is then condensed at the temperature corresponding to this pressure.

(4) That the water after condensation appears in the hot well at the temperature of condensation corresponding to the back pressure in the cylinder.

In an actual engine these conditions are only imperfectly realized owing to losses of energy due to radiation, conduction, leak, unresisted expansion, condensation, losses due to transference from the boiler to the cylinder, and from the cylinder to the condenser, and again to the hot well, all of which will be considered in detail later.

Therefore in considering the performance of a steam engine, we have to compare the actual quantity of work W obtained per pound of steam with the quantity U which is actually available between the pressure of formation and the pressure of condensation.

In making the comparison there is, however, some difficulty in fixing what ought to be considered the pressure of formation and the pressure of condensation in the Rankine Cycle with which the actual performance is to be compared.

Considering the motive-power circuit as a whole, the pressure of formation should be fixed at the pressure at which the steam is produced, namely, the pressure corresponding to the load on the safety valve; and the pressure of condensation at the pressure corresponding to the lowest temperature in the circuit, namely, the temperature of the feed water. If the steam is superheated then the temperature of superheat in the superheater itself is the temperature from which to reckon U.

The practical objection to this is that the imperfections of the boiler part of the motive-power circuit, and also of the condenser part, are not distinguished from the imperfections of the engine part.

So that, with temperatures chosen as above, the standard which would be applied to a combination consisting of a good engine and a bad boiler and a bad condenser, would be equally applied to a plant including a good boiler and a bad engine providing that the boiler pressure and the feed temperature were respectively the same in each case. There are, therefore, reasons of a practical and a commercial nature for calculating the efficiency of a part only of the whole motive-power circuit.

A Committee[1] of the Institution of Civil Engineers have recommended that the actual performance of a steam engine shall be compared with the performance of an ideal engine working on the Rankine Cycle in which the pressure of formation is to be taken equal to the pressure of the steam supplied to the actual engine, measured in the steam pipe near to, but on the boiler side of the stop valve; and the pressure of condensation in the ideal engine is to correspond with the pressure measured in the exhaust pipe of the actual engine outside but close to the engine.

In making the comparison in this way the inefficiencies of the other parts of the motive-power circuit are excluded.

The absolute efficiency of an actual engine can only be found experimentally by measuring the work done and the heat supplied. If W is the work done as measured by the indicator, and if H is the heat supplied as determined from the boiler feed or air-pump discharge, the absolute thermal efficiency is $\frac{W}{H}$.

There are, therefore, three efficiencies concerned in the performance of a steam engine which are of interest. These are

(*a*) the absolute thermal efficiency found experimentally $\frac{W}{H}$

(*b*) the absolute thermal efficiency of the ideal engine of comparison working between arbitrarily fixed limits of pressure, and therefore temperature, chosen as directed above . . . . $\frac{U}{Q}$

(*c*) the ratio of these two efficiencies . . . . $\frac{WQ}{HU}$

The third ratio is the efficiency of the actual engine relative to the standard engine of comparison, and is specifically named by the Committee "the efficiency ratio".

In applying the standard of comparison to a non-condensing engine, the pressure of the atmosphere fixes the lowest temperature to which the steam can fall. This temperature is about 100° C., and it should, therefore, be taken as the lowest temperature to be used in calculating U.

In the case of a locomotive the upper temperature is suitably fixed by the pressure of formation in the boiler itself, because

[1] Report of the Committee on the Thermal Efficiency of Steam Engines, *Proc. Inst. C.E.*, vol. 134, 278.

following the recommendations of the Institution Committee the pressure would have to be taken in the dome. A point in the exhaust pipe outside but near to the cylinder falls between the cylinder and the blast pipe. The pressure at this point is largely influenced by the speed of the engine, and is not, therefore, a very suitable place to make the measurement. All things considered the most suitable temperature to take as the lower limit from which to calculate U is that corresponding to the atmospheric pressure. The variation of this pressure is so small compared with the variation of the upper pressure, that for purposes of comparison it may be

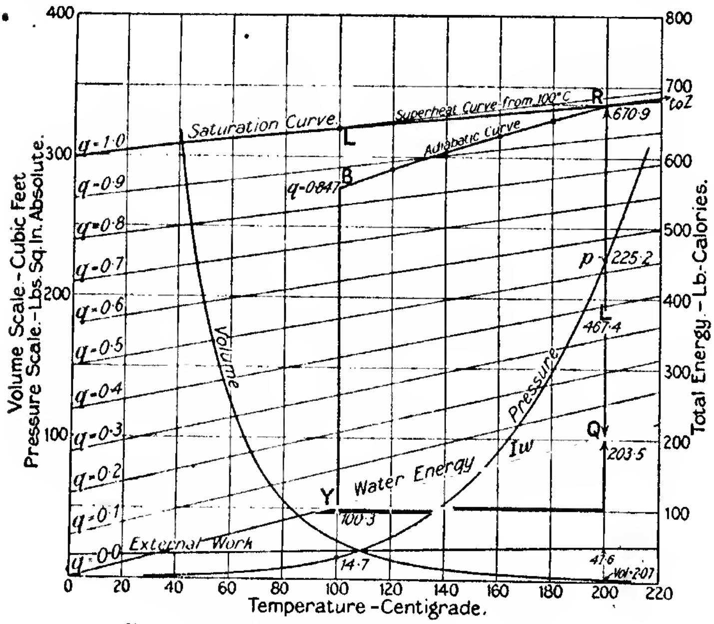


FIG. 62.—Diagram showing properties of 1 lb. of steam.

considered constant at 14·7 lbs. per square inch; and therefore the lower temperature may be considered constant at 100° C.

Standard efficiencies calculated from temperature limits fixed in this way are shown in Fig. 86, page 301.

56. Diagram showing the Properties of Steam.—The properties of a pound of steam are exhibited graphically in Fig. 62 where the pressure, the specific volume v of 1 lb. of dry saturated steam, the total energy I_s, the liquid heat h added to APw, that is the water energy I_w, the external work $AP(v - w)$, are severally plotted against the temperature centigrade.

The vertical distance between the total energy line for 1 lb. of dry saturated steam, that is the saturation curve, and the liquid energy line OYQ, represents the latent heat L. The vertical distance representing the latent heat is intersected by lines dividing it into 10 equal parts. These lines are lines of constant dryness. The value of q corresponding to each line is written against it.

Above the saturation curve are lines showing the increase of energy as the steam is superheated at constant pressure from 100° C. and also from 200° C.; only a small portion of the line falls on the diagram in the latter case. By the aid of this diagram the total energy of the steam corresponding to any assigned position of a state point may be read off on the scale to the right of the diagram.

Thus starting with water at 0° C., and tracing the formation of 1 lb. of steam superheated to 300° C., that is 100° C. above the temperature of formation, which is, let us say, 200° C., it will be seen that heat is absorbed, and that the state point moves along the liquid energy line until it arrives at Q, where the temperature remains constant and steam begins to form. After passing through the several degrees of dryness the state point reaches R, the position corresponding to complete dryness and saturation. As the supply of heat to the steam is continued the state point moves along the line RZ until it reaches a position where RZ produced is cut by a vertical through the temperature 300°. The total energy of the steam at the end of the process is 727 lb.-cals.

The energy absorbed from the beginning up to any stage of the process is found by referring the state point horizontally to the scale on the right of the diagram.

A horizontal line on the diagram is a line of constant total energy. If therefore steam changes from one state to another without doing work and without gain or loss of heat, the energy will remain constant, and the state point at the end of the change will be found on a horizontal line drawn through the initial position.

A change of state at constant total energy may be made by throttling the steam through a non-conducting porous plug or through a non-conducting tube of small bore. The change at constant energy is, however, only imperfectly realized because it is impossible to avoid some exchange of heat with the environment.

This throttling process may be illustrated on the diagram. Suppose a pound of dry saturated steam at 225 lbs. per square inch is throttled down to 14·7 lbs. per square inch. Draw a horizontal line through R to cut the line of constant pressure through L. A vertical through the point of intersection will cut the temperature scale at the temperature to which the steam is superheated after the process of throttling is finished. Problems of this kind are more easily followed on the corresponding diagram plotted with oblique co-ordinates as shown in the next Section.

The state of the steam through the Rankine Cycle can easily be followed on the diagram.

Starting at the point Y, the temperature of condensation, the total

energy of the water is represented by the ordinate 100-Y. The state point then moves along the water energy path as the water is heated until a point Q is reached corresponding with the pressure of formation and then along the vertical path to R as the latent heat is added. Arriving at R the steam is then expanded adiabatically to the lower temperature corresponding to the pressure of condensation along the adiabatic path RB. The steam is wet at the end of this process. From B to the initial point Y heat energy is removed by condensation and the cycle is complete when sufficient heat has been removed to bring the state point to Y. Methods are given above on pages 190 and 191 for calculating the dryness after adiabatic expansion or alternatively for calculating the total energy after adiabatic expansion. The point *B* can be fixed by either method.

The diagram shown in Fig. 62 should be plotted by every student of the subject since it is a graphical representation of the Steam Tables. The constant pressure lines in the superheated region are rather crowded together in this form of diagram. They can, however, be opened out by using oblique co-ordinates in the way described in the next section.

57. The Total Energy Temperature Diagram.—A diagram useful in the solution of many practical problems is obtained by plotting vertically the Temperature against the Total Energy set out along an axis inclined 135° to the vertical axis, as shown in Fig. 63. The diagram is in fact the diagram of Fig. 62 sheared down in order to open out the network in the superheated region.

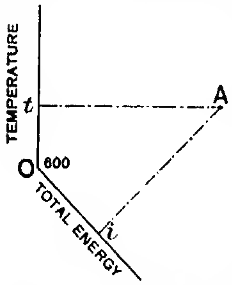


FIG. 63.—Axes of the total energy temperature diagram.

In this diagram lines of constant temperature are horizontal and lines of constant total energy are inclined 45° to the vertical.

It is convenient to select the origin O to correspond with zero on the ordinary centigrade scale and 600 on the energy scale, energy being measured in pound-calories.

For example, let A be a state point on the diagram. Draw a horizontal through A to cut the vertical axis in *t*, and draw a line through A at 45° to the vertical to cut the oblique axis in *i*. Then *t* is the temperature, and *i* is the total energy corresponding to the state A.

The most convenient form in which to arrange the diagram is shown in Fig. 64, where a horizontal line OI is drawn through O and an energy scale IQ inclined 45° to the vertical is drawn above this line. Lines at 45° through the divisions of the scales OI or IQ are lines of constant total energy.

The total energy corresponding to any state point is found by drawing a line through the point at 45° to cut either the horizontal scale OI or the oblique scale IQ.

A line joining points giving the same reading on the two scales is

a line of constant total energy. Therefore by using the two scales in this way a set-square can be placed across the diagram at 45° to the vertical axis, though it is generally more convenient to project to one or other of the scales by a set-square working from a T-square, the diagram being pinned down to a drawing board.

AB is the boundary curve or curve of saturation. It is plotted from the tables. AB is the path along which a state point would move if steam, dry and saturated at O, remained dry and saturated as the temperature changed to B. Further, projecting the point A and B

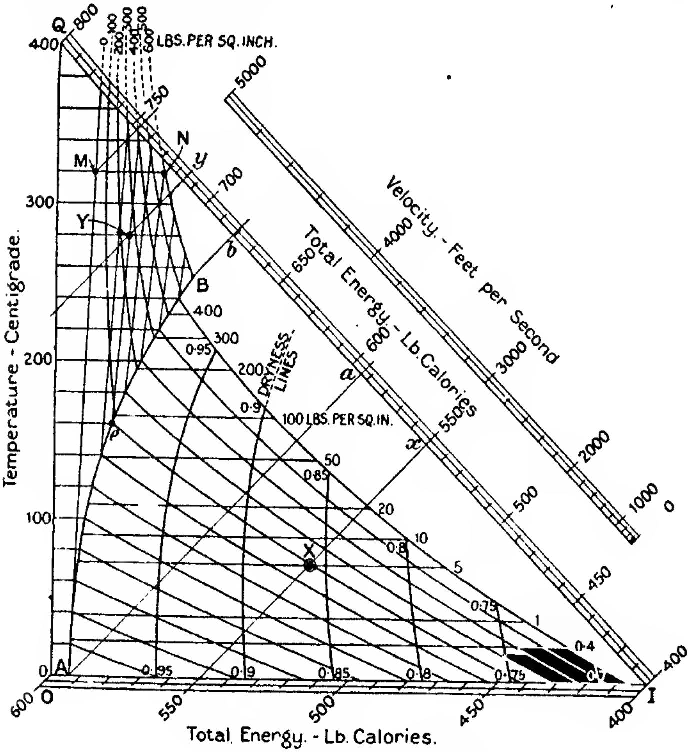


FIG. 64.—Total energy temperature diagram (Dalby).

to the energy scale, the length ab on the energy scale gives the heat energy which must be added to the steam as the change takes place in order to maintain it in the dry and saturated condition. The actual value of ab is found by taking the difference of the scale readings at b and a.

To the left and above AB is the superheated region of the diagram.

To the right of AB is the wet region.

It will be seen that both regions are formed into a network by intersecting families of lines.

In the wet region the horizontal lines are lines of constant pressure; the lines following the general direction of AB are lines of constant dryness; and the curves originating in the boundary curve AB and sloping down to the right like expansion curves are adiabatic curves.

Let X be the position of any state point in this wet region. A line at 45° through it cuts the scale IQ in x. Then from the diagram it will be seen that—

The total energy of the steam is	550 lb.-cals. per lb.
The pressure is	5 lbs. per sq. in.
The temperature is, by reference to the scale	72° C.
The dryness is	0·86.

In the network to the left of AB the horizontal lines are lines of constant temperature; the lines curving and closing together as they ascend from the boundary AB are lines of constant pressure and they meet in the boundary line the corresponding horizontal lines of constant pressure crossing the wet region; the curves sloping off to the right as they ascend are adiabatic curves.

Let Y be the position of a state point in this superheated region. A line at 45° through Y cuts the scale IQ in y. Then from the diagram it will be seen that—

The total energy of the steam is . . .	716 lb.-cals. per lb.
The temperature of superheat is . . .	280° C.
The pressure of formation is	260 lbs. per sq. in.

These two examples show how the diagram can be used to get the properties of a pound of steam when sufficient data are given to enable a state point to be located on the diagram.

The diagram, however, can be used for other purposes than this.

Let the state point move from the position Y to the position X along the adiabatic path YeX. Then the length xy on the total energy scale represents the difference of the total energies in the two states, that is $I_1 - I_2$, and this represents U, the work which is available between these two states.

Moreover, as the state changes the condition as regards superheat temperature and pressure to the left of the boundary line, and the state as regards pressure and dryness when the point has crossed the boundary line into the wet region, can be read off for any position of the point in the adiabatic path YeX.

The value of a good vacuum can be easily studied from this diagram. As the point X moves along the adiabatic curve eX to lower temperatures the increase in available energy as the pressure falls is read off directly, and it will be seen how largely the available energy is increased by small reductions of pressure when the pressure is low.

For example, it will be seen from the diagram that a reduction of

pressure from 1 lb. per square inch to 0·4 lb. per square inch along the adiabatic eX corresponds to 23 lb.-cals. per pound.

The total energy at e is, either from the Steam Tables or from the diagram, 667. The total energy if the condenser pressure is 1 lb. per square inch is 500 lb.-cals.

Therefore, U = 667 − 500 = 167 lb.-cals. per pound. A reduction of the condenser pressure to 0·4 lb. per square inch makes 23 lb.-cals. more available, giving a total of 190 lb.-cals. per pound. So that a reduction of pressure of 0·6 lb. per square inch at this pressure increases the available energy U nearly 14 per cent.

The adiabatic curves are drawn on the diagram close enough together to enable an adiabatic to be sketched through any given state point.

It is instructive to follow in detail the plotting of the total heat temperature diagram because of the insight it gives into the properties of steam. The method is as follows:—

1. Set out a temperature scale along OQ and a total heat scale along IQ. Number the scales so that the origin O reads zero on the ordinary Centigrade scale and 600 on the energy scale.

2. With the data given in Steam Table 1, page 738, plot the saturation curve AB.

3. Draw the lines of constant pressure.

In the wet region these are horizontal. In the superheated region the group is to be obtained by means of equation (5), page 175, which is

$$I = 0{\cdot}4772T + 464 - \frac{144}{1400}\left\{\frac{13}{3}c - 0{\cdot}016\right\}p$$

When $p = 0$

$$I = 0{\cdot}4772T + 464, \text{ a straight line}$$

Choose any temperature, say 320°, and calculate the value of I when $p = 0$. For the temperature chosen, I = 747. Find 747 on the scale IQ and project it at 45° to meet the temperature line 320° in M. This fixes one point on the line of limiting pressure. A second point is fixed by putting T = 273 in the above equation together with the value of p given in Table 2, page 742, namely, 0·0892. This second point is very close to A. The line of limiting pressure is then drawn through these two points.

The form of the equation shows that along a line of constant temperature equal increments of pressure correspond to equal increments of energy, since the co-efficient of p is constant when T is constant, c the co-aggregation volume being a function of the temperature only.

Values of the co-efficient of p corresponding to intervals of pressure of 10 lbs. calculated from—

$$\frac{144}{1400}\left\{\frac{13c}{3} - 0{\cdot}016\right\}10$$

are given in the last column of Steam Table 3.

For the temperature 320° the value is 0·38. So that for an

interval of 600 lbs. per square inch, the range of the scale, the length on the energy scale is $0{\cdot}38 \times 60 = 22{\cdot}8$. But the value of I at M is 747. Therefore the scale reading for the pressure 600 lbs. per square inch and the temperature 320 is $747 - 22{\cdot}8 = 724{\cdot}2$. Project 724·2 at 45° to meet the temperature line 320 in N. Then MN is the distance on the diagram corresponding to a range of pressure of 600 lbs. per square inch. This length is then to be divided into appropriate subdivisions to form a pressure scale.

In a similar way a pressure scale is formed on other temperature lines. Curves drawn through corresponding points of these scales form the family of constant pressure lines in the superheated region. These curves should intersect the saturation curve AB at points corresponding to the saturation temperatures. Horizontals through these intersecting points complete the family of curves by a series of constant pressure lines in the wet region.

In the large diagram supplied with this book the temperature lines are not drawn below the saturation curve. A set of constant pressure lines are drawn instead, because in the wet region pressure lines are generally more convenient for dealing with practical problems; moreover, the temperature scale is an even one, so that the temperature at any point in the wet region can be read from a scale placed vertically on the diagram.

4. Draw the adiabatic curves.

The general expression for I, equation (4), page 175, which applies only to the superheated region, may be written—

$$I = \left\{K_p^o - \frac{144(n+1)cp}{1400T}\right\}T + \frac{144bT \cdot cp}{1400c \cdot T} + 464$$

But on page 193 it is shown that $\frac{cp}{T}$ is a constant along an adiabatic curve.

Let D stand for $\frac{144cp}{1400T}$. Then neglecting the small term involving b,

$$I = \{K_p^o - (n+1)D\}T + 464$$

This equation shows that, with the approximation chosen, the adiabatics are, in the superheated region, straight lines with different slopes, the slope of any one line being determined by the value of the constant D. Hence to draw an adiabatic, first calculate D for some assigned temperature and then use the equation to find another point on the line.

For example, draw the adiabatic through the point 200° on the saturation curve. Taking the necessary data from the tables—

$$D = \frac{144 \times 0{\cdot}1909 \times 225{\cdot}22}{1400 \times 473{\cdot}1} = 0{\cdot}00933$$

Multiplying this by $n + 1 = \frac{13}{3}$, and reducing the term in the brackets—

$$I = 0{\cdot}4368T + 464$$

The point 200° on the saturation curve is, of course, one point on the adiabatic. A second point is found by calculating I for a selected value of T, as for instance, 320° = 593·1° absolute, with which value I = 723.

In the wet region points on the adiabatic corresponding to a series of temperatures may be calculated by equation (3), page 191. Values of G are given in Table 2, page 742, to facilitate the evaluation of this equation.

Continuing the illustration of the method with reference to the adiabatic passing through the point 200° on the saturation curve into the wet region, the equation is—

$$I = 1{\cdot}543T - G$$

where 1·543 is the adiabatic constant, found from Table 2 for 200° on the saturation curve, by adding ϕ_w to $\frac{L}{T}$, and G is the quantity tabulated in the last column of Table 2. When T is assigned, the corresponding value of G is selected from the last column of Table 2, and I can then be computed. Thus find I when the temperature falls to 50° C = 323·1° absolute. From Table 2, G = 4·3 at 50°. Therefore

$$I = 1{\cdot}543 \times 323 - 4{\cdot}3 = 494$$

Project this reading from the scale IQ or IO, at 45° on to a horizontal through 50° on the temperature scale, and a point on the adiabatic curve is fixed.

A sufficient number of points are fixed in this way, and then the curve is drawn through them.

In the large diagram adiabatic curves are drawn through points on the boundary curve corresponding to 10° intervals of temperature.

5. Lines of constant dryness.

Continue the temperature lines to the right across the boundary curve AB. Considering one of these horizontal lines, find the latent heat from the tables corresponding to the temperature it represents, and then step out on the energy scale intervals of $\frac{1}{10}$ of the latent heat or any smaller or greater fraction thought desirable and project the points so found to the temperature line. Join corresponding points lying on the succession of temperature lines by a smooth curve of constant dryness.

58. The Entropy Temperature Diagram.—Let A′A, B′B (Fig. 65) represent any two adiabatic curves on a pressure-volume diagram. Then it has been shown, page 161, that the heat taken in by the working agent during a change of state from any point, as A on AA′, to any point, as B on BB′, along a path of any shape, as AB, is represented by the area contained by the path AB and the indefinitely prolonged adiabatics through its end points A and B, and that the work done by the working agent during the change is represented by the area under the path AB and the verticals through its end

points. The work area is definite, but the area equivalent to the heat received is indefinite. It can, however, be made definite by taking advantage of the property involved in the second law, and specially demonstrated on page 162, under the heading Entropy, that between any two adiabatics the ratio of heat received Q to the temperature at which it is received T is a constant for all isothermal paths connecting the adiabatics, and that in general for any path with varying temperature

$$\int\frac{dq}{T} = \text{constant} = \frac{Q}{T}$$

Since, therefore, the position of the starting point A is immaterial, and the shape of the path is immaterial, consider a simple isothermal path A′B′. Then if Q units of heat, taken in at the constant temperature T, move the point from A′ to B′, $\frac{Q}{T}$ is the increase of this ratio, or the increase of entropy, as it is called, for all paths between the adiabatic, and is the constant in the equation above. For example, suppose that 37 units of heat received at a temperature of 370° absolute move the point from A′ to B′, then the increase of entropy is $\frac{37}{370} = 0{\cdot}1$. Draw two vertical lines (Fig. 65) at the distance $\frac{Q}{T} = 0{\cdot}1$ units apart; then they will represent the two adiabatics in the P.V. diagram, and will establish a division on a scale of entropy. The scale interval might equally have been derived from another point on the first adiabatic; for example, a point at 570° temperature. Then it would be found that to move the point from A to B at this higher temperature would require 57 units of heat, since $\frac{Q}{570}$ must equal $\frac{1}{10}$ for this particular pair of adiabatics.

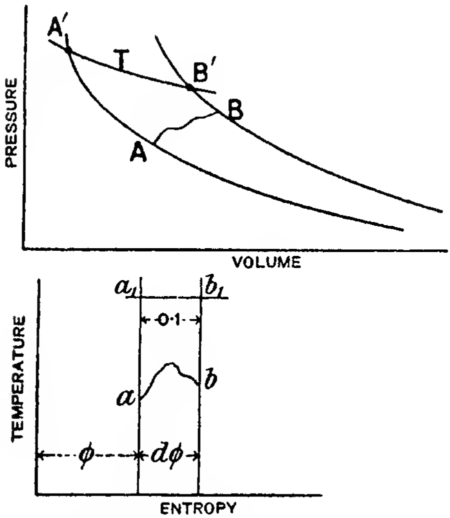


FIG. 65.—Derivation of entropy scale.

Any one of the properties of steam may now be plotted against this entropy scale. In particular, a useful diagram is obtained by

plotting the temperature. Then the horizontal axis is the axis of zero temperature, and the vertical axis the axis of zero entropy. The isothermal path A′B′ in the pressure-volume diagram becomes a horizontal line $a'b'$ in the new diagram, whilst the adiabatics of the pressure-volume diagram have become the vertical divisions of the entropy scale.

The area contained by the path $a'b'$ and the vertical adiabatics through the end points is then the definite area representing the heat taken in during the isothermal change A′B′. Similarly, the area under the curve ab is the definite area representing the heat taken in during the change along the corresponding arbitrary path AB.

The vertical distance between the end points of the path is the change of temperature during the process. There is no area, however, on a new diagram which represents the work done during the change.

The entropy temperature diagram can, of course, be derived directly from the definition of entropy given above, namely, that

$$d\phi = \frac{dq}{T}$$

hence

$$dq = Td\phi$$

From this it will be seen that the small quantity of heat received during a change is measured by the product of temperature and the variation of the entropy. The product $Td\phi$ represents the element of an area. The integral, therefore, namely,

$$Q = \int_A^B Td\phi$$

is the area included between a curve of temperature plotted against the entropy, the ordinates through the end points of the curve, and the axis of zero temperature on the absolute scale.

The vertical axis of the entropy temperature diagram may pass through any selected origin. It is usual to place it so that increase of entropy is measured from zero on the ordinary centigrade scale.

The horizontal axis must, however, always pass through the absolute zero of temperature.

Summarizing, the general properties of an entropy temperature diagram are: that an adiabatic change is represented by the movement of the state point in a vertical line; an isothermal change is represented by a movement of the point horizontally; that is to say, adiabatic curves are vertical straight lines, and isothermal curves are horizontal straight lines; and that the heat taken in during any change of state along any shaped path, as from a to b, is represented by the definite area under the path and the ordinates through its end points.

There is no area on the diagram which represents the work done during a change of state.

The path traced out on the diagram by the state point during the

formation of a lbs. of steam at constant pressure up to any degree of superheat can be traced out as follows :—

Select 200° as the temperature of formation.

Since entropy is measured from zero on the centigrade scale, entropy is zero when the temperature is 273°. This fixes the point A in the diagram, Fig. 66. Using the tables to find corresponding values of the temperature and the entropy for water ϕ_w, plot the curve AB, the distance 473–B being the entropy of water at 200° C.

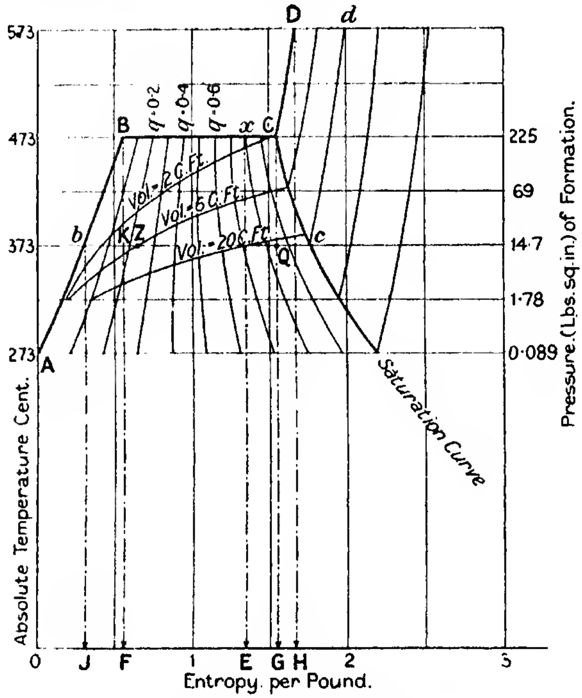


FIG. 66.—The entropy temperature diagram.

Then set out BC to represent the increase of entropy during evaporation at the constant temperature 200°. During this process the heat taken in is L, the latent heat. The increase of entropy is, therefore, $\frac{L}{473}$ units. Values of $\frac{L}{T}$ are given in both Steam Tables 1 and 2. Finally, plot the curve CD representing the increase of entropy during superheating. This can be plotted from the equation (23), Section 48, page 181, or from the simpler approximate equation

$$\phi_s + K_p\int\frac{dT}{T} = \phi_s + K_p\log_e\frac{T_3}{T_1} \quad . \quad . \quad . \quad . \quad (1)$$

In this equation K_p is a value of the specific heat derived from Table 3 to correspond with the pressure and range of superheat; ϕ_s is the entropy of the dry saturated steam at the temperature T_1, and T_3 is the temperature to which the steam is superheated

An approximate expression, based on the assumption that the specific heat of water is constant and equal to unity, can be used to calculate points on the part of the curve AB. With this assumption, the increase of entropy as the water is heated from T_2 to T_1 is

$$\phi_{w_1} - \phi_{w_2} = \int \frac{dT}{T} = \log_e \frac{T_1}{T_2}$$

Therefore, by using the tables or approximate methods of calculations, a curve ABCD in the entropy temperature diagram can be drawn, which represents the movement of the state point from A during the formation of a pound of steam superheated to any temperature.

The area ABFO under the part of the curve AB represents the heat taken in during the warming of the water.

The area BCGF under the part of the curve BC represents the heat taken in during the process of evaporation at 200° C., namely, the latent heat, 467·4 lb.-cals.

The area CDHG under the part of the curve CD represents the heat taken in during the process of superheating at constant pressure from 200° to 300° C.

The whole heat taken in as the point moves from A to D is the sum of these areas.

If the process of evaporation is stopped when only, say, 0·8 of the latent heat has been communicated, then the increase of entropy during evaporation is $0{\cdot}8 \frac{L}{T}$, an increase represented by the length Bx and given generally by $\frac{qL}{T}$, where q is the dryness fraction. The area BxEF is the heat received during the process of partial evaporation.

Adiabatic expansion from the state point C is represented by the vertical straight line through C.

The entropy temperature diagram for steam is formed by drawing curves similar to ABCD for a series of values of the formation temperature. The locus of the point C is a boundary curve between the wet region and the superheated region of the diagram. If the state point moves along this curve, a change is taking place in such a way that the steam remains always dry and saturated, and the heat absorbed during the change is the area under the curve and between the ordinates through the end points of the path marking the initial and final positions of the state point.

The locus of the point B is the curve representing the entropy of water. The curve through B and the curve through C merge into one another at the critical temperature where the latent heat, and therefore the distance BC $= \frac{L}{T}$, vanishes.

A state point x anywhere within these bounding curves denotes that the steam is wet, and the value of the dryness fraction is found by drawing a horizontal line through the state point to cut the boundary curves in points B and C, and then calculating the ratio $\frac{xB}{BC}$.

If the horizontals between the boundary curves are divided each into ten equal parts, curves drawn through corresponding points are curves of constant dryness. The diagram, Fig. 66, is shown divided in this way by curves which enable the dryness fraction to be read off by inspection for any position of the state point.

Again, during a change of entropy BC the volume changes $(v - w)$ cub. ft., or with sufficient accuracy v cub. ft., and the change is made uniformly as heat is added, so that if only half the latent heat is added, the change of volume is only $\frac{1}{2}v$. Let v be any particular volume, 6 cub. ft., for example. Then if a series of points, as Z, be taken on successive horizontals, such that

$$bZ : bc = 6 : v = q : 1$$

a curve through them will be a curve of constant volume 6 cub. ft. When lines of constant dryness are drawn on the diagram, the point Z is easily located from q, and q is calculated from

$$q = \frac{V}{v}$$

The volume v is found from the steam tables corresponding to any horizontal, that is any temperature on which Z is to be plotted.

Three constant-volume curves are shown in the figure. Thus, an entropy temperature chart consists fundamentally of a series of entropy curves like ABCD, plotted from the steam tables or from suitable expressions, with the addition of lines of constant dryness and lines of constant volume. A complete chart of this kind has been published by Captain Sankey.[1]

The development of the diagram in connection with the solution of problems is mainly due to MacFarlane Gray.[2]

A Carnot Cycle on the diagram between the temperatures BC and bc, starting at the upper temperature, is represented by the movement of the state point from the initial position B along the isothermal BC to C during the reception of heat, and then down the adiabatic line from C to the lower temperature Q, followed by the movement of the point from Q to a point K vertically below B during the rejection of heat at the lower temperature, and finally a movement from this point K vertically to B corresponding to adiabatic compression from the lower to the higher temperature. The point traces out the boundary of a rectangle BCQK. During the movement of the point from B to C the heat absorbed is represented by the rectangle BCGF. During the movement of the point from Q to K the heat rejected is

[1] Captain Sankey, "The Energy Chart," Albert Frost & Sons, Rugby, 1905.
[2] MacFarlane Gray, *Proc. Inst. Mech. E.*, 1889, 411.

represented by the area of the rectangle QKFG. The difference, represented by the area BCQK, has been transformed into work. The efficiency of the cycle is the ratio of the areas of the rectangles BQ and BG, that is BK : BF.

Notice that as adiabatic expansion proceeds along CQ the steam gets wetter, and that the final wetness is given by the ratio $\frac{oQ}{bc}$.

Notice also that in order to secure that adiabatic compression shall bring the state point back to B, condensation must be stopped at K when the wetness is $\frac{bK}{bc}$, always assuming that such a process is possible.

The Rankine Cycle for the same temperature limits is represented by a movement of the state point from the initial condition of water at b along bB during the warming process, along BC during the process of evaporation, along CQ during complete adiabatic expansion to the back pressure corresponding to the lower temperature, and along Qb during the process of complete condensation.

The heat taken in during the whole cycle is represented by the area bBCGJ. The heat rejected to the condenser is represented by the area QbJG. The heat transformed into work is the area bBCQ. The efficiency is the ratio of these two areas $\frac{b\text{BCQ}}{b\text{BCGJ}}$.

Comparing these areas with those concerned in the Carnot Cycle, it will be seen that the Rankine Cycle is not quite so efficient, because of the effect of the sloping line bB along which heat is taken in at gradually increasing temperatures, but always below the maximum temperature. In the Carnot Cycle no heat is used to raise the temperature from the lower to the higher value. It is done by adiabatic compression. In the Rankine Cycle heat is used for this purpose at an average temperature about midway between the lower and the higher temperature. And none of the heat added during the process of warming the feed water can be used between the two temperatures of the engine as efficiently as if it were all added at the higher temperature.

The consideration of these two cycles has brought out this point, that although there is no area on the diagram which represents the work done during a change of state, yet when the state point describes a closed curve the area enclosed represents the difference between the heat received and the heat rejected during the cycle, and if all the processes are reversible this area is equivalent to the heat transformed into work during the cycle.

When the point moves round the boundary of the area in the clockwise direction, the work done is positive, corresponding to work done by expansion of the working agent; when it moves in the counter-clockwise direction, the work done is negative, as in compression of the working agent.

The entropy itself returns to the same value at the initial point

of the cycle, so that on the whole there has been no gain nor loss of entropy. This is expressed by the relation that for reversible cycles

$$\int \frac{dQ}{T} = 0$$

This relation may be regarded also as another expression of the second Law of Thermodynamics.

59. The Mollier Diagram.—This diagram, Fig. 67, is constructed by plotting the total energy against the entropy. The

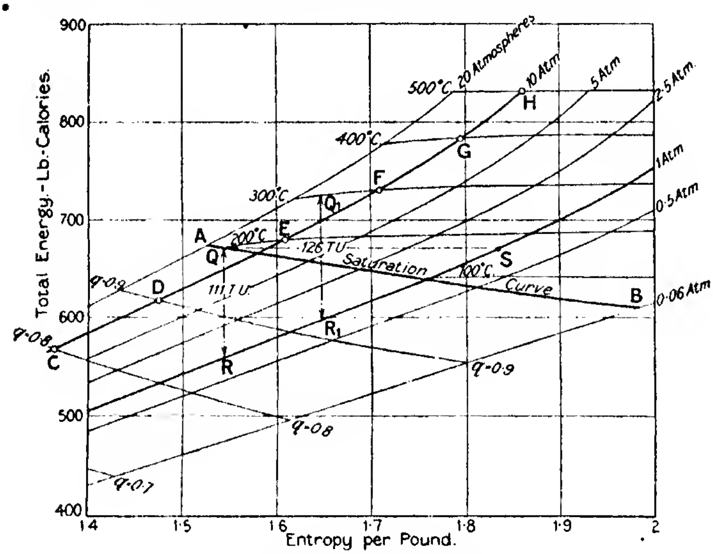


FIG. 67.—Total energy-entropy diagram.

saturation curve AB is first plotted from the tables. Lines of constant pressure intersected by lines of constant dryness form a network in the wet region below AB, whilst above it the lines of constant pressure are intersected by lines of constant temperature.

A movement of the state point horizontally corresponds to a change made at constant total heat. A movement vertically corresponds to a change at constant entropy. Also since a reversible adiabatic change is a change at constant entropy a vertical movement also corresponds to an adiabatic change.

The network in the wet region may be plotted by calculating the co-ordinates of the intersections of the constant pressure and constant dryness lines as follows:—

In general

$$I = I_w + qL \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\phi = \phi_w + \frac{qL}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

To find points on a line of constant pressure P, first find from the tables the temperature corresponding to P, assign a value to q, and then by the aid of (1) and (2) calculate I and ϕ. The point plotted with these co-ordinates will lie on the pressure line P and also on the dryness line q. Thus let P be 10 atmospheres corresponding say to 150 lbs. per square inch, and let $q = 0{\cdot}8$. From the tables $T = 454{\cdot}4$, $I_w = 183{\cdot}6$, $\phi_w = 0{\cdot}514$, $\frac{L}{T} = 1{\cdot}0627$, $L = 482{\cdot}9$. With these data $I = 570$ and $\phi = 1{\cdot}364$, and C is the corresponding point on the diagram, Fig. 67. With the same data, except that q is $0{\cdot}9$, a second point on the 10-atmosphere line is plotted at D. Calculate the values of I and ϕ for a series of values of q, and the co-ordinates so found determine a series of points on the 10-atmosphere line, each point in the series being in addition a point on a line of constant dryness.

Plot a series of constant-pressure lines in this way, and then draw curves through points on them of the same dryness, and the network in the wet region is complete.

Values of I' for various values of P and T in the superheat region are given in Table 3. It remains to find the corresponding values of ϕ.

This can be done by calculating a series of values of ϕ along a line of constant pressure for a series of assigned values of T. From equation (22), page 181, writing a subscript s for the condition of dry saturated steam,

$$\phi - \phi_s = K_p^0 \log_e \frac{T}{T_s} - R \log_e \frac{P}{P_s} - nA\left(\frac{cP}{T} - \frac{c_s P_s}{T_s}\right) \quad . \quad (3)$$

With the pressure constant $P = P_s$ and the second term in (3) vanishes, and P comes outside the bracket in the third term. The values of ϕ_s, T_s, and c_s, are those corresponding to the saturation temperature belonging to the constant pressure P.

For P = 10 atmospheres, taken equal to 150 lbs. per square inch,

$$T_s - 454{\cdot}4, \phi_s = 1{\cdot}5765, c_s = 0{\cdot}2184.$$

With these values equation (4) becomes

$$\phi = 1{\cdot}5765 + 0{\cdot}4772 \log_e \frac{T}{454{\cdot}4} - 51{\cdot}4\left(\frac{c}{T} - 0{\cdot}00048\right) \quad . \quad (4)$$

The value of c for the assigned temperature T is calculated from

$$c = 1{\cdot}192\left(\frac{273{\cdot}1}{T}\right)^{\frac{10}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For example, with T = 400 = 673·1° absolute, c from (5) is 0·0588,

and ϕ from (4) is 1·784. Also, from Table 3, I′ = 781·5; therefore the co-ordinates of the point at the intersection of the constant pressure line 10 atmospheres and the constant temperature line 400, are

$$I' = 781 \cdot 5$$
$$\phi = 1 \cdot 784$$

This point is shown at G, Fig. 67.

Points E, F, and H, corresponding to temperatures 200, 300, and 500 on the 10-atmosphere line, are found in a similar way.

The line through the points CDEFGH is thus the complete locus of a point moving from the wet region into the superheated region at the constant pressure of 10 atmospheres.

Other pressure lines are found in a similar way, and then joining points on them at the same temperature the network in the superheat region is complete.

The outstanding feature of this diagram is that the vertical distance between any two points represents the available energy U when the steam passes from the state represented by the upper point through a Rankine Cycle, the lower temperature of which corresponds with the lower point.

For example, the initial condition of dry saturated steam at a formation temperature of 200° is shown by the point Q on the diagram, Fig 67. If condensation in the Rankine Cycle takes place at one atmosphere pressure, a vertical from Q on to this line intersecting it in R defines a length QR, which represents on the energy scale to the left the value of U.

With the conditions assumed, U scales 111 lb.-cals. If the steam is supplied to the engine after being superheated at constant pressure from 200° to 300° C., the initial state point is at Q_1 and the final point at R_1. The available energy U is by measurement 126 lb.-cals.

The horizontal line QS shows a change at constant total heat. The initial condition is dry saturated steam at 200° C. In the final condition the steam is at a pressure of 1 atmosphere, and is superheated to 160° C.

Mollier has also plotted a diagram of Pressure and Total Heat.

60. Irreversible Processes.—When a temperature difference exists work can be done. The maximum quantity of work which can be done is expressed by the second law, namely,

$$W = Q\frac{dT}{T}$$

This law requires that in the cycle of operations through which the working agent passes all the changes of state shall be made by reversible processes.

In general changes of state take place by irreversible processes and the work obtained from a given temperature difference is less than the amount defined by the second law.

For example, steam passing through the admission valve of an

engine always falls to some extent in pressure, and therefore in temperature, by throttling. So that at the outset of the cycle of pressure-volume changes in the cylinder, part of the temperature difference established by the heating and the condensing circuits is used in merely causing the steam to flow into the cylinder. No external work is done, and part of the temperature fall is sacrificed.

Again, steam passing through a steam turbine is heated by the frictional resistances opposing its flow. Heat has already been converted into mechanical energy to produce the velocity of flow, and this mechanical energy is in part re-converted into heat by friction. This is an irreversible process.

Again, two fluids at different temperatures mix together and gradually come to a common temperature. The original conditions at the instant of mixing cannot be recovered. Therefore, the process is irreversible.

The freezing of an overcooled liquid is an irreversible process. An example relating to the irreversible process of the condensation of supersaturated steam is given on page 599.

A general principle operating in many of these irreversible processes is that when the work obtained from a given temperature difference is less than the amount defined by the second law, heat energy equivalent to the loss appears in the working agent at the end of the process. Heat is in fact added to the working agent internally, equivalent to the work of the unutilized temperature difference.

A reversible change of state made adiabatically involves two conditions. One, that during the change, the working agent neither gains nor loses heat to an external source; the other that the entropy is constant during the change.

If heat is added internally as a result of irreversible processes, the first condition is fulfilled but the second is not, because the addition of heat internally increases the entropy.

For a reversible process an adiabatic curve is a curve of no heat transmission and also a curve of constant entropy, that is, an **isentropic curve.** For an irreversible process the adiabatic curve cannot be drawn, because the processes cannot be followed in detail during a change of state. In general, after an irreversible process, only the final state of the working agent is determinate.

Consider, for example, the simple case of adiabatic throttling. The final state of the working agent after throttling from one pressure to a lower pressure without exchange of heat with its environment is the result of a change made at constant total energy. Given the initial conditions, the final state can be found at once from the total energy temperature diagram, by drawing a line at 45° through the initial state point to cut the line corresponding to the pressure to which the working agent is throttled. This final point is a point on an unknown curve representing an irreversible adiabatic process. No intermediate points can be determined, so that no curve can be drawn for the process.

The problem may be further illustrated with the aid of the entropy temperature diagram.

Let A, Fig. 68, represent the initial position of a state point. Let the steam fall by throttling from a temperature T to a temperature t a little below it, through the small range $\delta T = T - t$. The unused temperature difference is δT. In a reversible cycle the external work ABCD could be obtained from this temperature difference. It is not obtained, however, and by the principle stated above, the heat equivalent of this work appears in the working agent at the end of the process. Therefore, take the point B′ such that $BB' \times t =$ the area ABCD. Then B′ is the state point at the end of the process showing clearly that the irreversible process of throttling has resulted in an increase of the entropy BB′. Analytically, the heat corresponding to the work area ABCD is $\frac{qL\delta T}{T}$. And this by the above principle is equal to $td\phi$.

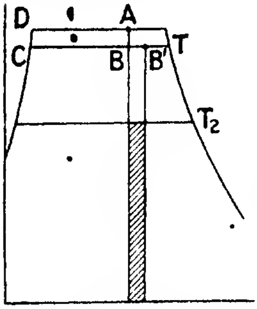


FIG. 68.—Increase of entropy caused by throttling.

Therefore $$d\phi = \frac{qL \times \delta T}{Tt}$$

The increase of entropy for small changes of temperature can be computed in this way.

Referring to the figure again, it will be seen that if the engine is condensing at the lower temperature T_2, the part of the rectangle below the lower temperature shows the thermodynamic loss caused by throttling.

When the drop of temperature is large, as in the case of throttling experiments through a porous plug or its equivalent, the analytical relations are somewhat different though the geometrical representation is the same.

Consider the case of steam flowing through a tube of small bore. No work is done as the steam flows through the tube, and assuming that there is no exchange of heat with the walls of the tube, the flow is adiabatic, and the flow is made at constant total heat. The expression for the change of total heat is, from equation (13), page 168

$$dI = dE + A \, . \, d(pv) = K_p dT - A\left\{T\left(\frac{dv}{dT}\right)_p - v\right\}dp$$

If the flow takes place at constant total heat, $dI = 0$.

So that $$K_p dT - A\left\{T\left(\frac{dv}{dT}\right)_p - v\right\}dp = 0$$

That is $$K_p \frac{dT}{dp} = A\left\{T\left(\frac{dv}{dT}\right)_p - v\right\}$$

The rate at which the temperature falls with the pressure, namely, $\frac{dT}{dp}$, is called the cooling effect, and it is one of the most important physical properties of the working agent. Call it C.

Substituting its value in the equation for total heat

$$dI = K_p dT - K_p C dp$$

and the total variation of total energy with pressure and temperature is known, provided that K_p and C can be measured.

For a perfect gas the cooling effect is zero. It is by observations on the cooling effect that Callendar has deduced the values of the co-aggregation volume for steam.

It has been shown above that in any reversible cycle the sum of the changes of entropy for once round the cycle is zero. With irreversible processes it is clear that at the end of the cycle the working agent will give more entropy to the cooling circuit than it received from the heating circuit because of the heat additions, and therefore the entropy additions, made during the cycle, additions which are equivalent to the work corresponding to the unutilized parts of the temperature difference established by the heating and the cooling circuit. The entropy discharged to the condenser is therefore greater than that received from the heating circuit.

Since in practice all processes are irreversible, there is always some fraction of a temperature difference inevitably sacrificed, and therefore entropy tends to increase, a general law first established by Clausius.

This general law has the extended meaning that all natural processes tend to the increase of entropy. If, for example, the earth were completely isolated from all sources of energy, that is to say, if it were an adiabatic planet, the general tendency of natural processes would be towards a general level of temperature, heat flowing from the hotter to the colder bodies, the entropy of the flow constantly increasing until in the limit, when all things had fallen to a general level of temperature, the entropy would have a maximum value. The only way to combat this general tendency is to put heat engines working, as far as possible, reversibly between every temperature difference which exists and so prevent the increase of entropy by transforming the heat flow produced by the temperature difference into mechanical energy.

CHAPTER IV

THE MOTIVE-POWER CIRCUIT—*continued*

THE ACTUAL PERFORMANCE OF STEAM ENGINES. LOSSES AND THE MEANS OF REDUCING THEM

61. Comparison of the Actual and the Ideal Pressure-Volume Diagrams. — The standard with which to compare the actual performance of a steam engine is the ideal performance of steam in a Rankine Cycle of changes between the limits of pressure measured in the steam pipe and in the exhaust pipe of the engine itself.

The ideal performance is computed from the conditions that the steam supplied to the cylinder per cycle falls from the upper pressure measured in the steam pipe to the lower pressure measured in the exhaust pipe by adiabatic expansion; that there is no compression and no clearance; that there are no losses by radiation, conduction, condensation, or leak, and that there is no loss by throttling.

A comparison between the actual and the ideal performance may be made in various ways. Amongst others, a comparison of the actual indicator diagram with the ideal pressure-volume diagram is particularly instructive, although in some particulars comparison is not strictly possible.

The area of an indicator diagram taken from an engine cylinder is always smaller than the area of the pressure-volume diagram representing the corresponding Rankine Cycle. This is illustrated in Fig. 69, where an actual indicator diagram is sketched along with the ideal pressure-volume diagram.

In the ideal diagram OP_1 is the pressure measured in the steam pipe, OP_2 is the pressure measured in the exhaust pipe, P_1V_1 is the volume corresponding to the weight of steam fed to the cylinder per cycle, and V_1V_2 is an adiabatic curve through V_1.

The actual indicator diagram is placed so that its atmospheric line coincides with the atmospheric line of the ideal diagram, and so that the tangent to the indicator diagram at right angles to the atmospheric line coincides with the pressure axis OP of the ideal diagram.

The reasons for so placing the indicator diagram will appear immediately.

The difference between the areas of the two diagrams represents cylinder losses.

These losses may be analysed in the way shown in the shaded areas. The areas B and D represent the losses due to wire drawing during admission and exhaust; the areas N and E the loss respectively due to compression and clearance; the area F the loss mainly due to condensation and leak; and the area G the loss due to incomplete expansion of the steam.

It will be observed that these areas encroach one upon the other, and that the magnitude of any particular loss depends upon the magnitude of other losses in a way which makes any analysis of the losses somewhat artificial. Nevertheless, the diagram serves to identify various losses, and to bring them all into comparison in a general way.

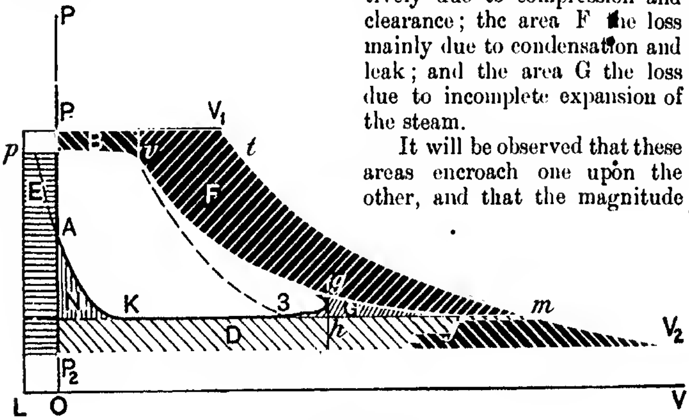


FIG. 69.—Comparison of the actual and the ideal P.V. diagrams.

62. Wire Drawing.—Wire drawing is the name used to describe the diminution of pressure, generally small, caused by the resistance to the flow of steam through the valves, ports, and passages leading to or away from the cylinder.

The pressure in the cylinder during the admission period is always less than the pressure in the steam pipe on the boiler side of the stop valve. The work corresponding to this difference is expended in setting the steam in motion when the valve opens, and in overcoming the frictional resistance against which the steam flows into the cylinder. This resistance varies roughly as the square of the velocity of flow, and it can be minimized by providing short straight passageways for the steam, and avoiding sharp bends. In fact, it is well to design the ports and passages as nearly as possible as if they were intended for the flow of water.

As the piston speed increases, the velocity of flow increases, and the pressure drop increases. Extreme cases of wire drawing are exhibited by indicator diagrams taken from locomotives running at high speeds.

In a particular case of a locomotive running at 20 miles per hour corresponding to a mean piston speed of 388 ft. per minute, the steam gauge pressure fell from 200 lbs. per square inch in the boiler to

187 lbs. per square inch at the beginning of the stroke, and this fell gradually during admission to 141 lbs. per square inch at cut off, which took place at 27 per cent. of the stroke. An increase of speed to 60 miles per hour, corresponding to 1164 ft. per minute *mean piston speed, but with* boiler pressure, regulator opening, and cut off unchanged, reduced the initial pressure to 177, and the pressure at cut off to 90 lbs. per square inch.

The area B, Fig. 69, shows the loss caused by wire drawing during the admission period: the area D, the loss during the exhaust period. The vertical difference between the back pressure line of the actual diagram and the back pressure line of the ideal diagram shows the pressure difference required to establish and maintain the flow from the cylinder to the condenser against the resistance in the exhaust pipe. This pressure difference operates during a relatively large change of volume, and care should be taken to minimize it as much as possible by the provision of exhaust openings of large area. In condensing engines the back pressure is greater than the pressure due to the temperature of the steam, because of the presence of air which is brought over by the steam in solution, or which leaks in when the pressure is lower than that of the atmosphere.

In locomotives the back pressure is necessarily increased by the blast pipe, but the steam passages leading to the actual orifice should be large with gradually changing areas towards the orifice, and they should be free from bends, in order that the pressure drop required to produce the blast is not unnecessarily increased by the resistance of badly designed passages. In the case of the locomotive, quoted above, the back pressure increased from 1·3 lbs. per square inch above atmospheric pressure at the lower speed to 10 lbs. per square inch at 60 miles per hour.

An event of the cycle is never sharply defined on the indicator diagram, because the gradual opening or closing of a valve produces throttling, which is shown by the rounding off of the junctions of the curves forming the diagram, and a consequent reduction of the area.

Throttling is increased by the presence of water with the steam. Water is generally found with the steam in the supply pipe. Its presence is due either to condensation or to priming. Condensation can be reduced by well lagging the steam pipes, and most of the water can be removed from the steam before it passes through the stop valve by means of a separator. In one kind of separator the steam is thrown into a vortex, and the water falls to the bottom of the chamber, whilst the steam passes away at the top.

The thermodynamic effect of throttling is discussed in Section 60, page 218. Throttling is an irreversible process, and although it tends to dry the steam, the heat produced by throttling, and which is responsible for the increase in dryness, is used to less advantage than if it were utilized to do work between the highest and lowest temperatures in the cycle.

63. Incomplete Expansion.—The area G, Fig. 69, represents the loss of work caused by the drop of pressure from g, the pressure at the end of expansion, to h, the back pressure.

There is no physical reason why expansion should not be continued until the pressure falls to the back pressure at j. There are, however, practical reasons which make a drop of pressure like gh desirable.

In the first place, the size of the cylinder would have to be increased considerably in order to provide a volume corresponding to the pressure at j, and the consequent increase of wall surface would increase the loss by radiation and condensation, and this loss would more than counterbalance the slight gain due to increased expansion. Moreover, even if there were a gain it would be too small to justify the increased cost of the larger cylinder. Secondly, there is no advantage to be gained in reducing the pressure by expansion below the pressure equal to the sum of the back pressure of the steam and the mean pressure required to run the engine without load. That is to say, the drop gh in the diagram should not be less than the pressure required to overcome the engine resistance.

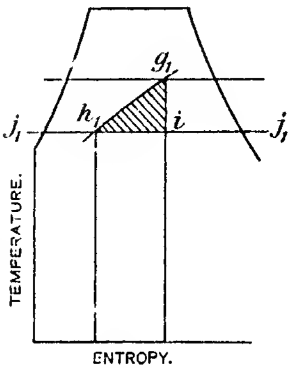


FIG. 70.

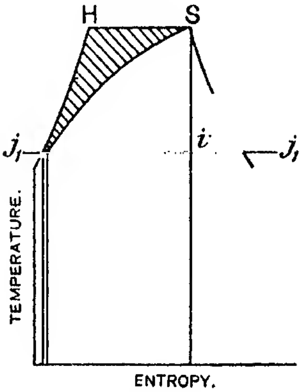


FIG. 71.

Loss due to incomplete expansion.

The thermodynamic loss due to incomplete expansion may be illustrated graphically on the entropy temperature diagram. Let Fig. 70 be an entropy temperature diagram for 1 lb. of steam, and suppose that the steam expands adiabatically, or, in fact, by any method down to the temperature g_1, corresponding to the point at which expansion is stopped and the drop to the back pressure begins. Let j_1j_1 be the temperature corresponding to the back pressure. Then the change of state from g_1 to the back pressure is made at constant volume. Let g_1h_1 be the constant volume curve through the point g_1. Then the area under g_1h_1 represents the heat passing to the condenser during the reduction of temperature corresponding to the drop of pressure gh in Fig. 69.

If expansion had been carried out adiabatically to the back pressure, the heat rejected to the condenser would have been the area under ih_1. The loss due to incomplete expansion is, therefore, represented by the cross-hatched area.

The diagram may be used in a very direct manner to illustrate the advantage of expansion.

For example, assuming no throttling, and that the steam is dry and saturated at the end of admission, and that just 1 lb. is admitted to the cylinder, the state point at the end of admission is represented by S, Fig. 71. If now the steam is not expanded at all, but is put into communication with the condenser immediately at the end of admission, which in this case is at the end of the stroke, the change of temperature from S to the condenser temperature j_1j_1 takes place along the constant-volume curve sketched through S, and the cross-hatched area represents the work done during the cycle with no expansion. With complete adiabatic expansion down to the back pressure, the work done during the cycle is represented by the complete area Sij_1H.

If the expansion curve does not follow the adiabatic law, the point i falls to the right or to the left of the position shown in the diagrams according as heat is taken in from or lost to the walls during expansion. The modification in the loss due to incomplete expansion is then shown by the modification of the area h_1g_1i, Fig. 70.

64. Clearance and Compression.—During the return stroke, the piston does work on the steam against the back pressure. Up to the point K, Fig. 69, where the exhaust valve closes, the back pressure is constant, but after this point the back pressure rises as the steam which is shut in the cylinder is compressed into the clearance space.

Let A, Fig. 69, be the point at which the admission valve opens. Then the area N represents the loss of work due to compression. It is a loss in the sense that without compression the quantity of work represented by the area N could have been done externally, so that for a given quantity of work per cycle the cylinder could have been smaller.

The result of compression is to fill the clearance space OL with steam at the pressure corresponding to A, so that when the admission valve is opened the weight of fresh steam flowing into the clearance space is only that which is necessary to bring the pressure up to the initial pressure at p.

If the point K is so chosen that the pressure at the end of compression is equal to the initial pressure, then, when the admission valve is opened, there is equilibrium between the steam in the steam chest and the steam in the clearance space, and no fresh steam flows into the clearance space.

When the pressure at the end of compression falls short of the initial pressure it is said to be incomplete.

When it is carried right up to the initial pressure it is said to be

complete. When the compression is complete the irreversible drop of pressure between the initial and the compression pressure is avoided.

The weight of steam in the clearance space may be regarded as a separate quantity expanding with the steam admitted up to the point of cut off, as indicated in Fig. 72, where expansion curves are drawn through the points a and c, corresponding respectively with the volume of the clearance space and the volume at cut off. The curves are assumed to follow the law PV = a constant, an assumption sufficiently accurate for the general consideration of the problem.

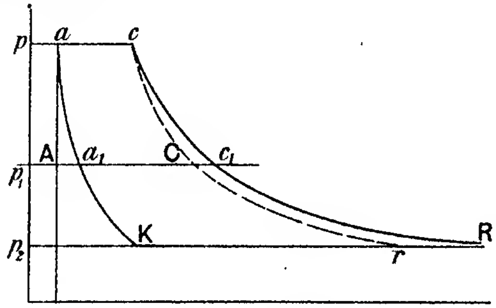


Fig. 72.—Cushion steam.

A horizontal at a lower pressure p_1 cuts these curves in the points a_1 and c_1. The volume p_1a_1 is the volume to which the clearance steam expands; p_1c_1 is the actual volume of the steam in the cylinder, and the difference a_1c_1 shows the volume corresponding to the steam admitted to the cylinder during the movement of the piston from the dead point to the point of cut off.

With complete compression this is the volume corresponding to the steam supplied up to the point of cut off, since none of the steam fed to the cylinder is required in the clearance space.

The final volume of the clearance steam is p_2K.

Still assuming complete compression, the exhaust valve is, on the return stroke, closed at K, and the clearance steam is compressed along its expansion curve back to the initial pressure in the steam chest.

In these circumstances the weight of steam in the clearance space may be identified as a particular mass which is always in the cylinder, and which is alternately expanded and compressed along the same curve.

This alternate expansion and compression of the clearance steam contributes nothing to the work done on the piston during the cycle, and theoretically no steam is drawn from the boiler once the clearance space has been filled. It may, therefore, be eliminated from the diagram.

It follows that if the volume a_1c_1 is set back to the vertical through

a, a point C is obtained on the expansion curve which the steam supply would have followed if there had been no cylinder clearance.

Finding a series of points like C and then sketching in the curve, a pressure-volume diagram, with the clearance steam eliminated, is constructed in a form suitable for comparison with the ideal pressure-volume diagram. If the compression is incomplete, the expansion curve is constructed from the compression curve corresponding to the actual weight of steam shut in the clearance space. Referring to Fig. 69 again, the dotted expansion curve through v is constructed in this way from the compression curve corresponding to the weight of steam shut in the clearance space. The continuation of the compression curve to the initial pressure is shown dotted. The difference of the areas is not materially altered by this correction, but the area F is increased and presents to the eye a truer picture of the loss due to condensation and leak.

Although theoretically the cushion steam once supplied to the clearance space requires no further supplement, yet practically there are losses due to leak and condensation which have to be made up.

With complete compression the effect of the clearance space on the thermodynamic efficiency of the engine is an indirect one. It increases the surface on which condensation of the fresh steam may take place, and it makes it necessary to provide a larger cylinder to perform a given quantity of work, and thus again acts indirectly to determine a larger wall surface per unit of power developed. This may be seen at once from Fig. 72. The volume of the steam at the end of expansion is R. If there had been no clearance it would have sufficed to make a cylinder with the volume corresponding to r. The increased volume rR is necessitated by the clearance volume, and is equal to p_2K.

When the compression is incomplete the compression curve does not coincide with the expansion curve of the clearance steam, and the loss of area due to compression is reduced. But the cylinder feed must be increased by the weight of steam required to bring the pressure in the clearance space up to the initial pressure, and this causes a thermodynamic loss by unresisted expansion, and the steam is thus used to less advantage than if it had been expanded along with the main supply from the highest pressure in the cylinder. The additional quantity of steam required depends upon the extent to which compression is carried.

For example, assume a back pressure of 15 lbs. per square inch; an initial pressure of 200 lbs. per square inch; and a clearance volume of 8 per cent. in a cylinder of 10 cub. ft. capacity. The clearance volume is then 0·8 cub. ft.

The weight of dry saturated steam required to fill a given volume at a given pressure can always be found from the tables, since—

$$\text{Weight required} = \frac{\text{Given volume}}{\text{Tabular volume corresponding to the given pressure}}$$

Using the tables, it will be found that the weight of steam required to fill 0·8 cub. ft. at 200 lbs. pressure per square inch is 0·345 lb.; at 15 lbs. per square inch it is 0·0305 lb. If there is no compression at all, the steam which must be supplied to fill the clearance space when the piston reaches the dead point is 0·375 lb. If compression is carried up to say 100 lbs. per square inch, the weight of steam required to fill 0·8 cub. ft. at this pressure is 0·18 lb., and the quantity of steam to be supplied is reduced to 0·165 lb. The actual steam supply would be, in each case, greater than these calculated quantities by amounts depending mainly upon condensation and leak.

Compression as actually carried out in the cylinder must not be confused with the ideal compression in the fourth process of the Carnot Cycle. In the actual cylinder, compression is applied to a mere fraction of the cylinder feed, out of contact with water, and the effect of compression is to change the pressure of the steam without any sensible condensation; in fact, the process is merely the compression of a gas. In the Carnot Cycle, compression is applied to the whole weight of steam and condensed water used in the cycle, and at the end of the process the whole of the steam has been condensed to water at the initial temperature.

The practical advantage of the process of compression in an actual cylinder is that the steam in the clearance space acts as an elastic cushion to assist in bringing the reciprocating parts to rest at the dead point, and then to aid in accelerating them in the next stroke. In a particular case, considered fully below, the force required to bring the reciprocating parts to rest increased from zero about the middle of the stroke to 35 tons at the dead point; the compression in the cylinder, though incomplete, was sufficiently high to balance this force at the dead centre, and so relieve the actual working parts of the strain which would otherwise have been brought upon them. Compression is in fact an essential process for the smooth working of the engine.

Summarizing, clearance directly influences the thermodynamic efficiency by increasing the quantity of steam required per cycle; by compression this increase is reduced, and may be avoided altogether if compression is complete. With complete compression the thermodynamic efficiency is indirectly influenced by the extent of the wall area in the clearance space itself and by the necessarily increased size of the cylinder required for a given power. Clearance thus influences the cost of the unit of power. Clearance provides a space for a cushion of steam which, by its alternate compression and expansion, aids in bringing the reciprocating parts to rest and in accelerating them along the next stroke, and makes high speeds and smooth working possible.

65. The Missing Quantity.—The cost of maintaining a continuous supply of fuel and water is the main cost of running a steam plant, and records of the rate of fuel and water supply are

amongst those which must necessarily be kept in connection with any steam plant.

The indicator enables the action of the steam to be examined in the cylinder and the power developed to be measured, and hence the water and fuel per unit of motive power to be calculated and the cost of the power unit to be estimated. These measurements are sufficient to enable the performances of engines to be compared in a practical way.

The weight of dry steam actually in the cylinder may be computed from the indicator diagram, and this is presumably the weight of steam supplied per stroke from the steam pipe.

If, however, a comparison is made between the weight of steam accounted for by the indicator diagram and the weight of steam actually supplied to the cylinder by the steam pipe, it is always found that the indicator diagram accounts for only a part of the supply. There is a quantity of steam missing, which in large and economical engines may amount to 20 per cent. of the whole supply; 30 per cent. is, however, a more usual value, and a loss of 70 per cent. may be exceeded in small engines.

Watt, after repairing a model of a Newcomen engine, set it to work, and he was so struck by the magnitude of the difference between the steam supply and the performance, that is by the magnitude of the missing quantity, that he started the investigations which resulted in the Watt engine of 1782.

In the Newcomen engine steam at atmospheric pressure was admitted to the cylinder, and when quite full cold water was injected to condense the steam. The cylinder itself was cooled by the process, and a large fraction of the next supply of steam was condensed in reheating the cylinder to a temperature at which steam could exist in it. The steam so condensed would be "missing" from an indicator diagram.

Watt soon laid down the principle that "in order to make the best use of steam it was necessary that the cylinder should be maintained always as hot as the steam which entered it," and he invented the separate condenser as a step towards realizing this principle. The cooling water in a separate condenser is never in contact with the walls of the steam cylinder, and the variation of cylinder wall temperature is thus greatly reduced, in consequence of which initial condensation and the missing quantity are also reduced. In order still further to maintain the temperature of the cylinder, Watt added the steam jacket.

It will be perceived that notwithstanding the imperfect scientific knowledge of the time—there was no science of thermodynamics and no knowledge of the mechanical equivalent of heat, and only an imperfect knowledge of the properties of steam—Watt proceeded to reduce the missing quantity of steam by a method as truly scientific as if he had lived 100 years later and had inherited the work of Carnot and Joule.

The loss caused by the missing quantity is presented graphically

by the comparison in Fig. 69. If all the steam supplied to the cylinder were present as steam at cut off, the state point would be at V_1. Allowing for throttling it might fall to t. The loss of work caused by the loss of steam is therefore represented by the area $vtm3$, it being remembered that the dotted curve $v3$ is the expansion curve which the steam present would have followed if the cushion steam had been eliminated.

The early engineers had no thermodynamic standard with which to compare their results. It is due to the genius of Carnot that we are able to prescribe accurate limits to the possible performance of steam, and hence to estimate the loss in given conditions of working and to realize what improvement is possible.

It is not possible to maintain the wall of the cylinder as hot as the entering steam, because as the temperature of the steam falls by expansion towards the exhaust pressure, the temperature of the wall falls also, and during the whole of the exhaust stroke the low-temperature steam is in contact with the wall; therefore, although the mean temperature of the wall does not fall by any means as low as the steam temperature, the time occupied by the stroke being too small to allow a temperature equilibrium to be established, yet when the fresh supply of steam enters the cylinder the wall of the clearance space is cooler than the steam; there is, therefore, a temperature gradient and condensation is inevitable. But, as will be shown immediately, the rate at which condensation can take place on the metal surface of the cylinder is not sufficiently great in normal conditions of working to account for the whole of the missing quantity. The experiments of Callendar and Nicolson have established beyond question that the total loss is due partly to condensation and partly to leak. In slide-valve engines probably the greater part of the missing quantity of steam leaks away past the valve directly into the exhaust pipe, and so does not pass into the cylinder at all.

Experience has shown that the missing quantity is reduced if expansion is carried out in a chain of two or three cylinders instead of in one cylinder, and that further reduction is made if the steam supplied is superheated. Compound expansion and the use of superheated steam will be considered in detail below.

Consider now the way in which the missing quantity is found from an indicator card and the measured steam supply, using for the purpose of exemplification the diagram shown in Fig. 73, for which the measured steam supply is 17·74 lbs. per minute at 102 revolutions per minute.

The diagram is calibrated by the addition of the pressure and the volume axis in the way explained in Section 2, Chapter I.

Let W be the weight of steam per stroke supplied through the steam pipe; let D be the part of this accounted for by the indicator diagram; and let w be the weight of steam shut in the clearance space when the exhaust valve closes.

Then the weight of steam which ought to be expanding in the

cylinder is W + w pounds. The weight of steam actually expanding is D + w pounds. And assuming w to be constant, the missing quantity is W − D pounds.

In order to find the weight of steam expanding, select a point on the expansion curve well after cut off, and then deduce the weight from the pressure and volume which it represents, by aid of the tables.

In Fig. 73, C is a point on the expansion curve well after cut off. The pressure at C is 35 lbs. per square inch, and the volume is 0·66 cub. ft. From the tables the volume of 1 lb. of dry saturated

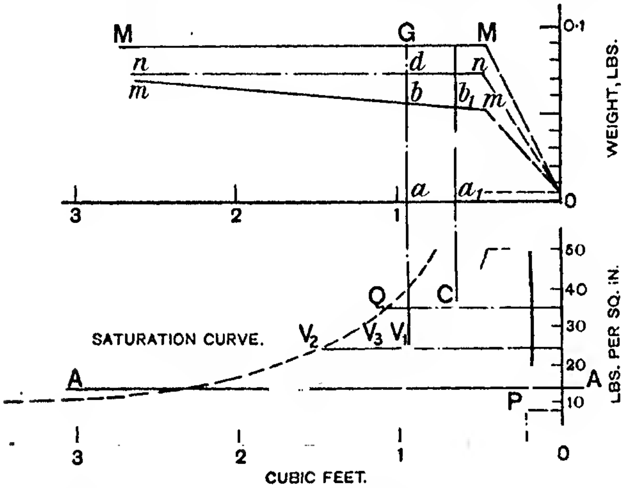


FIG. 73.—The missing quantity and the weight-volume diagram.

steam at 35 lbs. per square inch is 11·9 cub. ft. Therefore the weight of steam corresponding to the state point C is $\frac{0·66}{11·9} = 0·0554$.

The weight of the clearance steam is computed on the assumption that the compression curve is a saturation curve and therefore that w is constant, an assumption not strictly correct, but involving negligible error. Select a point P on the compression curve well after the closing of the exhaust valve. The pressure at P is 8 lbs. per square inch, the volume is 0·2 cub. ft.; the tabular volume of a pound of dry saturated steam at 8 lbs. per square inch pressure is 47·3 cub. ft; then the weight of steam shut in the clearance space is $\frac{0·2}{47·3} = 0·004 = w$.

Thus D, the weight accounted for by the diagram at the point C, is 0·055 − 0·004 = 0·051 lb.

But since steam is supplied at the rate of 17·74 lbs. per minute

when the speed is 102 revolutions per minute, and the engine is double-acting—

$$W = \frac{17 \cdot 74}{204} = 0 \cdot 087 \text{ lbs. per stroke}$$

Therefore W − D, the missing quantity, is 0·036 lb. at the point C. This is 41 per cent. of the supply.

The weight of steam which ought to be expanding in the cylinder is $W + w = 0 \cdot 091$ lb. The saturation curve corresponding to this weight is shown dotted in Fig. 73. Points like Q are plotted by selecting a pressure, and then by the aid of the tables calculating the corresponding volume for a weight of 0·091 lb. That is, the volume on the diagram corresponding to $W + w$ pounds of steam at a given pressure is equal to the tabular volume per pound corresponding to the pressure, multiplied by $W + w$.

Thus the pressure at Q is 35 lbs. per square inch, and therefore the volume is $11 \cdot 9\ (W + w) = 11 \cdot 9 \times 0 \cdot 091 = 1 \cdot 09$ cub. ft.

The horizontal distance between the expansion curve of the indicator diagram and the saturation curve shows the missing quantity by volume.

As expansion proceeds the action of the wall causes re-evaporation of the steam condensed during admission, so that the weight accounted for by the diagram increases towards the end of the stroke, and the missing quantity diminishes. The calculation of the weight of steam at any point on the expansion curve is carried out in the same way as for the point C.

It is better to show the missing quantity by weight rather than by volume, an incidental advantage of the diagram being that it can be drawn within the limits of the length of the indicator diagram. A weight-volume diagram is shown above the indicator diagram, Fig. 73. The weight of steam $W + w$ which ought to be in the cylinder after cut off is shown by the horizontal line MM. The weight which is in the cylinder at any point after cut off is given by ordinates to the curve *mm*. This curve is drawn through points found by plotting $D + w$ against the piston displacement. $a'b'$ is the value of $D + w$ corresponding to the point C.

The gradual diminution of the missing quantity brought about by re-evaporation as expansion proceeds is well brought out by this diagram.

If a curve could be added, like *nn*, showing how the loss is divided between leak and condensation, the diagram would show the condition of things in the cylinder completely. The way to plot such a curve, at least approximately, is the subject of the next few sections.

Meanwhile, assume that *nn* is correctly placed. Then an ordinate to it gives the weight of steam and water in the cylinder, and the dryness fraction of the steam can be calculated. Thus, when the piston is at *a* the weight of steam present as vapour is *ab*; the

weight of steam vapour and water condensed from the steam supply is ad; the dryness q is then $= \dfrac{ab}{ad}$. The leak is dG.

Or, on the lower diagram, where V_3 is the point corresponding to d, $q = \dfrac{\text{the volume corresponding to } V_1}{\text{the volume corresponding to } V_3}$. The leak by volume is V_3V_2.

66. The Wall Action and the Condensation Area.—The steam which flows into the cylinder when the admission valve is opened is hotter than the walls; therefore heat flows from the steam to the walls down the temperature gradient, in consequence of which some of the steam condenses and the walls are warmed. As the piston moves along from the dead point, condensation continues on the increasing wall surface, and the temperature of the wall surface increases as the temperature of the steam falls until a point is reached at which there is momentarily thermal equilibrium between the steam and the walls; after this point the wall is hotter than the steam; the temperature gradient is reversed and heat flows back from the wall into the steam and begins to re-evaporate the water condensed in the earlier part of the stroke. This thermal gradient continues during the whole of the return stroke and reverses again at a point just after the admission valve is opened. During each cycle there is thus a reversal of the temperature gradient between the steam and the wall.

The surface temperature of the metal is not uniform at any instant during the general rising and falling temperature of the walls. There are the local differences corresponding to a thermal gradient along the barrel, so that the time at which the temperature gradient reverses at a particular point in the surface depends upon the position of the element of the surface in which the point is situated. The elements of the surface near the end of the expansion stroke in a single-acting engine are exposed to the action of the steam for only a small fraction of the cycle and are probably always hotter than the steam in contact with them. An element in the clearance space would always show a reversal of the temperature gradient unless the walls were kept hot artificially by means of a steam jacket supplied with steam hotter than the steam admitted to the cylinder. Every element of the wall surface, however, wherever it be situated, passes through a cyclical variation of temperature which recurs with regularity during each successive revolution.

The variation of the temperature gradient at a particular element of the wall surface is shown by plotting on a time base a curve giving the temperature of the steam in the cylinder and a second curve giving the temperature of the metal surface of the element. The vertical distance between these curves is the temperature difference at the corresponding time, and, if the curves intersect, the points of intersection show the times in the cycle at which the temperature gradient reverses.

Such a pair of curves is drawn in Fig. 74, for a wall element in the cylinder cover of a slide-valve engine arranged to work single-acting at a speed of 73 revolutions per minute. The cylinder is 10½ ins. diameter and 12 ins. stroke.

The distance OX represents the time occupied by one revolution, and it is divided into 60 parts. OX is in fact a crank-angle base assuming uniform rotation, and each unit division on the scale represents the time required in seconds to turn through an angle of 6°.

The temperature corresponding to an ordinate of the curve *sss* is found from the pressure given by the corresponding ordinate in the indicator diagram.

The temperature of the wall element is plotted from observations made by means of a thermo-couple embedded in the wall element at a distance of 0·039 in. from the inner wall surface.

The diagram shows that whilst the steam temperature ranges through a variation of 67° C., the wall temperature at a depth of

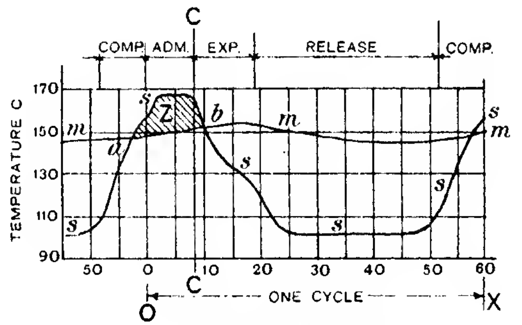


Fig. 74.—The condensation area.

0·039 in. ranges through only 2·2° C. The metal temperature curve *mmm* cuts the steam temperature curve in the points *a* and *b*. During the time from *a* to *b* the steam is hotter than the wall element; from *b* to *a*, measured along to the right, the wall element is hotter than the steam; hence it is fair to assume that condensation begins at *a* and ends at *b*, and that re-evaporation begins at *b* but cannot continue beyond *a*.

This is a Callendar and Nicolson diagram, and the significant fact which it brings out is that the variation of the temperature of the wall element is small compared with the variation of the temperature of the steam. Callendar and Nicolson made numerous observations of the temperature differences between the steam and surface elements differently situated in the walls. In every experiment the variation of surface temperature was only a mere fraction of the variation of the temperature of the steam.

The inference from these observations is, that the rate at which steam condenses on a metal surface is physically limited.

Callendar and Nicolson then assumed a simple and workable hypothesis, which may be called the **Law of Condensation**, namely, that:

"The rate at which steam condenses on a metal surface is proportional to the difference of temperature between the steam and the surface and is independent of the pressure."

This assumption makes the amount of condensation taking place on an element of surface per revolution proportional to the average excess temperature of the steam multiplied by the time during which the steam is above the temperature of the element of surface.

The average temperature difference G acting to produce condensation on the element of the surface corresponding to Fig. 74 is the average height of the shaded area Z.

The time during which G acts is the fraction $\frac{ab}{OX}$ of the time occupied by one revolution.

Let N be the speed in revolutions per minute; then the time occupied by one revolution is $\frac{60}{N}$ secs.

Therefore the time during which G acts is $\frac{ab}{OX}\frac{60}{N}$ secs., and this is equal to $\frac{ab}{N}$ secs. when the distance OX is divided into 60 parts.

Therefore the condensation produced per revolution by the average temperature difference G is proportional to $G.\frac{ab}{N}$; that is proportional to $\frac{Z}{N}$.

And the condensation produced per minute by the average temperature difference G is proportional to the area Z.

The area Z is called the **Condensation Area**.

A glance at Fig. 74 will show that the difference between the condensation area and the area cut off by the straight line giving the average temperature of the element of wall surface is small. Therefore the condensation area may be computed to a first approximation from the mean temperature of the wall element.

But the question is still open as to how much steam is condensed during the period a to b. It is the practical solution of this problem given in Callendar and Nicolson's paper[1] that makes their work so valuable an addition to engineering science.

The essential feature of their solution is the use of Fourier's method to calculate the flow of heat from the surface into the metal, from the observed temperature variation of a point just within the wall, and then the comparison of the quantity so found with the condensation area Z.

It was found that the ratio between the quantity of heat calculated in this way and the magnitude of the condensation area

[1] "Law of Condensation of Steam," *Proc. Inst. C.E.*, vol. 131.

Z was fairly constant over a wide range of speed and cut off, and that the average value was 0·61. This result shows that the law of condensation assumed may be accepted as correct at least to a first approximation. It follows that the difficult mathematical process, details of which are given in Section 69, page 243 below, which must be used to calculate the heat-flow into the walls, may be replaced by the simpler process of finding the condensation area Z and then multiplying it by the constant 0·61.

This method of comparison may be illustrated by quoting figures relating to a particular experiment. With cut off at 20 per cent. of the stroke, and a speed of 73 revolutions per minute, the observed range of temperature at a depth of 0·039 in. in the wall was 2·2° C., corresponding to a range on the surface of 3·33° C. Assuming the variation of surface temperature to be simple harmonic, that is, assuming that the curve *mmm*, Fig. 74, is a simple harmonic curve, the heat Q flowing into the cylinder wall is by calculation 0·86 lb.-cal. per square foot of wall surface per revolution. The corresponding condensation area measured 1·42 units. The ratio $\frac{Q}{Z} = \frac{0·86}{1·42} = 0·61$. In eighteen experiments carried out with different ratios of expansion and at different speeds and on variously situated wall elements, the ratio was nearly the same. Therefore, in any one of these experiments Q could have been found by merely multiplying the condensation area, expressed in the units specified above, by the constant number of 0·61.

One unit of the condensation area is proportional to the flow of heat produced by a temperature difference of 1° C. acting for one second. The comparison then yields the result that one degree difference of temperature between the steam and the surface will cause a condensation of 0·61 lb.-cal. per sq. foot per second, and thus 0·61 is an important physical constant. All the condensation areas in the comparison were, however, measured from the mean temperature of the wall element, and therefore differ slightly from the true condensation area. Correcting for this difference, Callendar and Nicolson give 0·74 as the true value of the ratio. Callendar has by an entirely different method[1] verified that a degree difference of temperature acting for a second will condense 0·74 lb.-cal. per sq. foot on a cast-iron surface. Therefore, if Z is the true condensation area, S the area of the surface of the element in square feet, Q the number of lb.-calories passing from the steam into the element per minute,

$$Q = 0·74ZS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Elaborate apparatus is required to obtain measurements from which the temperature of the wall element can be plotted, but the mean temperature of the wall element can be observed in a very simple and direct manner, because the temperature variation of the surface penetrates only a little way into the wall of the cylinder. Even when the speed is as low as 25 cycles per minute, a surface

[1] *Report of the British Association for the Advancement of Science*, 1897, 418.

variation of 5¼ degrees dies away to the average value of the temperature at a depth of about ¾ in., and at 500 cycles per minute the temperature is uniform at a depth of about 0·2 in. The mean temperature of an element of wall surface may, therefore, be measured by means of an ordinary thermometer placed in a hole drilled to within a small distance of the inner surface of the element and filled with mercury.

When the heat transfer is computed from a condensation area, measured from the mean temperature, the value of the constant found by comparing the mean area with the calculated value of the heat-flow should be used, namely, 0·61. In this case

$$Q = 0{\cdot}61ZS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For example, observations made on a wall element 0·6 sq. ft. area gave a mean temperature of 152° C. The condensation area measured 102·6 units. The heat absorbed by the wall element from the steam is then

$$Q = 0{\cdot}61 \times 102{\cdot}6 \times 0{\cdot}6 = 38 \text{ lb.-cals. per minute}$$

This result gives the whole quantity of heat absorbed by the element during a revolution. To estimate the heat absorption up to a given epoch of the cycle, say up to the point of cut off, draw an ordinate CC, Fig. 74, on the temperature-time diagram corresponding to this point, and then measure the condensation area Z', say up to this ordinate. That is to say, the condensation area is formed by the intersection of the steam temperature curve, the metal temperature curve, and the ordinate.

In order to compute the whole condensation during a revolution the whole wall surface exposed to the steam is divided into a number of elements over the surface of which the temperature variation is sensibly the same during the revolution, and then the process explained above is to be applied to each element. The whole condensation is then the sum of the condensations produced by each element of the wall surface.

Callendar and Nicolson estimated the condensation produced by the various elements of the wall surface of their experimental engine, and found that 90 per cent. of the whole condensation takes place on the clearance surface.

The whole condensation per minute may thus in practice be estimated with considerable accuracy from the clearance surface alone and from the condensation area determined by the average wall temperature of the cylinder cover.

67. Maximum Condensation per Cycle.—The condensation and the re-evaporation areas of the temperature-time diagram will be equal if the temperature *régime* is such that the mean temperature of the clearance surface, on which the greater part of the initial condensation takes place, is reduced to the mean temperature of the steam. If the temperature of the clearance surface falls below this,

evaporation will be incomplete, water will accumulate in the cylinder, which, when blown out through the cylinder cocks, is disposed of without drawing upon the walls for the heat which they gained during its condensation. The wall temperature, therefore, rises rapidly whilst water is being rejected from the cylinder.

The mean temperature of the steam is thus, assuming the law of condensation to be true, a natural minimum of temperature for the walls, and the corresponding condensation area is a maximum. Therefore the indicator diagram itself may be made to furnish a standard from which the maximum amount of condensation per revolution can be computed.

Callendar and Nicolson's rule for finding the maximum condensation per cycle possible for the variation of steam temperature shown by the pressures on an indicator card is as follows :—

Draw the temperature-time diagram corresponding to the indicator diagram, and rule across it a line representing the average temperature. The area above this line is the maximum condensation area corresponding to the cycle. The numerical value of the maximum condensation expressed in lb.-cals. per minute is 0·61 times this area when it is measured in degrees C. sixtieths of a cycle units.

The maximum area measured from the mean wall temperature differs from the true maximum area measured from the actual metal temperature cycle curve by small amounts depending upon the speed and the point in the cycle up to which the condensation is estimated.

Callendar and Nicolson give the following correcting factors for the points of cut off and release :—

$$a = \frac{1 + \sqrt{n}}{3 + \sqrt{n}} \text{ when condensation is measured up to cut off,}$$

$$a = \frac{\sqrt{n}}{3 + \sqrt{n}} \text{ when condensation is measured up to release,}$$

where n is the speed in revolutions per minute.

A further allowance may be made for the barrel surface exposed by adding to the clearance space S, the quantity

$$s = ldc$$

where l is the length of the stroke in feet,

d is the diameter of the cylinder in feet,

c is the fraction of the stroke at which cut off takes place.

This correction was devised by Callendar and Nicolson, after the examination of a large number of cycles of double-acting engines, where cut off was in the region of 50 per cent. of the stroke.

The maximum condensation possible in a given cycle may therefore be estimated from

$$Q = 0{\cdot}74aZ(S + s)$$

where Z is computed by the above rule.

The numerical factor 0·74 is used because aZ purports to be the true condensation area.

S is the area of the clearance surface at one end of the cylinder in square feet, and includes the barrel portion, the face of the piston, and the steam passage up to the admission valve.

To ascertain if condensation is taking place at the maximum rate in a particular engine, the temperature of the wall of the clearance space is measured by means of thermometers inserted in mercury-filled holes drilled in the wall. If the mean temperature is found to be equal to the average temperature of the steam as determined by the mean indicator diagram, it may be concluded that the limiting rate has been reached. The corresponding condensation area is then found from the indicator diagram in the way directed above.

The limiting rate is seldom reached in a cylinder supplied with dry steam if the cut off is fairly early, but it may be reached if the steam is wet.

When an engine has been running sufficiently long to enable a definite temperature *régime* to be established, the quantity of heat received by the wall during condensation is equal to the quantity drawn from the wall during evaporation.

Re-evaporation takes place at a lower temperature than condensation, and therefore the heat given to the wall per pound of steam condensed is greater than the heat drawn from the wall per pound of steam re-evaporated. If t_1 is the mean temperature of condensation, and t_2 the mean temperature of re-evaporation, the difference between the heat given to the wall and the heat drawn from the wall per pound of steam condensed and re-evaporated is approximately

$$t_1 + 540 - 0{\cdot}7t_1 - (t_2 + 540 - 0{\cdot}7t_2) = 0{\cdot}3\,(t_1 - t_2)$$

But if the heat balance is to be maintained this quantity of heat must be drawn from the wall, and it is probably withdrawn to evaporate the additional quantity of water produced by adiabatic expansion. In general, the water produced by adiabatic expansion is more than sufficient to deprive the walls by its evaporation of heat to the extent necessary for the preservation of the balance. Water brought in by wet steam is also available for re-evaporation.

When the heat balance is established, an excess of water over and above that required to maintain it is rejected from the cylinder through the cylinder cocks or gets away through the valves.

The heat balance may be described as stable or unstable, according as the quantity of condensation and re-evaporation of water reaches the limiting value or not. The limiting value is reached when the mean temperature of the metal cycle falls to the mean temperature of the steam. When there is sufficient water in the cylinder to draw from the wall by its evaporation the maximum quantity of heat, an increase in the amount of water present makes little or no difference to the heat balance, and therefore no difference to the initial condensation, and the condition is

stable. But when the water re-evaporated is not sufficient to require the limiting quantity of heat for its evaporation, the condensation increases or diminishes as the water present increases or diminishes, and the condition is thus unstable.

The peculiarity of the unstable condition is that a small increase in the quantity of water re-evaporated produces a large increase in the weight of water condensed. To illustrate this, suppose condensation to take place at the mean temperature t, and re-evaporation at the mean temperature t_1, then the heat available per pound of water condensed for the evaporation of water additional to the quantity condensed is $0{\cdot}3(t - t_1)$ lb.-cals. per pound. Re-evaporation requires L_2 units per pound, therefore

$$0{\cdot}3(t - t_1)W = wL_2$$

$$\frac{W}{w} = \frac{L_2}{0{\cdot}3(t - t_1)} = 36$$

$$\text{if } t = 150^\circ \text{ C.}, \; t_1 = 100^\circ, \text{ and } L_2 = 540$$

Thus, unless the limiting condition of condensation prevails in the cylinder, a change in the working conditions which causes a small increase in the quantity of water requiring re-evaporation may result in additional initial condensation of many times the amount of the increase.

The analysis of the action of the walls, the method of determining the condensation area, and the way to compute the amount of steam condensed from the temperature variation of the walls and the indicator diagram, and the way to compute the condensation limit corresponding to a given indicator card, briefly explained in this and the preceding section, are all to be found in Calendar and Nicolson's paper mentioned above. The methods of analyses described in the paper have opened up a new field of research in connection with steam engine economy. The fact that the missing quantity is so large compared with the possible quantity of condensation suggests that leak plays a much more important part in steam engine economy than has hitherto been suspected. Callendar proved by experiment that slide valves which are quite tight when standing leak when working. Considering the general distortion produced on valve and valve chest by temperature changes this is not to be wondered at.

The practical outcome of Callendar and Nicolson's work is that the temperature of the cylinder cover should form one of the data observed during an engine trial, so that the condensation area can be drawn and from it an estimate made of the condensation; then by comparison with the steam supply and the steam accounted for by the indicator diagram an estimate of the leak can be made. This method of analysis cannot but be fruitful in results to engine designers. The object to be aimed at is to arrive at forms of valves and cylinders, steam chests, and ports which will have no relative distortion as the temperature changes and so reduce the loss by leak.

68. *Application of the Method of Condensation Areas to an Engine Trial.*—Accepting the results of Callendar and Nicolson's analysis that the quantity of steam condensed per cycle can be computed from a condensation area measured from the mean temperature of the metal cycle in the cylinder cover, it is an easy matter to estimate with fair accuracy the loss by condensation in an engine trial, and therefore the loss by leak, when the total supply of steam to the cylinder is known.

As explained above, the true condensation area is somewhat smaller than the area measured from the mean temperature of the wall surface. This will be understood from Fig. 75, where the sloping curve *ab* shows part of the actual temperature curve of which AB is the mean. The curve generally rises across the loop it cuts off from the steam temperature curve, and the slope decreases as the speed increases. The difference between the area ACB and the true area *a*C*b* depends largely upon the speed.

Factors, depending upon the speed, which correct the area ABC to the true area *a*C*b* at cut off and release, have been given in the

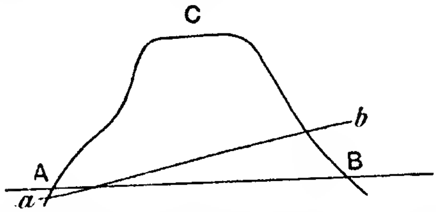


FIG. 75.—The condensation area and the metal cycle correction.

preceding section for the particular case of limiting condensation, but the metal cycle temperature curve depends upon so many variable conditions that factors of general application cannot very well be devised; but failing data for plotting the true curve *ab* and remembering that the correction is not large in many cases, the factors devised for the case of limiting condensation may be used for the purpose of obtaining an approximate correction.

Let the area in degree C. and sixtieths of a cycle measured from the mean temperature line AB be Z. Then the true area is approximately aZ, where a is a correcting factor given above. And the heat flow from the steam to the walls during the cycle is

$$Q = 0{\cdot}74aZ \text{ lb.-cals. per square foot per minute}$$

The condensation taking place on the barrel surface of the cylinder which is uncovered as the piston moves from the dead point to the point at which condensation ceases and re-evaporation begins, can be allowed for by adding to the area S of the clearance space an additional quantity s calculated from the product of the length of the stroke and the diameter of the cylinder and the fraction of the stroke at which cut off takes place.

As mentioned above, this correction was devised by Callendar and Nicolson for the case of double-acting engines, where cut off is in the region of 50 per cent. of the stroke, after the examination of a large number of cycles. The condensation may therefore be estimated from

$$Q = 0{\cdot}74aZ(S + s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Summarizing the various steps of the method :—

1. Plot on a time base, divided into sixtieths of a cycle, the steam-temperature curve corresponding to the pressures given by the indicator card.

2. Rule across the diagram a horizontal line giving the mean temperature of the cylinder cover as measured by thermometers placed in small mercury-filled holes drilled to within about quarter inch of the inner wall surface of the cover.

3. Measure the condensation area Z.

4. Calculate the factor a from

$$a = \frac{1 + \sqrt{n}}{3 + \sqrt{n}}$$ when condensation is measured up to cut off;

$$a = \frac{\sqrt{n}}{3 + \sqrt{n}}$$ when condensation is measured up to release. This factor is not to be applied unless the condensation area extends up to the release point.

5. Then the product $0{\cdot}74aZ$ gives the heat in lb.-cals. flowing per minute from the steam into the metal per square foot of wall surface in the clearance space.

6. Determine the area S of the clearance surface at one end, including the barrel portion, the face of the piston, and the steam passage up to the steam admission valve.

7. Add to this the additional quantity s found by multiplying the stroke by the diameter of the cylinder, both in feet, and by the fraction of the stroke at which cut off takes place.

8. Then the actual quantity of heat flowing from the steam to the walls is $0{\cdot}74aZ(S + s)$ lb.-cals. per minute up to the point of cut off or release according to the circumstances of the case.

9. This expression may be put into a form probably sufficiently accurate for all practical purposes

$$W = 0{\cdot}09aZ(S + s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where W is the number of pounds of water condensed per hour. If the engine is double-acting and the clearing surfaces at each end are equal, multiply this by 2. If the mean temperature of the cylinder cover cannot be measured, then the maximum condensation possible should be calculated by assuming a mean temperature which is the mean of the steam temperature cycle.

Fig. 76 shows an indicator diagram calibrated for pressure and volume together with the corresponding temperature-time diagram. The cylinder from which the diagram is taken is 15¾ ins. diameter;

22 ins. stroke; clearance at each end, 0·174 cub. ft; clearance surface at each end when the piston is on the dead point, 6·25 sq. ft. Speed, 92 revolutions per minute.

The positions of ordinates corresponding to equal time intervals of $\frac{1}{60}$ of a cycle are located on the indicator diagram, neglecting the obliquity of the connecting rod, by means of the semi-circle shown.

The observed mean temperature of the cylinder cover at the back end is 114° C., and the line through M is drawn across the tem-

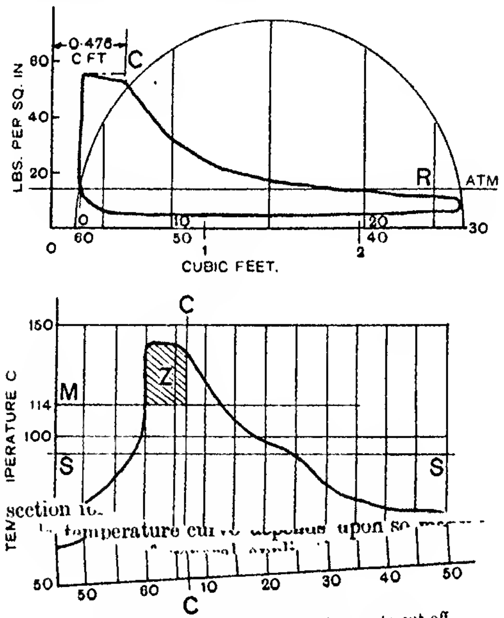


FIG. 76.—Estimation of condensation up to cut off.

perature-time curve at this temperature. The line SS shows the mean temperature of the steam. The vertical line CC corresponds with cut off. The area marked Z is the condensation area up to cut off. It measures 175 units.

The factor a for correcting this area to the metal cycle is

$$a = \frac{1 + \sqrt{92}}{3 + \sqrt{92}} = 0{\cdot}84$$

This is empirical and is only used in the absence of observation defining points on the true metal cycle curve.

The equivalent barrel surface

$$s = dlc = \frac{15\tfrac{3}{4} \times 22 \times 0{\cdot}25}{144} = 0{\cdot}34$$

So that $S + s = 6{\cdot}6$ sq. ft for each end. Then, using expression (2),

$$\left.\begin{array}{l}\text{Lbs. of steam condensed}\\ \text{per hour at each end of}\\ \text{the cylinder up to cut off}\end{array}\right\} = 0{\cdot}09 \times 0{\cdot}84 \times 175 \times 6{\cdot}6 = 87$$

The total estimated condensation per hour is therefore 174 lbs. The measured supply to the engine is 1080 lbs. of steam per hour.

The steam accounted for at cut off by the indicator diagram is 620 lbs. per hour.

The conditions at cut off may therefore be put in the following way:—

Accounted for by diagram	620	lbs. per hour
Accounted for by condensation . . .	174	,, ,,
Leak by difference	286	,, ,,
Total measured supply	1080	,, ,,

From the temperature-time diagram it would appear that the condensed steam should all be re-evaporated at release. The steam accounted for at release should therefore be 620 + 174 = 794 lbs.

Measuring from the diagram, the steam actually accounted for at the point R, where the pressure and volume scale respectively 11·3 lbs. per square inch and 2·44 cub. ft., is 786 lbs. per hour, a close agreement considering the elements of uncertainty in the data.

This example illustrates how the method may be applied to analyse the missing quantity into condensation and leak. Great care must be taken to determine the true mean temperature of the cylinder cover. The local difference of temperature in the clearance surfaces are greater the larger the engine, and it will be found that the temperature measured in a hole near the steam port will be higher than the mean. Holes should therefore be drilled in different positions in each cover in order to get a mean temperature from as many readings as possible.

Since the distortions of valve seatings, valves and sliding surfaces produced by temperature variation depend upon the actual form of the castings and upon considerations peculiar to each casting, it cannot be expected that any general law of leakage can be found. A general law of condensation is much more likely to be correct. The general accumulation of results of trials analysed in the way indicated will possibly suggest improvements in design which will reduce leakage, and anything which will do this will reduce the missing quantity and so improve the economy of the reciprocating steam engine.

69. The Calculation of the Flow of Heat into a Cylinder Wall assuming that the Temperature Variation on the Wall Surface is a Simple Harmonic Function of the Time.—Let AB be an infinitely extended surface exposed to temperature variation, and let T be

the temperature at a depth x measured perpendicularly from the surface. Consider two surfaces distant respectively x and $x + \Delta x$ from the surface AB.

Suppose the surface distant x to be maintained at the uniform temperature T, and that the surface distant Δx from it to be at the uniform temperature $T + \Delta T$, then the difference of temperature on the two surfaces Δx apart is ΔT.

In these circumstances the rate at which heat energy flows across unit area from one surface to the other surface varies directly as the difference of temperature and inversely as the distance between them, so that, if ΔQ is the heat flowing in time Δt, in the limit,

$$\frac{dQ}{dt} = -k\frac{dT}{dx} = \text{the rate of flow per unit area} \quad . \quad . \quad (1)$$

The minus sign is prefixed because T decreases as x increases. The constant k is the coefficient of conductivity. It was specially determined by Callendar and Nicolson for the cast-iron from which the engine cylinder of their experiment was made, and its average value is 5·4 when area surface is measured in square feet and the depth x is measured in inches.

Again, consider the region enclosed between two surfaces δx apart. Then the rate at which heat energy flows into the region between them across unit area of the surface distant x from the outer surface is from (1)

$$-k\frac{dT}{dx} = -ki$$

where i is written for $\frac{dT}{dx}$ and is the slope of the temperature-depth curve at the point where it crosses the surface. The rate at which heat energy flows out of the region across unit area of the second surface distant δx from the first surface is

$$-k\frac{dT}{dx} = -ki'$$

where i' is now the slope of the temperature-depth curve at the point where it crosses the second surface.

The rate at which heat energy is stored in the region is the rate at which the slope varies with x, that is

$$k\frac{di}{dx} = k\frac{d^2T}{dx^2}$$

The quantity of heat stored in a mass δx wide, of unit area in the direction of flow is

$$k\frac{d^2T}{dx^2}\delta x$$

The quantity of heat stored in the region is also equal to the rate at which the temperature increases with the time, that is $\frac{dT}{dt}$,

multiplied by the mass included between the planes per square foot of area, and again multiplied by the specific heat of material. The mass enclosed between the planes per unit area is $D\delta x$ lbs. Then the rate at which energy is stored is

$$sD\frac{dT}{dt}\delta x$$

where s is the specific heat of the material.

Equating these two rates

$$\frac{d^2T}{dx^2} = \frac{sD}{k}\frac{dT}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is Fourier's equation for the flow of heat in a direction perpendicular to a surface AB of infinite extent over which there is a cyclical variation of temperature of such a character that at any one instant the temperature all over the infinite surface is uniform.

Assume now that the cyclical variation of temperature T_0 on the outside surface AB, where $x = 0$, is a simple harmonic function of the time given by

$$T_0 = a \sin pt$$

Then with this assumption the solution of equation (2) is

$$T = a\epsilon^{-mx} \sin (pt - mx) \quad . \quad . \quad . \quad . \quad (3)$$

In this equation T is the temperature at depth x at time t, when there are n cycles per minute; a is a constant depending upon the amplitude of the temperature variation on the surface, whilst m is a constant depending upon the physical properties of the material through which the heat is flowing and upon the cyclical speed n. When n is given in revolutions per minute, m is calculated from

$$m = \sqrt{\frac{\pi n D s}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

D in the units employed is 36·6, corresponding to a density of 440 lbs. per cubic foot, whilst s, the specific heat, is 0·123, and as mentioned above $k = 5{\cdot}4$.

When pt is increased by the amount 2π the lag just equals one period, and the wave length of the flow is thus given by

$$mx = 2\pi$$

from which

$$x = \frac{2\pi}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The range of temperature is so reduced at the depth of a wave length that Callendar and Nicolson take the wave length as the practical limit to the penetration of the heat wave into the wall, so that at the depth of a wave length the temperature variation differs insensibly from the mean value.

The maximum range of temperature at a given depth is the maximum value which T can have by equation (3). It is clearly a

maximum when $\sin(pt - mx) =$ unity. The range of temperature at any point at depth x from the surface is then

$$T = a\epsilon^{-mx} \text{ degrees} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The rate at which heat flows across unit surface is given by equation (1). The value of the temperature T in terms of the depth x is given by equation (3). Differentiating (3) in order to find the value of $\frac{dT}{dx}$ in equation (1)

$$\frac{dT}{dx} = -ma\epsilon^{-mx}\sin(pt - mx) - ma\epsilon^{-mx}\cos(pt - mx)$$

At the surface where $x = 0$, this becomes

$$\frac{dT}{dx} = -ma(\sin pt + \cos pt) = -ma\sqrt{2}\sin\left(pt + \frac{\pi}{4}\right)$$

Therefore substituting this value of $\frac{dT}{dx}$ in (1)

$$dQ = -kma\sqrt{2}\sin\left(pt + \frac{\pi}{4}\right)dt \quad . \quad . \quad . \quad . \quad (7)$$

The flow is cyclical, and as much heat flows into the wall during the cycle as flows out. Therefore to find the flow in, integrate (7) between 0 and half the periodic time. The value of t for half the periodic time is $\frac{1}{2N} = \frac{\pi}{p}$. Therefore

$$Q = -kma\sqrt{2}\int_0^{\frac{\pi}{p}} \sin\left(pt + \frac{\pi}{4}\right)dt$$

From which

$$Q = \frac{kma\sqrt{2}}{p}$$

Multiplying numerator and denominator by Ds, and writing 2πN for p,

$$Q = \frac{kmaDs}{\sqrt{2}\pi nDs} = \frac{amDs}{m^2\sqrt{2}}$$

$$Q = \frac{3{\cdot}18a}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Imagine the infinite surface AB to be turned into a cylinder wall. The conditions of distribution remain unaltered, and these equations may therefore be used to trace out the temperature variations in a cylinder wall and also in the flat cylinder cover with an approximation sufficiently near to the truth to give a clear insight of the wall condition.

Bringing the results together, the index coefficient m is from (4)

$$m = 1{\cdot}62\sqrt{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The wave length, taken as the depth at which the temperature variation settles down to the mean temperature, is from (5)

$$x = \frac{2\pi}{1{\cdot}62\sqrt{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The range of temperature at any depth x from the surface is from (6)

$$\mathrm{T} = a\epsilon^{-1{\cdot}62\sqrt{n}\,.\,x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The quantity of heat flowing across the surface into the wall is from (8)

$$\mathrm{Q} = \frac{3{\cdot}18a}{1{\cdot}62\sqrt{n}} \text{ lb.-cals. per square foot per cycle} \quad . \quad . \quad (12)$$

And finally the quantity of heat flowing into the wall per square foot per minute is

$$\mathrm{Q} = \frac{3{\cdot}18an}{1{\cdot}62\sqrt{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In these expressions n is the cyclical speed in revolutions per minute; x is the depth measured in inches; and a is a constant depending upon the range of temperature on the surface.

This constant a can be calculated when the range of temperature T′ at a particular depth x' is known. Callendar and Nicolson measured this range at different depths and in different conditions of working. The use of these equations may be illustrated by a numerical example.

In a particular experiment the range of temperature variation was measured to be 2·2° C. at a depth of 0·039 in. from the surface of the metal when the speed was 73·4 revolutions per minute.

With these data $\sqrt{n} = 8{\cdot}57$, and the index in equation (11) becomes

$$1{\cdot}62 \times 8{\cdot}57 \times 0{\cdot}039 = 0{\cdot}541$$

and the range T = 2·2; therefore solving (11) for a

$$a = 2{\cdot}2\epsilon^{0{\cdot}541}$$

From which

$$\log a = \log 2{\cdot}2 + 0{\cdot}541 \log \epsilon$$

giving

$$a = 3{\cdot}78^\circ \text{ C.}$$

The general equation for this range on the surface is therefore

$$\mathrm{T} = 3{\cdot}78\epsilon^{-1{\cdot}62\sqrt{n}\,.\,x}$$

Therefore a range of temperature of 2·2° C. at a depth of 0·039 inch corresponds to a range of 3·78° C. at the surface. This is therefore the variation of the temperature of the inner surface of the cylinder corresponding to the temperature variation of the steam in contact with the surface. The only assumption made to obtain this

result is that the temperature variation on the surface is a simple *harmonic function* of the time.

Again, with this range at the surface, from (12), the quantity of heat flowing into the wall is

$$Q = \frac{3\cdot78 \times 3\cdot18}{13\cdot9} = 0\cdot864 \text{ lb.-cals. per cycle}$$

And from (13) this is equal to $0\cdot864 \times 73\cdot4 = 63\cdot4$ lb.-cals. per minute.

The wave length is from (10)

$$x = \frac{2\pi}{13\cdot9} = 0\cdot452 \text{ in.}$$

The variation of temperature at this depth may be calculated from (11)

$$T = 3\cdot78\epsilon^{-13\cdot9 \times 0\cdot452}$$

from which $T = 0\cdot00712$, thus verifying the assumption made that at the depth of the wave length the temperature variation has practically subsided to the mean temperature.

This example may be continued to illustrate the determination of the constant of the condensation area.

At the time the data used in the previous example were measured, namely, a temperature variation of 2·2° C. at a depth of 0·039 in. at 73·4 revolutions per minute, it was observed that the mean temperature of the wall was 149° C. Using this temperature in connection with the steam temperature curve plotted from the indicator diagram, it was found that the condensation area measured 1·43 degree C. sixtieths of a cycle units. But the quantity of heat flowing into the wall as calculated above is 0·864 lb.-cal. per square foot per cycle. Therefore

$$\frac{Q}{Z} = \frac{0\cdot864}{1\cdot43} = 0\cdot604$$

The average value of this ratio from 18 experiments in different conditions was found to be 0·61.

The actual form of the metal temperature cycle is not simple harmonic. To judge of the influence of the form of the cycle on the heat-flow, Callendar and Nicolson assumed an extreme arbitrary form and analysed it into a Fourier series of 12 terms, and then calculated the corresponding temperature range on the wall surface from the observed range at 0·04 in. They found that the temperature range was greater than the range for the corresponding simple harmonic cycle found as above, but that the quantity of heat Q flowing into the walls was practically the same as if calculated from a simple harmonic cycle distribution of temperature. The form of the cycle has therefore only a small influence on the quantity of heat flowing per cycle, and for the practical purpose of calculating the heat-flow into the walls per revolution the simple

harmonic distribution of temperature may be assumed without involving serious error.

Professor Cotterill[1] applied Fourier's method in his investigation of the action of the sides of a cylinder, but at the time there were no data from which the amplitude a could be determined. Callendar and Nicolson, using the most delicate electrical methods of measuring temperature, supplied the data necessary to determine the variation of metal temperature actually produced by a given variation of steam temperature in the cylinder, and in addition they devised the method of condensation areas, a method which will be invaluable to engineers in the practical investigation of the actual behaviour of steam in the cylinder. It was truly said in the discussion at the Institution of Civil Engineers that the paper was really a gift to engineers from the laboratory.

70. Influences which modify the Magnitude of the Missing Quantity.—The ratio of expansion has a marked effect on the missing quantity. Theoretically the greatest efficiency is obtained when the steam falls from the initial to the back pressure by adiabatic expansion alone as in the Rankine Cycle. The number of expansions necessary to reduce the pressure in this way is relatively large. The expansion necessary is given by the ratio $\frac{q_2V_2}{V_1}$, where q_2 is the dryness fraction after adiabatic expansion from the initial pressure corresponding to the initial volume V_1 to the back pressure corresponding to the volume V_2, both V_1 and V_2 being the volumes found in the tables.

For example, 1 lb. of dry saturated steam at an initial pressure of 150 lbs. per square inch occupies a volume of 3·04 cub. ft. The dryness fraction after adiabatic expansion to a pressure of 20 lbs. per square inch is 0·88, and V_2, the tabular volume at this pressure, is 20·07 cub. ft. The necessary ratio of expansion is therefore

$$\frac{20\cdot07 \times 0\cdot88}{3\cdot04} = 6$$

And if the back pressure is 2 lbs. per square inch, the necessary ratio of expansion is 46.

It is found experimentally that as the ratio of expansion is increased the missing quantity is increased. The partial explanation of this is that with the increased range of expansion there is a greater range of temperature variation on the cylinder walls, and the initial condensation is increased per square foot of wall area; but the factor most influential in augmenting the missing quantity is the increased wall surface which necessarily accompanies an increase in the expansion ratio.

Consider the figures worked out above. Adiabatic expansion from 150 to 20 lbs. pressure requires a cylinder whose final

[1] "The Steam Engine," Cotterill. E. F. Spon, 1890.

volume is about 18 cub. ft. per pound. If expansion is carried adiabatically from 150 to 2 lbs. the cylinder volume is q_2V_2 = about 140 cub. ft. per pound. If there had been no expansion at all, the cylinder volume would have been 3 cub. ft. only. Wall surfaces corresponding to 18 cub. ft. and 140 cub. ft. are therefore the necessary accompaniments of the adiabatic expansion of 1 lb. of dry saturated steam from 150 to 20 lbs. and from 150 to 2 lbs. per square inch back pressure respectively.

The increased wall surface means increased condensation and also increased leak, because the leak away depends upon the linear extent of the valve and piston peripheries, both of which increase as the size of the cylinder increases.

The gain resulting from increased expansion is therefore reduced by initial condensation and leak, both of which increase as the expansion ratio increases, and there is for each cylinder a particular ratio of expansion at which the gain is balanced by the loss. The ratio of expansion at which equality of gain and loss occurs is not sharply defined and as a rule the ratio of expansion may be varied considerably without causing much change in the steam supply to the cylinder per I.H.P. hour.

These points may be illustrated by citing the results of some experiments made by Mr. Bryan Donkin on a small single-cylinder vertical surface condensing engine.[1] Cylinder, 6 ins. diameter × 8 ins. stroke.

A series of experiments was made at about 218 revolutions per minute with an initial pressure of 65 lbs. per square inch (gauge pressure 50 lbs. per square inch). The back pressure was about 5 lbs. per square inch. These are the results obtained:—

Ratio of expansion	1·8	2·6	3·7	4·8	6·0	6·8	
Feed water per I.H.P. hour . .	39·9	38·7	41·2	41·0	44·6	45·6	lbs.

In this case the moderate ratio of 2·6 appears to be the most economical, notwithstanding that the ratio of adiabatic expansion from 65 to 5 lbs. is nearly 10. The engine is a small one, and therefore the wall action is great in proportion to the volume of the steam. As the cylinder increases in size the relative effect of the wall action decreases, the most economical ratio of expansion is increased, and the steam supply per I.H.P. hour is less affected by changing the ratio of expansion.

A steam jacket has a beneficial influence especially when applied to small cylinders. To be effective, the jacket should be supplied with steam at the boiler pressure, and every precaution must be taken to prevent the accumulation of condensed steam in the jacket. The jacket acts to maintain a high mean temperature of the metal cycle, and thus reduces the condensation area, and indirectly it may act to reduce the leakage by promoting a more uniform temperature of all the parts.

[1] *Proc. Inst. Mech. E.*, Oct. 1892, 467.

In the trials quoted above the cylinder was provided with a jacket, but steam was not turned into it.

The effect of turning steam into the jackets is shown by the following results. The "feed water per I.H.P. hour" includes the steam used in the jackets.

Jacketed Trials.

Ratio of expansion	1·8	2·6	3·7	4·8	6·0	6·8
Feed water per I.H.P. hour . .	30·7	28·8	28·4	25·2	26·7	27·2 lbs.

Comparing these two sets of figures, the reduction of the missing quantity by the use of a hotter cylinder is remarkable. The best ratio of expansion is now increased from 2·6 to 4·8. The records of many experiments primarily intended to establish the value of the steam jacket are to be found in the Reports[1] of the Research Committee of the Institution of Mechanical Engineers.

Records of a series of experiments at various ratios of expansion, condensing and non-condensing, with and without steam jackets, and with saturated and superheated steam, will be found in a later paper by Mr. Bryan Donkin.[2]

It appears from the Reports of the Institution of Mechanical Engineers that the saving of steam produced by a steam jacket is greater than the steam required by the jacket. The economy is considerable in the case of small cylinders, but is less pronounced as the size of the cylinder is increased.

Accepting Callendar and Nicolson's conclusions that the greater proportion of the steam condensed is condensed on the clearance surface, it would appear that jackets applied to the covers only would be more effective than jackets applied to the cylinder barrel only. Cover jackets alone with a well-lagged cylinder barrel should therefore give good results.

Speed affects the condensation area by reducing the temperature range of the metal cycle, and it would be expected that an increase of speed would result in an increase of efficiency. This is clearly shown by the Willans experiments.[3] In a simple engine the consumption of steam at an initial pressure of 130 lbs. per square inch was reduced from 23·7 lbs. per I.H.P. hour to 20·3 lbs. by the mere increase of the speed from 131 to 405 revolutions per minute, and in the condensing trials with steam at an initial pressure of 90 lbs. per square inch an increase of speed from 116 to 401 revolutions per minute reduced the consumption from 20 to 17·3 lbs. per I.H.P. hour.

In general the speed can be varied considerably without making a great change in the economy.

The increased efficiency produced by increased speed may be illustrated by reference to a set of experiments made by Prof. Goss on a locomotive and recorded in "High Steam Pressures in Locomotive Service," 1907.

[1] *Proc. Inst. Mech. E.*, 1889, 1892, 1905. [2] *Ibid.*, 1895.
[3] Willans, *Proc. Inst. C.E.*, March, 1888; April, 1893.

The experiments also further illustrate the point that the best economy is obtained by moderate ratios of expansion, and also that both speed and ratio of expansion may be changed considerably without causing much variation in the economy.

The engine was fitted with two cylinders each 16 ins. diameter and 22 ins. stroke. Clearance averaged 7·6 per cent. of the effective volume. Boiler pressure 180 lbs. per square inch by gauge. The cylinders were non-jacketed, but were lagged. The table shows the feed water per I.H.P. hour.

TABLE 16.—Showing the Influence of Speed and the Influence of the Ratio of Expansion on the Economy.

Revolutions per minute.	Steam per I.H.P. hour. Number of expansions. 5	4	3·2	2·6
98			25·44	25·91
146	26·54	25·36	24·62	24·61
194	25·89	24·08	23·68	25·85
245	26·61	24·43	24·87	

The most economical condition of working is at 194 revolutions per minute with an expansion ratio of 3·2.

Following a horizontal line, it will be seen that at 98 revolutions per minute 3·2 expansions were better than 2·6.

At 146 revolutions per minute 3·2 expansions give about the same economy as 2·6, but the loss exceeds the gain if the expansion is increased to 4, and the loss is greater still at 5 expansions.

At 194 revolutions per minute a ratio of 3·2 is still the best, whilst at 245 revolutions per minute the range may be increased to 4 with economy.

Considering the results vertically, it will be seen that there is a gain of economy with increase of speed up to 194 revolutions per minute, but at the highest speed the economy falls off, probably because the effect of wire-drawing is increasing rapidly.

In this set of experiments the indicated horse-power varied from 235 at 146 revolutions per minute with 5 expansions to a maximum of 609 at 194 revolutions with 2·6 expansions.

The indicated horse-power corresponding to the most economical conditions was 524.

Prof. Goss also established the influence of the initial pressure on the economy of the engine by running trials with boiler pressures ranging from 120 to 240 lbs. per square inch. The results for normal conditions of running are given below.

Boiler pressure by gauge.	Steam per I.H.P. hour in pounds.
120	29·1
140	27·7
160	26·6
180	26·0
200	25·5
220	25·1
240	24·7

71. Compound Expansion.—The range of expansion may be considerably increased without proportionately increasing the missing quantity if the expansion is carried out in a series of stages in separate cylinders. Or put in another way, the missing quantity

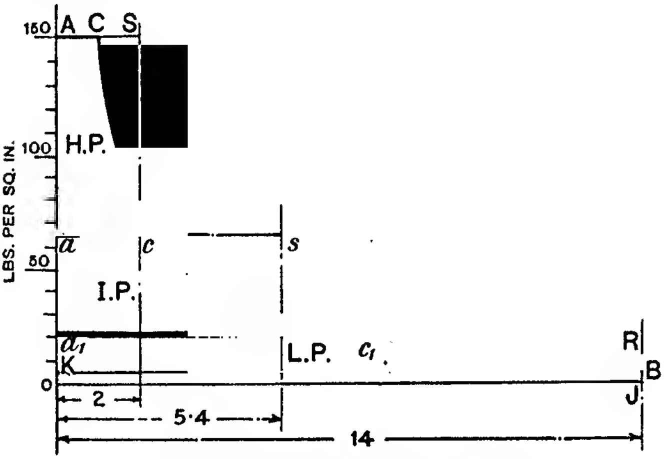


FIG. 77.—Compound expansion in three stages.

corresponding to a given range of expansion in a single cylinder is reduced if the expansion is carried out in stages in separate cylinders.

Although losses are incurred by the transference of the steam from cylinder to cylinder in the series as expansion goes on, yet these losses are small in comparison with the gain in efficiency when the initial pressure is high.

The division into three stages of the expansion of 1 cub. ft. of steam from an initial pressure of 150 lbs. per square inch to a final volume of 14 cub. ft. is illustrated in Fig. 77. The expansion curve is drawn from the relation PV = a constant, so that the pressure at the end of expansion is 10·6 lbs. per square inch. The ratio of expansion is 14, a ratio considerably greater than the best ratio for a single cylinder.

Assuming a back pressure of 5 lbs. per square inch, the diagram

ACRBK is the hypothetical indicator card corresponding to 14 expansions of a cubic foot of steam from an initial pressure of 150 lbs. per square inch in a single cylinder, omitting the clearance steam and neglecting small losses.

The whole expansion is divided into three stages, and the pressure difference in each stage is usually chosen so that the three areas into which the hypothetical diagram for the whole expansion is divided are equal. If so divided the indicated horse-power of each cylinder of the series is the same. But if desired the pressure stages may be arranged to secure equal temperature difference in each stage. The practice in regard to an ordinary triple-expansion engine for the mercantile marine when the initial pressure is about 150 lbs. per square inch is to make the cylinder volumes in the proportions

$$\text{H.P.} : \text{Int.P.} : \text{L.P.} = 1 : 2{\cdot}7 : 7$$

Using these ratios for the purposes of illustration, on the scale of Fig. 77, they correspond to volumes of 2, 5·4 and 14 cub. ft.

The volume of the high-pressure cylinder is thus 2 cub. ft., and it is represented by AS.

The volume of the intermediate cylinder is 5·4 cub. ft., and is shown by *as*.

The volume of the low-pressure cylinder OJ remains unaltered, and is 14 cub. ft.

Drawing vertical lines across the diagram corresponding to these volumes, it will be seen that the cut off in the high-pressure cylinder is at 50 per cent.; in the intermediate cylinder at 45 per cent.; and in the low-pressure cylinder at 52 per cent.

The drop at the end of expansion is, in the high-pressure cylinder, 10 lbs., in the intermediate cylinder, 5 lbs., and in the low-pressure cylinder, 5 lbs.

The difference between the back pressure in the high-pressure cylinder and the initial pressure in the intermediate cylinder is about 3 lbs. This represents the loss of pressure due to transference of the steam from the one cylinder to the other. Similarly, there is a transference drop between the intermediate and low-pressure cylinder.

The diagram losses are blackened in the figure.

It will be seen from the figure that the pressure stages between the initial pressure and the final back pressure are determined by the drop at the end of expansion in each cylinder. A drop is desirable as explained above, and the pressure in any cylinder should not be allowed to fall below the back pressure added to the engine resistance.

It will be clear from the figure that the division may be varied considerably, and the volume ratios may be varied considerably without affecting the total power.

The fixed quantities on which the total power depend are the volume of the low-pressure cylinder, the volume at cut off in the

high-pressure cylinder, the initial pressure, and the back pressure in the low-pressure cylinder.

The ratio of expansion in a compound engine is measured by the ratio of the volume of the low-pressure cylinder (including clearance) to the volume of the steam at cut off in the high-pressure cylinder.

Measuring from the diagram the pressure stages are from the initial pressure of 150 lbs. per square inch to a back pressure of 68 lbs. per square inch in the high-pressure cylinder; from an initial pressure 65 lbs. per square inch to a back pressure of 23 lbs. per square inch in the intermediate cylinder; and from an initial pressure of 20 to a back pressure of 5 lbs. per square inch in the low-pressure cylinder.

These pressure differences, namely, 82, 42, and 15 lbs. per square inch, correspond to temperature intervals of 32°, 35°, and 36° C. The division determined by the volume ratios chosen and the pressure drops at the end of expansion determine an almost equal division of the whole temperature range between the three cylinders.

Comparing together a simple and a compound engine of equal power, in each of which the ratio of expansion, the initial and the condenser pressures are the same, it will be seen that—

(1) The cylinder of the simple engine is equal in volume to the low-pressure cylinder of the compound engine;

(2) The temperature difference in each cylinder of the compound engine is reduced to a fraction of the whole temperature range in the single cylinder of the simple engine, thus reducing the loss of steam by condensation;

(3) The pressure difference in each cylinder of the compound engine is reduced to a fraction of the whole pressure range in the single cylinder of the simple engine, thus reducing the loss by leak.

It is only the leak from the low-pressure valve and piston which ultimately passes into the condenser as a loss, inasmuch as leak through the high-pressure valve or piston passes into the steam chest of the intermediate cylinder; and leak from the intermediate valve and piston passes into the low-pressure valve chest. In the example considered above the result of compounding is to reduce the temperature range from 109° C. to about one-third of this in each cylinder, and to reduce the pressure range from 145 lbs. to 15 lbs. in the low-pressure cylinder.

The leaking peripheries of the valves would be larger in the case of the low-pressure cylinder than in the case of the valves fitted to a cylinder of equal volume in which high-pressure steam was used directly, so that the reduction of pressure in the low-pressure cylinder obtained by compound working is partly compensated for by the increase in the length of the leaking periphery, but on the whole it may be expected that there would be a smaller loss by leak with the small pressure difference found in the low-pressure cylinder of a compound engine than there would be with the large pressure difference in the corresponding single cylinder of a simple engine.

The economy achieved by compound working is therefore due partly to the reduction of initial condensation and partly to the reduction of the loss by leak. The only practicable way to get an idea of the proportion of the whole loss caused by either one of these causes is to estimate the condensation in each cylinder by Callendar and Nicolson's method of condensation areas and then find the loss by leak as a difference, the total missing quantity being found from the indicator cards and an independent measurement of the feed water in the way illustrated in Section 68, page 240.

The back pressure against which the steam in the high-pressure cylinder is discharged and the initial pressure of the supply to the intermediate cylinder, are in Fig. 77 shown constant. These conditions can only be realized when a receiver is placed between the two cylinders many times larger in volume than the volume of the intermediate cylinder. In practice the pipe connecting the exhaust orifice of the high-pressure cylinder with the steam chest of the intermediate cylinder often acts as the receiver, and even when this is enlarged to form a receiver of greater capacity the volume compared with the volume of the intermediate cylinder is relatively small. Consequently, the pressure in the receiver rises and falls as it receives the exhaust steam from the high-pressure cylinder or is drawn upon to supply steam to the intermediate cylinder. The pressure can only remain constant if the rate at which the receiver receives steam from the one cylinder is equal to the rate at which it supplies steam to the other cylinder. This condition is never realized because the admission valve to the intermediate cylinder and the exhaust valve of the high-pressure cylinder are not open during the same part of the cycle.

When the exhaust valve of the high-pressure cylinder is closed and the admission valve to the intermediate cylinder is open, the pressure in the receiver falls; and the pressure rises during that part of the revolution where the exhaust valve of the high-pressure cylinder is open and the admission valve of the intermediate cylinder is closed. There is thus a cyclical variation of pressure in a receiver between two cylinders, and this may be recorded by an ordinary indicator, the drum being driven from the crosshead of either of the cylinders. The range of variation of cyclical pressure depends upon the volume of the receiver, the valve settings, and upon the angle between the two cranks belonging to the cylinders.

Similar remarks apply to the receiver between the intermediate and the low-pressure cylinders.

In practice therefore the blackened areas on the illustrative diagram would be irregular.

It is advisable to keep the receiver as small as possible consistent with a moderate range of the cyclical variation of pressure within it, in order to reduce the surface from which radiation can take place and on which condensation of the steam occurs. Care must be taken to thoroughly lag and to thoroughly drain a receiver.

Sometimes means are taken to add heat to the steam as it passes through the receiver in order to ensure a supply of dry steam to the next cylinder. The receiver is then often called a re-heater. Thermodynamically this is not a good way of working, because heat is added at a temperature below the highest temperature in the cycle and full advantage is not taken of the temperature fall established by the boiler and condenser. In many cases small receivers well lagged and drained are more efficient than larger receivers arranged for re-heating.

The distribution of power between the several cylinders of a compound engine is finally made by the adjustment of the cut off in each cylinder separately. If the cut off in a cylinder is reduced, a reduced volume of steam is drawn per cycle from the receiver; but inasmuch as the weight flowing through the cylinder will not be sensibly altered, since the weight of steam flowing through the engine depends upon the cut off in the high-pressure cylinder, the reduced supply is received at a higher pressure, and consequently more work is done in the cylinder per cycle. But this is not all. The increase of receiver pressure increases the back pressure against which the higher-pressure cylinder works, and the work done per cycle is therefore reduced. Thus the effect of reducing the cut off in a cylinder forming one of a series is twofold—it increases the work done in the cylinder and reduces the work done in the cylinder above it. Pushing the matter to a limit, it will be seen that if the cut off in the low-pressure cylinder is reduced until the range of expansion is the same as the total range through the engine, the cylinders and receivers above it in the series become a mere extension of the steam supply pipe, and all the power is developed in the low-pressure cylinder. On the other hand, if the cut off in the lower-pressure cylinder is put late in the stroke, the back pressure in the cylinder above is so reduced that all the work is thrown on to the cylinder above and the lower-pressure cylinder tends to become merely an exhaust pipe, either to the cylinder below it or to the condenser if it is the last in the series.

A compound engine has the mechanical advantage that the distribution of power between two, three and sometimes four cylinders enables them to be arranged so that the variation of the forces acting on the framing of the engine is reduced, and a turning effort on the crank shaft more nearly uniform can be obtained than is possible if the power is developed in one or more cylinders each supplied with steam at the initial pressure and exhausting into the condenser. To use an electrical analogy, in a compound engine the cylinders are placed in series across the pressure difference established by the boiler and condenser, whilst in a multi-cylinder simple engine of equal power they are placed across in parallel, so that each cylinder works between the full range of pressure and the full range of temperature. The advantage of reducing the pressure range will be fully considered in the chapter dealing with crank-effort diagrams.

72. Example. The Combination of Diagrams. The advantage gained by compound expansion is directly shown by the following trial, which was made by the author on the experimental engine of the City and Guilds Engineering College. The engine is a two-cylinder cross compound, with a receiver between the high- and low-pressure cylinders; and the cranks are at right angles. These are the cylinder *dimensions*:—

High-pressure cylinder: Diameter, 8·73 ins.; stroke, 22 ins.; effective volume, 0·762 cub. ft.; clearance 0·091 cub. ft.

Low-pressure cylinder: Diameter, 15·76 ins.; stroke, 22 ins.; effective volume. 2·45 cub. ft.; clearance volume, 0·172 cub. ft.

The object of the trials was to compare the steam consumption when working with nearly the same range of expansion, first, when the expansion is carried out in a single stage in one cylinder; secondly, when the expansion is carried out in two stages in two cylinders.

The large cylinder alone was used for the single-stage expansion. The cut off was set at 0·12 of the stroke, and allowing for clearance, this corresponds with 0·476 cub. ft. of steam in the cylinder at cut off, and a real ratio of expansion of 5·6.

A typical indicator diagram taken during this trial is shown in Fig. 76.

Typical high- and low-pressure diagrams, taken during the compound trial, are shown calibrated in Fig. 78. The data for the volume calibrations are given above. The barometer during both trials remained steady at 29·1 ins., giving a pressure of 14·28 lbs. per square inch. The calibration of each diagram is done in the way explained in Section 2, Fig. 5. The final volume of the steam in the low-pressure cylinder is 2·62 cub. ft. And from the calibrated high-pressure diagram the volume at cut off is nearly 0·4 cub. ft. The ratio of expansion secured in the compound trial is therefore 6·5. This is greater than in the simple trial, but it is sufficiently near to enable a comparison of the steam consumption to be made.

The following are the results:—

UNJACKETED—(Oct. 27, 1911).

	Simple.	Compound.
Ratio of expansion	5·6	6·5
Speed: Revs. per minute	92	95
Indicated horse-power	39·06	39·1
Air pump discharge: lbs. per hour	1080	854
Receiver drainage: „ „ „	—	35
Total supply to H.P. cylinder: lbs. per hour	—	890
Steam per I.H.P. hour	27·7	22·8

Saving by compound working, 5·9 lbs per I.H.P. hour, equal to 21 per cent.

An engine of the same size and power built for ordinary purposes would probably show a better economy, because there is a larger area of pipe surface in the experimental engine than would usually be found in practice, and hence there is a greater loss by radiation.

The barrels of each cylinder and the back cylinder covers are provided with steam jackets, but these trials were carried out without any steam in them. A comparative set of trials was made with

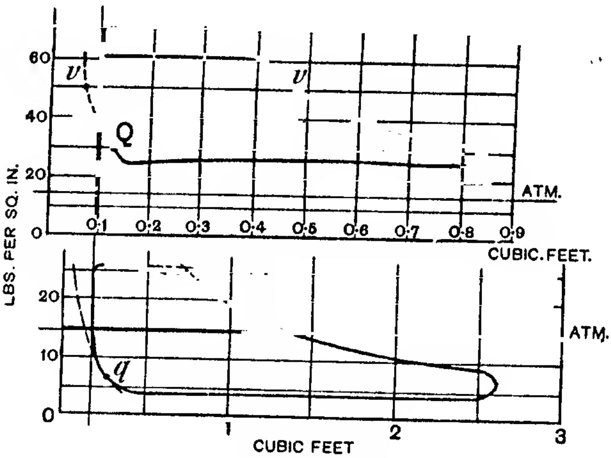


FIG. 78.—Calibrated H.P. and L.P. diagrams.

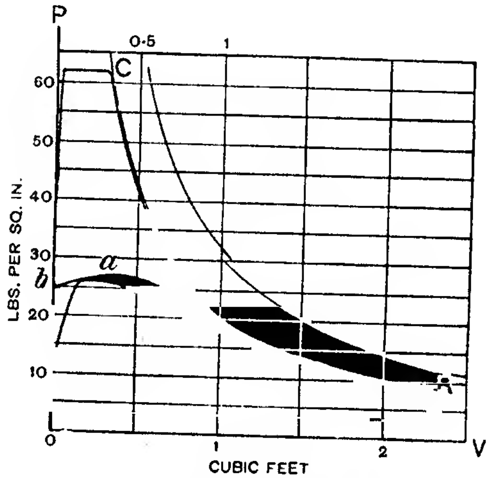


FIG. 79.—Combination to same scale of diagrams of Fig. 78.

steam at the boiler pressure in all the jackets in each case. There is a distinct saving due to jacketing, as will be seen from the following results :—

JACKETED TRIALS—(Nov. 17, 1911).

	Simple.	Compound.
Ratio of expansion	5·3	6·5
Speed: Revs. per minute	90	89
Indicated horse-power	44·3	39·5
Air pump discharge: lbs. per hour	1012	700
Jacket drainage: " "	58	80
Receiver drainage: " "	—	26
Total steam supply: " "	1070	806
Steam per I.H.P. hour	24·1	20·4

The saving by compound working is not so great as in the unjacketed trials, being 15 per cent. as against 21 per cent.

The saving due to the jacket in the simple engine is 3·6 lbs. of steam per I.H.P. hour, and in the compound engine it is rather less—2·4 lbs. per I.H.P. hour.

In this comparison the ratio of expansion in the simple trials is greater than the most economical for the single cylinder, and in the compound trials it is less than the most economical for the two cylinders. Greater economy would be shown in the simple trial with a ratio of about 4, and in the compound trials with a ratio of about 10.

Professor Mellanby [1] published the results of an elaborate series of experiments which he made on the experimental engine at the Manchester School of Technology, in order to determine the economy due to jacketing.

Five trials were made at each of the following ranges of expansion: 25; 18; 12·3; 8·1. The best results were obtained at expansions ranging between 14 and 19 when the ends of both the high-pressure and the low-pressure cylinders were steam jacketed together with the barrel of the high-pressure cylinder only. With these conditions the consumption ranged from 16·95 to 17 lbs. of steam per I.H.P. hour. With no steam in any of the jackets, the consumption ranged between 18 and 18·2 lbs. per I.H.P. hour, whilst the expansion ranged from 11 to 17. The ranges of expansion given in each case were the most economical. The consumption per I.H.P. hour increased if the expansion ratios were taken outside the limits given in either direction.

The trials were all made at 60 revolutions per minute. The engine is a two-cylinder cross compound. The high-pressure cylinder is $11\frac{1}{2}$ ins. diameter and 36 ins. stroke, and is fitted with a Corliss valve gear. The low-pressure cylinder is 20 ins. diameter and 36 ins. stroke, and is fitted with slide valves and Meyer expansion plates. Each jacket is supplied direct with steam from the boiler.

The two diagrams shown in Fig. 78 are drawn to the same scale of pressure and volume in Fig. 79 with the clearance steam eliminated. The combination is done as follows:—

(1) Draw the pressure and volume axes OV, OP, Fig. 79. It

[1] *Proc. Inst. Mech. E.*, June, 1905: "Steam Jacketing".

will generally be found convenient to use the scales of the low-pressure diagram for these axes.

(2) Select points Q and q on the compression curves after the compression has well begun, and draw through them the expansion curve of the clearance steam. The dotted curves through Q and q, Fig. 78, are drawn to the relation PV = a constant, this being sufficiently accurate to represent the expansion curve of the clearance steam. See Section 64 above.

(3) Replot each diagram to the axes OV, OP, Fig. 79, using for the volume at any particular pressure the intercept between the dotted curve and the expansion curve, Fig. 78. For example, when the pressure is 50 lbs. per square inch, the volume of the steam in the high-pressure diagram to be set out from OP in the combined diagram along the 50-lbs. line is vv, corresponding to 0·41 cub. ft.

The actual volume of steam at cut off is 0·33 cub. ft. after the clearance steam has been eliminated, and this is the point C in the combined diagram.

An expansion curve CR is drawn through C; from the equation PV = a constant, to serve as a standard with which to compare the expansion curves of the two parts of the diagram.

The expansion curve of the high-pressure cylinder isindistinguishable from the dotted curve CR. The expansion curve of the low-pressure diagram falls below it. It must be remembered, however, that the actual weight of steam fed to the low-pressure cylinder is less than that fed to the high-pressure cylinder by the amount condensed in the receiver between the cylinders, which in this case amounted to 35 lbs. per hour. Allowing for this correction, the agreement between the actual expansion curves in the combined diagram and the hyperbola through C is remarkably close.

The modification of the shape of the area representing the loss due to transference in comparison with the uniform pressure lines assumed in the explanatory diagram, Fig. 77, is well shown in the combined diagram, Fig. 79. There is a slight overlapping shown cross hatched to be set against the blackened area.

The increase of the back pressure as the high-pressure exhaust steam flows into the receiver is shown by the rising of the back-pressure line of the diagram towards a maximum in the region where the admission valve of the low-pressure cylinder opens, after which the pressure falls as steam flows out into the larger cylinder. This shape of the area is typical of engines with cranks set at 90°.

The cut off in the low-pressure cylinder is much earlier than would be inferred from the standard curve, owing to the loss of steam in the receiver and owing to the losses in pressure in the nature of drop between the cylinders.

The overlap of the back-pressure line of the high-pressure cylinder and the admission line of the low-pressure cylinder is brought about by the difference of phase between the points on the two curves. For instance, the point a shows approximately the

position on the back-pressure line where steam begins to flow from the receiver into the low-pressure cylinder; the corresponding point on the admission line is at *b*. Admission and back pressures measured along the same ordinate are not simultaneous in point of time. The phase difference depends upon the angle between the cranks.

A saturation curve is drawn in Fig. 79 for the actual steam supply to the high-pressure cylinder, namely, 890 lbs. per hour, equal to 0·078 lb. per stroke, and this is broken as it passes into the low-pressure region to correspond with the steam supply to the low-pressure cylinder, namely, 854 lbs. per hour, equal to 0·075 lb. per stroke.

73. The Compound Marine Engine.—In the year 1874 the marine engine[1] had developed into a vertical direct-acting surface-condensing engine, in which the steam expanded by two stages from initial pressures ranging between 50 and 100 lbs. per square inch. The introduction of three-stage expansion from an initial pressure of 150 lbs. per square inch, introduced by Dr. Kirk in 1874 and 1881, resulted in such marked economy that this type of engine soon became general both in the mercantile marine and in warships. Even to-day the bulk of marine engines afloat are three-stage expansion engines using steam at an initial pressure of 150 to 160 lbs. per square inch, though in large ships and in special cases this pressure is often exceeded and the number of expansion stages is increased.

The economy of the triple-expansion engine, using steam at about 150 lbs. per square inch in comparison with two-stage expansion from initial pressures ranging from 70 to 120 lbs. per square inch, is established by the Reports of the Research Committee on Marine Engine Trials, a summary of which is given by Professor Beare[2] in the *Proceedings* for 1894.

The main results of these trials are given in Table 17.

The steam consumption averaged about 21 lbs. per I.H.P. hour in the case of the two-stage expansion engines, along with a fuel consumption ranging from 2·32 to 2·9 lbs. of coal per I.H.P. hour.

The steam consumption averaged about 14 lbs. per I.H.P. hour in the three-stage expansion engines, rejecting the high consumption of the *Tartar*, which was probably due to priming since the fuel consumption ranged with the other two engines tried. The fuel consumption varied from 1·46 to 2 lbs. of coal per I.H.P. hour.

These results establish a steam economy of about 33 per cent. in favour of the triple compound engines, along with a fuel economy

[1] I am indebted to Mr. Druitt Halpin for a reference to a paper by J. Grantham, M.Inst.C.E., on "Ocean Steam Navigation," in the *Proc. Inst. Civil E.*, vol. 29, 1870. In this paper will be found interesting historical particulars of engine performance at that date.

[2] "Abstract of Results of Experiments on six Steamers and conclusions drawn therefrom in regard to the efficiency of Marine Boilers and Engines," by Professor T. H. Beare, *Proc. Inst. Mech. E.*, 1894. Part I.

TABLE 17.—STEAM ENGINE TRIALS.

	2nd class cruiser *Minerva.*	2nd class cruiser *Hyacinth.*	R.M.S. *Saxonia.*	3rd class cruiser *Medusa.*	3rd class cruiser *Medea.*	Cargo coaster *Fusi Yama.*	G.E.R. passenger *Colchester.*	Paddle Mail Steamer, Dover and Ostend, *Ville de Douvres.*	Cargo and passenger *Meteor.*	Cargo *Tartar.*	Cargo *Iona.*	Single cylinder uniflow engine constructed by Sulzer.
Displacement (tons)	5600	5600	22,580	2800	2800	2175	1675	1090	2090	2250	4430	—
Number of engines	2	2	2	2	2	1	2	1	1	1	1	—
Type of engine	Vertical triple expansion.	Vertical triple expansion.	Vertical quad. expansion.	Vertical triple expansion.	Vertical triple expansion.	Vertical compound expansion.	Vertical compound expansion.	Inclined compound expansion.	Vertical triple expansion.	Vertical triple expansion.	Vertical triple expansion.	Single cylinder uniflow.
Number of cylinders per eng.	3	4	4	3	3	2	2	2	3	3	3	1
Diam. of H.P. cylinder (ins.)	33	26	29	25	25	27·35	30	50·12	29·37	26·03	21·88	48·3
,, I.P. ,, ,,	49	42	1st 41½ 2nd 59	43	43	—	—	—	44	42	34	—
,, L.P. ,, ,,	74	2, each 48″	84	74	74	50·3	57	97·12	70·12	68·95	56·95	—
Stroke	3′ 3″	2′ 6″	4′ 6″	3′ 3″	3′ 3″	2′ 9″	3′ 0″	6′ 0″	3′ 11·94″	3′ 6″	3′ 3″	3′ 11·2″
Type of boiler	Single-ended cylindrical	Belleville	Single-ended cylindrical	Water-tube Durr type	Water-tube Yarrow, large tube type	Single-ended cylindrical	Double-ended cylindrical	Single-ended cylindrical	Double-ended cylindrical	Double-ended cylindrical	Single-ended cylindrical	—
Boiler pressure, by gauge (lbs. per sq. inch)	148	277	199	230	229	71·64	95·5	120·64	160·1	158·2	179·58	170
Pressure in H.P. valve chest	136	220	192	—	—	65	76	104	149	136	157	—
,, ,, exhaust pipe	—	—	—	—	—	3	3	5	3	2	1	—
,, ,, condenser	—	—	—	—	—	2·32	2·51	4·72	2·73	1·7	0·7	—
Speed. Revs. per min.	128	171	78	—	—	55·59	86	36·82	71·78	70	61·1	110
Total I.H.P.	8132	10,180	9099	7346	7556	3713	1979·7	2977	1994	1057·4	645·4	1610
Steam (lbs.) per I.H.P. hour: Main engines	16·30	15·42	13·47	14·47	15·06	—	—	—	—	—	—	—
Jackets	0·36	—	—	—	—	—	—	—	—	—	—	—
Auxiliary engine	2·20	1·72	0·86	1·50	1·41	—	—	—	—	—	—	—
Total	18·86	17·14	14·33	15·97	16·47	21·17	21·73	20·77	14·98	19·83	13·35	19·8
Make-up water per I.H.P. hour	0·35	0·45	0·26	—	—	—	—	—	—	—	—	—
Heat supplied to main engine (p. min.) (including jackets), reckoned from exh. temp. Lb.-cals.	1,377,777	1,637,222	1,251,666	1,101,111	1,168,333	78,333	438,000	60,666	29,366	↑	89,500	—
Thermal efficiency of engine per cent.	13·9	14·6	17·2	15·7	15·2	11·2	10·7	11·7	16·1	Priming	17·1	20
Thermal efficiency of Rankine engine per cent.	24·8	28·7	27·4	28·3	28·3	19·45	20·3	20·3	25·3		29·7	31·4
Efficiency ratio	56·1	50·9	62·8	55·5	53·7	57·8	52·8	57·6	63·6	↓	57·6	64
Coal per I.H.P. hour (lbs.)	2·42	2·11	1·29	2·06	2·0	2·66	2·9	2·32	2·01	1·77	1·46	—
Reference to trial	Admiralty Committee on Naval Boilers, Reports: 1902	1902	1902	1904	1904	1890	*Proceedings of Institute of Mech. Engrs.*: 1890	1892	1889	1890	1891	—

which is more variable owing to differences in the boilers with which the engines were associated. Comparing the fuel consumption of the best triple-expansion engine with that of the best two-stage expansion engine, the difference is $2·32 - 1·46 = 0·86$ lb. of coal per I.H.P. hour in favour of the triple, a saving of 37 per cent. Comparing the worst three-stage with the worst two-stage, there is a difference of $2·9 - 2 = 0·9$ lb. of coal per I.H.P. hour, or 30 per cent. saving in favour of the triple expansion engine.

Details of more recent trials by a Committee of the Admiralty are given in the table.

The three stages of the expansion in the ordinary triple-expansion engine are usually carried out in three cylinders placed side by side in the order, H.P.—Intermediate—L.P., but in some engines this order is varied, as in the *Iona* of the marine engine trials above referred to, where the high-pressure cylinder is placed in the middle.

In large engines the volume required for the last stage of the expansion is so great that two low-pressure cylinders are generally provided, so connected that the exhaust steam from the intermediate cylinder flows into them simultaneously. That is to say, the two low-pressure cylinders are placed in parallel between the intermediate cylinder and the condenser. The engine then becomes a three-stage expansion engine with four cylinders, and when the type was first introduced the usual sequence was H.P.—Intermediate—L.P.—L.P.

This type of engine has the advantage that it can be well balanced, because, unless there are at least four cranks, the reciprocating parts cannot be balanced amongst themselves. Crank angles can be found for which four sets of reciprocating parts are in balance, the inner pistons being thickened in order to bring them to the weight required for the conditions of balance.

A modification of this sequence of cylinders was introduced in the engines of the *Innisfallen*, built by Messrs. Wigham and Richardson in 1895, in order to reduce the thickening of the pistons to a minimum. The *Innisfallen* arrangement is L.P.—Intermediate—H.P.—L.P.

This arrangement brings the reciprocating masses of the engine, as they would ordinarily be designed, nearer to the weights required for dynamical balance, and the addition of mass merely to satisfy these conditions is reduced to a minimum.

This type of engine may therefore be regarded as the best for a three-stage expansion engine which is required to run at a high speed without producing vibration, without limitation regarding size. It is a suitable type for comparatively small engines as well as for large engines. It is relatively more costly to build in smaller sizes than the three-cylinder triple expansion engine of equal power, but it has the advantage that it can be properly balanced, whilst the reciprocating masses of the three-crank engine cannot be balanced amongst themselves unless the cranks are mutually at 180°, an arrangement which is inconvenient and which in fact loses the advantage of the near approach to uniformity of turning moment

possessed by a triple-expansion engine with three cranks mutually at 120°.

The engines of s.s. *Deutschland*, built in 1900, furnish an example of a four-stage expansion engine supplied with steam at about 200 lbs. per square inch (225 lbs. per square inch boiler pressure). There are two engines developing together a total of 35,000 I.H.P.

Each of the two engines develops 17,500 H.P. distributed between six cylinders. The first stage of the expansion is done in two high-pressure cylinders, the second and third stages each in a single cylinder, and the fourth stage is carried out in two low-pressure cylinders. The flow of steam is therefore from the boiler through two high-pressure cylinders in parallel, and then through two intermediate cylinders in series, and finally through two low-pressure cylinders in parallel. There are, however, only four cranks. Each of the two inner cranks is driven by the combined action of one high-pressure cylinder placed vertically above one low-pressure cylinder, and each outer crank is driven by an intermediate cylinder. This arrangement brings the heavy reciprocating masses in the middle and the lighter mass on the outside, just the distribution required by the conditions of dynamical balance. The crank angles and the masses of the reciprocating parts are adjusted so that the engine is balanced on the Yarrow-Schlick-Tweedy system.

There are six sets of valve gear, one for each cylinder. The cylinder sizes are:—

2 high-pressure cylinders, each 3 ft. diameter and 6 ft. stroke.
1 intermediate cylinder, 6·1 ft. diameter, 6 ft. stroke.
1 intermediate cylinder, 8·6 ft. diameter, 6 ft. stroke.
2 low-pressure cylinders, each 8·84 ft. diameter, 6 ft. stroke.

The corresponding volume ratios are 1 : 2 : 4 : 8·7.

These dimensions give some idea of the size to which the reciprocating marine engine has developed.

Recent advances have been in the direction of the substitution of the steam turbine for the reciprocating engine, wholly in the case of warships and in many mercantile ships, and partly, as in the case of the recent White Star steamers, where a steam turbine is used in combination with reciprocating engines. These developments will be discussed more fully in the chapter on steam turbines.

74. The Compound Locomotive.—The development of the compound locomotive began in 1876 with the construction at the Creusot Works of an engine designed by Mr. Anatole Mallet for the Bayonne and Biarritz Railway Company.

In this engine steam was expanded in two stages from an initial pressure of 140 lbs. per square inch by gauge, in two cylinders placed side by side and coupled to cranks at right angles. The volume of the high-pressure cylinder was 0·72 cub. ft.; that of the low-pressure cylinder 2 cub. ft. The volume ratio is thus 1 : 2·8.

The economy which resulted led to rapid development and the

compound locomotive became a standard type of engine on the Continent and in America.

In England compound locomotives for regular service have been built only by the London and North-Western, the Great Eastern, the North-Eastern, and the Midland Railway Companies, although odd engines for experimental purposes have been built and tried on most of the larger railways.

The first compound locomotive in Great Britain was built at Crewe in 1878, and after further tentative experiments Mr. F. W. Webb designed the three-cylinder compound locomotive associated with his name. The "Experiment" class, as the first 30 were called, expanded steam from a boiler gauge pressure of 150 lbs. per square inch in two stages; the first stage in two high-pressure cylinders in parallel, placed outside the frames and connected to the trailing axle of the two driving axles provided; the second stage in one low-pressure cylinder, placed between the frames and connected to the leading driving axle. The driving wheels were 6 ft. 6 ins. in diameter, but no coupling rods were used. The engine was essentially a two-cylinder outside single, and a single cylinder inside single, concentrated into one engine.

Owing to the absence of coupling rods there was no definite phase relation between exhaust from the high-pressure and admission to the low-pressure cylinder, so that the receiver pressure was quite arbitrary. The low-pressure cylinder was 26 ins. diameter and 24 ins. stroke, and the volume ratio H.P. : L.P. = 1 : 2·64.

Many engines of this type were afterwards built with variations in the diameter of the driving wheels and in the cylinder proportions, the later engines having a boiler pressure of 175. lbs. per square inch, a low-pressure cylinder 30 ins. diameter and 24 ins. stroke; a volume ratio, H.P. : L.P. = 1 : 1·99, and driving wheels 6 ft. 3 ins. diameter. In 1897 Mr. Webb designed a four-cylinder compound expanding from 200 lbs. per square inch in two stages. The engine was essentially a pair of cross compound engines placed side by side, each engine being connected to a pair of cranks at 180° apart, and each 180° crank pair being placed at right angles. The two low-pressure cylinders were inside the frame, each 20½ ins. diameter and 24 ins. stroke; and the two high-pressure cylinders were outside the frame. The volume ratio was H.P. : L.P. = 1 : 1·82.

The Worsdell and Von Borries compound locomotive, many of which were built for the Great Eastern Railway, the North-Eastern Railway, and for railways abroad, is a two-stage, two-cylinder engine, similar in general design to an ordinary engine, but with one of the two cylinders enlarged to make a low-pressure cylinder. The dimensions of the cylinders of the first engine of the kind built at Stratford in 1885 were: high-pressure, 18 ins. diameter; low-pressure, 26 ins. diameter; and the common stroke 24 ins. In other respects the engine was an ordinary four-coupled express passenger engine, with driving wheels 7 ft. diameter, and a boiler pressure of 160 lbs. by gauge. The cylinders in a 7 ft. 6 ins. single engine built for the North-Eastern Railway Company in 1890 were:

high-pressure, 18 ins. diameter; low-pressure 28 ins. diameter, with a common stroke of 24 ins. The volume ratio is thus 1 : 2·42. In this latter engine the boiler pressure was 175 lbs. per square inch.

The compound locomotive has been scientifically developed in France, the land of its birth, to a state of high efficiency, and the two-cylinder simple engine using saturated steam is an obsolete type for large engines. The introduction in 1898 of the De Glehn engine by the late M. Du Bousquet on the Nord Railway marked an epoch in the history of compound locomotives. The De Glehn compound locomotive is a two-stage expansion four-cylinder engine, with two high-pressure cylinders placed outside the frames, connected to the trailing of the two driving axles; and with two low-pressure cylinders placed inside the frames and connected to the leading driving axle. Each of the two axles is thus independently driven. The wheels are coupled, but the rods have only to transmit small differences of power between the two axles, their chief duty being to maintain the phase difference between the axles. The inside cranks on the leading axle are placed at right angles, and each outside crank is at 180° with the inside crank nearest to it. In this way the reciprocating masses of the engine are balanced for forces, but not for horizontal couples. (This matter is discussed in Chapter IX.) The turning moment presents the same general characteristic as with two cranks at 90°, it is neither better nor worse. Each axle, in fact, is driven by two cranks at 90°, but even in cases where the four cylinders drive on to one axle the turning moment is the same in character as with two cranks at 90°. Each cylinder is provided with a separate valve gear of the Walschaerts type. The actual gear of one of these engines is illustrated in Fig. 206, Chapter X.

Engines of this kind were put on the service between Paris and Calais, a service remarkable both for the high average rate of speed and for the punctuality with which the trains were run.

The engines, which were designed in 1901, were of the Atlantic type, so far as the wheel arrangements were concerned, there being a leading bogie, four coupled wheels each 80 ins. diameter, and a pair of trailing wheels, as indicated by the figures 4—4—2.

The cylinders were: high pressure, 13·375 ins. diameter; low pressure, 22 ins. diameter; the common stroke being 25·2 ins. The volume ratio is thus 1 : 2·7. The working pressure was 228 lbs. per square inch. The De Glehn engine has gradually developed in size and power, and its efficiency has further been increased by the addition of a superheater. The De Glehn engine running on the Paris-Calais service in 1913 is a six-coupled engine of the Pacific type; that is to say, a leading bogie is followed by a group of six coupled wheels each 80 ins. diameter, and these again are followed by a pair of small trailing wheels, as indicated by the group figures 4—6—2. The boiler pressure is still 228 lbs. per square inch, but the steam is superheated in a Schmidt superheater. The cylinders are: high pressure, 16·375 ins. diameter; low pressure, 25·625 ins. diameter; and the common

stroke 26·375 ins. The grate area is 33·9 sq. ft., and the engine weighs 85 tons.

Full particulars of the dimensions and the performance of French compound locomotives up to the year 1904 will be found in a paper[1] by Professor Edouard Sauvage, the distinguished consulting engineer to the Western Railway of France. M. Sauvage states that for the consumption of equal weights of fuel the compound locomotive can haul a train one-third greater in weight than the ordinary simple engine, or can haul a train of equal weight at a greater speed, and in many cases can haul a greater weight at a greater speed. In the discussion of the paper M. Du Bousquet stated that records taken over a period of eighteen years showed that the four-cylinder compound locomotive on the Nord Railway consumed about 17 per cent. less fuel than the ordinary simple engines, and that increased cost for oil was only small, and that the cost of maintenance was about 1·2 pence per mile greater than for the simple engines. M. Anatole Mallet contributed to the discussion, and stated that whilst in 1879 there were only a dozen compounds of small size, there were in 1904 more than 18,000, including the largest engines built. Mr. Churchward stated that the De Glehn compound "La France," purchased by the Great Western Railway Company, had exerted a pull of 2 tons on the drawbar at a speed of 70 miles per hour, and gave some particulars of a comparative experiment between "La France" and a simple engine with two cylinders, each 18 ins. diameter and 30 ins. stroke, which had shown the same result.

S. M. Vauclain introduced a successful type of compound locomotive in the United States in 1889. The high- and the low-pressure cylinders were cast together, and the piston rods belonging to a pair were both coupled to one cross head, and the distribution of steam to both cylinders was done by one piston valve operated by a link motion.

Later, Vauclain introduced a four cylinder-compound driving on to four cranks arranged in two 180° pairs at right angles.

The Mallet Articulated Compound Locomotive, first introduced by Mallet on the Continent in 1889, is now being rapidly developed in the United States. This locomotive consists of one long boiler carried on two separate driving trucks. Each driving truck has a pair of guiding truck wheels followed by a group of coupled wheels, and is provided with a pair of cylinders and the necessary driving machinery. The boiler is fixed to the rear driving truck, but the leading driving truck is free to turn and to move laterally under the boiler as the engine runs on a curve. The sequence of the wheels in the two trucks is reversed, so that there is a small pair of wheels leading, followed by a group of coupled wheels, and then in the rear driving truck a group of coupled wheels, followed by small trailing wheels. The steam pipes connecting the cylinders are all articulated to give the necessary freedom as the driving trucks follow the track.

[1] "Compound Locomotives in France," Professor E. Sauvage, *Proc. Inst. Mech. E.*, 1904.

A Mallet Articulated Compound Locomotive, built in 1912 by the Baldwin Locomotive Company for the St. Louis Iron Mountain and Southern Railway Co., is one of the most powerful locomotives in the world. The total weight of the engine is 195 tons. Each driving truck consists of a guiding truck and eight coupled wheels. The guiding truck wheels are 30 ins. diameter, and the driving wheels 55 ins. diameter. The general arrangement of the wheels is therefore 2—8——8—2. The trailing truck is driven by two high-pressure cylinders placed outside the frames, which exhaust through articulated pipes to the two low-pressure cylinders which drive the leading truck. The high-pressure cylinder is 26 ins. diameter and the low-pressure 40 ins. diameter, the stroke being 32 ins. in each case. The volume ratio is thus: H.P. : L.P. = 1 : 2·4. This engine is provided with a feed-water heater in the front section of the boiler, and a superheater in the main boiler section. The grate area is 84 sq. ft. The total heating surface is 5763 sq. ft., and this includes 1230 sq. ft. in the feed-water heater and 890 sq. ft. in the superheater. The boiler pressure is 200 lbs. per square inch. The tractive force of this engine is 42 tons.

The economy of a compound locomotive is not so marked as the economy of the compound marine engine or the compound land engine. This is due partly to the fact that the locomotive is a non-condensing engine, and the range of temperature and pressure is smaller than in the condensing engines, and partly to the fact that once a marine engine is started it generally runs at full load for days at a time, whilst the load on a locomotive is changing almost from minute to minute. The trials at the St. Louis Exhibition show that the simple engines tried on the testing plant required from 24 to 28 lbs. of steam per I.H.P. hour according to the mode of working, whilst the compound locomotives tested required from 19 to 27 lbs. of steam per I.H.P. hour. The minimum consumption was reduced to 16·6 lbs. per I.H.P. in the case of the one compound engine tested, which was provided with a superheater. These results are discussed in more detail below.

Another point of difference between the marine and the locomotive compound engine is that a marine engine is started light, and the load comes on gradually as the speed is increased. In the locomotive the start has to be made against the full load, and perhaps with the high-pressure slide valves in the central position over the ports, so that steam cannot get into the engine at all. Starting difficulties are avoided by the provision of starting valves, by means of which the high-pressure exhaust is led to the blast pipe directly, whilst steam at a reduced pressure is admitted from the boiler directly to the low-pressure cylinder, so that both the high-pressure and the low-pressure cylinders can be simultaneously charged with steam in order to bring a maximum turning moment on to the driving axles. Once started, the valves are put out of action, often automatically, and the low-pressure cylinder receives its supply of steam from the

high-pressure cylinder. In this way the engine starts as a simple engine and then automatically becomes a compound engine.

The compound engine is almost abandoned in England. Recent developments are all in the direction of four-cylinder simple engines, using superheated steam. The three-cylinder Smith compounds on the Midland Railway have probably proved the most successful type of compound locomotive in this country. Many are at work, and a brief description of the original design will serve to bring out many points which are peculiar to the compound locomotive.

There are three cylinders, one high-pressure between the frames connected to a central crank on the driving axle, and two low-pressure in parallel outside the frames connected to the same driving axle. The three cranks on the driving axle are set, the two outer at 90°, the middle at 135° with each outer crank.

One of the difficulties in connection with the working of compound

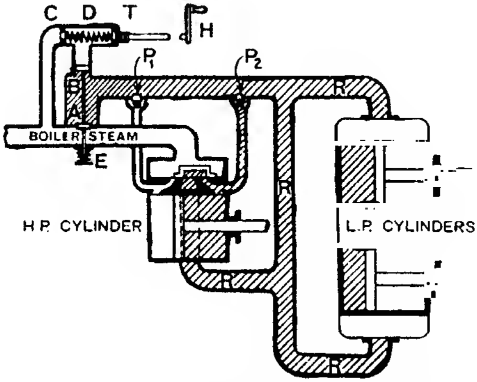


FIG. 80.—Diagram of cylinders for Smith Compound Locomotive.

locomotives at high speeds is, that as the cut off is reduced in the high-pressure cylinders the receiver pressure falls and the low-pressure cylinders take considerably less than their share of the work. The characteristic feature of the Smith design is the way in which the receiver pressure is controlled. Within defined limits the driver can decrease or increase the receiver pressure at will. If it is increased, more work is thrown on the low-pressure cylinders; if it is decreased more work is thrown on the high-pressure cylinder. If the cut off in the high-pressure cylinder should be so early that the receiver pressure falls below the minimum value fixed for the receiver, then high-pressure steam is automatically admitted direct from the boiler without the intervention of the driver. This is called by the inventor the "semi-compound" way of working. The general arrangement of the valves and cylinders by which this control is brought about is shown diagrammatically in Fig. 80. In the actual engine the three cylinders are on a line across the engine, and

not displaced laterally as shown in the diagram. The steam in the receiver is shown cross-hatched. The boiler steam can get into the receiver RRR through the valve A. The valve A and the small piston B are rigidly connected by one spindle, which passes outside and is loaded with a spring E. A valve C gives admission to boiler steam above the piston B. Suppose that the driver by turning the handle H has compressed the spring D on to the valve C so that the boiler steam cannot open it, there will then be no pressure acting on the piston B; the boiler steam will then lift the valve A and pass into the receiver at a pressure determined by the spring E, which may be arranged to throttle the pressure down to 30 or 40 lbs.

Next assume that the driver has turned back the handle H so far that the spring D exerts no pressure on the valve C. Boiler steam will pass through C and exert a pressure on the piston B and so oppose the opening of the valve A; if the areas B and A are in the ratio 1 to 2, steam at about half the boiler pressure will pass through A into the receiver. Therefore the pressure at which A passes steam into the receiver can be varied between the minimum fixed by the area B with C wide open and the maximum fixed by the spring E with C closed, by the mere adjustment of the compression of the spring D, which the driver can do at any moment from the cab. Once the compression of the spring D is fixed, the action of the valve A is automatic.

If the engine should happen to stop in the position where the high-pressure valve covers both the steam ports, when steam is turned on to start again, although it cannot get into the high-pressure cylinder it passes through A and into the low-pressure cylinders, and the engine starts with the low-pressure cylinders alone. Unless, however, some special provision is made, the steam in the receiver will tend to reverse the high-pressure engine, because it has free access to one face of the piston through the exhaust port, the other face, in the position of the slide valves assumed, having only atmospheric pressure on it. This difficulty is avoided by putting the receiver into communication with the other side of the piston through the non-return valves P_1 and P_2, so that for the moment the high-pressure piston is in equilibrium and the low-pressure pistons are left to start the engine without opposition.

Actual drawings of the details of the valves together with particulars of the fine performances of an engine of this type, built for the Midland Railway by the late Mr. Samuel Johnson, will be found in *Engineering* for Feb. 6, 1903.

The low-pressure cylinders in this engine are each 21 ins. diameter and 26 ins. stroke, whilst the high-pressure cylinder is 19 ins. diameter and 26 ins. stroke. The volume ratio is thus 1 : 2·34. The boiler pressure is 195 lbs. per square inch by gauge, and the four coupled wheels are each 7 ft. diameter. The system can be equally applied to four cylinder engines.

The details of the engines of this class on the Midland Railway

have been simplified by Mr. Fowler. Steam is admitted direct to the receiver through a small pipe so connected to the regulator that the supply is started when the regulator starts to open, but it is cut off when the regulator is about quarter open, and remains cut off during positive movement of the regulator. The valve A is omitted.

Mr. Ivatt made comparative trials[1] between a four-cylinder two-stage compound; a four-cylinder combined compound or simple; and a two-cylinder simple engine running in the same service on the Great Northern Railway, and found that, reckoning the cost of coal, oil and repairs, the compound engines had little advantage over the simple engine on the particular service selected for experiment.

There are many papers relating to the subject of compound locomotives, amongst which may be mentioned: "The Compound Locomotive: history of compounding as applied to locomotives," by E. Worthington, *Proc. Inst. C.E.*, vol. 96; "Compound Locomotives," R. H. Lapage, *Proc. Inst. Mech. E.*, 1889; "Compounding of Locomotives Burning Petroleum Refuse in Russia," by T. Urquhart, *Proc. Inst. Mech. E.*, 1890; "The Development of the Compound Locomotive in England," W. E. Dalby, *The Engineering Magazine*, vol. 27, April and September, 1904; "Compounding and Superheating in Horwich Locomotive," by G. Hughes, *Proc. Inst. Mech. E.*, 1910, Part II.

75. The Use of Superheated Steam.—It has long been known that the missing quantity is reduced by the use of superheated steam. Superheated steam was, in fact, used in earlier days, when steam pressures were lower, and there is a paper by John Penn relating to the subject in the *Proceedings of the Institution of Mechanical Engineers* for the year 1859. Its use was advocated by Hirn, and his influence is seen to-day in the greater development of the method in South Germany. The mechanical difficulties associated with hot steam and the trouble in connection with lubrication led temporarily to its abandonment, though the economical advantages of superheated steam were quite recognized. The coming of metallic packing and more particularly the possibility of obtaining lubricating oil which will stand the high temperatures to which superheating is carried and will still lubricate, have removed some of the earlier difficulties. The renaissance of superheated steam in practice is due largely to Herr Schmidt.

The economy resulting from the use of superheated steam is most strikingly shown by experiments on a small engine, because the wall action and the initial condensation are relatively great. A series of trials at different loads and speeds of a small engine indicate generally the conditions in which economy is likely to result from superheating.

Professor Ripper[2] in 1897 made a series of careful trials on a

[1] "Note on Road Trials of Three Express Passenger Engines, carried out on the Great Northern Railway in 1906," H. A. Ivatt, *Proc. Inst. Mech. E.*. May, 1907.

[2] W. Ripper, "Superheated-Steam Engine Trials," *Proc. Inst. C.E.*, vol. 128.

small horizontal single-acting non-condensing steam engine, known as a Schmidt motor, in order to compare the relative advantages of a saturated steam supply with a superheated steam supply in various conditions of load, initial pressure, and superheat.

Quoting one set of experiments made at about 13 I.H.P., the results in the following table indicate a remarkable decrease in the steam fed to the engine per I.H.P. hour as the temperature of the steam supply increases, the pressure meanwhile being practically constant and in the region of 100 lbs. per square inch (absolute).

TABLE 18.—SUPERHEATED STEAM TRIALS. DATA FROM PROFESSOR RIPPER'S PAPER.

I.H.P.	Pressure of steam entering the engine. Lbs. per sq. in.	Temperature of the steam entering the engine. Degrees C.	W. Weight of steam condensed per I.H.P. hour. Lbs.	Number of expansions.	Speed. Revolutions per minute.
13·3	101·7	164·6	39·62	3·42	177
13·33	98·5	218·2	33·8	2·67	180
13·47	98·6	305·0	23·36	2·54	175
13·49	99·45	342·0	20·08	2·59	176

The steam in the first experiment was dry and saturated.

Comparing the steam per indicated horse-power-hour, using saturated steam, namely, 39·62 lbs., with the steam per indicated horse-power-hour, using steam superheated to 342° C., the saving is practically 50 per cent.

The heat carried to the engine per pound of steam is, however, greater with superheated than with saturated steam, and in order to estimate the true value of the saving, the work done per pound of superheated steam supplied to the engine must be compared with the work U of the corresponding engine of comparison working between the same limits of pressure and the corresponding limits of temperature, the upper temperature limit being the temperature to which the steam is superheated. The upper temperature limit is given in the third column of the table above. Column 1 in the comparative Table 18A shows the work done per pound of steam condensed per I.H.P hour expressed in lb.-calories, and is calculated from the expression $\frac{1414}{W}$, in which W is the weight of steam per I.H.P. hour given in the fourth column of Table 18.

The total energy of the superheated steam per pound I′ is given in column 2. It is found directly from Steam Table 3, page 744.

Column 3 shows the total energy I_2 after the steam has expanded adiabatically to 14·7 lbs. per square inch. It is found from the energy-temperature diagram by tracing the adiabatic path from the initial to the final temperature in the way already explained above, or from the Mollier diagram.

The difference is the work U in the Rankine engine of comparison.

The heat supplied to the engine per pound of steam is shown in column 5, and is the same both for the actual and the Rankine engine, since the lower temperature of condensation is the same for both engines. It is the difference $I - I_w$ equal to $I' - 100$. The efficiencies of the actual and the Rankine engine, together with the efficiency ratio, are shown respectively in the last three columns of the table.

TABLE 18A.—DEDUCTIONS FROM DATA TABLE 18.

Lb.-calories equivalent to the work done per pound of steam. $\frac{1414}{W}$	Total energy of the steam per pound. I'.	Total energy after adiabatic expansion to 14·7 lbs. per sq. in. I_2.	Lb.-calories per pound of steam supplied to the Rankine engine. U.	Heat supplied per pound of steam. $I' - 100$.	Thermal efficiency of the actual engine.	Thermal efficiency of the Rankine engine.	Efficiency ratio.
					Per cent.	Per cent.	
35·65	662	582	80	562	6·3	14·2	0·45
41·90	692	607	85	592	7·1	14·3	0·49
60·40	735	639	96	635	9·5	15·1	0·63
68·40	754	650	104	654	10·4	15·9	0·66

It will be seen that whilst the actual engine increases in efficiency from 6·3 to 10·4 per cent., the Rankine engine of comparison increases from 14·2 to 15·9, an increase of 11·6 per cent. as against an increase of 65 per cent. in the actual engine.

The efficiency caused by superheating increases therefore at a greater rate than the ideal efficiency of the engine of comparison.

The reason why the actual engine shows such a rapid gain of efficiency is that superheated steam is not such a good conductor of heat to the metal boundaries as saturated steam, and that it probably does not leak away so rapidly owing to the absence of water films and to its greater viscosity. Whatever be the reason, the fact remains that actual engines increase in efficiency at a much greater rate than the corresponding engine of comparison.

Professor Ripper demonstrated in his paper that for the small engine he used, a high degree of superheat is necessary to ensure that the steam shall be dry at cut off, and that the greater the ratio of expansion the higher must be the superheat to secure this. For example, with 2·58 expansions a superheat of 120° C. gave dry steam at cut off, but with 4·07 expansions it requires 153° C. to get the same result. The proportionate saving by the use of superheated steam is not so great with large engines as with small engines, but nevertheless remarkable economy is shown when superheated steam is supplied to large engines, especially if the temperature to which the steam is superheated is high.

In a trial made by Sir Alfred Ewing on a two-cylinder compound condensing Schmidt engine developing 184 I.H.P., the consumption of steam per I.H.P. hour was reduced from 17·2 lbs. with saturated

steam to 10·4 lbs. with steam superheated to 292° C., the boiler pressure in each case being 140 lbs. per square inch. The steam was actually superheated to 396° C., but 104° of this was lost in a re-heater between the cylinders through which the steam passed on its way from the boiler to the high-pressure cylinder. In the saturated steam trial, the missing quantity at cut off in the high-pressure cylinder was about 30 per cent. of the steam supply, but in the superheated trial the steam remained superheated during the whole of the expansion in the high-pressure cylinder, becoming wet at cut off in the low-pressure cylinder. The results of the trial of a single-cylinder Sulzer engine supplied with steam superheated to 283° C. are shown in the last column of Table 17, page 263. This engine is described more particularly in Section 79.

A superheater consists of a bundle of tubes, immersed in hot furnace gas, through which the steam passes on its way from the boiler to the cylinder. Other things being the same, equal temperature differences produce a smaller flow of heat across superheater heating surface than across boiler heating surface, because the dry, semi-superheated or superheated steam offers a greater resistance to the flow of heat than water. To be effective, the superheater must be placed where the furnace gas is hottest, and it must have heating surface large enough in area to ensure the transmission of heat in sufficient quantity to produce the superheat desired.

The actual quantity of heat required to produce a given temperature of superheat from a pound of dry saturated steam is relatively small compared with the quantity required to produce the dry saturated steam itself, yet heating surface must be provided in proportion considerably greater than the heat quantities themselves, for equal differences of temperature between the furnace gas and the steam, and the furnace gas and the water.

For example, referring to Steam Table 3, page 744, the total energy of a pound of steam at 200 lbs. per square inch, superheated to 350°, is 755 lb.-cals., whilst the total energy when the steam is dry and saturated is 669 lb.-cals. The heat added during the process of superheating is thus 86 lb.-cals. The heat added during the formation of dry saturated steam is to the heat added during its subsequent superheating as 1 : 0·13. Heating surface considerably in excess of 13 per cent. of the total boiler heating surface must be added in order to secure this degree of superheating. An example of a water-tube boiler fitted with a superheater is shown in Fig. 50, together with some results obtained when fired with oil fuel. In the first of the trials in the table, page 139, steam at 242 lbs. pressure per square inch is superheated 50°, that is to a temperature of about 253° C. From Steam Table 3, page 744, $I' = 700$ and $I_s = 671$, so that the heat added is 29 lb.-cals., equivalent to about 4 per cent. of the heat added to produce dry saturated steam. The superheater surface provided in the boiler is, however, 23 per cent. of the boiler heating surface.

The use of superheated steam in connection with locomotives is developing rapidly. Schmidt in 1897 applied a superheater to a

locomotive, and securing the co-operation of Herr Garbe of the Prussian States Railways, proceeded to a series of experiments and trials which ultimately established that clear saving results from the use of superheated steam even in the exacting and varied conditions of locomotive service. So definite were the results that the practice of fitting superheaters extended rapidly in Germany and on the Continent and America and afterwards in England. In 1908 over a thousand locomotives on the Prussian States Railways were fitted with superheaters. The leading locomotive engineers of this country are developing superheaters suited to the special conditions of the traffic on English railways.

A concise account of the development of the use of superheated steam on the Continent, together with illustrations of different kinds of superheaters and the results of many trials in service, will be found in the reprint of a course of lectures which was delivered at the Institution of Civil Engineers by Professor E. Sauvage[1] at the invitation of the University of London, in 1910.

In the first arrangement designed by Schmidt for locomotives some of the tubes were replaced by a relatively large flue tube, in which the bundle of small steel tubes forming the superheater was placed. Later, the superheater was placed in the smoke-box with special flue tubes to lead hot gas to it. In the arrangement now generally known as the Schmidt System the upper rows of tubes are replaced by two or three rows of larger tubes, generally about 5 ins. inside diameter, in each of which is placed a superheater element consisting of a tube about 1 in. inside diameter bent four times on itself and connected at the smoke-box end to the steam collector. The collector receives saturated steam from the boiler and delivers superheated steam to the cylinders. The collector is made in one casting and may be likened to two long parallel corridors between which lies a row of rooms or cells. Calling one corridor the boiler corridor, and the other the superheated steam corridor, and numbering the cells in order, every odd-numbered cell opens into the boiler corridor, and every even-numbered cell opens into the superheated steam corridor. The opening between any cell and its corridor is made by taking the whole of the cell wall away. The boiler steam pipe opens into the centre of the boiler corridor, and the odd-numbered cells are therefore all of them filled with saturated steam. Communication between an odd-numbered cell and the even-numbered cells lying on either side of it is made through superheater elements. The steam gets superheated as it flows through the long narrow passage of the element in the flue tube, and so all of the even-numbered cells are filled with superheated steam. Communication between an odd and the even-numbered cells between which it lies is generally made through six superheater elements, three to the cell on the one side and three to the cell on the other side. It will be understood, of course, that in an actual collector the levels of the

[1] "Lecture on Superheating on Continental Locomotives," E. Sauvage. London: University of London Press, 1911.

odd and the even cells need not be the same; that the cells may communicate with their corridors through the floor or through the roof, and that the corridors may be arranged one over the other with alternately inclined communicating ways; but however the arrangement may be made, the essential idea is the same, namely, that of a row of cells communicating, the odd-numbered ones with a common supply pipe, and the even-numbered ones with a common delivery pipe, or *vice versâ*, adjacent cells communicating with each other through superheater elements.

A Schmidt superheater applied to an express passenger engine of the London and North-Western Railway[1] is shown in Fig. 81. C is the collector; F, F, F are the large flue tubes; E, E, E are superheater elements; D is the steam pipe supplying saturated steam to the boiler corridor; S and S are the steam pipes through which the superheated steam flows from the superheated steam corridor to the cylinders. It will be seen that communication between adjacent cells of the collector is made through three elements, one element from each of the three rows of large flue tubes.

Each superheater element extends to within a few feet of the fire-box tube plate, and each is free to expand since it is fixed only to the collector end and each is held centrally in the flue tubes by distance pieces. The temperature of the tubes of the superheater element is high. It may be roughly estimated as the mean of the temperature of the furnace gas without it and the temperature of the superheated steam within it.

An essential feature of a locomotive superheater is the provision made to prevent the overheating of the superheater elements when steam is shut off. It will be seen that in Fig. 81 a damper M is supplied for this purpose. The damper is sometimes connected to the regulator, so that when steam is shut off the flow of hot gas through the flue tubes is prevented, and so danger of overheating the elements within them is avoided.

Another method of preventing the flow of hot gas is to blow a small jet of steam along each tube from the smoke-box end when the steam is shut off, and dispense with the damper altogether. Still another way which is being developed is to cause steam to circulate through the superheater elements and through the cylinders when the regulator is closed.

A few particulars of the engine shown in Fig. 81 are given herewith.

Heating Surfaces:

159 1¾-in. tubes	1160·9	sq. ft.
25 5¼-in. flue tubes	486·3	,,
Fire-box	171·2	,,
Total heating surface of boiler	1818·4	,,
24 sets of superheater elements	413·6	,,
Total heating surface of boiler and superheater	2232·0	,,

[1] The author is indebted to Mr. Bowen Cooke, Chief Mechanical Engineer of the London and North-Western Railway, for particulars of this engine.

The heating surface of the superheater elements is 25 per cent. of the heating surface of the boiler tubes, including the flue tubes;

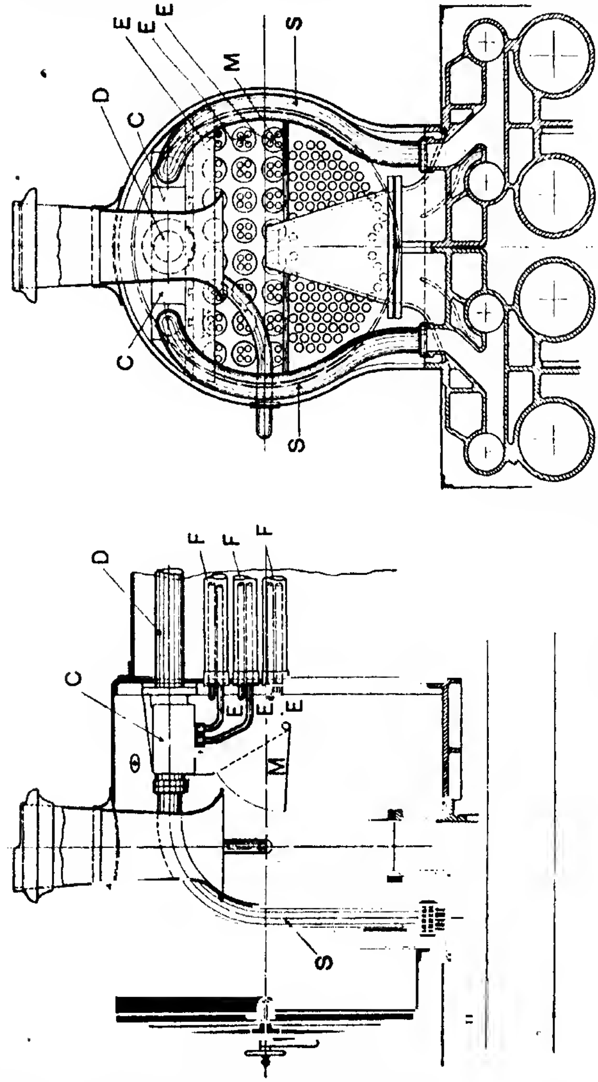


FIG. 81.—Superheater. London and North-Western Railway.

23 per cent. of the total boiler heating surface; and 18·5 per cent. of the total boiler and superheater surface.

The grate area is 30·5 sq. ft., so that there are 13·5 sq. ft. of superheating surface per square foot of grate, and 2·4 sq. ft. of superheater heating surface per square foot of fire-box heating surface.

Boiler pressure, 175 lbs. per square inch.

Four cylinders, each 16 ins. diameter and 26 ins. stroke.

Driving wheels, 6 ft. 9 ins. diameter.

Radial truck wheels, 3 ft. 3 ins. diameter.

Weight of engine in working order, 77 tons 15 cwt.

Weight of engine and tender, 117 tons.

Type, 4—6—0.

In favourable conditions of running, that is, at full power with few stops, a saving of 20 per cent. has been realized by the use of superheated steam in a simple engine. In a comparative trial on the P.L.M. Railway of France, between two large engines of the 4—6—2 type, one a four-cylinder simple fitted with a superheater, the other a four-cylinder compound using saturated steam, there was a saving of 13 per cent. of coal per I.H.P. hour, and 15 per cent. of water per I.H.P. hour in favour of the simple engine.

It seems pretty well established by the records of experiments that a saving of 15 per cent. can be realized by the use of superheated steam in a simple engine, except where the conditions of working are unfavourable, as, for example, where the service requires frequent stops.

By the use of superheated steam, the power of a locomotive may be increased, because the cut off in a cylinder receiving superheated steam must be set later than it would be if the cylinder were receiving saturated steam at the same rate through the steam pipe. Instead, however, of setting the cut off later, it is better to provide the volume required by increasing the size of the cylinder, and so maintain an early cut off and a reasonable ratio of expansion. This is what is usually done, and it will be found that locomotives intended for heavy service and fitted with superheaters are fitted with larger cylinders than would be the practice for the same boiler supplying saturated steam.

The tubes forming the superheater elements should be kept small, in order to furnish a large surface in proportion to the cross section; at the same time the cross section must not be too small, otherwise the drop of pressure between the boiler and the cylinders will be excessive. Professor Sauvage states in the lectures referred to above that the sectional area of the superheater elements should be proportioned to allow the steam to flow through them at about 160 ft. per second in order to avoid excessive drop; at the same time, it is certain that the greater the velocity of flow, the better the heat transmission. The size of the tubes used in many Schmidt designs have an average inside diameter of about 1 in. And the proportion between the boiler heating surface and the superheater surface is 78 to 22.

Lubrication of the valves and piston becomes of primary importance when superheated steam is used, and engines are usually

fitted with a mechanical lubricator to ensure a proper distribution of oil in the cylinder and valve chest. There are many special problems brought into prominence when superheating is adopted. The mechanical distortion of the parts produced by the much higher temperatures in the engine requires consideration, and the design should tend to symmetry of form in order that the change of form produced by the high temperature may be symmetrical.

Stumpf has designed a locomotive engine in which symmetry of heat-flow is aimed at also. The cylinder is longer than usual and is provided with exhaust ports at the centre, which the piston uncovers at the end of the stroke. The piston is deep in order to make this mode of working possible. Steam is admitted at each end of the cylinder by separate valves, and thus the ends of the cylinder are always hotter than the centre. Heat flows then symmetrically from ends to centre, and the entering steam flows into a clearance space which is probably hotter than in the case of the ordinary method of distribution.[1]

For detailed information relating to the subject, the following papers may be consulted:—

"Superheaters applied to Locomotives on the Belgian State Railways," M. J. B. Flamme, *Proc. Inst. Mech. E.*, 1905, No. 3.

"Compounding and Superheating in Horwich Locomotives," G. Hughes, *Proc. Inst. Mech. E.*, 1910, No. 2.

76. Expansion Curves and Heat Exchange.—Owing to the exchange of heat between the steam and the cylinder walls and to the loss of steam by leak, the expansion curve shown on an indicator diagram follows no regular law, but it can generally be represented with sufficient accuracy by an equation of the form PV^n = a constant.

Normally the expansion curve on an indicator diagram lies above the corresponding adiabatic curve and in the region of the saturation curve drawn through the point of cut off, because the steam receives heat from the walls as the expansion proceeds towards release. Any steam leaking through the admission valve tends to still further lift the expansion above the adiabatic curve; in fact, actual expansion curves often approximate to the rectangular hyperbola PV = a constant.

In any case, between the limits of an adiabatic curve and a rectangular hyperbola through the same point of cut off, the influence of the form of the expansion curve on the work area is small as will be seen from Fig. 82, where the lower curve is an adiabatic, the upper curve a rectangular hyperbola, and the middle curve a saturation curve, all representing the expansion of 1 lb. of dry saturated steam from an initial pressure of 200 lbs. per square inch according to their several laws.

For design purposes the rectangular hyperbola PV = a constant may therefore be used without serious error, and it may also be

[1] "Die Gleichstrom Dampflokomotive," *Zeit. Verins deutsch. Ing.*, Dec. 10, 1910, 2089.

drawn through the cut off point of an actual indicator diagram to serve as a standard with which to compare the actual expansion curve; because if the actual curve lies much above it, it is pretty

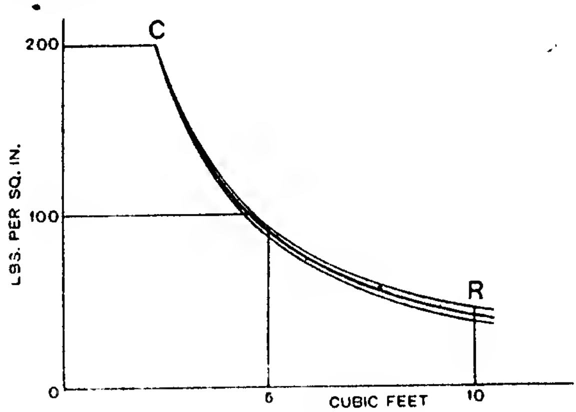


FIG. 82.—Comparison of expansion curves.

certain that steam is leaking in; and if much below it, that steam is leaking out of the cylinder.

Assuming that there is no leak, it is an easy matter to calculate the quantity of heat Q flowing between the steam and the walls during the whole period of expansion or during any part of it.

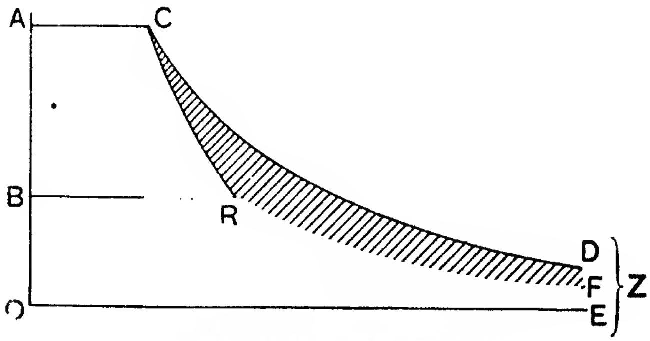


FIG. 83.—Heat exchange with walls during expansion.

Let P_1, V_1, I_1 respectively be the pressure, the volume, and the total energy of 1 lb. of steam at C, Fig. 83, and let P_2, V_2, I_2, be the corresponding values at R, and let W be the work corresponding to the area ACRB; then Q, the heat flowing between the steam

and the walls during the expansion of the steam from C, along the curve shown, to R, is

$$Q = I_1 - I_2 - \frac{W}{J} \text{ lb.-cals.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If Q is − the flow is from the walls to the steam.
If Q is + the flow is from the steam to the walls.
This is proved as follows :—

Let CR, Fig. 83, be part of any expansion curve.
Through C draw the adiabatic curve CD.
Through R draw the adiabatic curve RF.

Then, as shown on page 164, the shaded area represents the heat flowing from the steam to the walls during the change of state from C to R along the path CR.

This area is equal to—

Area OACDZEO − area OBRFZEO − area ACRB . (2)

The area OACDZEO is by definition, Section 45, page 160, the total energy I_1 of the steam in the state C.

The area OBRFZEO is the total energy I_2 of the steam in the state R.

The area ACRB is the work W.

Substituting these values in (2), the proposition in (1) is established.

If the point R is below the adiabatic through C, the area Q is positive and the steam does the external work W during expansion, and in addition rejects heat energy Q to the walls.

If the point R is above the adiabatic curve through C, the area Q is negative and the steam during expansion does the external work W, and in addition receives the heat Q from the walls.

If the point R is on the adiabatic curve through C, the area Q vanishes and the steam during expansion does the work W, and neither receives nor rejects heat to the walls.

The numerical value of W may be determined from an indicator diagram in two ways:—

(1) Measure the area directly from the diagram.

(2) Assume that the expansion curve can be expressed by PV^n = a constant, and then with data derived from the diagram find a suitable value of n, and then calculate W from $\dfrac{n(P_1V_1 - P_2V_2)}{n - 1}$ since this expression is the integral $c\int_{P_2}^{P_1} v \cdot dp$, which represents the work area.

In the particular case of the rectangular hyperbola where $n = 1$, $W = P_1V_1 \log_e \dfrac{P_1}{P_2}$.

The application of the method of logarithmic plotting to the

expansion curve on a diagram will show if it is suitably represented by an equation of the form PV^n = a constant. Thus, taking the logarithms of each side,

$$\log P + n \log V = \log c$$

which may be written $$y + nx = C$$

This is the equation of a straight line, so that if pairs of values of P and V are measured from the calibrated indicator diagram, and then for each pair the logarithm of P is plotted for y vertically against the logarithm of V for x horizontally, the points will lie on a straight line if the expansion curve follows the assumed law.

Having found the best straight line, its slope gives the value of n, and the intercept C, on the vertical axis, gives the value of $\log c$, so that both c and n can be determined from the graph.

Logarithmic plotting is facilitated by the use of logarithmic squared paper.

Assuming no leak, the method of calculation may be illustrated by applying equation (1) to find the quantity of heat received by the steam per pound from the walls during expansion along a saturation curve. The saturation curve is represented approximately by the equation

$$PV^{\frac{16}{15}} = 490$$

P is here in pounds per square inch and V is in cubic feet. The equation becomes therefore

$$Q = I_1 - I_2 - \frac{\frac{16}{15}(P_1V_1 - P_2V_2)144}{J(\frac{16}{15} - 1)} \quad . \quad . \quad . \quad (3)$$

Since expansion is along a saturation curve, the values of I_1, I_2, V_1 and V_2 are taken direct from the tables.

If the initial pressure is 200 lbs. per square inch and the final pressure 20 lbs. per square inch, this equation becomes,

$$Q = 669{\cdot}7 - 642{\cdot}8 - \frac{16(200 \times 2{\cdot}32 - 20 \times 20{\cdot}07)144}{1400}$$

$$= 26{\cdot}9 - 103{\cdot}7 = -76{\cdot}8 \text{ lb.-cals. per pound of steam}$$

The minus sign shows that the flow is from the walls to the steam.

As a second example calculate Q when 1 lb. of steam, initially dry and saturated, expands from 200 lbs. per square inch to 20 lbs. per square inch along the curve PV = a constant.

I_1 is found direct from the tables, and is 669·7 lb.-cals. The final volume is from the equation

$$V_2 = \frac{P_1V_1}{P_2} = \frac{200 \times 2{\cdot}32}{20} = 23{\cdot}2 \text{ cub. ft.}$$

A reference to the tables shows that the volume of a pound of dry saturated steam at 20 lbs. per square inch pressure is 20·07, which is less than the terminal volume calculated. The steam is therefore superheated, and the temperature to which it is superheated must be calculated before I_2 can be found from the tables.

The temperature corresponding to any pressure and volume is given implicitly in the characteristic equation for steam, but the equation is difficult to solve for the temperature. A simpler relation of sufficient accuracy may be used if the superheating is not very great by neglecting the term for the co-aggregation volume and the term w and writing the relation in the form of a perfect gas,

$$PV = RT$$

the constant R being calculated for each case from the tabular volume, pressure and temperature corresponding to the given final pressure. Thus, with the data of the example, when the pressure has fallen to 20 lbs. per square inch, and therefore the temperature to 382° C. absolute, the tabular volume is 20·07 cub. ft. So that

$$R = \frac{20 \times 20{\cdot}07}{382} = 1{\cdot}05$$

Using this constant for the calculation of the temperature when the volume is 23·2 cub. ft.

$$T = \frac{20 \times 23{\cdot}2}{1{\cdot}05} = 442° \text{ absolute}$$

corresponding to 169° C.

A reference to Steam Table 3, page 744, will show that at this temperature and at a pressure of 20 lbs. per square inch, $I_2 = 672$ lb.-cals. The value of $\frac{W}{J}$ is in this case

$$\frac{144P_1V_1}{1400}\log_e\frac{P_1}{P_2} = \frac{144 \times 200 \times 2{\cdot}32 \times 2{\cdot}3}{1400} = 109$$

Therefore $\quad Q = 669 - 672 - 109 = -112$ lb.-cals.

The negative sign shows that the flow is from the walls to the steam.

In the case where the terminal volume V_2, calculated from $V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}V_1$ is less than the saturated volume corresponding to the terminal pressure, the steam is wet at the end of expansion, and the value of q_2 is calculated from $\frac{V_2}{V_t}$, where V_t is the tabular volume corresponding to the terminal pressure P_2. Knowing P_2 and the dryness q_2 the corresponding state point can be located on the energy-temperature diagram, and then I_2 can be read off.

There is an alternative way of calculating the heat-flow Q, namely, by using the general relation,

$$Q = \text{change of internal energy} + \text{external work.}$$

The method used above is equivalent to (changing the signs all

through, in order to get $I_2 - I_1$ which is strictly the change of total energy)

Q = change of total energy + work area ACRB, Fig. 83.

It will be found that the method explained is generally more convenient for steam calculations.

Hirn,[1] who was the first to make use of rational principles to analyse the actual results of engine trials, and many workers after him, employed the alternative method to calculate the heat exchanges between the steam and the walls during the whole cycle of changes shown by the indicator diagram. The assumption that there is no leak, however, makes "Hirn's Analysis" of little practical value in the analysis of actual trials, though it is a valuable method theoretically, since it is founded on true fundamental principles. Its application is tedious, and, moreover, with the assumption of no leak, the heat exchanges between the walls and the steam can be shown in a simpler manner by means of the entropy-temperature diagram. A full account of Hirn's analysis as developed by his followers, together with many examples of its use, will be found in Peabody's "Thermodynamics of the Steam Engine".

77. Cylinder Volume for given Power and Speed.—Let D be the area of an indicator diagram in which pressure is measured in pounds per square inch and volume in cubic feet. Then the work done per minute is

$$144DN \text{ ft.-lbs.}$$

N is here the number of times the diagram is described per minute, that is, the number of cycles per minute, so that,

N = revolutions per minute × 2, for a double-acting engine.
N = the revolutions per minute for a single-acting engine.

The work per minute is also 33,000 I.H.P.

Therefore $$144DN = 33{,}000 \text{ I.H.P.}$$

from which $$D = \frac{33000 \text{ I.H.P.}}{144N} \qquad (1)$$

an expression from which the area of the indicator diagram can be calculated when the indicated horse-power which the engine is to develop is given, together with the speed.

Again, consider the indicator diagram shown in Fig. 84. There are four pressures which directly influence the area, namely,

p_1, the initial pressure,
p_2, the pressure at the end of expansion,
p_3, the back pressure,
p_4, the pressure at the end of compression.

[1] *Bull. Soc. Ind. Mulhouse*, from 1877.

The net area of the diagram is

$$abcdef = jbch + hcdg - ifeg - jafi$$

And this again is given by

$$\text{net area} = \int_{p_2}^{p_1} v \, . \, dp + V_2(p_2 - p_3) - \int_{p_3}^{p_4} v \, . \, dp - C(p_1 - p_4) \quad (2)$$

C is the clearance volume ja: put it equal to fV_2. Assume that both the expansion curve and the compression curve are rectangular

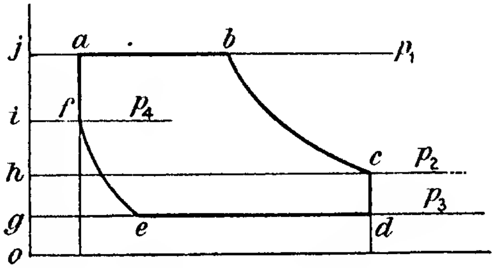


Fig. 84.—Four pressures which determine the shape of an indicator diagram.

hyperbolas, pv = a constant; then, since for the expansion curve $pv = p_2V_2$, and for the compression curve $pv = p_4fV_2$,

$$\int_{p_2}^{p_1} v \, . \, dp = p_2 V_2 \log_e \frac{p_1}{p_2}$$

$$\int_{p_3}^{p_4} v \, . \, dp = p_4 f V_2 \log_e \frac{p_4}{p_3}$$

Substituting these values in (2); equating it to ηD, and then solving for V_2, we have

$$V_2 = \frac{\eta D}{\left\{p_2 \log_e \frac{p_1}{p_2} + (p_2 - p_3)\right\} - f\left\{p_4 \log_e \frac{p_4}{p_3} + (p_1 - p_4)\right\}} \quad (3)$$

The factor η is used with D found from (1) to enlarge the area in order to allow for wire-drawing and the consequent rounded corners of the indicator diagram.

The volume V_2 is the volume swept through by the piston in one stroke plus the clearance volume fV_2. Therefore

$$\text{Effective volume} = V_2(1 - f) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

When the effective volume is calculated, the diameter of the cylinder can then be found, providing that the stroke is fixed.

The pressure drop between the boiler and the initial pressure in the cylinder varies with the type of boiler used and with the speed. p_1 may provisionally be taken 10 per cent. below the boiler pressure, though this percentage drop is often exceeded, especially with high-speed engines supplied by water-tube boilers in which the boiler pressure is sometimes intentionally made considerably higher than the pressure required at the engine. p_2, the pressure at the end of expansion, depends upon the ratio of expansion. It should never be allowed to fall lower than the sum of the back pressure p_3 + the pressure required to overcome engine resistance. There is no advantage gained by making the drop from p_2 to p_3 too small, as pointed out above in Section 63 when discussing incomplete expansion. p_3, the back pressure, depends upon the type of engine. With a non-condensing engine it may be fixed at 3 lbs. above the atmosphere in ordinary conditions of working, so that $p_3 = 18$ lbs. per square inch. In the case of condensing engines it should be taken 5 lbs. above the condenser pressure. p_4, the pressure at the end of compression, may advantageously be fixed equal to the initial pressure, though other considerations of a practical kind modify the problem. The kind of valve gear, the speed of running, the clearance volume, all influence this pressure. The particular values to be assigned to the four pressures, therefore, is a matter which can only be settled by the conditions of each particular problem.

The form of expression (3) shows clearly the influence on the cylinder volume both of the clearance volume and the drop at the end of expansion. For a given power and speed the numerator of (3) is constant. The denominator is the difference between two quantities in brackets; the first is a function of the ratio of expansion and the drop at the end of expansion; the second is a function of the clearance volume and the compression ratio.

Let x be the fraction of the total volume V_2 at which compression begins. Then the volume at the beginning of compression is xV_2 cub. ft., and the pressure is p_3. At the end of compression the volume is fV_2, and the pressure is p_4. Therefore

$$p_4 fV_2 = p_3 xV_2$$

giving

$$\frac{f}{x} = \frac{p_3}{p_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This shows that as f gets smaller the pressure at the end of compression p_4 gets larger. It should not increase in value beyond p_1 the initial pressure of the steam, therefore $\frac{p_3}{p_1}$ is the limiting value of the ratio for a given back pressure p_3.

Also x cannot have a value greater than unity corresponding to compression beginning at the beginning of the return stroke. It follows that as f approaches zero, the pressure at the end of compression approaches an infinite value. With small clearance volumes, relief valves must be provided to allow the steam to escape

in case the conditions of working change so that p_3 has a greater value than that for which the clearance is suitable. Relief valves are usually provided in any case to allow water to escape from the cylinder.

Diagrams taken from locomotives at high speeds sometimes show a terminal compression higher than the initial pressure. This is because at high speeds the cut off is reduced, and this cannot be done without at the same time increasing x; also p_3 tends to increase, so that with f fixed in value, it will be seen from equation (5) that p_4, which varies directly both with x and with p_3, increases also.

Let k be the fraction of the stroke at which compression begins, measured from the dead point corresponding to the minimum volume. Then

$$k = \frac{x - f}{1 - f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

from which k can be found when x is given or *vice versâ*.

Again, the volume at cut off is V_1. The ratio of expansion is then $\frac{V_2}{V_1} = r$, say, giving $V_1 = \frac{V_2}{r}$.

Let c be the fraction of the stroke at which cut off takes place, then

$$c = \frac{\frac{1}{r} - f}{1 - f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and the apparent ratio of expansion as measured from the indicator card, equal to ρ say, is

$$\rho = \frac{1}{c} = \frac{1 - f}{\frac{1}{r} - f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Solving for r, the real ratio of expansion,

$$r = \frac{1}{c - cf + f} = \frac{\rho}{1 - f + f\rho} \quad . \quad . \quad . \quad . \quad (9)$$

Finally the clearance volume is usually stated as a fraction of the volume swept out by the piston. Let F be this fraction, then the relation between F and f is

$$F = \frac{f}{1 - f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and

$$f = \frac{F}{1 + F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If, for example, the clearance is stated to be 10 per cent. of the effective volume, f, from (11), is 0·091.

The mean pressure corresponding to the diagram area ηD is found by dividing it by the effective volume $V_2(1 - f)$. Thus

$$\text{Mean pressure} = \frac{\eta D}{V_2(1 - f)} \quad . \quad . \quad . \quad . \quad (12)$$

The value of f to be chosen depends upon the size of the cylinder and upon the kind of steam distributing valves used with it. In large cylinders fitted with Corliss valves or with drop valves, f ranges from 0·02 to 0·06. In cylinders fitted with slide valves, f ranges from 0·07 to 0·1 when the cylinders are about 18 ins. diameter and 26 ins. stroke, a size usual in locomotives; but if piston valves are used the range of f is generally from about 0·1 to 0·16. These values apply also to the high-pressure cylinders of marine engines, the lower values being taken with the larger cylinders. For large low-pressure cylinders f ranges from 0·05 to 0·1 for either slide valves or piston valves.

EXAMPLE.—Find the volume of the cylinder of a double-acting non-condensing engine to give 1000 I.H.P. when the speed is 100 revolutions per minute, from the following data:—

Boiler pressure, 150 lbs. per square inch by gauge = 165 absolute.
Pressure at end of compression, 75 lbs. per square inch.
Ratio of expansion, 3.
Diagram factor, η = 1·1.
Clearance factor, f = 0·1.

From (1) $$D = \frac{33000 \times 1000}{144 \times 200} = 1145$$

The numerator of (3) is then 1·1 × 1145 = 1260.

P_1 = 150, allowing for wire drawing between the boiler and the cylinder.

P_2 is therefore 50 lbs. per square inch, the ratio of expansion being 3.

P_3 = 18, allowing 3 lbs. drop between exhaust pressure and atmospheric pressure.

P_4 = 75.

Then from (3)

$$V_2 = \frac{1260}{(55 + 32) - 0{\cdot}1(106 + 75)} = \frac{1260}{69} = 18{\cdot}25 \text{ cub. ft.}$$

This is the total cylinder volume when the stream is compressed in the clearance space to half the initial pressure.

The effective volume by equation (4) is 18·25(1 − 0·1) = 16·4 cub. ft. If there is no clearance, and therefore no compression, f = 0, and the denominator reduces to 87, giving V_2 = 14·5 This is the minimum volume possible for the power and speed.

With f = 0·1 and no compression, the second bracketed term in the denominator is 13 and the denominator reduces to 74, giving V_2 = 17·1. Finally, with f = 0·1 and with full compression, the second bracketed term in the denominator becomes 32 and the denominator reduces to 55, so that V_2 = 23 cub. ft.

These volumes in order of magnitude are: 14·5, no clearance and no compression; 17·1, clearance 10 per cent. of V_2, but with no compression; 18·25, same percentage clearance, but with half compression; 23, same percentage clearance, but with full compression.

This example sufficiently illustrates the effect of clearance and compression on the size of the cylinder.

The cut off in the cylinder takes place at the same fraction of the stroke for all values of the volume found above, because, as shown by equation (8), ρ depends upon the clearance fraction f and the true ratio of expansion only. In this example the cut off is

$$\rho = \frac{r(1 - f)}{1 - rf} = \frac{3(1 - 0\cdot1)}{1 - 0\cdot1 \times 3} = 0\cdot385$$

that is 39 per cent. of the stroke.

It is important to determine the value of ρ, because this is the fraction of the stroke at which cut off must take place in order to secure the true ratio of expansion required.

An interesting special case arises in connection with the Unaflow Cylinder, a drawing of which is shown in Fig. 171. The peculiarity of the engine in connection with the present subject is, that compression begins almost at the beginning of the return stroke of the piston, and the fraction of the stroke at which it begins cannot be varied. Let x be 0·9, giving that compression begins when the piston has swept out only $\frac{1}{10}$ of the maximum volume V_2 on the return stroke. Also let the pressure at the end of compression be fixed at 153 lbs. per square inch, a pressure which should be taken about 25 to 30 lbs. lower than the initial pressure of the steam. Then from (5)

$$f = \frac{0\cdot9 p_3}{153} = \frac{p_3}{170}$$

Suppose now that the engine is condensing, and that p_3, the pressure at which compression begins, is 1 lb. per square inch. Then the clearance volume must be $\frac{1}{170}$ of the maximum volume V_2 to ensure that the pressure at the end of compression is 153 lbs. per square inch. There is no difficulty in designing the cylinder for this small clearance, because in this engine the ports are in the cylinder cover and the clearance can therefore be reduced to any value required.

An engine designed with this small clearance may have to start without a vacuum, in which case p_3 is 15 lbs. per square inch. With this value of p_3

$$f = \frac{15}{170} = \frac{1}{11}, \text{ say}$$

This is a clearance volume 16 times as large as that which is required when p_3 is 1 lb. per square inch. The difficulty is got over by the provision of supplementary clearance space in the cylinder cover, which at starting can be put into communication with the cylinder by opening out at each end valves which normally act as relief valves.

When a cylinder of this kind is used for a non-condensing engine, sufficient clearance must be permanently provided to prevent over-compression.

In the case of a locomotive, for example, in which the pressure at

the beginning of compression may be as much as 20 lbs. per square inch at high speeds, the clearance with the above limitation of the pressure at the end of compression is,

$$f = \frac{20}{170} = 0{\cdot}12 \text{ approximately,}$$

or from (10) about $13\frac{1}{2}$ per cent. of the effective volume of the cylinder.

An example may be taken to further illustrate these points.

Find the size of a cylinder for the data of the previous example but with the added conditions that compression is invariable and begins when the piston has swept out $\frac{1}{10}$ of the maximum volume, and that the pressure at the end of compression is 150 lbs. per square inch.

A value of f must first be calculated for these added conditions. Thus with the data given in the example above

$$f = \frac{0{\cdot}9 \times 18}{150} = 0{\cdot}108$$

The expression (3) reduces to

$$V_2 = \frac{1260}{(55 + 32) - 0{\cdot}108(318 + 0)} = 24 \text{ cub. ft.}$$

The effective volume of the cylinder is $V_2(1 - f) = 21{\cdot}4$ cub. ft. From (7) cut off is at about 25 per cent. of the stroke, since r, the real ratio of expansion, is 3, and f is given above.

78. Cylinder Volumes. Compound Engines.—The determination of the volumes of the cylinders of a compound engine involves two problems:—

(1) The calculation of the size of the low-pressure cylinder for given conditions of power and speed, and for a given ratio of expansion. By ratio of expansion is here meant the whole ratio fixed upon for the engine, and not the ratio in the low-pressure cylinder alone. If, for example, the steam is to be expanded 15 times, then the ratio of expansion to be used in this problem is 15.

(2) The determination of the volume of the cylinder in which the steam is partially expanded before flowing into the low-pressure cylinder.

The first problem is solved in the way just explained and illustrated for a simple engine, with, however, a slight modification in the process of calculation, to be explained presently.

The second problem is indeterminate, because there are so many ways in which the whole expansion may be divided between the cylinders without sensibly changing the efficiency of the engine as a whole.

It is generally desirable to divide the power equally between the

cylinders unless there is some reason for an unsymmetrical distribution, as for example in the case of a four-cylinder balanced engine where the crank angles are unequal.

In whatever proportion the total power is distributed, the division should be made so that in each cylinder the pressure at the end of expansion is higher than the back pressure. This pressure difference or drop multiplied by the volume of the cylinder is the work done in the cylinder against a constant resistance equal to the drop, and this work may be regarded as done against the approximately constant resistance of the machinery of the line of parts to which the cylinder is connected. If the work done against machinery resistance is equally shared by the cylinders, the product of drop and cylinder volume is constant for each cylinder and equal to b, say. The determination of the volume ratios in order to secure

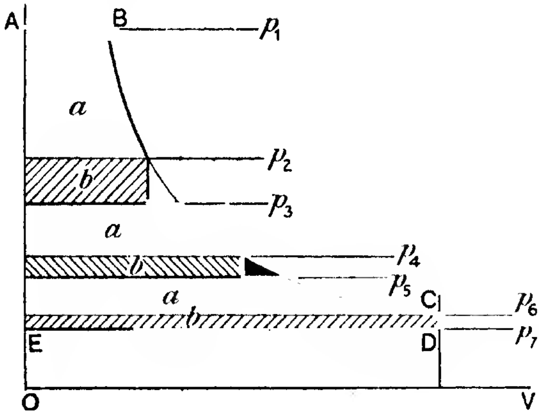


FIG. 85.—Cylinder volumes. Compound engine.

equality in drop-work and at the same time equal distribution of power between the cylinders is a definite problem which can be solved when the initial pressure in the high-pressure cylinder and the back pressure in the low-pressure, and the ratio of expansion are given.

A complication is introduced in the problem, however, by the fact that the clearance factor is different for each cylinder, but the difficulty is avoided by neglecting clearance altogether and afterwards adjusting the volume of each cylinder to allow for a clearance appropriate to the size of the cylinder and the type of valve used and the ratio of compression chosen.

Stating the problem in a definite form it stands thus:—

Find the volume ratios between the cylinders of a compound engine so that there is equality of power and equality of drop-work between them, having given the initial pressure in the high-pressure

cylinder, the back pressure in the low-pressure cylinder, the ratio of expansion, and assuming that the expansion curve is a rectangular hyperbola and that there is no clearance.

It is further assumed that the receiver volumes are so large that the pressure in them remains constant, so that the back pressure of any one cylinder is the initial pressure of the cylinder next below it. Receiver drop is, however, allowed for by increasing the hypothetical back pressure in the low-pressure cylinder.

Consider the division of power between three cylinders, as shown in Fig. 85, where ABCDEA is the whole diagram divided by the pressures p_3 and p_5 into three parts. The shaded areas, marked b, show the drop-work in each part; the unshaded areas a show the remaining part of each diagram. The total area of each individual diagram is thus $(a + b)$. Since the total area of each diagram must be the same in order to secure equal distribution of power, and since the drop-work is to be equal in each diagram, it follows that the areas marked a must be equal in each diagram.

Let c be the constant of the expansion curve BC, then the upper area a is equal to $\int_{p_2}^{p_1} vdp = c \log_e \frac{p_1}{p_2}$, with similar expressions for each of the diagrams. And since these are to be equal

$$c \log_e \frac{p_1}{p_2} = c \log_e \frac{p_3}{p_4} = c \log_e \frac{p_5}{p_6}$$

that is

$$\frac{p_1}{p_2} = \frac{p_3}{p_4} = \frac{p_5}{p_6} = \lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

And since the shaded areas are to be equal

$$(p_2 - p_3)v_2 = (p_4 - p_5)v_4 = (p_6 - p_7)v_6 = b$$

Also

$$v_2 = \frac{c}{p_2}\,;\; v_4 = \frac{c}{p_4}\,;\; \text{and } v_6 = \frac{c}{p_6}$$

Substituting these values for the volumes, and rearranging the terms

$$\frac{p_3}{p_2} = \frac{p_5}{p_4} = \frac{p_7}{p_6} = \frac{c - b}{c} = \delta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Then using alternately the ratios in (1) and (2)

$$p_1 = p_2\lambda = p_3\frac{\lambda}{\delta} = p_4\frac{\lambda^2}{\delta} = p_5\frac{\lambda^2}{\delta^2} = p_6\frac{\lambda^3}{\delta^2} = p_7\frac{\lambda^3}{\delta^3} \quad . \quad . \quad (3)$$

From which

$$\left(\frac{\lambda}{\delta}\right)^3 = \frac{p_1}{p_7} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

And the volume at cut off = 1.

Volume of high-pressure cylinder $= \dfrac{p_1}{1} = \lambda$

Volume of intermediate cylinder $= \dfrac{p_1}{p_4} = \dfrac{\lambda^2}{\delta}$

Volume of low-pressure cylinder $= \dfrac{p_1}{p_6} = \dfrac{\lambda^3}{\delta^2}$

Divide through by λ and then the volume ratios are

Cut off.		High-pressure cylinder.		Intermediate.		Low-pressure cylinder.	
$\dfrac{1}{\lambda}$	:	1	:	$\dfrac{\lambda}{\delta}$	:	$\left(\dfrac{\lambda}{\delta}\right)^2$	. . . (5)

Therefore the volume ratios are in a geometrical progression with $\dfrac{\lambda}{\delta}$ for the common multiplier.

The cut off in the high-pressure cylinder takes place at the volume $\dfrac{1}{\lambda}$

$$\text{The whole ratio of expansion} = \frac{\lambda^3}{\delta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{The pressure in first receiver} = p_1\left(\frac{\delta}{\lambda}\right) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{The pressure in second receiver} = p_1\left(\frac{\delta}{\lambda}\right)^2 \quad . \quad . \quad . \quad . \quad (8)$$

Although the ratio $\dfrac{\lambda}{\delta}$ is immediately determined from (4), and from it the volume ratios, the value of λ must be calculated in order to find the volume in the high-pressure cylinder at cut off, $\dfrac{1}{\lambda}$. From (2)

$$\delta = \frac{p_7}{p_6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the drop in the low-pressure cylinder is given, then $p_6 = p_7 +$ drop. If, however, the ratio of expansion is given and not the drop, then

$$p_6 = \frac{p_1}{r}.$$

So that

$$\delta = \frac{rp_7}{p_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

And using (4)

$$\lambda = r\left(\frac{p_7}{p_1}\right)^{\frac{2}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

These expressions may be generalized for any number of stages. Thus, let there be n stages, then calling the volume of the high-pressure cylinder unity, the cylinder volumes are in the ratios

$$1 : \frac{\lambda}{\delta} : \left(\frac{\lambda}{\delta}\right)^2 \; . \; . \; . : \left(\frac{\lambda}{\delta}\right)^{n-1} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\text{The ratio } \frac{\lambda}{\delta} = \left(\frac{\text{Initial pressure in high-pressure cylinder}}{\text{Back pressure in low-pressure cylinder}}\right)^{\frac{1}{n}} \quad (13)$$

$$\lambda = r\left(\frac{\text{Back pressure, low-pressure cylinder}}{\text{Initial pressure, high-pressure cylinder}}\right)^{\frac{n-1}{n}} \quad (14)$$

where r is the ratio of expansion.

$$\text{Ratio of expansion} = \frac{\lambda^n}{\delta^{n-1}} = \left(\frac{\text{Initial pressure}}{\text{Back pressure}}\right) . \lambda^{n-1} \quad (15)$$

$$\text{Pressure in the } m\text{th receiver} \quad p_1\left(\frac{\delta}{\lambda}\right)^m \quad . \quad . \quad (16)$$

In these expressions the initial pressure is the average pressure likely to be shown by the admission line of the indicator diagram in the high-pressure cylinder.

The back pressure is the pressure likely to be shown by the exhaust line in the low-pressure cylinder increased to allow for pressure drops in the receivers. When actual diagrams from a compound engine are combined, and a standard expansion curve is drawn through the cut-off point in the high-pressure cylinder, it will usually be found that the expansion curves of the intermediate and of the low-pressure diagram fall below the standard curve. If therefore the back pressure is suitably increased, a rough allowance is made for receiver losses because the effect is to lift each diagram nearer to the standard curve. Therefore, for purposes of a preliminary design, the back pressure to be used in the expression is

$$\left\{\begin{matrix}\text{condenser}\\ \text{pressure}\end{matrix}\right\} + \left\{\begin{matrix}\text{Drop between cylinder}\\ \text{and condenser}\end{matrix}\right\} + \left\{\begin{matrix}\text{Allowance for}\\ \text{receiver drop.}\end{matrix}\right\}$$

Failing actual data of the drop, an allowance of 2 lbs. may be made for the low-pressure receiver, and 3 lbs. for the intermediate receiver, though the drop often exceeds this largely.

The final adjustment of the cylinder volumes may be made as follows :—

From equation (3), page 286,

$$V_2 = \frac{A}{B - C}$$

where A is written for ηD, and B and C for the functions in the denominator, C being the function which depends upon the clearance only. If clearance is neglected

$$V_0 = \frac{A}{B}$$

Eliminating A from these two equations,

$$V_2 = \frac{V_0 B}{B - C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

But $$B = p_2 \log_e \frac{p_1}{p_2} + p_2 - p_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

for the high-pressure cylinder, and its value can be calculated for each of the cylinders with the data already obtained in connection with the determination of the volume ratios.

Also
$$C = f\left(p_c \log_e \frac{p_c}{p_3} + p_1 - p_c\right) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

in which f is the clearance factor and p_c is the pressure at the end of compression. A value of C can be calculated for each cylinder when the values of f and p_c are chosen.

Therefore the volume V_2 can be calculated from the volume V_0 found on the assumption of no clearance.

EXAMPLE.—Find the volume ratios, neglecting the effect of clearance, for a three-cylinder three-stage compound engine from these data.

Initial pressure in high-pressure cylinder, 155 lbs. per square inch.

Back pressure in low-pressure cylinder, 3 lbs. per square inch.

Back pressure to be used in estimating, 8 lbs. per square inch, allowing 2 lbs. for the low-pressure receiver and 3 lbs. for the high-pressure receiver.

Ratio of expansion, 15.

From equation (13), with $n = 3$

$$\frac{\lambda}{\delta} = \sqrt[3]{\frac{155}{8}} = 2{\cdot}7$$

From (13) the volume ratios are

	High pressure.		Intermediate pressure.		Low pressure.
	1	:	2·7	:	$2{\cdot}7^2$
that is	1	:	2·7	:	7·3

From equation (14)

$$\lambda = 15\left(\frac{8}{155}\right)^{\frac{2}{3}} = 2$$

Therefore cut off in high-pressure cylinder = ½ high-pressure volume

From equation (16),

$$\text{Pressure in high-pressure receiver} = \frac{155}{2{\cdot}7} = 58{\cdot}5 \text{ lbs. per sq. in.}$$

$$\text{Pressure in low-pressure receiver} = \frac{155}{7{\cdot}3} = 21{\cdot}5 \text{ lbs. per sq. in.}$$

From the relation in (3)

$$\left.\begin{array}{r}\text{Pressure at end of expansion}\\ \text{in high-pressure cylinder}\end{array}\right\} = \frac{155}{2} = 77 \text{ lbs. per sq. in.}$$

$$\left.\begin{array}{r}\text{Pressure at end of expansion}\\ \text{in intermediate cylinder}\end{array}\right\} = \frac{58{\cdot}5}{2} = 29{\cdot}5 \quad ,,$$

$$\left.\begin{array}{r}\text{Pressure at end of expansion}\\ \text{in low-pressure cylinder}\end{array}\right\} = \frac{21{\cdot}5}{2} = 10{\cdot}7 \quad ,,$$

Thus all the pressures are known.

The next step is to determine the volume of the low-pressure cylinder, neglecting clearance, for the power and the speed required.

Let the indicated horse-power to be developed be 650, and the speed 60 revolutions per minute. Diagram factor 1·1. Then from equation (1), page 285,

$$1 \cdot 1 D = 1366$$

And from equation (3), page 286,

$$V = \frac{1366}{10 \cdot 34 \log_e 15 + 10 \cdot 34 - 8} = 45 \text{ cub. ft.}$$

Then the corresponding volume of the intermediate cylinder is 16·67 cub. ft., and of the high-pressure cylinder 6·17 cub. ft.

These have now to be corrected for clearance. The following scheme shows the values of f selected as appropriate together with the pressures, including the selected pressures at the end of compression:—

Cylinder.	Initial pressure.	Pressure at end of expansion.	Back pressure.	Pressure at end of compression.	f.
	lbs. per sq. in.	lbs. per sq. in.	lbs. per sq. in.	lbs. per sq. in.	
H.P.	155	77	58	155 full comp.	0·11
I.P.	58	29	21	50	0·092
L.P.	21	10·5	8	12	0·07

Then from equation (18), page 295, for the high-pressure cylinder,

$$B = 77 \log_e 2 + 77 - 58 = 72 \cdot 7$$

From (19), page 296,

$$C = 0 \cdot 1\left(155 \log_e \frac{155}{58}\right) = 16 \cdot 7$$

Therefore, from equation (17), page 295

$$\left.\begin{array}{l}\text{Volume of high-pressure} \\ \quad \text{cylinder}\end{array}\right\} = \frac{6 \cdot 17 \times 72 \cdot 6}{55 \cdot 9} = 8 \text{ cub. ft.}$$

In a similar way it is shown that the corrected volumes of the intermediate and low-pressure cylinders are respectively 20 cub. ft. and 49 cub. ft.

The real volumes are then

	High pressure.		Intermediate pressure.		Low pressure.
	8 cub. ft.		20 cub. ft.		49 cub. ft.
Their ratios are	1	:	2·5	:	6·1

The effective volumes, $V(1 - f)$, are

	High pressure.		Intermediate pressure.		Low pressure.
	7·1 cub. ft.		18·2 cub. ft.		45·5 cub. ft.
Their ratios are	1	:	2·55	:	6·4

These relations show the effect of clearance and compression in modifying the real volume ratio between the cylinders.

The cut-off in the high-pressure cylinder, from equation (7), page 288, with $r = 2$, is 44 per cent. of the stroke.

This analysis is easily extended to engines where a stage of the expansion is carried out in two cylinders connected in parallel still with the condition of equal distribution of power between the total number of cylinders and equal drop-work in each.

In the three-stage four-cylinder engine in which the low-pressure cylinder is divided into two of equal volume, the condition of equal distribution of power requires that

$$c \log_e \frac{p_1}{p_2} = c \log_e \frac{p_3}{p_4} = 2c \log_e \frac{p_5}{p_6}$$

Therefore

$$\frac{p_1}{p_2} = \frac{p_3}{p_4} = \left(\frac{p_5}{p_6}\right)^2 = \lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

And combining this with the ratios in (2), page 293

$$\frac{\lambda^{\frac{5}{2}}}{\delta^3} = \frac{p_1}{p_7} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

From (10), page 294
$$\delta^3 = r^3\left(\frac{p_7}{p_1}\right)^3$$

Therefore

$$\lambda = r^{\frac{6}{5}}\left(\frac{p_7}{p_1}\right)^{\frac{4}{5}} = \left(\text{ratio of expansion}\right)^{\frac{6}{5}} \times \left(\frac{\text{Back pressure}}{\text{Initial pressure}}\right)^{\frac{4}{5}} \qquad (22)$$

λ and δ are thus determined, and the calculation is continued as in the general case. In a similar way the distribution of power between the cylinder may be made in any proportion by altering the area a in each cylinder, keeping the area b constant. The proportional numbers will appear as powers in the λ equation.

The results obtained from this analysis are to be regarded as giving a basis for a preliminary design. The cut off in each cylinder is to be afterwards adjusted to suit the receiver volumes and the crank angles, in order to obtain a drop at the end of expansion in each cylinder as nearly as possible equal to the value corresponding to some suitably chosen area, b, Fig. 85, page 292.

79. Efficiencies Obtained in Practice.—The efficiencies of a number of locomotive engines are plotted in Fig. 86 against the initial pressure by gauge. The efficiency of the Rankine engine of comparison for a constant lower temperature of 100° C. is plotted against the upper pressure used in calculating the efficiency, and the curve AB is drawn through the points so found. The vertical region between AB and the axis is divided by lines, so that each ordinate is divided into ten equal parts. These lines enable the efficiency ratio corresponding to any plotted experiment to be read off. For example, with an initial pressure of 120 lbs. by gauge, equal to 130 lbs.

absolute, and a back pressure corresponding to 100° C., the efficiency of the Rankine engine is PR, equal to 0·16 approximately. The actual efficiency of an engine supplied with steam at *120 lbs.* per square inch by gauge is, say, 0·122, corresponding to point *X* in the diagram. Then the efficiency ratio of the engine is, reading from the diagram, about 0·76.

Points 1 to 6 in the diagram relate to the experiments made by Prof. Goss to determine the effect of using high-pressure steam. They were all made on the same engine, "Schenectady," and full particulars of the trials are recorded in "High Steam Pressures in Locomotive Service," published by the Carnegie Institute of Washington, in 1907. Point No. 1 is the average of 12 experiments; No 2, the average of 11; No. 3, the average of 13; No. 4, the average of 12; No. 5, the average of 11; and No. 6, the average of 8 experiments. Point No. 7 is the average of 5 experiments carried out on the London and South-Western Railway, with an express passenger engine, type 4—4—0, running in service.[1]

Point 8 is the average of 2 experiments made by Donkin[2] and Kennedy on a small engine belonging to the Great Eastern Railway.

Point 9 is the average of 3 experiments on three different engines tested in service on the North-Eastern Railway.[3]

The remaining points are plotted from results recorded in "Locomotive Tests and Exhibits," published by the Pennsylvania Railroad Co. in 1905, and relate to the series of trials carried out on the testing plant at the St. Louis Exhibition.

Point No. 10.	2-8-0.	Simple.
Point No. 11.	2-8-0.	Simple.
Point No. 12.	2-8-0.	Four-cylinder cross-compound.
Point No. 13.	2-10-2.	Four-cylinder tandem compound.
Point No. 14.	4-4-2.	Four-cylinder De Glehn compound.
Point No. 15.	4-4-2.	Four-cylinder compound.
Point No. 16.	4-4-2.	Four-cylinder compound.
Point No. 17.	4-4-2.	Four-cylinder compound superheater.

The efficiency ratio of the superheat engine cannot be read off from the plotted lines. It must be compared with the corresponding efficiency of the Rankine engine using superheated steam, two efficiency curves for which are plotted, one for a temperature of 250° and a second, CD, for 300° C.

Thus, if the experiment X related to an engine supplied with steam superheated to 300° C., its efficiency ratio is the ratio XP : SP.

The superiority of the engine supplied with superheated steam, No. 17, over all the others is manifest, and the superiority of the compound engines over the group of simple engines is clear. The compound efficiencies form a constellation about the efficiency 0·13,

[1] Adams and Pettigrew, "Trials of an Express Locomotive," *Proc. Inst. C.E.*, vol. 125.

[2] Donkin and Kennedy, "Experiments on Steam Boilers," *Engineering*, 1897.

[3] W. M. Smith, *Proc. Inst. Mech. E.*, 1898.

TABLE 19.—The Absolute and the Relative Thermal Efficiencies of Certain Locomotives.

No. of point on the diagram.		Efficiency.		
1	Testing plant	0·086	Average of 12 experiments at 120 lbs. sq. in. gauge press.	Data recorded in "High Steam Pressures in Locomotive Service," by W. F. Goss. Published by the Carnegie Institute of Washington, 1907.
2	,,	0·096	,, 11 ,, 140 ,, ,,	
3	,,	0·100	,, 13 ,, 180 ,, ,,	
4	,,	0·097	,, 12 ,, 200 ,, ,,	
5	,,	0·099	,, 11 ,, 220 ,, ,,	
6	,,	0·101	,, 8 ,, 240 ,, ,,	
7	On the road .	0·11	,, 5 ,, 170 ,, ,,	Data recorded in "Trials of an Express Locomotive," by Adams and Pettigrew, *Proc. Inst. C.E.*, vol. 125, 1895-1896.
8	,, .	0·085	,, 2 ,, 122 ,, ,,	Deduced from Kennedy and Donkin's Trials on a G.E.R. Locomotive. Data recorded in "Experiments on Steam Boilers," published by *Engineering*, 1897.
9	,, .	0·094	,, 3 ,, on 3 different engines at 175 lbs. sq. in.	Deduced from data in "Results of Recent Practical Experience with Express Locomotive Engines". W. M. Smith, *Proc. Inst. Mech. Eng.*, October, 1898.
10	Testing plant at St. Louis Exhibition.	0·104	Experiment giving max. economy, 198 lbs. sq. in. gauge press. Type of engine, 2–8–0, simple.	"Locomotive Tests and Exhibits," published by the Pennsylvania Railroad Co., Philadelphia, 1905.
11		0·102	,, ,, 199 ,, ,, ,, ,, 2–8–0, simple.	
12		0·123	,, ,, 209 ,, ,, ,, ,, 2–8–0, 4-cylinder cross-compound.	
13		0·116	,, ,, 213 ,, ,, ,, ,, 2–10–2, 4-cylinder tandem compound.	
14		0·127	,, ,, 215 ,, ,, ,, ,, 4–4–2, 4-cylinder De Glehn compound.	
15		0·125	,, ,, 222 ,, ,, ,, ,, 4–4–2, 4-cylinder compound.	
16		0·121	,, ,, 222 ,, ,, ,, ,, 4–4–2, 4-cylinder compound.	
17		0·129	Experiment at max. economy. Superheat, 82° C. 204 ,, ,, ,, ,, 4–4–2, 4-cylinder compound with superheater.	

with an efficiency ratio of about 0·63. These results show that the locomotive engine, within the limitations of temperature imposed on it, is an economical engine and compares with engines of any other type quite favourably. Marine engines of large power seldom realize greater efficiency ratios than the locomotive, as will be seen from Table 17, page 263, where a number of authoritative results are

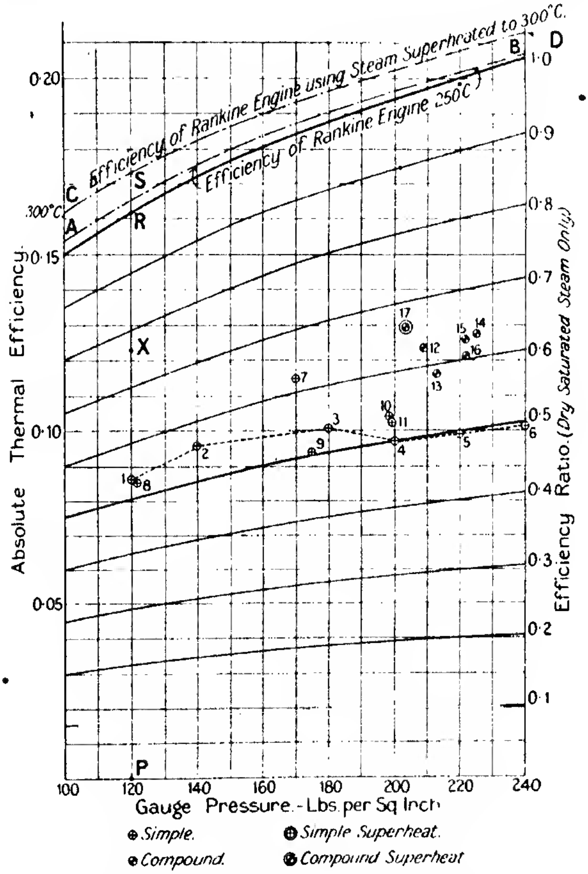


FIG. 86.—Diagram showing the efficiencies of certain locomotives.

brought together for comparison. These results cannot so conveniently be plotted in a diagram like those relating to non-condensing engines, because the back pressure in the exhaust pipe against which their several efficiencies are reckoned is variable. The table, however, gives all particulars of their performance. The results in the first group were obtained by the Committee appointed by the Admiralty to

report on the different types of boilers used in the Navy, and the results of all the boiler trials, together with the engine trials, will be found recorded in the Blue Books quoted in the Table. There is a mine of information in the Reports of this Committee relating to the performance of marine engines and boilers, and some of the boiler results have been quoted above in Table 10, page 122.

The data of the second group in Table 17, page 263, are reduced from trials of marine engines in service made by the Research Committee of the Institution of Mechanical Engineers, records of which are distributed through various volumes of the *Proceedings*. A summary of all the trials by Professor Beare will be found in Part I. of the *Proceedings of the Institute of Mechanical Engineers* for 1894.

The curves Fig. 87 show the results obtained by the use of super-

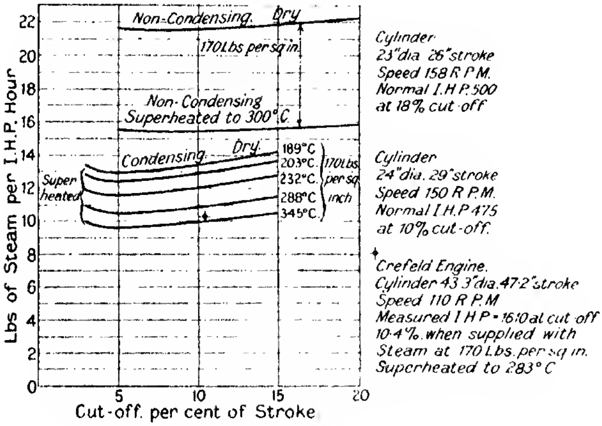


Fig. 87.—Steam consumption. Sulzer single-cylinder Una-flow double acting steam engine.

heated steam in a Sulzer single-cylinder steam engine built with Una-flow cylinders.

The author is indebted to Mr. Carl Sulzer for the data from which the curves have been constructed. The cylinder of an engine of this kind is fully described below. Briefly, there is an admission valve in each cylinder cover, and an exhaust port at the centre of the cylinder, which is closed and opened by the piston itself. All of the steam supply flows through the cylinder cover jackets on its way through the admission valves to the cylinder. The barrel is unjacketed. By this arrangement the ends of the cylinder are kept hot, and the flow of heat is always in one direction along the cylinder barrel, namely, from the hot ends to the relatively cool centre. Hence the term "Una-flow" as applied to this cylinder. The clearance surface at the ends, on which most of the condensation takes place, is by this arrangement kept hotter than in the usual

type of engine, because it is never exposed to the flow of a stream of cool exhaust steam; and further the entering steam does not flow into the cylinder through a passage which has just been cooled down by exhaust steam flowing out of the cylinder, as is the case when a cylinder is designed to work with a slide valve.

Referring to Fig. 87, it will be seen that there are two groups of curves. The first group consists of two curves only, the upper of which shows the results obtained non-condensing with dry saturated steam at 170 lbs. pressure per square inch by gauge, and the lower curve of the pair shows the results obtained when the steam is superheated to 300° C. The nominal horse-power of this engine is 500.

The lower group of curves in the figure exhibit the results obtained with a condensing engine of about 475 H.P. supplied with steam at 170 lbs. per square inch by gauge with various degrees of superheat. The upper curve of the group gives the results for dry saturated steam, and each curve below gives the results obtained when the steam is superheated to the particular temperature written against it. A consumption of 9¼ lbs. of steam per I.H.P. hour is obtained with steam superheated to 345° C. when the cut off is about 5 per cent. of the stroke. The effect on the economy of the degree of superheat is shown clearly by these curves. The curves show also the effect on the economy of a variation in the cut off and bring out the point that although there is one cut off at which the economy is greatest, yet this point can be varied considerably without producing much variation in the economy.

The result of a test made on a Sulzer single-cylinder condensing engine built with Una-flow cylinders, and erected in a spinning factory, is shown by the cross on the diagram. The engine developed 1610 I.H.P. at 110 revolutions per minute when supplied with steam at 170 lbs. pressure by gauge and superheated to 283° C., and the consumption measured was 10·3 lbs. of steam per I.H.P. hour.

CHAPTER V

CONDENSING PLANTS AND THE COOLING CIRCUIT

80. Introduction.—The cooling circuit is linked with the motive-power circuit in the condenser. The water flowing through the part of the cooling circuit in the condenser condenses the steam, and establishes a pressure in the part of the motive-power circuit within the condenser corresponding to a temperature somewhat above the mean temperature of the circulating water. In the limit, with the flow in the motive-power circuit and in the cooling circuit infinitely slow, the temperature of condensation and the temperature of the cooling water would be identical, and the pressure established would correspond with the temperature of the cooling water. In practice there is a temperature difference, a difference necessary to form a heat-fall so that heat may flow through the plant in sufficient quantity.

A condensing plant is in general added to a steam engine in order to reduce to as low a value as possible the back pressure against which the piston works, since without a condenser the back pressure cannot be lower than the atmospheric pressure. The steam is in contact with its liquid both in the boiler and in the condenser, and therefore the pressure difference between the boiler and the condenser is determined by the definite temperature range of the plant. Moreover, the greater the temperature difference the greater is the possible thermal efficiency, and therefore the addition of a condenser always increases the possible thermal efficiency of the whole plant, though in particular cases the addition may be commercially undesirable, because the cost of maintaining the flow of the large quantity of cooling water may more than balance the gain due to an increase of thermal efficiency.

81. Surface Condensers and Air Pumps.—The elements of a surface condensing plant are the condenser, the air pump, and the circulating pump. A surface condenser connected with an air pump is shown in Fig. 88.

The condenser is generally formed of a steel or cast-iron barrel, A, with flanged ends, to which are bolted tube plates B, B, and end covers E, E. Tubes run from plate to plate, and they are secured at each end by a simple packing gland of the kind shown in the figure. The tube plates are further secured by the longitudinal stays D, D. In long condensers the tubes are supported and held at the correct

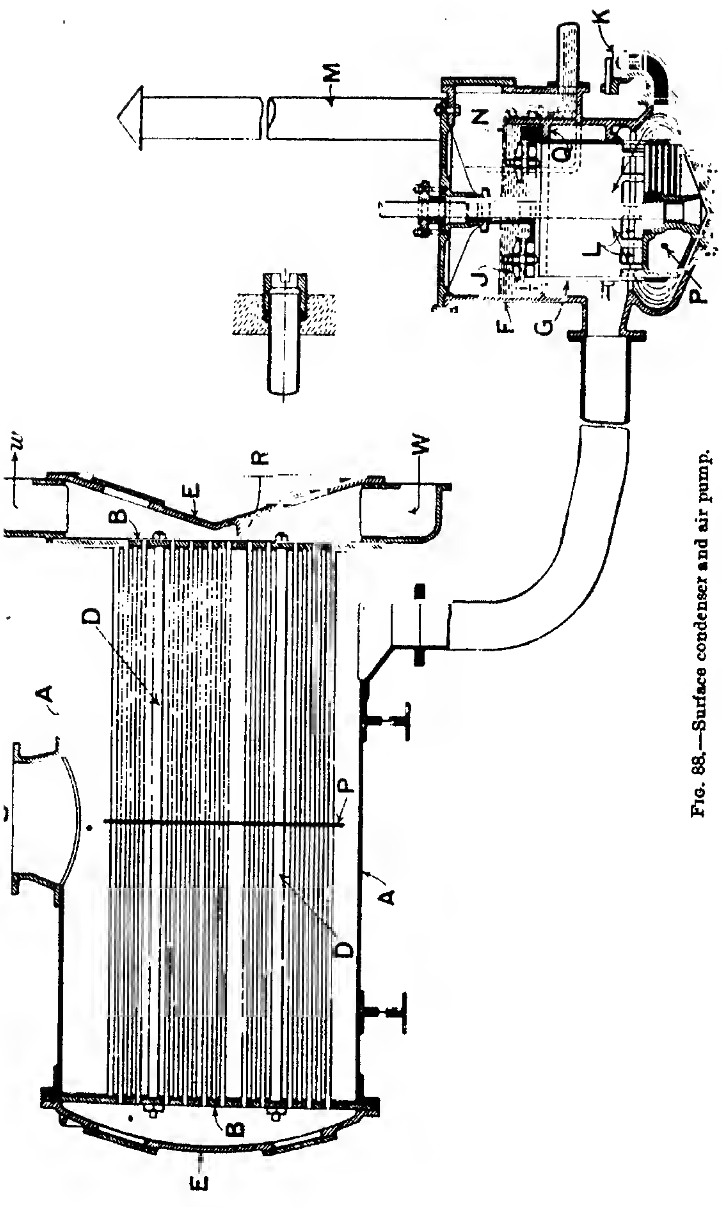


FIG. 88.—Surface condenser and air pump.

pitch by an intermediate plate P, through which the tubes pass. In the illustration steam enters at the top of the condenser at S, and is *condensed* as it winds its way amongst the tubes through which the *cooling water is flowing*. Directing or baffle plates are added to large condensers in order to ensure that the steam is properly distributed amongst the tubes. A condenser is described by Prof. Weighton[1] in which the steam space is divided into a series of separate compartments, all in communication with one another, but so arranged that the condensed steam in any one compartment drained away into the air pump without flowing over the tubes below it.

The end covers E, E are so designed that when bolted in place the circulating water, which enters at W, is compelled to pass first through the lower tubes and then back again through the upper tubes to the exit at *w*. This flow is determined by the rib R, which divides the water space at the right end into two compartments, and so prevents flow between the entry and the exit except through the tubes. So far as the water is concerned this is a "double flow" condenser. Ribs are sometimes arranged to cause a treble or even a quadruple flow. The "effective length" of tube is the length through which the circulating water is compelled to flow between its entry and exit. The condensed steam drains into the air pump.

In general the elements of an air pump are the foot valves, the bucket piston with its valve, and the head valves. The foot valves are the suction valves to the pump, the head valves are the delivery valves, and the action is that of an ordinary bucket pump. The charge of air, water, and condensed steam is drawn into the pump through the foot valves on the upward stroke of the bucket piston. The piston passes through the charge on its next downward stroke, the foot valve closing and the valves in the piston opening to allow this to take place. In the next upward stroke the charge is lifted by the bucket piston and forced through the head valves into the hot well.

These elements are illustrated in Fig. 91, page 318, where foot valves are indicated at L, the bucket piston with its valves is shown by K, and head valves are shown at V.

An Edwards air pump is shown in Fig 88. In this air pump separate foot valves and bucket valves are dispensed with and only head valves are required. The functions of the foot valves and the bucket valves are performed by a solid piston and a fixed liner with ports in it.

The pump consists of a barrel casing F, fitted with a top cover and gland; a liner G dropped into this casing and secured to an inside flange Q; a conical piston P; a plate for the head valves, J being one of them, and an atmospheric relief valve K.

The condensed steam drains into the pump and collects in the

[1] "The Efficiency of Surface Condensers," by Prof. R. L. Weighton, *Trans. Inst. Naval Arch.*, 1906.

conical base of the casing. It will be seen that there is a ring of openings or ports at the lower end of the liner G marked L.

When the piston is in its lowest position these ports are fully open, and free communication is established between the condenser and the space between the upper face of the piston and the head valves. Imagine the piston to be in its highest position. Condensed steam drains into the conical space at the bottom of the casing, and air and vapour accumulate in the lower part of the casing and liner. As the piston descends its conical under surface makes a gradual entry into the accumulated water and displaces it upwards, the curved sides of the casing guiding the water through the open ports L, L, into the liner. The piston in its next upward stroke closes the ports L, L, and forces the charge of air, vapour, and water above it through the head valves. The lip of weir N ensures that the head valves shall always be well covered with water, so as to seal them from the air. The charge passes through this layer of water, the vapour and air passing away through the pipe M, whilst the condensed steam flows over the weir and away into the hot well. If the pressure in the pump should for any reason exceed that of the atmosphere the valve K lifts, and so prevents damage.

It will be understood that in a pump of this kind the charge above the piston stands at the same pressure as that which exists in the pipe connecting the pump with the condenser, and no pressure head is required to lift the foot valves or bucket valves.

82. Heat Rejected in the Condenser per pound of Steam.—The pressure in the cylinder during the exhaust stroke is greater than the pressure in the condenser, but the flow of steam from the cylinder to the condenser takes place at approximately constant total energy. The steam is, in fact, throttled from the pressure shown by the exhaust line to the pressure corresponding to the temperature of the condenser. The heat energy to be removed in the condenser per pound of steam flowing into it from the cylinder is therefore the latent heat of the steam corresponding to its state, and to the pressure shown by the exhaust line of the indicator diagram. The whole discharge into the condenser, however, includes not only the steam flowing into it from the cylinder, but also the steam and water leaking directly into it through the valves. The state of the steam is therefore unknown at entry to the condenser, and, consequently, it is not possible to calculate accurately what is the latent heat which the cooling circuit must carry away per pound of steam supplied to the engine. It may, however, be arrived at quite accurately enough for all practical purposes by calculating the total energy of the steam at the temperature of condensation corresponding to the temperature in the condenser, and allowing for the reduction of the condensed steam to the temperature of the air-pump discharge. An allowance may be made for the loss of heat in other ways by making the calculation with an arbitrarily selected dryness fraction. Thus, let t_0 be the temperature of the water to which the steam is reduced,

measured by a thermometer placed in the stream as it flows from the air pump. Let t be the temperature of condensation. Then, with a dryness fraction q, the heat rejected per pound of water discharged by the air pump is

$$I_w + qL - I_{wa} \text{ lb.-cals.}$$

or, approximately,

$$t - t_a + L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

since the difference between corresponding values of t and h are negligible, and q may be taken equal to unity for design purposes.

For example, if the temperature of the steam flowing into the condenser is 70° C., corresponding to 4½ lbs. per square inch back pressure, and if the temperature of the condensed steam as it issues from the air pump and falls into the hot well is 60° C., then, assuming that $q = 1$, the heat rejected to the cooling water per pound of steam flowing in the condenser is by (1)

$$70 - 60 + 557 = 567 \text{ lb.-cals.}$$

Generally it may be assumed that the condensation of 1 lb. of steam corresponds to the rejection of 600 lb.-cals. in the condenser, the bulk of which is carried away by the cooling water, radiation, however, accounting for a small proportion.

The quantity of cooling water required depends upon the type of condenser used. In a surface condenser of the kind shown in Fig. 88 the quantity is large, because the rise of temperature of the water is relatively small.

Let t_1 be the temperature of the circulating water as it enters the condenser, and t_2 the temperature as it leaves it, and let W pounds of water be forced through the condenser tubes by the circulating pump per pound of steam condensed, then the heat taken up by the cooling water is

$$W(t_2 - t_1) \text{ per lb. of steam condensed.}$$

Then, generally for a surface condenser

$$W(t_2 - t_1) = I_w + qL - I_{wa} \quad . \quad . \quad . \quad . \quad (2)$$

or with the approximation mentioned above

$$W(t_2 - t_1) = 600 \text{ lb.-cals.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If, for example, the circulating water enters at 10° C. and this temperature increases to 30° C. as the water flows through the tubes, then, assuming that each pound of steam rejects 600 lb.-cals. to the cooling water, W = 30 lbs. If the engine is using 15 lbs. of steam per I.H.P. hour, the circulating pump must therefore force through the tubes 450 lbs. of cooling water per I.H.P. hour, and if the total horse-power is 1000 this corresponds to 450,000 lbs. per hour. The difficulty of obtaining sufficient condensing water is often very considerable. The temperature of the water entering the condenser is determined by the temperature of the source of supply. The temperature as it leaves depends upon the quantity of water pumped through

the condenser, but in no circumstances can its temperature be higher than the temperature of the steam corresponding to the pressure in the condenser, because if in the limit this temperature be reached there would be no temperature difference between the steam and the cooling water to cause a flow of heat from the one to the other.

83. Jet Condensers.—The quantity of cooling water is considerably reduced by the use of a jet condenser, the form of condenser originally applied to the steam engine by James Watt. The condenser is simply a cast-iron or steel chamber of any suitable form into which the steam flows from the engine cylinder to meet and mingle with a jet of water sprayed directly into the condenser. There are no tubes. The condensed steam and the cooling water used to condense it flow away to the air pump together. More work is thrown on to the air pump in this case, because it has to remove not only air vapour and condensed steam, but also the cooling water injected into the condenser to condense the steam.

In calculating the quantity of cooling water required in this case, it is to be observed that the final temperature of the cooling water is the same as the temperature of the condensed steam because they mingle together in one stream. Equation (2), page 308, then becomes

$$W(t_a - t_1) = I_w + qL - I_{wa} \quad . \quad . \quad . \quad . \quad (1)$$

or with the approximation mentioned above

$$W(t_a - t_1) = 600 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the air-pump discharge is 40° C. and the temperature of the cooling water is 10° C. when it enters the condenser, then W, the weight of cooling water required per pound of steam condensed, is 20 lbs., a quantity comparing favourably with the 30 lbs. which would be required for surface condensation.

The jet condenser is only used in land installations where a plentiful supply of clean fresh water is obtainable, because the condensing water and the condensed steam are delivered into the hot well from which the boiler feed is drawn. Surface condensers are always used with marine engines, in order to avoid the introduction of salt water into the boilers.

84. Cooling Ponds and Cooling Towers.—The problem of obtaining an adequate supply of cold condensing water, either for a jet or for a surface condenser, often presents formidable difficulties unless the plant be placed near a river or canal from which water may be drawn and into which it may be discharged after passing through the condenser. There is, of course, no difficulty at sea. In towns the cost of taking water from the mains and discharging it into the drains after passing through a condenser is usually prohibitive. If sufficient space can be found for a cooling pond, a pond into which the circulating water is discharged, and from which it is drawn, the

cost of the circulating water is reduced to the cost of restoring the loss of water by evaporation from the surface of the pond. In manufacturing districts cooling ponds or tanks are sometimes placed on the tops of factory buildings. The area of the surface of the cooling pond must be large enough to ensure a transfer of heat to the atmosphere with sufficient rapidity to keep the pond cool, so that the water supplied to the condenser, and which is drawn from the cooling pond, may not have an unduly high temperature.

An alternative method is to build a tower over a tank or basin, and to fill the interior of the tower with what may be called a cubical network of wood laths and timbers, or something equivalent. The circulating water from the condenser is discharged at the top of the tower, and it trickles down over the interior obstructions into the tank at the bottom. Air flows up through the tower as through a chimney, and the intimate mingling of the air and the water produced by the cubical network in the tower results in a rapid cooling of the water as it trickles down into the lower tank. This lower tank is the supply tank for the circulating pump. Water passes from this tank into the circulating pump through the condenser into the top of the cooling tower and back again into the tank. The cooling circuit is complete and is a closed one. Many cooling towers may now be seen about our large towns generally in connection with electric supply plants. For a given power the ground area required for a cooling tower is very much less than the ground area required for a cooling pond.

The water in a pond or in a tower is cooled mainly by evaporation into the air. Make-up water must be supplied continuously to compensate for this evaporation.

The rate at which water will evaporate into air depends upon the proportion of steam already contained in the air. In a space devoid of air, evaporation from the surface of water would be very rapid and steam would be produced at the pressure corresponding to the temperature of the water. Evaporation into dry air is rapid, but much slower than in a space devoid of air, but as the air gets more and more charged with steam the rate of evaporation gets slower and slower; until, finally, when the air is saturated evaporation stops altogether and a condition of equilibrium is reached which is only disturbed by a variation of temperature or pressure. Increase the temperature of air already saturated and evaporation will begin again and will continue until another condition of equilibrium is reached corresponding to the new temperature. Decrease the temperature of air already saturated and the steam in it in excess of the quantity proper to the new and lower temperature is precipitated as fog, or, if it is cold enough, as snow. Although the term "saturation" is used in connection with the air it should strictly be used as applied to the space occupied by the air; the temperature which determines the conditions of equilibrium is not necessarily the temperature of the air, but the temperature of the space occupied by the air. All that the air does is to modify the rate at which evaporation takes place. The

weight of vapour which fills a given space at a given temperature is practically the same whether air or any other vapour is there or not. The calculation of the weight of steam which saturates a given volume at a given temperature is useful in connection with many engineering problems, and it is therefore given in some detail in the next section.

85. On the Evaporation of Water into and the Condensation of Water from Air.—The calculation of the weight of steam which can saturate a given space and thus form with air simultaneously occupying that space an invisible mixture is based on the laws of the mixture of gases and vapours, and in particular on the law known as the "Law of Partial Pressures," which may be stated as follows:—

A mixture of a gas and a vapour at a common temperature will exert on the boundary containing it a pressure which is equal to the sum of the pressures which each element of the mixture would exert separately if it alone occupied the space.

This means that in a mixture of air and steam the pressure exerted by the steam is the same as the pressure it would exert if there were no air present, and the pressure exerted by the air is the same as it would exert if there were no steam present.

Let a vessel, V cub. ft. in volume, contain a mixture of W lbs. of air and x lbs. of steam at the common temperature T° C. absolute. The pressure P exerted by the air alone can be calculated from the characteristic equation $PV = 96WT$, and is

$$P = \frac{96WT}{V} \text{ lbs. per square foot} \quad . \quad . \quad . \quad . \quad (1)$$

Again, assuming that there is a small amount of water in the vessel so that the steam is in contact with its liquid, the pressure p which the steam exerts on the boundary depends upon the temperature T only, and its magnitude can be found from the steam tables.

Finally, the actual pressure exerted on the boundary, that is the pressure which would be shown by a gauge applied to the vessel arranged to indicate absolute pressure, is

$$P + p$$

The pressure P and p are the partial pressures referred to in the law stated above, and their sum is the actual pressure. It should be fully understood that if the temperature remains unaltered the sudden annihilation of the air present would have no sensible effect on the steam pressure, but would of course reduce the total pressure to p; or the sudden annihilation of the steam present would have no sensible effect upon the pressure of the air present but would reduce the total pressure to P.

The weight of steam, x, required to fill, or saturate, a volume v at temperature T is $\frac{v}{V_t}$ lbs., where V_t is the volume of 1 lb. of dry

saturated steam at the temperature T as given in the steam tables, so that

$$x = \frac{v}{V_t}$$

The total weight of gas in the mixture is thus $(W + x)$ lbs.

By way of example, find the pressure in a condenser of 10 cub. ft. capacity in which the condensed water is at the uniform temperature of 30° C. and, in addition to the vapour corresponding to this temperature, there is present $\frac{1}{10}$ of a pound of air.

Neglect the volume of the condensed water present in the condenser.

The absolute temperature in the condenser is 303° C. From equation (1) the pressure due to the air is

$$\frac{96 \times 0{\cdot}1 \times 303}{10 \times 144} = 2{\cdot}02 \text{ lbs. per square inch}$$

From the tables, the pressure of steam at 30° C. is 0·6 lb. per square inch. The actual pressure in the condenser is therefore 2·62 lbs. per square inch.

From the tables the actual volume of 1 lb. of steam at 30° C. is 526 cub. ft., so that the weight of steam in a volume of 10 cub. ft. is $\frac{10}{526} = 0{\cdot}019$ lb.

The weight of the mixture is therefore 0·119 lb.

If the mixture of air and steam in a vessel be so conditioned that the temperature is identical with the atmospheric temperature, and that the pressure which it exerts on the vessel is equal to the pressure of the atmosphere, the mixture will stand in equilibrium with the surrounding atmosphere, if the walls of the vessel containing it are annihilated, supposing always that the surrounding atmosphere is saturated with steam. If the surrounding atmosphere is not saturated, the steam in the volume imagined free will gradually diffuse into the dry air. The conditions of the mixture of air and steam which constitutes the atmosphere can be examined by assuming that a sample is for the moment enclosed in a boundary, and that the mixture enclosed exerts a total pressure equal to the barometric pressure, and that it is at the same temperature as the atmosphere. The new condition implied in this assumption is that the sum of the partial pressures is given and is equal to the barometric pressure.

The solutions of these two problems are especially useful, namely, find the weight of steam required to saturate 1 lb. of dry air at a given temperature, and find the heat contained by the mixture.

Let B be the barometric pressure; let P be the pressure exerted by the air, and p the pressure exerted by the steam, and let the common temperature be T. By the law of partial pressures,

$$P + p = B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The steam pressure p is known from the steam tables, since the temperature T is given; the pressure of the air in the mixture is

therefore $144(B - p)$ lbs. per square foot. From the characteristic equation for air, the volume V of 1 lb. of dry air at the temperature T is

$$V = \frac{96T}{144(B - p)} \text{ cub. ft.} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Also V_t, the volume of 1 lb. of dry saturated steam, is found from the tables. Therefore x, the weight of steam required to saturate 1 lb. of dry air at the temperature T at the given barometric pressure is

$$x = \frac{96T}{144(B - p)V_t} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

In this expression B and T are observed; p and V_t are then found from the table for the temperature T, and the pressures B and p are expressed in lbs. per square inch.

The total heat carried by 1 lb. of dry air saturated with x lbs. of steam can be calculated from this result. The specific heat of air may be taken equal to 0·24 at constant pressure, and the latent heat of steam is L. Then reckoning above 0° C., the total heat energy of the mixture of 1 lb. of dry air and the x lbs. of steam required to saturate it is

$$H = 0{\cdot}24t + x(I_w + L) \text{ lb.-cals.} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

EXAMPLE.—Find the weight of steam required to saturate 1 lb. of dry air, and the total heat of the mixture when the temperature is 30° C., and the barometer reads 30 ins.

From the steam tables the pressure p, corresponding to 30° C. is 0·6 lb. per square inch, and V_t is 526.

Using these values in (4), $x = 0{\cdot}027$ lb.

Substituting x in (5), and finding the values of I_w and L from the tables for the given value of T, it will be found that $H = 23{\cdot}66$ lb.-cals., of which 7·2 belong to the air, and the remainder, namely, 16·4, to the steam associated with the air.

The curves,[1] Fig. 89, are drawn through points found by similar calculations using a series of values of the temperature, and with the constant value 14·7 lbs. per square inch for the atmospheric pressure. For example, in Fig. 89

TA is the heat carried by 1 lb. of dry air at 57° C.

AS is the heat carried by the weight of steam necessary to saturate 1 lb. of dry air at 57° C.

TS is the total heat carried by the mixture.

Ordinarily air is not completely saturated with steam. The most expeditious way to find how much steam is contained in the air is first to find the "Dew point" by means of a wet and dry bulb hygrometer. By this instrument the air is cooled down to a temperature at which the steam present is sufficient to saturate it, and this temperature is shown by the condensation of steam on the

[1] Similar curves were drawn by the author for the Report of the Committee on the Efficiency of Internal Combustion Engines published in the *Minutes of the Proceedings of Civil Engineers*, vol. 163, Session 1905-1906.

cooled bulb of the instrument. Knowing this temperature the problem may be solved by means of the curves in Fig. 89. Thus, if t is the temperature as found above, find the point on the horizontal axis representing it, and draw an ordinate ts to cut the curve in s, and through s draw a line parallel to the air line OA to cut an ordinate TS drawn through the point representing the actual temperature of the atmosphere in S_1. Then TS_1 represents the heat contained in 1 lb. of the actual air, and the weight of steam associated with this 1 lb. is less than the weight required to saturate it in the proportion of as to AS. Therefore water will continue to evaporate into it.

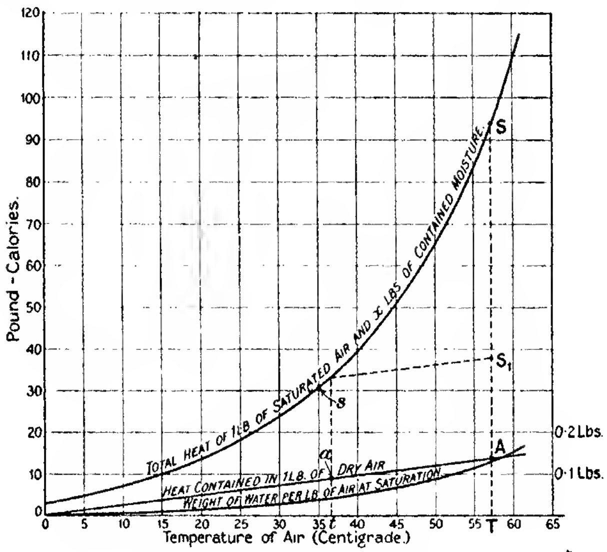


FIG. 89.—Total heat contained in 1 lb. of air at different degrees of moisture, and the weight of moisture present.

The curve shows that the action of a cooling tower depends upon the state of the atmosphere. If the atmosphere is warm and dry, the water evaporates quickly, and the water is therefore rapidly cooled. The nearer the air is to saturation the slower the cooling. Also the colder the atmosphere the slower the cooling, though a cold dry atmosphere is more effective than a wet warm air. The actual cooling of water both in a cooling pond and in a tower is effected mainly by this process of evaporation, though radiation, conduction and convection also aid in the transfer of heat from the water to the air.

The mixture in an engine condenser may be regarded as an atmosphere which is always saturated. And just as the presence of air reduces the rate of evaporation, so the presence of air reduces the rate of condensation in a condenser.

86. Influence of Air in the Condenser on the Pressure in the Condenser.—The actual pressure in a condenser is the sum of the pressures due to the steam, and the pressure due to the air present. The pressure due to the steam depends upon the temperature of the condensed water only, and can be found from the steam tables when the temperature of the water is assigned. The pressure due to the air depends upon the temperature, and also upon the weight of air in the condenser. The curves in Fig. 90 show the influence of a small quantity of air on the condenser pressure in relation to the temperature of condensation.

The steam pressure, plotted against the temperature, is shown by the thick curve. The condenser pressure could be read off from this curve for an assigned temperature of condensation if no air were present. The pressures in the condenser due to the presence

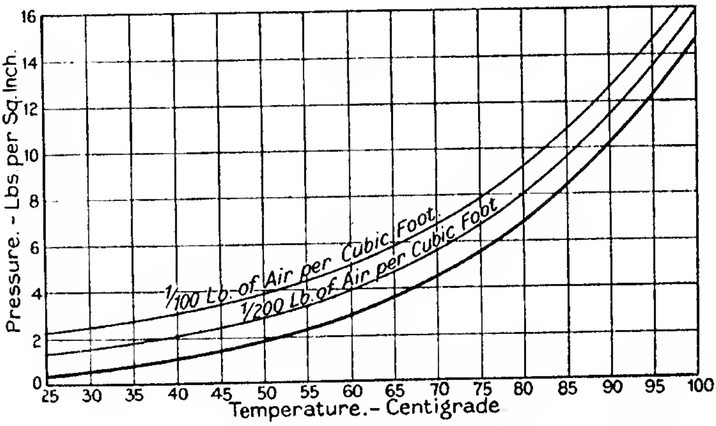


FIG. 90.—Influence of air in the condenser on the pressure in the condenser.

of $\frac{1}{200}$ and $\frac{1}{100}$ of a pound of air per cubic foot of condenser volume are shown by the curves plotted above it from the steam curve as a base. It will be observed that the increase of pressure is practically 1 lb. at 25° C., increasing to 1·3 lbs. at 100° C., when $\frac{1}{200}$ of a pound of air per cubic foot is present. The pressure is calculated from the characteristic equation $PV = 96WT$, substituting the assigned weight per cubic foot for W, and unity for V. The pressure is a linear function of the temperature in these circumstances, but for approximate calculations the air pressure in the condenser may be taken at 2 lbs. per square inch above the steam pressure due to the temperature for every $\frac{1}{100}$ of a lb. of air present per cubic foot of volume.

At a condensation temperature of 50° C., the steam pressure is about 1·8 lbs. per square inch, and the air pressure due to the presence of $\frac{1}{200}$ of a lb. of air per cubic foot of condenser volume, is about 1 lb. per square inch, roughly about half the steam pressure,

At 30° condensation temperature, the steam pressure is about 0·6 lb. per square inch, and the air pressure would be about twice this amount.

These considerations show the necessity of providing specially for the removal of air from condensers in cases where "high vaccua" or low condenser pressures are required.

Although the pressure through a condenser is constant, the temperature varies, being highest at the top where the exhaust steam enters, and lowest at the bottom. The condenser pressure will have a value corresponding to a temperature intermediate between these two temperatures. The temperature of the hot well as measured by a thermometer placed in the air-pump discharge will be lower than the lowest temperature in the condenser.

The ratio between the actual vacuum in a condenser and the vacuum corresponding to the temperature of condensation is sometimes called the Vacuum Efficiency. It is necessary to state which particular temperature is taken to be the temperature of condensation. Sometimes the temperature of the steam at the top is taken, and sometimes the temperature of the hot well. The lower the temperature selected the greater the vacuum with which to compare the actual vacuum. If the temperature of the steam at entry to the condenser is taken, then the vacuum corresponding to it may be less than the actual vacuum, and the efficiency is greater than unity. On the other hand, if the temperature of the hot well is taken, then the corresponding vacuum in general is greater than the actual vacuum, and the efficiency is less than unity. In both cases the result depends mainly upon the quantity of air in the condenser.

Thus, referring again to the curves, Fig. 90, if the condensation temperature is assumed to be 50° C., the temperature of the steam at entry to the condenser, the corresponding vacuum referred to a 30 in. barometer would be

$$\frac{(14{\cdot}7 - 1{\cdot}8)30}{14{\cdot}7} = 26{\cdot}3 \text{ inches}$$

If there were $\frac{1}{250}$ of a pound of air present per cubic foot of condenser volume, the actual vacuum would be

$$\frac{(14{\cdot}7 - 2{\cdot}8)30}{14{\cdot}7} = 23{\cdot}3 \text{ inches,}$$

assuming the temperature through the condenser to be constant. Then the vacuum efficiency would be $\frac{23{\cdot}3}{26{\cdot}3} = 0{\cdot}89$.

If there were no air present and the actual vacuum corresponded to a lower temperature of 45° C. the corresponding pressure would be 1·4 lbs. per square inch, equal to a vacuum of 27·2 ins., and the vacuum efficiency would be 1·17. This value would gradually fall as air accumulated either by leak or through an air pump working at too low a speed.

On the other hand, if the air-pump discharge temperature fell to 40° C., the corresponding pressure would be 1 lb. per square inch, and the vacuum 28·4 ins., and the vacuum efficiency would be $\frac{27 \cdot 2}{28} = 0 \cdot 97$, and this would fall as air accumulated.

Whichever temperature be selected from which to calculate the vacuum of comparison, the efficiency found may be regarded as a figure of merit in relation to the air pump and the tightness of the condenser.

87. **Wet and Dry Air Pumps.**—The ideal condition of working is to maintain the temperature of the water formed by the condensation of steam at the temperature of condensation. But some cooling below this is advantageous for the removal of air, because it increases the density of air and more air is therefore packed into the air pump per stroke. The ideal condition is approached in practice by using two pumps, one the "wet pump" to remove the condensed water, and the other the dry pump to remove the air, the air being cooled in the dry pump by an independent circulation of water. In this way the air is removed at a lower temperature than the condensed water, and the condensed water appears in the hot well at a higher temperature than it would have appeared had the cooling been carried out in one air pump cylinder.

In cases where a specially good vacuum is required, as when a condensing plant is working with a steam turbine, special attention must be given to the removal of the air, and it is quite usual in such cases to employ wet and dry pumps to secure a good vacuum.

The arrangement of wet and dry pumps adopted by G. & J. Weir, Limited, is shown in Fig. 91. The cross-section of the surface condenser is pear-shaped, and the lower part only is shown in the diagram. The inlet to the dry air pump is placed immediately below the bottom of the condenser at D. The air drawn through this pipe is cooled by injecting water in the space below the foot valves. It is then, by the bucket piston B, delivered through the head valves into the upper chamber of the dry air pump, from which it passes, through the pipe E, into the space below the head valves of the wet air pump A, through the spring-loaded valve F, which is loaded so that the pressure in the dry pump is about 5 lbs. per square inch when the pressure in the condenser is about 1 lb. per square inch. The difference of pressure between the upper chamber of the dry pump and the space between the foot valve is sufficient to cause the cooling water to flow down the pipe H and through the injection water cooler against the hydraulic resistances of the cooler.

The wet pump A then delivers the water, vapour and air passing through its foot valves, together with the air delivered by the pipe E, through its head valves and through the common air-pump discharge. When starting, the valve G is opened and a supply of injection

water is drawn in to the dry pump from the hot well of the wet pump. Normally G is closed. It will be seen that the injection water circulates through the cooler and dry air pump without chance of getting re-aerated by contact with the outside air.

The condensed steam is pumped at approximately the steam temperature in the condenser, whilst the air is pumped at the temperature determined by the cooling water flowing into the dry air pump. The two pumps are driven by one steam cylinder placed in line with one of the pumps, the second being driven from a rocking lever attached through links at one end to the common piston rod of the steam cylinder and pump cylinder, and at the other end to the piston rod of the second pump.

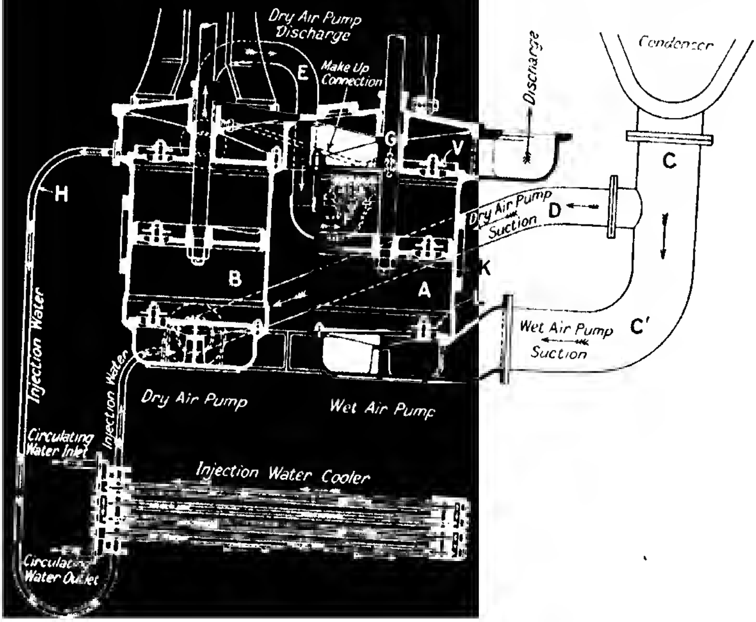


FIG. 91.—Wet and dry air pumps.

88. Types of Condensing Plants.—The surface condenser and air pump, or the jet condenser and air pump, are the types in most general use, the former universally for marine engines and the latter generally though not by any means universally for land installations. With each there is a pump to circulate the water through the tubes of the condenser or to inject water into the condenser. The air pump may be driven directly by the main engines, or by a side lever connected to one of the main engine cross-heads, but for use with high-speed engines, and even for many slow speed engines, the air pump is separately driven. The air pump and condenser then form

an independent unit, with which is sometimes included the circulating or injection pump, though it is more usual to have an independently driven circulating pump of the centrifugal type.

An independent condensing plant with a separately driven circulating pump or injection pump enables the condenser pressure to be easily controlled because the speed of the air pump and the speed of the circulating pump can be separately regulated.

There are several other types of condensing plant in use, each type having advantages in special circumstances.

The **Barometric Condenser** consists primarily of a water barometer. Imagine a pipe closed at the upper end standing vertically in a tank of water. Suppose the pipe to be 40 ft. high. The water will rise in the pipe until its upper level stands at about 34 ft. from the level of water in the tank. It is in fact a water barometer. The space in the pipe above the free surface will be filled with water vapour at a pressure corresponding to the temperature. If, for instance, the temperature is 15° C. the pressure will be about 0·25 lb. per square inch, corresponding to a vacuum of 0·45 in. of mercury.

If the exhaust pipe from a steam engine is carried up and opens into the space at the top of the barometer tube and a supply of cold water is sprayed into the space, the steam will be condensed as fast as it arrives, and the condensed steam and the injection water will together fall on top of the column of water, but there will be no accumulation, because a quantity will pass out of the pipe into the tank at the bottom equal to the quantity falling on to the column at the top. The water-level remains at a constant value depending upon the pressure of the external atmosphere. If no air were brought over with the steam, the pressure in the space would correspond with the pressure of steam at the temperature of the space.

One way to remove the air is to bring the steam in at the top through a nozzle and arrange the injection water so that it meets the combined jet of steam and air as it leaves the nozzle, the kinetic energy of the motion being sufficient to cause a current down the leg of the barometric tube of sufficient magnitude to carry away the air. The arrangement is, in fact, equivalent to an exhaust steam injector driving the air down the tube and into the hot well against the atmospheric pressure. Another way is to connect a "dry air pump" to the top of the tube and bring the exhaust pipe in at the side of the upper space, the injection water being sprayed in to meet the steam. Sometimes a cooler is added, so placed that the air withdrawn from the top passes around a bank of tubes through which the cold injection water is flowing on its way to the air pump, so that by cooling it its density is increased before it arrives at the air pump.

Barometric condensers are to be seen in connection with many electric power stations.

In the **ejector condenser** a stream of cold water is forced through a nozzle into a second nozzle which surrounds it and which forms the termination of the exhaust pipe. There is a nozzle facing the

exit of the second nozzle, but reversed. The combination forms an injector operated by water in which the energy of the water jet and the steam condensed with it form a combined jet of water and air with sufficient energy to flow out against the atmospheric pressure into the hot well. In some forms a succession of exhaust steam nozzles surround the water jet to secure a more efficient condensation of the steam.

In towns where space cannot as a rule be obtained for either cooling ponds or cooling towers **evaporative condensers** are sometimes used. The steam from the exhaust pipe is led through a bank of tubes placed either vertically or horizontally in the open air, generally on the roof of a building, and water trickles over the outside of these tubes falling into a tank placed below. The water is evaporated at the pressure corresponding to the atmospheric temperature, and in evaporating cools the bank of tubes and condenses the steam circulating within them. The condensed steam is pumped out together with the air and vapour by an air pump to which the tubes are connected. Cooling is effected chiefly by the evaporation of the water trickling over the tubes, but partly by radiation and convection. In practice about 1 lb. of water is evaporated per pound of steam condensed.

Generally, the working of an evaporative condenser cannot be controlled easily. The rate of evaporation depends so much on the state of the atmosphere that constant conditions of working cannot be maintained. For a description of the practical details of condensers of this class a paper, "Evaporative Condensers," by Mr. H. G. V. Oldham, published in the *Proceedings of the Institution of Mechanical Engineers*, 1899, should be consulted.

89. Rate of Condensation in a Surface Condenser.—The experiments[1] of Callendar and Nicolson, considered in Sections 66 and 67 above, show that the rate at which steam condenses on a metal surface depends chiefly upon the difference of temperature between the steam and the surface, and is nearly proportional to the temperature difference for such ranges of pressure as are common in practice. From the results of experiments made at 100° C. and also at 300° C., they found that heat is transferred from the steam to the metal surface at the rate of 0·74 lb.-cal. per square foot of surface per second per degree difference of temperature between the steam and the metal surface. The metal surface itself is kept at a constant temperature by the water which flows across the opposite side of it. Therefore, in a surface condenser, the rate of condensation of the steam on an element of the tube surface is conditioned by two temperature falls, namely, the fall from the steam temperature to the temperature of the tube, and the fall from the tube temperature to the temperature of the circulating water flowing through the tube.

[1] *Proc. Inst. C.E.*, vol. 131, p. 18. Report of the British Association for the Advancement of Science, 1897, p. 418.

The presence of air in the condenser greatly retards the rate of condensation, and Stanton[1] has shown that at the low pressure in a condenser the rate is also influenced by the density of the steam. Stanton concludes that there is a linear relation between the temperature of the steam and the temperature of the tube surface for a given rate of condensation and speed of air pump, of the form

$$T = mT_0 + c$$

where T is the temperature of the steam in the condenser,
T_0 is the temperature of the tubes,
m is a constant, and
c is a constant depending upon the speed of the air pump.

This result means that the rate of heat transmission from the steam to the metal surface is reduced as the density is reduced, because a greater temperature difference is required to maintain a given rate of condensation.

The rate of condensation found by Callendar and Nicolson may, however, be regarded as the ideal limiting rate in a space devoid of air since the rate has been determined both at 300° C. and 100° C. If this rate could be maintained in a condenser a temperature difference between the steam and the tube surface of 4° C. only would determine a rate of heat transmission of nearly 3 lb.-cals. per square foot per second, corresponding to the condensation of about 18 lbs. of steam per square foot of tube surface per hour.

In practice from 6 to 10 lbs. of steam are condensed per square foot of tube surface per hour according to the quantity of circulating water flowing through the tubes. There are few recorded results from which the actual temperature of the condenser tubes can be found, so that the temperature fall from the steam to the tubes in actual condensers working under practical conditions is hardly known.

The experiments of Dr. Stanton[1] indicate that in an actual condenser the rate of condensation is about 0·12 lb.-cals. per square foot per second per degree difference of temperature with a condenser pressure of 1·66 lbs. per square inch, and that this rate was exceeded when the condenser pressure was increased and was diminished when the pressure was reduced.

The temperature of the tubes was inferred in these experiments from general equations which are given in the paper.

As the quantity of circulating water is increased the temperature fall between the steam and the tube is increased, and the fall from the tube to the water temperature is increased. Stanton has shown experimentally that the heat transmission from the tube to the water is increased for a given quantity flowing by increasing the velocity of flow through the tubes. Thus small tubes and high velocity of flow tend to an increased rate of heat transmission, and therefore a more efficient use of the condensing water, inasmuch as

[1] "The Efficiency and Design of Surface Condensers," T. E. Stanton, *Proc. Inst. C.E.*, vol. 136, p. 321.

a smaller quantity of water is required to produce a given rate of condensation. This, of course, involves a corresponding increase in power required to drive the circulating pump.

Messrs. Allen & Co. have carried out an extended series of experiments at Bedford in connection with surface condensers, and the diagrams given in their paper summarize the results obtained in a practical way, and show the gradually increasing cost of a reduced condenser pressure.[1]

Reference should also be made to Prof. Weighton's experiments with a surface condenser.[2] Recent developments in rotary air pumps used with condensing plants are described in a paper by A. E. Leigh Scanes,[3] and a general review and discussion of condensing plants will be found in a paper by W. E. Storey,[4] reprinted in *Cassier's Magazine* for February, 1915.

[1] "Surface Condensing Plants and the Value of the Vacuum Produced," R. W. Allen, *Proc. Inst. C.E.*, vol. 161, p. 169.

[2] "The Efficiency of Surface Condensers," Prof. Weighton, *Trans. Inst. Naval Arch.*, 1906.

[3] "Modern Condensing Systems," *Proc. Inst. Mech. E.*, Feb., 1913.

[4] "Methods of Condensation," Birmingham Association of Engineers, April, 1914.

CHAPTER VI

Characteristic Energy Diagrams for Simple and Compound Locomotives

90. Data. Definition of x, the Independent Variable.—The preceding chapters indicate that the processes of energy transference and energy transformation in a steam engine are related to one another in exceedingly complex ways. The characteristic diagram has been devised by the author to show these relations in a comprehensive manner for any plant through the whole range of its power. The diagram is probably most useful in studying the complex problem of the locomotive, and therefore in describing the diagram reference will be made to the locomotive. In order to obtain data for plotting the curves which constitute the diagram for a particular locomotive, it is necessary to measure certain quantities experimentally during a period of two to three hours' steady running. It is quite impossible to obtain steady conditions of running on the road for anything like the time required. Changing direction, changing speed, changing gradients, changing weather conditions all operate to destroy the steady character of the working which must be maintained in order to obtain a reliable set of simultaneous values of the several quantities concerned.

The steady conditions required for accurate measurement can, however, be obtained on a locomotive testing plant of the kind established at Swindon, by the Great Western Railway, at Purdue University, or at the St. Louis Exhibition, in connection with the trials inaugurated by the Pennsylvania Railroad Company.

Many valuable data have been accumulated by the use of these plants. Details of a comprehensive set of experiments made on the engine "Schenectady" by Prof. Goss are given in "High Steam Pressure in Locomotive Service". Details of complete trials on eight different engines at the St. Louis Exhibition are given in "Locomotive Tests and Exhibits," published by the Pennsylvania Railroad Company in 1905. There are no published reports containing similar data for British locomotives.

The first thing to be done in order to show energy transference and transformation curves on a diagram is to choose one of the quantities concerned as an independent variable. The quantity chosen is the rate at which energy is transferred across the boiler heating surface from the heating circuit to the motive-power circuit,

or, more shortly, the flow of heat energy across the heating surface. This quantity is denoted by the symbol x in the sequel, and it is measured numerically by the number of lb.-calories transferred per minute.

The second thing to be done is to compute the values of the following five quantities from the experimental data for a series of values of x :—

(1) Rate at which heat energy is transformed into mechanical energy in the cylinder. This is measured by the indicated horse-power.
(2) Rate at which energy is given to the heating circuit, measured by the number of lb.-calories which would be obtained by perfect combustion from the average weight of coal shovelled into the fire-box per minute.
(3) Rate at which energy is actually produced by combustion in the heating circuit found from the experimental data in the way exemplified in Section 28, Chapter II.
(4) Rate at which heat energy is transformed into mechanical energy in the cylinder measured by the indicated horse-power expressed in thermal units.
(5) Pound-calories required per indicated horse-power minute, added for convenience of comparison with other trials.

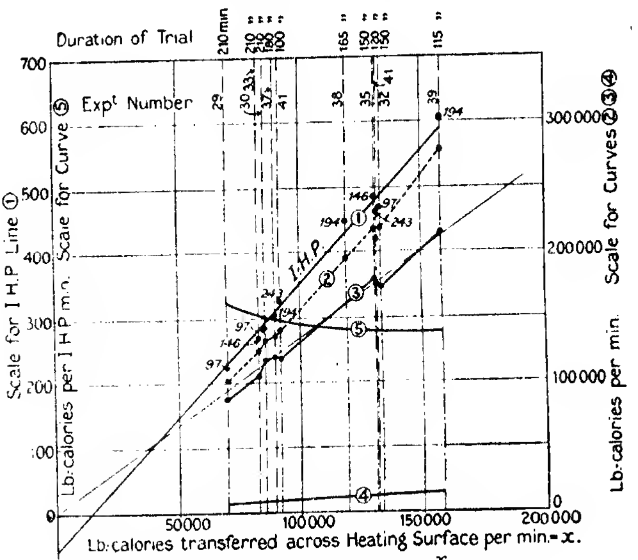
The rate at which heat energy is transformed into mechanical energy is here measured in two ways, namely, by the indicated horse-power, and by indicated horse-power expressed in lb.-calories per minute. A curve of indicated horse-power plotted against x is useful, as will be seen below. A curve of indicated horse-power, expressed in lb.-calories per minute, is comparable with the curves corresponding to 2, 3, and 4.

The third thing to be done is to plot each of the five quantities in one diagram on an x base, and draw smooth curves through the points so obtained.

It is to be understood that data corresponding to each value of x in the selected series must be obtained from observation taken during a trial where a condition of steady running is maintained during a period of two to three hours.

A condition of steady running is maintained when the crank axle revolves at constant speed, and when at the same time the flow of heat across the heating surface from the heating to the motive-power circuit is constant. The heat energy x, which flows across the heating surface, does not all flow into the cylinder. There are losses due to leaks and to the safety valves which are greater the greater the boiler pressure. A correction for these losses would not make any material difference in the energy diagram. It must be remembered, however, that the lb.-calories per I.H.P. minute, as plotted in curve 5, means the lb.-calories transferred across the heating surface per I.H.P. minute.

91. Examples of Characteristic Energy Diagrams.—Fig. 92 shows a characteristic energy diagram derived from data corresponding to a group of trials made at a boiler pressure of 200 lbs. per square inch by gauge, and recorded in "High Steam Pressures in Locomotive Service". The leading dimensions of the engine are given in Table 19. The small dots near the indicated horse-power line, marked No. 1 in the diagram, indicate the actual values of the measured indicated horse-powers, and the figures near the dots give



Curve (1) Straight line representing the I.H.P. Eq.is. $x/245 - 60$

" (2) Lb-calories corresponding to coal shovelled into firebox per min.

" (3) Lb " produced by combustion per min

" (4) Lb. " transformed into mechanical energy per min. $= x/104 - 1416$ I.H.P. in T.U.

" (5) Lb " per I.H.P. minute $= x/(x/245 - 60)$

Efficiency of transmission = 75 per cent

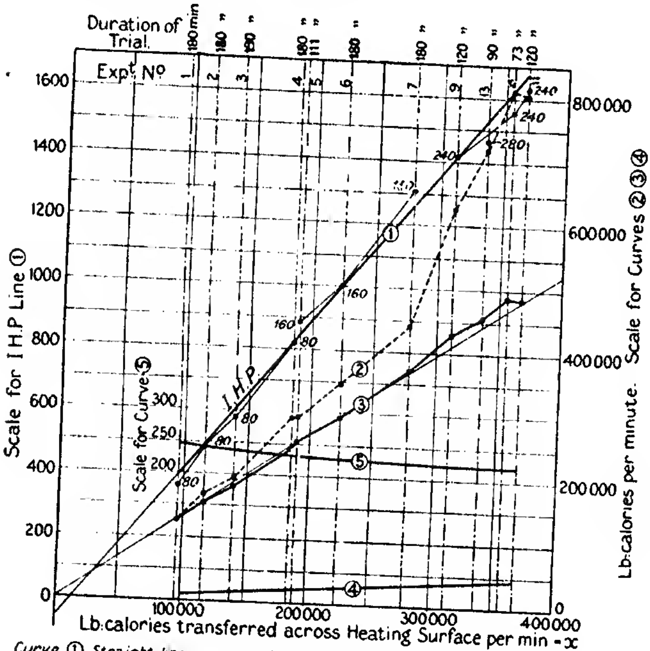
FIG. 92.—Characteristic energy diagram. Simple locomotive.

the corresponding speed of the crank axle in revolutions per minute.

Fig. 93 shows a diagram corresponding to the data given in the Pennsylvania Railroad Company's report above mentioned, obtained from trials on a compound locomotive, No. 600, built by the Baldwin Locomotive Company. The leading dimensions of this engine are given on page 327. The small dots and the figures near the indicated horse-power line give respectively the actual value of the measured horse-power and the corresponding speed of the crank axle in revolutions per minute.

In each figure a chain-dotted vertical line is drawn through the value of x, corresponding to the steady conditions of a trial, and the duration of a trial is written at the top of each of these lines.

It will be seen by counting the number of chain-dotted verticals in each diagram that the diagram Fig. 92 is constructed from ten separate trials, and that the diagram Fig. 93 is constructed from eleven separate trials.



Curve ① Straight line representing the I.H.P. Eq. is. I.H.P. = x/212 − 60
" ② Lb-calories corresponding to coal shovelled into firebox per min.
" ③ Lb " produced by combustion per min.
" ④ Lb " transformed into mechanical energy per min = x/9 − 1416
" ⑤ Lb " per I.H.P. minute = x/(x/212 − 60)
Efficiency of transmission = 75·5 per cent approx.

FIG. 93.—Characteristic energy diagram. Compound locomotive.

Similar diagrams, together with twelve others, were published in *Engineering*, August 19, 1910, the fourteen diagrams there given showing the characteristic energy diagram for each of the eight engines tested at the St. Louis Exhibition, and for each of the six groups of trials made by Prof. Goss on the engine "Schenectady," each group being at a different boiler pressure.

92. The Indicated Horse-power is a Linear Function of x.— A remarkable result is brought out in these diagrams. It will

TABLE 19A.—ENGINES TRIED ON THE TESTING PLANT OF THE PENNSYLVANIA RAILROAD COMPANY AT THE ST. LOUIS EXHIBITION, 1904.

No. of Engine at the Trials.	Owners.	Type.	Cylinders.	Boiler Pressure. By gauge.	* Total Heating Surface.	Grate Area.	Diameter of Driving-Wheels.	+ Total Weight.	+ Adhesion Weight.	Type of Valve.	Type of Valve-Gear.
				lbs. per sq. in.	sq. ft.	sq. ft.	in.	tons.	tons.		
100	Pennsylvania Railroad Company. No. 1499. Built at the Juniata shops of the company, 1904	2-8-0 simple	Outside, 22 in. by 28 in.	205	2482·26	49·2	56	86·70	77·21	Richardson balanced.	Stephenson.
200	Lake Shore and Michigan Southern Railway Company. No. 734. Built at Brooks Locomotive Works, 1900	2-8-0 simple	Outside, 21 in. by 30 in.	200	2541·22	33·76	63	80·80	72·80	Allen-Richardson.	Stephenson.
300	Michigan Central Railway Company. No. 585. Built by American Locomotive Company, 1902	2-8-0 cross-compound	Two outside, 23 in. by 32 in., 35 in. by 32 in.	210	2819·20	49·43	63	84·40	73·40	Low-pressure Allen-Richardson; high-pressure piston.	Stephenson.
400	Atchison, Topeka, and Santa Fé Railway System. No. 929. Built at the Baldwin Locomotive Works, 1903	2-10-2 tandem compound	Four outside, 19 in. by 32 in., 32 in. by 32 in.	225	4306·13	58·40	56·5	127·6	104·4	Piston.	Stephenson.
500	Pennsylvania Railroad Company. No. 2512 (De Glehn). Built by the Société Alsacienne de Constructions Mécaniques, Belfort, France, 1904	4-4-2 compound	Two high-pressure outside, 14·18 in.: two low-pressure inside, 23·7 in.; stroke, 25·25 in.	225	2656·48	33·39	80	73·2	39·23	High-pressure balanced slide; low-pressure slide.	Walschaerts.
600	Atchison, Topeka, and Santa Fé Railway System. No. 535. Vauclain compound, built at the Baldwin Locomotive Works, 1904	4-4-2 compound	Two high-pressure inside, 15 in.; two low-pressure outside, 25 in.: stroke, 26 in.	220	2902·05	48·36	79	90	44·20	Piston.	Stephenson.
700	Built by the Hannoversche Maschinenbau-Actien-Gesellschaft. No. 628, 1904	4-4-2 compound, fitted with superheater	Two high-pressure inside, 14·16 in.: two low-pressure outside, 22 in.; stroke, 23·62 in.	200	1753·15 including superheater	29·06	78	59·6	29·2	High-pressure piston: low-pressure Allen balanced.	Walschaerts, modified by Von Borries.
800	New York Central and Hudson River Railroad Company. No. 3000. Built by the American Locomotive Company, 1904	4-4-2 compound	Two high-pressure inside, 15½ in.: two low-pressure outside, 26 in.; stroke, 26 in.	220	3000	49·9	79	89·25	49·1	Piston.	Stephenson.

Engine "Schenectady," on which the experiments were made which are recorded in "High-Pressure Steam in Locomotive Service".

	Purdue University	4-4-0 simple	Outside, 16 in. by 24 in.	120, 160, 180, 200, 220, 240	1322	17·0	69·25	48·7	27·2		

* Based on the fire-side of tubes.

+ Weights taken with water at second gauge-cock and with normal fire.

be seen in Figs. 92 and 93 that the dots representing the indicated horse-power fall very nearly on a straight line in each case, notwithstanding the wide variations in speed and cut-off in the cylinder at which the different experiments were made. A reference to the fourteen diagrams published in *Engineering* will show that this result may be accepted as practically true for every one of the engines tested. The engines tested were different in type and in size, and were made by different makers, some in America and some in Europe, as will be seen from the particulars given in Table 19A, page 327.

The indicated horse-power line corresponding to the eight engines tried at the St. Louis Exhibition, and the six groups of trials on "Schenectady" above mentioned, are plotted in one diagram in Fig. 94 for the purpose of comparison. The limits of each set of trials are indicated in every case. The lines produced all pass below the origin. Therefore, it may be taken as generally true, at least to a first approximation, that if x is the rate at which heat energy is transferred across the heating surface of a locomotive boiler, then, up to a definite limit

$$\text{I.H.P.} = ax - b$$

where a and b are constants, and the engine is worked with the regulator at a fixed opening, and the power of the engine is regulated by varying the cut-off in the cylinder.

The graphic comparison in Fig. 94 establishes some interesting facts. It will be noticed that the indicated horse-power lines belonging to engines 700, 300, 800, 600, 500, fall together in a group having practically the same slope, and differing not greatly in actual vertical position.

The common characteristic of these engines is compound expansion of the steam, though the mechanical arrangement of the cylinders and the details of the design are different in each case.

The two simple engines tried at St. Louis, Nos. 100 and 200, also yield indicated horse-power lines which fall pretty well together and in the same region of the diagram as the group of six lines from the engine "Schenectady," each line of the group corresponding to a different boiler pressure.

It follows without doubt, considering the large number of accurate experiments upon which the lines of the diagram are based, that in *steady conditions* of working a compound locomotive is more economical than a simple engine, since a greater rate of working can be obtained for a given flow of heat across the heating surface.

Again, it may be noticed that the best engine in the compound group is fitted with a superheater.

With regard to the engine "Schenectady" the indicated horse-power lines at 180 and 200 lbs. per square inch boiler pressure are almost identical. There is a decided gain in steam economy, as the steam pressure is increased from 120 through 160 to 180 lbs. per square inch, but the diagram indicates that, so far as the engine "Schenectady" is concerned, there is little gain in steam economy after the pressure is increased above 180 lbs. per square inch by gauge.

93. Standard Indicated Horse-power Lines for Steady Running.—The indicated horse-power lines corresponding to engines 300 to 800 fall so close together, notwithstanding differences of boiler pressure, differences of size, and differences of the details of construction, that it would almost appear that a mean indicated horse-power line may be used generally as a standard line for compound locomotives of the types represented in the St. Louis trials. A similar remark applies to the simple engines. Measuring from the diagram, the constants for these mean standard lines have the values given in the following equations. Compound engines

$$\text{I.H P.} = \frac{x}{212} - 60$$

Simple engines, in which the boiler pressure is from 180 to 220 lbs. per square inch by guage,

$$\text{I.H.P.} = \frac{x}{250} - 60$$

The symbol x here stands for the number of lb.-calories transferred across the boiler heating surface from the heating to the steam circuit per minute. There are not sufficient data available to allow standard lines to be deduced for compound engines fitted with superheaters or for simple engines fitted with superheaters.

94. General Discussion of Curves 2, 3, 4, 5, in the Energy Diagram.—The ordinates of curve No. 2 represent the rate at which energy is given to the heating circuit as measured by the number of thermal units which would be obtained by the perfect combustion of the average quantity of coal shovelled into the fire-box per minute. The ordinates to curve No. 3 show the rate at which heat energy is actually produced in the heating circuit by the combustion of fuel. The calculation of this quantity for a given value of x is a difficult matter. The calculation has been exemplified in Chapter II., Section 28. It will be seen in each diagram that as x increases curve No. 2 slopes upwards at a greater rate than curve No. 3, showing that energy must be supplied at a continually increasing rate as the flow is increased. This is a consequence of the condition that as the power required from the engine increases, the corresponding increased amount of fuel must be burnt upon a grate of constant size, therefore the velocity of the air through the fuel bed increases and there is an increasing loss of unburnt small fuel, carried through the tubes and away out of the chimney top.

In each diagram curve No. 3 approximates to a straight line passing through the origin, the thin line shown being drawn for the purpose of comparison. If the curve were actually a straight line through the origin, the efficiency of transmission, measured by ratio of the heat transferred across the heating surface per minute to the heat produced by combustion per minute, would be a constant. That the efficiency differs surprisingly little from a constant value is

shown by the close approximation of the curves marked 3 with the straight line passing through the origin. It will be seen that the efficiencies of transmission in Figs. 92 and 93 are 75 and 76½ per cent. respectively. The efficiencies of transmission shown on the fourteen energy diagrams published in *Engineering* vary between 75 and 77 per cent. This shows that not only is the efficiency of transmission approximately constant for a particular engine, but that it is approximately constant, and in the neighbourhood of 77 per cent., for all locomotive engines of the type used in the experiments, when the boilers are clean.

The vertical difference between the curves 2 and 3 indicate the furnace loss. This increases rapidly as x increases, and this increase is a common feature in all the 14 diagrams published in *Engineering*, Aug. 19, 1910.

The curve begins, as a rule, to turn upwards when the rate of combustion is in the region of 100 lbs. of coal burnt per square foot of grate per hour. The turning-point cannot, however, be generalized, because it will depend so much upon the kind of coal used. The coal used in the St. Louis trials was of good quality, with a calorific value of about 8000 lb.-cals. per pound (14,400 B.T.U. per lb.). The average calorific value of the coal used in Prof. Goss' experiments on "Schenectady" was slightly lower.

The ordinates of curve No. 4 are reduced from the corresponding ordinates of curve No. 1 by the multiplying factor $\frac{33000}{1400}$, which brings "horse-power" to the equivalent "lb.-calories per minute". Or more directly, using the general expression for the I.H.P., curve No. 4 may be plotted from

$$y = \{ax - b\} \frac{33000}{1400}$$

This reduces to

$$y = \frac{x}{9} - 1416 \text{ for curve 4, Fig. 93,}$$

and to

$$y = \frac{x}{10{\cdot}4} - 1416, \text{ for curve 4, Fig. 92.}$$

Reducing the standard equations of Section 93, page 329, in this way, there results the convenient expressions,

$$\left.\begin{array}{ll}\text{for compound engines} \quad . & \dfrac{x}{9} - 1400 = \\[2ex] \left.\begin{array}{l}\text{for simple engines with} \\ \quad\text{180 lbs. sq. in. boiler} \\ \quad\text{pressure by gauge} \quad .\end{array}\right\} & \dfrac{x}{10{\cdot}6} - 1400 = \end{array}\right\} \begin{array}{l}\text{lb.-calories transformed} \\ \text{into mechanical energy} \\ \text{per minute in the} \\ \text{cylinders of a loco-} \\ \text{motive.}\end{array}$$

If the line representing these equations passed through the origin, the ratio of transformation would be constant, but passing, as it does, below the origin, the ratio increases as x increases. If

$x = 100000$ the energy transformed is approximately at the rate of 9711 lb.-cals. per minute for the standard compound engine, and 8220 for the standard simple engine, the percentage transformation being respectively 9·7 and 8·2 per cent. of the flow of heat across the heating surface. If $x = 200000$, the compound engine transforms 20822 lb.-cals. per minute, and the simple engine 17850, being respectively 10·4 per cent. and 8·9 per cent. of the heat-flow. Hence, between the great range of furnace action represented by a transference of 100000 and 200000 lb.-cals. per minute, the range of transformation varies by less than 1 per cent. in usual types of locomotive, thus showing how nearly constant the efficiency of transformation, that is, the absolute thermal efficiency, of a locomotive remains through all the conditions of working.

The ordinates to curve 5 are found by dividing x by the corresponding horse-power. Or the curve may be plotted from an equation of the form

$$\text{Lb.-cals. per I.H.P. minute} = \frac{x}{ax - b}$$

Curve 5, Fig. 93, and Curve 5, Fig. 92, are plotted from an equation of this form. The constants used were $b = 60$ and $a = 212$ for the simple engine, and 250 for the compound engine.

These constants may be used for general calculations, with fair approximation giving

$$\left.\begin{array}{ll} \text{for compound locomotives . . . .} & = \dfrac{x}{\frac{x}{212} - 60} \\[2ex] \text{for simple locomotives, with at least 180 lbs. per sq. in. boiler pressure by gauge . . . .} & = \dfrac{x}{\frac{x}{250} - 60} \end{array}\right\} \text{lb.-calories per I.H.P. minute.}$$

The following figures of performance are calculated from these equations :—

Compound engines, when $x = 100000$; lb.-cals. per I.H.P. min. = 243.
Simple engines, when $x = 100000$; lb.-cals. per I.H.P. min. = 294.
Compound engines, when $x = 200000$; lb.-cals. per I.H.P. min. = 226.
Simple engines, when $x = 200000$; lb.-cals. per I.H.P. min. = 270.

95. Maximum Value of x and the Intensity of Transmission, and the rate at which Energy can be introduced into the Cylinders.—The size of the boiler limits the total quantity of energy transmitted across the heating surface per minute. The limit of the quantity of heat transmitted, corresponding to a boiler of given size, cannot be strictly defined. In practice the average maximum intensity of transmission is about 100 lb.-cals. per minute per square foot of heating surface, with locomotive boilers of the usual proportions. In the St. Louis experiments the maximum intensity of transmission ranged from 72 to 133 lb.-cals. per minute, and in

the experiments recorded in "High Steam Pressure in Locomotive Service," from 100 to 128 lb.-cals. per minute.

The maximum rate of heat transmission recorded in "Trials of an Express Locomotive," is 132,500 lb.-cals. per minute across a total heating surface of 1358·6 sq. ft., giving an intensity of transmission of 98 lb.-cals. per square foot per minute. The intensity of transmission recorded for the three trials made by Kennedy and Donkin on a Great Eastern locomotive, tested stationary and afterwards in service on the road, varied from 50 to 56 lb.-cals. per minute per square foot of heating surface, which was 859 sq. ft. The engine was not, however, working at its maximum power in any of the experiments, because the rate of fuel combustion was only varied from 28 to 35 lbs. of coal per square foot of grate per hour. For design purposes it may be considered that the average maximum rate of transmission in a locomotive boiler should be limited to 100 lb.-cals. per minute per square foot of total heating surface.

The maximum intensity of transmission is very much greater than the average intensity. The heating surface in the fire-box exposed to the direct radiation of the incandescent carbon and flame transmits heat at a far higher rate than heating surface at the smoke-box end of the tubes. In the illustrative example worked out on page 126, the intensity of transmission by radiation alone was in the limit 1000 lb.-cals. per square foot of fire-box heating surface per minute. If in practice even half of this rate is obtained, there is still to be added the transmission in the fire-box due to conduction and convection. It is probable, therefore, that the intensity of transmission is in the region of 1000 lb.-cals. for the fire-box heating surface exposed to the fire.

Several experiments have been made from time to time with the object of measuring directly the intensity of transmission at various parts of the heating surface. Messrs. Dewrance and Wood[1] in 1842 divided the water space between the tube plates of a locomotive boiler into six equal compartments by means of vertical partitions, and found that though the intensity of transmission in the first compartment was equal to that in the fire-box, the intensity in the second compartment was only one-third of that in the first compartment, and in the remaining compartments it was practically nothing.

Graham[2] in 1860 divided a cylindrical boiler set in brickwork into four compartments by vertical transverse plates, each compartment being open to the atmosphere, and found that 67 per cent. of the total heat transmitted took place in the first compartment, 18 per cent. in the second compartment, 8 per cent. in the third compartment, and 5½ per cent. in the fourth compartment.

Couche[3] in 1876 made some experiments on a locomotive boiler

[1] "Experiments on a sub-divided Locomotive boiler, Manchester and Liverpool Railway," *Trans. Inst. Naval Arch.*, vol. 3, p. 122.

[2] "Experiments on the evaporation of Divided Boilers," *Proc. Manchester Lit. Phil. Soc.*, vol. 15, p. 8.

[3] "Experiments on the Chemins de Fer du Nord locomotives," *Chemins de Fer (voie et meteriel roulant)*, vol. 3, p. 32.

divided into five compartments by means of vertical tube plates, and the experiments show that the intensity of transmission per square foot of surface was from two to three times greater from the fire-box section than from the first tube section according to the draft, which ranged from $\frac{3}{4}$ in. to $3\frac{1}{2}$ ins. vacuum in the smoke-box.

All these experiments tend to confirm the important part played by radiation in the transmission of heat. As the rate of combustion is increased the temperature in the fire-box rises, and the consequently larger proportion of heat transmitted by radiation counterbalances the loss due to the furnace gases leaving at a higher temperature.

The ratio of tube surface to fire-box surface in locomotives of the usual type varies between wide limits according to the type of construction and the nature of the fuel to be burnt. Successful engines have been built for burning good coal where the ratio varies from 8 to 18, a good average value for modern practice being 12.

The straight line representing the indicated horse-power in the energy diagram cannot be continued indefinitely with any truth, as x increases indefinitely. A value of x is reached at which the line begins to fall towards x, so that equal increments of x do not correspond to equal increments of power. The value of x at which this occurs is not very definite. The steam carrying the heat energy to the cylinder has to make its way through the narrow entry offered by the steam port, an entry which is gradually opened and again gradually closed by the slide valve. The weight of steam which can be introduced into the cylinder per unit of time depends upon the velocity of flow through the steam port, the area of the port, and the density of the steam. The velocity and the area are continually changing so that it is difficult to compute the actual velocity, or even the average velocity of flow through the ports. An examination of the results of experiments shows that about 0·75 lb. of steam may be introduced per second per cubic foot of total cylinder volume without causing appreciable deviation from the straight line I.H.P. law, providing the cut-off is not greater than 0·3 of the stroke. If the cut-off is between 0·3 and 0·5 of the stroke, then the weight should not exceed 0·5 lb. per cubic foot per second. Of this quantity it may be estimated that 30 per cent. is lost in initial condensation and leakage so that the weight appearing as steam and accounted for by the indicator diagram would correspond to about 0·5 lb. of steam per second per cubic foot of cylinder volume up to 30 per cent. cut-off, and about 0·35 lb. per cubic foot per second for a cut-off between 30 and 50 per cent. of the stroke.

For compound engines it is probable that these rates should not be exceeded; in fact, considering that the steam has to be transferred from the high-pressure to the low-pressure cylinder through restricted passage ways and steam ports in addition to the introduction into the high-pressure cylinder, it would be better to work with lower values in relation to the volume of the high-pressure cylinders.

CHAPTER VII

Dynamics of the Steam Engine

96. Introduction.—The power developed and the resistance overcome are matched together at the crank shaft of a reciprocating engine, or the motor shaft of a steam turbine. The steam, whether through the agency of pistons and connecting rods or through the agency of the blades of a steam turbine, makes an effort to turn the shaft against the resistance which the engine is designed to overcome. The effort to turn the shaft is measured by a **turning couple.** This turning couple is also referred to as the **crank effort** or the **torque** acting on the shaft.

Whatever be the nature of the resistance against which the shaft is turned, whether it be the resistance of a screw propeller, or the resistance of the machinery of a mill coupled to the engine shaft by ropes, belts, or gearing, or the resistance to the rotation of an armature in a magnetic field, or the resistance to the motion of a train, the resistance at any instant may be reduced to a **resisting couple** acting at the shaft to oppose the action of the turning couple.

When the average speed of rotation of the shaft is constant from revolution to revolution, the average value of the turning couple is equal to the average value of the resisting couple; but if their actual values be compared at a particular instant of a revolution it will be found that in general they are unequal.

Whenever inequality exists a couple equal to the difference accelerates or retards the speed of the crank shaft according as the turning couple is greater or less than the resisting couple. The acceleration of the speed of the crank shaft produced by the accelerating couple depends upon the mass which must of necessity be accelerated when the speed of the shaft itself is accelerated. If L is the turning couple, R the resisting couple, and A the accelerating couple, then at every instant

$$L = R + A$$

When the average speed is constant, the turning couple is in general alternately greater and less than the resisting couple, in consequence of which the accelerating couple A is alternately positive and negative, and consequently acts to increase the speed of the shaft and then to diminish it, thus causing a cyclical variation of speed or an oscillation of speed about the mean speed. The amplitude of this oscillation is controlled by a flywheel.

When through some sudden change of load an inequality is produced between the average turning couple and the average resisting couple, the mean speed will change and will continue to change until by some means or other an exact equality between the average values is re-established. A governor is a mechanism designed to regulate the steam supply so that this equality of average effort and resistance is re-established automatically with as small a change of mean speed as possible. As will be seen below, a governor cannot regulate the steam supply to meet a new condition of loading without a permanent change in the mean speed, but a well-designed governor keeps the variation of mean speed within narrow limits notwithstanding that the load varies between wide limits.

The general course of the investigation is first to consider the relation between the turning couple and the steam pressure on the indicator diagrams, in the course of which it will be necessary to consider at some length the effect of the inertia of the moving parts. When the curves of crank effort and of resistance are drawn, the couple causing angular acceleration of the crank shaft can be immediately found, and problems connected with the cyclical variation of speed and the design of a flywheel to keep this variation within defined limits follow immediately.

Finally, problems relating to governor design and speed regulation are considered.

There are other dynamical problems connected with the steady running of a steam engine, but these are reserved for a chapter specially devoted to the subject of the Balancing of Engines.

97. Relation between the Pressure in the Cylinder and the Turning Couple on the Crank Shaft.—The relation between a pressure in the cylinder and the corresponding turning couple on the crank shaft is obtained from the principle of the conservation of energy, which applied to this problem, may be stated in the form:—

$$\left.\begin{array}{l}\text{The rate at which}\\ \text{work is done on}\\ \text{the piston}\end{array}\right\} = \left\{\begin{array}{l}\text{the rate at which}\\ \text{energy is stored in}\\ \text{the moving parts}\\ \text{of the mechanism}\end{array}\right\} + \left\{\begin{array}{l}\text{the rate at which}\\ \text{work is done on}\\ \text{the crank shaft.}\end{array}\right.$$

Any one of these rates may be negative.

The moving parts of the mechanism comprise the piston and all parts connected to and moving with it, the connecting rod, the crank axle, and all connected to and moving with it. An expression giving accurately the rate at which energy is stored in these parts as the speed of rotation changes would be exceedingly complex. An expression near enough for all practical purposes is obtained by making the assumption that the crank axle rotates with uniform angular velocity, and that the connecting rod, whose motion is even then complex, is replaced by two masses, one attached to and moving with the crank pin, the other attached to and moving with the cross-head.

The part of the mass of the rod to be added to the reciprocating

mass is found in this way. Weigh the rod and let W be its weight. Find the position of the mass centre of the rod by balancing it on a knife edge, or by any other suitable method, and let x be the distance of the mass centre of the rod from the centre of the big end, and let l be the length of the rod centre to centre. Then the part to be added to the reciprocating mass is

$$\frac{Wx}{l} \text{ lbs.}$$

and to the crank-pin mass $\frac{W(l - x)}{l}$ lbs.

It will be seen that these amounts correspond with the reactions produced on supports placed respectively under the small end and the big end centres of the rod, freely supporting it as a beam.

The reciprocating mass of a line of parts of a steam engine consists of a piston, a piston rod, a cross-head, and a part of the connecting rod $\frac{Wx}{l}$. Let M be the mass of the reciprocating parts, called briefly the piston mass. M is equal to $\frac{W}{g}$ when W is in lbs.-wt. Then if a is the acceleration of the piston in feet per second per second, Ma is the accelerating force in lbs.-wt. If v be the speed in feet per second at which the piston is moving, then the rate at which energy is stored is Mav ft.-lbs. per second.

Again, let A be the area of the cylinder, p the steam pressure acting in the cylinder at the instant when the piston velocity is v, and p_1 the value of the corresponding back pressure, then, neglecting the area of the piston rod, the rate at which work is done on the piston is

$$A(p - p_1)v \text{ ft.-lbs. per second}$$

Finally, let L be the corresponding couple acting on the crank shaft, and let ω be the angular velocity of the shaft, then the rate at which work is done against the crank shaft resistance is $L\omega$ ft.-lbs. per second. The energy equation is then

$$A(p - p_1)v = Mav + L\omega \quad . \quad . \quad . \quad . \quad . \quad (1)$$

that is

$$A(p - p_1) - Ma = \frac{LV}{rv} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

since ω, the angular velocity of the crank, is equal to the velocity of the crank pin V divided by the crank radius r. This is an instantaneous equation, and with the assumption of uniform velocity, and the further assumption of an indicator diagram repeating without sensible change, the variable quantities in the equation pass periodically through the same values as the crank passes through the same angular position. For a given value of the crank angle, p and p_1 are found from the indicator cards, a is calculated from the speed of the crank shaft and the dimensions of the gear, and the ratio V to v is calculated from the dimensions of the gear alone.

98. The Velocity and the Acceleration of the Piston.—The velocity and the acceleration of the piston may be found analytically or graphically.

Exact analytical expressions for the velocity and for the acceleration are complex and are rarely used, but simple approximate expressions are easily found. The graphical methods are theoretically exact, but the accuracy of the result is limited by the accuracy with which the diagrams can be drawn and scaled.

1. *The Analytical Method.*

In principle this method is simple. An expression is found giving the displacement x of the piston from some fixed point, in terms of the time, and this expression differentiated with regard to the time gives the piston velocity, and differentiated twice with regard to the time gives the acceleration of the piston.

Let Fig. 95 represent the configuration of the crank and connecting-rod mechanism when the crank angle has the value θ. Let

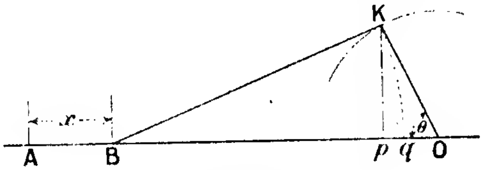


FIG. 95.—Crank and connecting rod.

$OK = r$, be the crank radius; and let $BK = l$, be the length of the connecting rod. Let A be the dead point the more remote from the crank shaft O. It is convenient to use this point as the datum from which to measure the displacement x of the piston.

With centre B and radius BK draw the arc Kq.

Then

$$x = AO - BO$$
$$= AO - (Bq + Op - pq) \quad . \quad . \quad . \quad . \quad (1)$$

But by the well-known property of chords intersecting in a circle, $(2BK - pq)pq = pK^2$, and since pq is small with regard to BK we may neglect $(pq)^2$, and thus,

$$pq = \frac{pK^2}{2BK} = \frac{r^2 \sin^2 \theta}{2l} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also since it is assumed that the crank revolves with uniform angular velocity ω,

$$\theta = \omega t$$

where t is the time in seconds from the instant when the crank position corresponds with $x = 0$.

Substituting in (1), $(r + l)$ for AO, $r \cos \omega t$ for Op, l for Bq and the value of pq given in (2), it becomes,

$$x = r + l - \left(r \cos \omega t + l - \frac{r^2 \sin^2 \omega t}{2l}\right)$$

that is

$$x = r - r \cos \omega t + \frac{r^2 \sin^2 \omega t}{2l} \quad . \quad . \quad . \quad . \quad (3)$$

The velocity of the piston, namely, $\frac{dx}{dt}$ is then

$$v = \frac{dx}{dt} = \omega r\left(\sin \theta + \frac{r}{2l} \sin 2\theta\right) \quad . \quad . \quad . \quad . \quad (4)$$

and the acceleration is

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \omega^2 r\left(\cos \theta + \frac{r}{l} \cos 2\theta\right) \quad . \quad . \quad . \quad (5)$$

These are approximate expressions because of the approximation introduced by equation (2) in respect of the magnitude of pq.

It is easy to see from Fig. 95 that the exact value of x is,

$$r + l - r \cos \theta - \sqrt{l^2 - r^2 \sin^2 \theta} \quad . \quad . \quad . \quad (6)$$

The value of the velocity v is then,

$$\frac{dx}{dt} = \omega r\left\{\sin \theta + \frac{r \sin 2\theta}{2\sqrt{l^2 - r^2 \sin^2 \theta}}\right\} \quad . \quad . \quad . \quad (7)$$

The value of the acceleration is,

$$\frac{d^2x}{dt^2} = \omega^2 r\left\{\cos \theta + \frac{rl^2 \cos 2\theta + r^3 \sin^4 \theta}{(l^2 - r^2 \sin^2 \theta)^{\frac{3}{2}}}\right\} \quad . \quad . \quad (8)$$

By means of expressions (4) and (5) both the velocity and the acceleration of the piston corresponding to any assigned crank angle θ can be calculated when the constant angular velocity ω of the crank and the respective lengths of the crank and connecting rod are given.

Since the velocity V of the crank pin is ωr it will be seen that the approximate velocity ratio $\frac{v}{V}$ between the piston and the crank pin is from (4)

$$\frac{v}{V} = \left(\sin \theta + \frac{r}{2l} \sin 2\theta\right) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

2. *Graphical Methods.*

PROP. 1.—*To find the Velocity of the Piston when the Velocity of the Crank Pin is given.*

Let the mechanism diagram in Fig. 96 represent the configuration of the crank and connecting-rod mechanism corresponding to any angle θ.

Take any origin V and draw Vk to represent the velocity of the crank pin K. Its direction is at right angles to the crank OK and its length represents to scale the given velocity. Draw Vb to represent the direction of the velocity of the piston. Through k draw a line at right angles to the connecting-rod cutting Vb in b. Then Vb represents the magnitude of the velocity of the piston to the scale on which Vk represents the velocity of the crank pin. Also kb represents the velocity of the point B relative to K.

If Vk is made equal to unity then the length Vb represents the velocity ratio between the piston and the crank pin, namely, $\frac{v}{V}$.

Proof.—The velocity of the point B is the resultant of the velocity of the point K and the velocity of B relatively to K. The only velocity that B can have relatively to K is in a direction at right angles to BK, since with K fixed the link BK is only free to turn about K as a centre. The magnitude of the relative velocity of B is fixed by the fact that the direction of the resultant Vb is always along the line of stroke OB. Therefore the construction fixes the magnitude of Vb also.

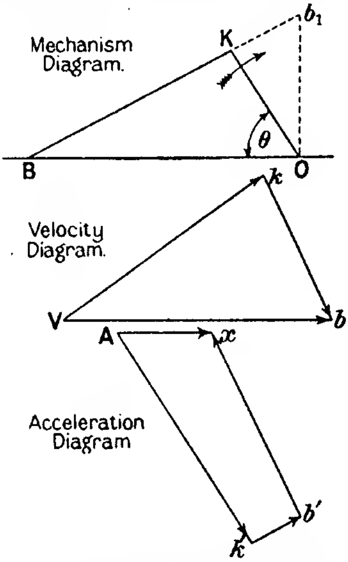


FIG. 96.—Mechanism; velocity diagram; acceleration diagram.

This velocity diagram may be combined with the diagram of the mechanism itself, although when this combination is made, the freedom to select the scale of the velocity diagram is sacrificed. Thus move the velocity diagram so that V lies over O in the diagram of the mechanism and Vk coincides with the crank direction OK, and imagine the scale of the diagram to be so adjusted that the point k coincides with the point K. Then kb will lie along the direction of the connecting rod, produced if necessary, and the velocity of the piston, Vb, will appear as a line through O at right angles to the line of stroke, its length being determined by the point b_1, where its direction intersects the direction of the connecting rod.

The scale of the velocity diagram is now fixed by the relation that the crank pin velocity ωr is represented by a line which also represents the radius OK to scale. Therefore if the piston velocity Ob_1 is measured with the scale used to draw the mechanism, the numerical value of the velocity is the length so found multiplied by the angular velocity ω of the crank.

Corollary.—In this diagram it will be evident that the velocity ratio between the piston and the crank pin is given by the ratio $Ob_1 : OK$.

PROP. 2.—*To Find the Acceleration of the Piston when the Angular Velocity of the Crank is uniform.*

The acceleration of the piston is readily found from the acceleration diagram of the mechanism corresponding to the configuration determined by a stated crank angle.

The drawing of the diagram depends upon a kinematic principle regarding motion in a plane which may be stated thus :—

When a link KB, Fig. 96, moves in its own plane and the acceleration of any point K in it is given in magnitude and direction, the acceleration of any other point B is the vector sum of the acceleration of K, the radial acceleration of B about K as axis, and the tangential acceleration of B about K as axis.

With the assumption that the crank moves uniformly the acceleration of the point K is ω^2KO in the direction KO. The magnitude of the radial acceleration of B about K is $\frac{u^2}{BK}$, where u stands for the velocity of B relative to K. This velocity is found from the velocity diagram, and in Fig. 96 it is represented by kb. Scaling this off $\frac{u^2}{BK}$ can be calculated. The tangential acceleration of B is known only in direction. The whole acceleration of B is in the direction of the line of stroke. With these data the acceleration diagram can be drawn.

Choose any origin A. Set out Ak' parallel to the crank direction to represent ω^2KO to any convenient scale. Set out $k'b'$ in the direction from B to K and equal to $\frac{u^2}{BK}$ calculated by aid of the velocity diagram. Draw $b'x$ at right angles to the rod KB to represent the direction of the tangential acceleration of B about K, and through A draw Ax parallel to the line of stroke to meet $b'x$ in x. Then Ax is the acceleration of the point B, and this is the acceleration of the piston.

For example, in Fig. 96 the rod BK is 1·85 times the crank KO, which is, say 1 foot in length. Let it be assumed that the crank revolves uniformly 300 times per minute. Its angular velocity is 31·4 radians per second, and this squared is 986. Then the velocity of K is 31·4 ft. per second, and the radial acceleration of K is ω^2KO = 986 ft. per second per second in the direction KO.

In the velocity diagram Vk represents 31·4 ft. per second, and scaling, kb will be found to represent 20·1 ft. per second = u. So that $\frac{u^2}{BK}$ = 218 ft. per second per second.

Starting from any origin A draw Ak' parallel to KO to represent 986; $k'b'$ parallel to the rod BK to represent 218. Ax, parallel to the stroke, meets $b'x$ at right angles to the rod in x and thus

determines Ax, the magnitude of the acceleration of the point B. This scales 380 ft. per second per second.

This method of drawing a velocity diagram and an acceleration diagram for the connecting-rod mechanism has been explained because of its application to mechanisms in general, and for this reason is well worth study. A more extended explanation, together with diagrams corresponding to such a complicated mechanism as a Joy Valve Gear, are given in the author's "Valves and Valve Gear Mechanisms," published by E. Arnold.

There are several constructions for the particular problem of finding the acceleration of the piston in the connecting-rod mechanism, the general feature of which is that the velocity and acceleration

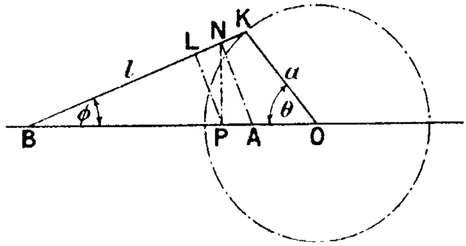


FIG. 97.—Bennett's construction.

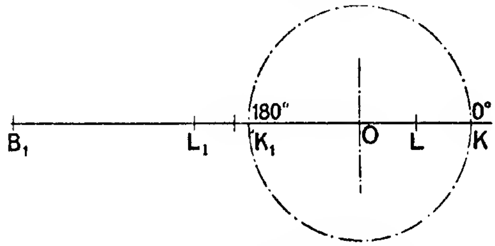


FIG. 98.—Form of diagram at dead points.

diagrams are combined with the mechanism diagram. Freedom to select the scale of the acceleration diagram is sacrificed by this combination, but on the other hand simple constructions are obtained. The best known of these constructions is perhaps that of Klein, but the simplest and most convenient from a drawing office point of view is probably the following one, which was invented by Mr. G. T. Bennett, of Emmanuel College, Cambridge, and was communicated to the author in August, 1902.[1]

Let KO, Fig. 97, be the crank and KB the connecting rod. On the connecting rod take a point L, such that

$$KL \times KB = KO^2$$

[1] First published in the second edition of "The Balancing of Engines," by Prof. W. E. Dalby. London: E. Arnold, 1906.

The point L may be fixed by dropping a perpendicular from O on to the rod KB when the crank is at right angles to the line of stroke.

Then, the crank standing at any angle with the line of stroke, draw LP at right angles to the connecting rod, PN at right angles to the line of stroke, and, finally, NA at right angles to the rod. AO is the acceleration of the point B to the scale on which KO represents the acceleration of the crank pin K.

It will be noticed that AOKN is really an acceleration diagram, like the acceleration diagram in Fig. 96, in which KO represents the acceleration of K, NK the radial acceleration of B about K, NA the tangential acceleration of B about K, and AO the acceleration of the piston. The diagram can be drawn in this position on the mechanism when the point N is fixed, and since NK $= \frac{u^2}{\text{BK}}$ its magnitude can easily be found from the velocity diagram.

At the dead points, when the connecting rod and therefore the point L is in the line of stroke, the distance between the points L and O gives the acceleration. Thus, in Fig. 98, LO and L_1O respectively represent the accelerations when the piston is passing through the dead points.

Proof.—Let θ be any crank angle Fig. 97, and let ϕ be the corresponding angle between the connecting rod and the line of stroke. Also let a be the crank radius, and l the length of the connecting rod Then—

$$l \sin \phi = a \sin \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Differentiating this with regard to the time t, and writing $\dot{\phi}$ for $\frac{d\phi}{dt}$, $\dot{\theta}$ for $\frac{d\theta}{dt}$, and assuming $\dot{\theta}$ constant and equal to ω—

$$l\dot{\phi} \cos \phi = a\omega \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eliminating θ from equations (1) and (2)—

$$\dot{\phi}^2 = \omega^2 - \frac{l^2 - a^2}{l^2} \cdot \omega^2 \cdot \sec.^2\phi \quad . \quad . \quad . \quad . \quad (3)$$

Differentiating this again with regard to the time—

$$\ddot{\phi} = - \frac{l^2 - a^2}{l^2} \cdot \omega^2 \cdot \frac{\sin \phi}{\cos^3 \phi} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Again, the position of the cross-head B is given by

$$x = a \cos \theta + l \cos \phi \quad . \quad . \quad . \quad . \quad : \quad (5)$$

Differentiating this with regard to the time, the velocity of the cross-head towards O is

$$- \dot{x} = a\omega \sin \theta + l\dot{\phi} \sin \phi \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Differentiating this velocity with regard to the time, the acceleration is

$$- \ddot{x} = a\omega^2 \cos \theta + l\ddot{\phi} \sin \phi + l\dot{\phi}^2 \cos \phi \quad . \quad . \quad (7)$$

Substituting the values of $\dot{\phi}^2$ and $\ddot{\phi}$ from (3) and (4)

$$-\ddot{x} = \omega^2\left(a\cos\theta + l\cos\phi - \frac{l^2 - a^2}{l}\sec^3\phi\right) \quad . \quad . \quad (8)$$

In the construction given above—

$$OB = a\cos\theta + l\cos\phi$$

and from the relation KB . KL = a^2, that is, $l(l - BL) = a^2$

$$BL = \frac{l^2 - a^2}{l}$$

Therefore

$$BP = \frac{l^2 - a^2}{l}\sec\phi$$

$$BN = \frac{l^2 - a^2}{l}\sec^2\phi$$

and finally

$$BA = \frac{l^2 - a^2}{l}\sec^3\phi$$

Therefore OB − BA = AO is seen by expression (8) to represent the acceleration of the point B to the scale on which KO represents the acceleration of the crank pin K.

99. **The Calculation of the Turning Couple L.**—EXAMPLE. The crank of an engine revolves uniformly 300 times per minute, and the driving and the back pressures in the cylinder are 180 and 20 lbs. per square inch respectively at the instant when the crank passes through the angle $\theta = 30°$. Calculate the corresponding value of the turning couple on the crank, having given that the diameter of the cylinder is 18 in., the length of the connecting rod four times the length of the crank, that the crank is 1·2 ft. radius, and that the reciprocating masses weigh 700 lbs.

At 300 revolutions per minute $\omega = 31{\cdot}4$ radians per second,

$$\omega^2 = 986$$

From equation (2), page 336,

$$L = \frac{rv}{V}\{A(p - p_1) - Ma\} \text{ ft.-lbs.}$$

From equation (5), page 338,

$$a = \omega^2 r(\cos 30° + \tfrac{1}{4}\cos 60°) = 1170 \text{ ft. per sec. per sec.}$$

From equation (9), page 338,

$$\frac{v}{V} = (\sin 30° + \tfrac{1}{8}\sin 60°) = 0{\cdot}608$$

And A the area of the cylinder is 254 sq. in., therefore

$$L = \{254(180 - 20) - \tfrac{700}{32} \times 1170\}\ 1{\cdot}2 \times 0{\cdot}608 = 10{,}940 \text{ ft.-lbs.}$$

Both the acceleration a and the velocity ratio $\frac{v}{V}$ might equally well have been found by the graphical methods explained above.

100. The Determination of the Turning Couple Graphically.—Returning to equation (2), page 336, it will be seen that the left side is the resultant force which, by means of the mechanism, produces the turning couple on the crank. It is the total pressure acting on the piston, namely, $A(p - p_1)$ lbs., minus (or plus if a is negative) the force required to accelerate the reciprocating mass, namely, Ma. Let this resultant pressure be represented by the symbol P, that is to say

$$P = A(p - p_1) - Ma = \frac{LV}{rv} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Then the construction (Fig. 99) may be used to find the corresponding value of L.

From K, the crank-pin centre, set out Kx to represent the force P; draw xy at right angles to the line of stroke to cut the direction of the connecting rod produced if necessary in y. Then xy measured to the force scale and multiplied by r, the crank radius, is equal to the magnitude of the turning couple L in foot-pounds.

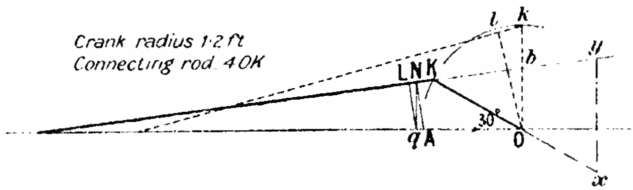


FIG. 99.—Construction for the turning couple L.

Proof.—From the diagram

$xy : Kx = Ob : OK = v : V$ Corollary. Prop. 1, page 340.

That is $xy : P = v : V$

Therefore $xy = \frac{Pv}{V} = \frac{L}{r}$ from (1) above

Therefore $L = xy \times r$

Since r is constant, the variation of the couple L is represented by the variation of xy.

101. Application of the Graphical Methods of Sections 98 and 100 to find the Magnitude of the Turning Couple L.—As will be understood from the previous sections, the value of the turning couple L may be found by several alternative methods. The most expeditious method for drawing-office purposes is to combine Bennett's construction for the acceleration with the method given in Section 100, above.

The steps in the process are enumerated below in connection with the data of the example of Section 99, page 343.

1. Set out the crank and connecting-rod mechanism with the crank at right angles to the line of stroke as shown dotted in Fig. 99, page 344.

2. From O drop a perpendicular Ol on to the connecting rod, thus fixing the point l.

3. Set out the mechanism in the position corresponding with the given crank angle, in this case 30°, and mark off the position of L on the rod. Thus make $KL = kl$.

4. Draw Lq at right angles to the rod, qN at right angles to the line of stroke, and NA at right angles to the rod, thus fixing the point A.

5. AO represents the acceleration of the piston on the same scale that KO represents the acceleration $\omega^2 r$ of the crank pin.

Measure AO. In the example it scales 1·19 ft. At the given speed, 300 revolutions per minute, $\omega^2 = 986$.

Therefore, the acceleration a of the piston at the instant when the crank is passing through the angle 30° is $1{\cdot}19 \times 986 = 1173$ ft. per second per second.

6. P can now be calculated from the given data, thus

$$P = A(p - p_1) - Ma = 254(180 - 20) - \tfrac{700}{32} \times 1173 = 15{,}050 \text{ lbs.}$$

7. Set out Kx to any convenient scale to represent 15,050 lbs., and draw xy at right angles to the line of stroke.

8. Measure xy with the scale used to set out Kx. It measures 9100.

9. Then L, the instantaneous magnitude of the turning couple, is $9100 \times 1{\cdot}2 = 10{,}940$ ft.-lbs.

102. Crank Effort Diagrams.—A diagram showing the turning couple acting on the crank plotted against the crank angle is called a crank effort diagram. The name is also applied to the diagram in which the force acting at right angles to the crank arm at the crank pin is plotted against the corresponding position of the crank pin in its path. The form of the curve is in each case the same. But in the first case the ordinate of the diagram represents the couple L in foot-pounds or foot-tons, and the horizontal distance represents the corresponding crank angle in radians; and in the second case the ordinate represents the force F, found by dividing the couple L by the radius of the crank r, and horizontal distances represent fractions of the path of the crank pin. In each case the area of the diagram represents the work done per revolution.

Points on the curve of crank effort may be fixed by calculating values of the turning couple L corresponding to a series of crank angles by the method of Section 101, but it is more instructive and generally more convenient to plot curves corresponding to

(1) The net pressure acting on the piston $(p - p_1)$ A;

(2) The pressure required to accelerate the reciprocating mass, namely, Ma;

(3) L, the turning couple.

Such a series of curves is shown in Figs. 100 and 101, together with the pair of indicator cards (Fig. 100) from which the curves are derived.

The indicator diagrams shown at the top of Fig. 100 were taken from a single-cylinder horizontal condensing engine driving the machinery of a mill. The piston is fitted with a tail rod equal in diameter to the piston rod, which is 9 ins. diameter, and the cylinder is 52 ins. diameter, so that the effective area of each side of the piston is practically 2060 sq. ins. The stroke is 6 ft., and the connecting rod is 17·5 ft. centre to centre. The reciprocating parts weigh 8·125 tons. Speed, 60 revolutions per minute.

The first thing to do is to find the positions of the piston corresponding to 24 equiangular positions of the crank in order to locate 24 ordinates on the indicator diagrams corresponding to 24 equiangular crank positions, as shown by the vertical lines numbered 1 to 24 in Fig. 100.

The intercept on one of these ordinates between the steam line AB of one diagram, and the back pressure line CD of the other diagram, gives the value of $(p - p_1)A$ for the corresponding position of the piston, and is the magnitude of the corresponding ordinate to curve No. 1, Fig. 100. Curve No. 1 is plotted from the axis 0–24 by means of ordinates found in this way. The shaded part of the indicator diagrams show the regions within which these intercepts lie during the stroke when the piston rod is moving out of the cylinder towards the crank shaft.

Curve No. 2 represents the force required to accelerate the piston masses. The magnitudes of the acceleration a are found by Bennett's construction for the several equiangular positions of the crank taken, and these magnitudes multiplied by $\frac{8{\cdot}125}{g}$ give a series of ordinates from which the curve is plotted.

The curves are continued to the right to show the variations of the quantities in the succeeding stroke, so that the whole base line 0–24 represents the movement of the piston during one revolution of the crank. Sometimes the curves are drawn as if the diagram Fig. 100 were folded on itself about the vertical line EE, in which case the two parts of the acceleration curve coincide.

In a vertical engine the force P is increased by the weight of the parts during the down stroke, and diminished equally during the up stroke. Allowance can be made for this in the diagram by plotting curve No. 1 as for a horizontal engine, and then shifting the axis 0–24 down through a distance equal to the weight of the reciprocating mass measured on the scale of the diagrams. Plot curve No. 2 from this new axis.

The vertical width of the shaded area between curves 1 and 2 represents the varying value of P during a revolution. It represents,

in fact, $(p - p_1)A - Ma$. It will be noticed that in the neighbourhood of points 10 and 23 the curves cross, indicating that the effective

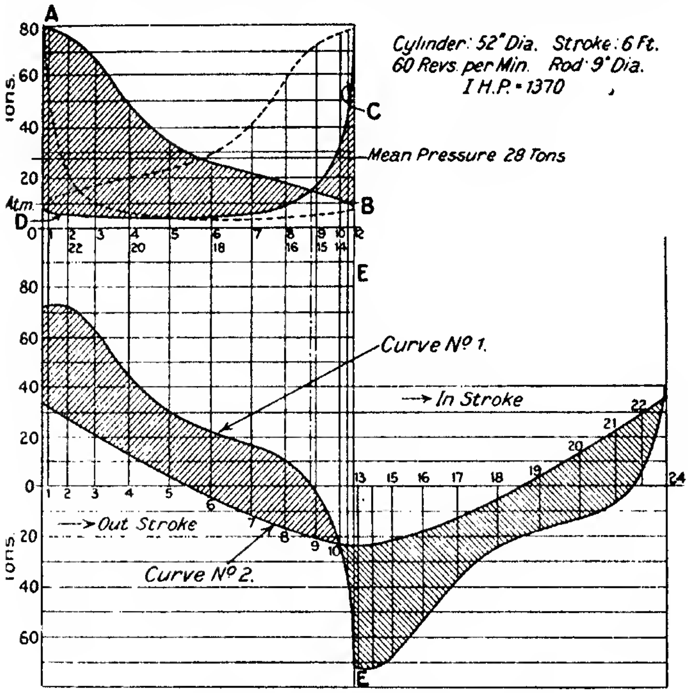


FIG. 100. – Indicator cards and curves of net pressure.

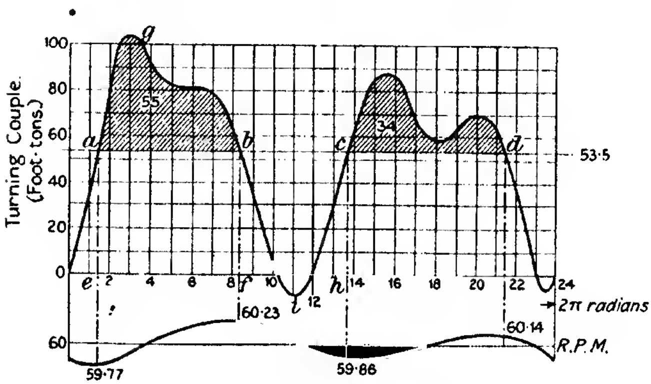


FIG. 101.—Crank effort diagram.

driving force P vanishes and changes sign. At these points the piston ceases to drive the crank, and the crank commences to drive the piston, and continues doing so until the dead point is reached, where the direction of motion of the piston changes, so that it can move in the direction in which steam pressure in the cylinder is tending to drive it. The reversal of pressure at the points of intersection causes a reversal of thrust to take place all through the mechanism. The pressures both between the big end and the crank journal, and between the small end and the cross-head pin change from one side of the brass to the other. The slide blocks change from one bar to another. These changes are accompanied by a knock if there is slack either in the bearing or between the slide bars and the blocks.

The effect of inertia, as will be seen from the curves, is to make the driving force P more uniform during the stroke. At the beginning of the stroke a force of 35 tons is required to set the reciprocating masses in motion, but this force rapidly diminishes towards the centre of the stroke, as the acceleration of the piston diminishes and changes sign when the acceleration becomes negative, until finally it increases to a negative value of about 25 tons in order to bring the piston to rest at the next dead centre, and to set it in motion again in the contrary direction along the next stroke at the end of which 35 tons is necessary to bring the piston to rest, and to start it again on a repetition of the cycle. So that at the beginning of the stroke, where the steam pressure is greatest, the accelerating force is subtracted; whilst at the end of the stroke where the steam pressure has fallen by expansion to a much smaller value, the accelerating force which is now a retarding force is added. The inequality between the steam pressures at the beginning and end of the stroke is therefore reduced by the inertia forces. At starting, the speed is low and therefore the inertia correction is small, the machinery has to withstand the stresses corresponding to the full range of the inequality in the steam pressures, but as the speed increases this inequality is gradually reduced as the inertia correction increases.

Applying the construction of Section 101, page 344, to the 24 values of P corresponding to the equiangular positions of the crank, the values of the couple L are obtained from which the crank effort curve Fig. 101 is plotted.

In the diagram the base is taken to represent one revolution, and the vertical ordinates represent the turning couple. The base may be taken equal to the length of the path described by the crank pin, in which case the vertical ordinates represent the couple L, divided by the crank radius r, that is the tangential force applied by the connecting rod to the crank pin. It is immaterial which method is used.

It will be seen that the turning couple rapidly increases from zero at the dead point to a maximum value in the third crank position, and then gradually falls in value until at the place corresponding

to the crossing of curves No. 1 and 2, Fig. 100, it vanishes as the force P vanishes, and becomes negative from that point onward to the dead point. Along the next stroke the curve is slightly different in form owing to the effect of the obliquity of the connecting rod.

The area between the crank effort curve and the axis represents the work done during a revolution. The shaded area between the curves 1 and 2, Fig. 100, also represents the work done during the double stroke. But the inertia correction does not affect the work done during the double stroke, so that this shaded area is equal to the area between curve No. 1 and the axis, and this area is equal to the product of the mean pressure in the cylinder, the area of the piston, and twice the length of the stroke. Let p_m be the mean pressure in the cylinder during the double stroke; L_a the average turning couple during a revolution; A the area of the cylinder; and l the stroke, then, neglecting frictional losses,

$$p_m A 2l = L_a \cdot 2\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$L_a = \frac{p_m A l}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The mean pressure in the cylinder during the double stroke multiplied by the net area is 28 tons, so that the mean turning effort is

$$L_a = \frac{28 \times 6}{\pi} = 53{\cdot}5 \text{ ft.-tons.}$$

This result may be used to check the accuracy of the crank effort curve, since its average ordinate should have this value.

No allowance has been made in drawing the crank effort curve for the energy absorbed in overcoming the frictional resistance of the mechanism between the piston and the crank shaft. In other words, the brake horse-power is assumed to be equal to the indicated horse-power. This is not, of course, strictly correct, but is generally sufficiently accurate for most of the purposes for which crank effort curves are required. If necessary a suitable reduction of P may be made before using it to calculate the couple, but inasmuch as the correction is different from point to point along the stroke, it is almost impossible to apply the correction accurately. Probably the best way to apply a correction is to draw the diagram in the way explained above, and then alter the vertical scale so that unity on the old scale reads 0·9 on the new scale, or whatever fraction is assumed to be the mechanical efficiency of the engine.

103. The Flywheel.—When investigating the action of a flywheel, the resisting couple against which the crank shaft is turned is usually assumed to be constant, and equal to the average magnitude of the turning couple.

With this assumption the line *abcd*, which in Fig. 101, page 347,

shows the average value of the turning couple, also represents the resisting couple against which the shaft is driven.

An examination of the figure will show that the inequality between the turning couple and the resisting couple ranges between positive and negative values of considerable magnitude. At the dead point the turning couple is zero, hut it rapidly increases in value as the crank turns, passing through equality with the resisting couple between crank positions Nos. 1 and 2, and reaches a maximum value at crank position 3, after which it falls through equality with the resisting couple between positions 8 and 9 to a negative value at 11, from which it changes to zero at the dead point 12, and passes through similar variations during the following stroke.

There is instantaneous equality between the turning couple and the resisting couple four times during the cycle, namely, at the crank positions corresponding to the points *a*, *b*, *c*, and *d*.

The work done by the couple as it drives the crank through an angle θ is represented by the area contained between the ordinates defining the angle on the diagram, the axis, and the part of the curve intercepted between the ordinates. Thus in Fig. 101 the work done by the couple as the crank turns from the angle defined by the ordinate *ea* to the angle defined by the ordinate *fb* is represented by the area *eagbf*.

Of this area the part *eabf* represents the work done against the resisting couple corresponding to the load on the crank shaft the remainder of the area, shown cross-hatched, and representing 55 ft.-tons, shows the work done against the resistance to acceleration of the moving parts connected with the crank shaft. Work expended in overcoming the resistance to acceleration appears as kinetic energy in the masses accelerated. Therefore the kinetic energy stored in the moving parts as the crank passes through the position corresponding to *b*, is greater than the kinetic energy stored in the position corresponding to *a*, by an amount represented by the cross-hatched area, which, in the particular example under consideration, is 55 ft.-tons. Also the speed at *b* is greater than the speed at *a*. How much greater depends upon the inertia of the parts accelerated. A flywheel is added to the shaft in order to provide a mass sufficiently great to keep the speed variation within small limits. The energy stored in the moving parts is usually negligible in comparison with the energy stored in a flywheel, so that the speed variation may be estimated from the change of the stock of kinetic energy in the flywheel alone. With this assumption the relations are simple, since the store of kinetic energy in a revolving flywheel is given by the simple expression

$$\frac{I\omega^2}{2}$$

where I is the moment of inertia of the flywheel about the axis of rotation, and ω is its angular velocity in radians per second.

Let ω_1 and ω_2 be the angular velocities of the shaft at the crank positions corresponding respectively to a and b, and let ΔE be the work represented by the cross-hatched area, a quantity usually called the "Fluctuation of Energy," since it is the quantity of energy which flows into the flywheel as the crank turns from the angle a to b and out again later in the cycle; then

$$\frac{I\omega_2^2}{2} - \frac{I\omega_1^2}{2} = \Delta E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

That is

$$\frac{I(\omega_2 + \omega_1)}{2}(\omega_2 - \omega_1) = \Delta E \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But the change of speed is always small compared with the mean speed; therefore, the mean speed in the cycle is given with sufficient accuracy by ω, and therefore ω may be substituted for

$$\frac{\omega_2 + \omega_1}{2}$$

so that

$$I\omega(\omega_2 - \omega_1) = \Delta E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which the change in speed may be calculated when ΔE, I, and ω are known.

The moment of inertia of the flywheel in the example selected above is 185 $\left(\frac{\text{ton-feet}^2}{g}\right)$ units.

The mean speed ω is one revolution per second, that is, 6·28 radians per second. The increase in speed corresponding to a fluctuation of energy of 55 ft.-tons is therefore

$$\omega_2 - \omega_1 = \frac{55}{185 \times 6{\cdot}28} = 0{\cdot}047 \text{ radian per second}$$
$$= 0{\cdot}0076 \text{ revolution per second}$$

With the assumption made above, namely, that the mean speed is the average of the maximum and minimum speeds, this corresponds to a variation of speed above and below the mean speed of 0·0038 revolution per second or 0·23 revolution per minute. The form of speed curve is roughly indicated on the diagram Fig. 101, page 347, below the crank-effort curve. The variation of speed, reckoned from the corresponding cross-hatched area in the next stroke, is smaller because ΔE is smaller.

In designing a flywheel to keep the speed within assigned limits, the greatest value of ΔE found in a revolution must of course be taken.

The relations given in equation (3) can be thrown in a more convenient form for design purposes. Multiply each side by $\frac{\omega}{2}$, and

remembering that the energy stored at the mean speed ω is given by $\frac{I\omega^2}{2} = E$,

$$\frac{\omega_2 - \omega_1}{\omega} = \frac{\Delta E}{2E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

from which
$$E = \frac{\Delta E}{2q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where q is put for the ratio $\frac{\omega_2 - \omega_1}{\omega}$. This ratio, namely, the ratio of the variation of speed to the mean speed, is called the **coefficient of fluctuation of speed,** or sometimes the **coefficient of unsteadiness.** Equation (5) shows that the energy which a flywheel stores at the mean speed of the crank shaft is equal to one-half the fluctuation of energy ΔE divided by the coefficient of unsteadiness.

The value of q, to be selected in designing a wheel, varies between $\frac{1}{20}$ and $\frac{1}{150}$, according to the purpose for which the engine is to be used. Small values of q should be taken for engines set to drive spinning machinery or electric-lighting dynamos, the higher values being used where speed variation is not of great moment. The value of q found in the example above, namely, 0·0076, may be regarded as typical of the best practice for engines driving spinning machinery.

The value of ΔE depends upon the number of cylinders coupled to the crank shaft and the angles between the cranks, and also upon the cut-off in each cylinder, and to the form of the curve showing the resisting couple.

Its value can only be found by drawing the crank-effort curve for assigned conditions, and then measuring the alternately positive and negative areas cut off by the curve of resistance. The greatest of these areas gives ΔE. The combination of the crank-effort curves from several cylinders into one curve is considered below. Meanwhile assume that a value ΔE can be assigned, and that a flywheel is to be designed for a given value of the mean speed and the coefficient of unsteadiness, with the added condition that the linear velocity of the flywheel rim must not exceed a limiting value v feet per second.

The energy which a flywheel must store to secure a given value of q, with an assigned value of ΔE, can be calculated from (5), and then I, the moment of inertia of the wheel, can be found. Or I may be calculated direct from equation (3).

The energy stored by the boss and the arms of a flywheel is small compared with the energy stored in the rim, and therefore the rim only need be considered when making preliminary or approximate calculations.

The moment of inertia of a flywheel rim weighing W tons may be calculated with sufficient accuracy from

$$I = \frac{Wr^2}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where r is the mean radius of the rim.

The energy stored in the rim is then given by

$$E = \frac{Wr^2\omega^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The velocity of the rim at the mean radius is ωr, and since this must not exceed a specified limiting speed v, W, the weight of the rim, can be calculated from

$$W = \frac{2gE}{v^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For example, if $\Delta E = 55$ ft.-tons, and q is to be $\frac{1}{150}$ when the mean speed is one revolution per second, equal to $\omega = 6{\cdot}28$ radians per second, the energy which must be stored at the mean speed calculated from (5) is 4125 ft.-tons. If the limiting velocity of the rim is fixed at 70 ft. per second, a speed which should not be exceeded by cast-iron wheels built up of segments and separate spokes, the weight of the rim is, from (8), 54 tons. To this must be added the weight of the spokes and the boss to obtain the actual weight of the wheel.

The mean radius of the rim is, from the relation $\omega r = v = 70$, 11·15 ft.

A more exact expression for the moment of inertia of a flywheel with n arms, which allows for the energy stored in the arms, can be found as follows. Assume each arm to be uniform in section, and to extend from the centre of the wheel to the mean radius, so that its length is equal to the mean radius r. And let w be the total weight of the n arms including the boss, so that the weight of each arm and its proportion of boss is $\frac{w}{n}$. The moment of inertia of one arm turning about one end is easily shown to be $\frac{wr^2}{3ng}$, and therefore the moment of inertia of the n arms is $\frac{wr^2}{3g}$. Adding this to the moment of inertia of the rim, the moment of inertia of the wheel is

$$I = \frac{r^2}{g}\left(W + \frac{w}{3}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In order, in the course of design, to use this expression, some assumption must be made regarding the ratio between W and w. A provisional assumption of $w = \frac{1}{2}W$ may be made for a heavy built-up wheel. With this assumption $I = \frac{7Wr^2}{6g}$, and equation (8) becomes

$$W, \text{ the weight of the rim, } = \frac{6}{7} \cdot \frac{2gE}{v^2} \quad . \quad . \quad . \quad (10)$$

A comparison of this equation with equation (8) shows that the weight of the rim, allowing for the effect of the arms and boss weighing in the aggregate half as much as the rim itself, is only $\frac{6}{7}$ of the weight found from (8), where the arms are neglected. With the

data given above the weight of the rim from (10) is 46 tons, as compared with 54 tons calculated from (8). Still assuming that the arms and boss weigh one-half the rim, the total weight of the wheel found from (8) is 81 tons, whilst from (10) it is 69 tons.

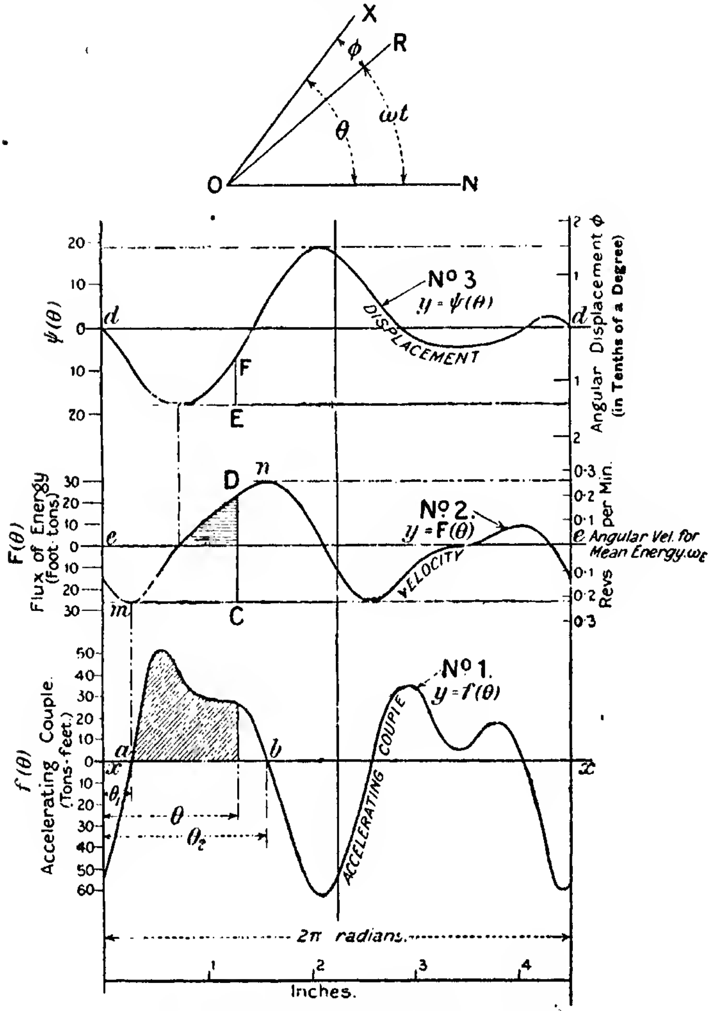


FIG. 102.—Flywheel. $f(\theta)$; $F(\theta)$ and $\psi(\eta)$ plotted.

104. General Theory of the Flywheel.—The preceding investigation enables the size of a flywheel to be found for given

values of ΔE and q, but it gives no information relating to the motion of the flywheel. The investigation in this section is due to Prof. Henrici, and it gives complete information of the motion of the wheel.

Whatever be the shape of the crank-effort curve, or the curve of the resisting couple, providing both are given, a curve can be constructed from them which represents the turning couple producing acceleration as a function of the crank angle θ, since an ordinate of this curve corresponding to any crank angle is the algebraic difference between the ordinates of the curve of crank effort and the curve of resistance at that crank angle.

For instance, curve No. 1, Fig. 102, is such a curve derived from Fig. 101, page 347, where the resisting couple is assumed to be constant.

Let $f(\theta)$ represent the accelerating couple given by this curve. It is clear that since it is alternately positive and negative the motion of the wheel will be alternately accelerated and retarded, and that the motion of the actual flywheel may be regarded as an oscillation superposed upon a uniform rotation equal to the average angular velocity of the wheel.

Let OR be a radius of reference rotating with the uniform mean angular velocity of the wheel, ω; so that the angle between it and the initial line ON is ωt. Let OX be a radial line in the actual wheel; and let ϕ be the variable angle between OX and OR, and θ the variable angle between OX and ON.

Consider the relations when the radius OX stands at an angle θ with the initial line. The energy actually stored in the wheel is $\frac{I\dot{\theta}^2}{2}$, where $\dot{\theta}$ is written for $\frac{d\theta}{dt}$.

The energy stored in the wheel at the mean velocity ω is $\frac{I\omega^2}{2}$.

The difference must be equal to the work done by the varying accelerating couple, namely,

$$\int_{\theta_1}^{\theta} f(\theta)d\theta = F(\theta) = \text{the shaded area above } ab \text{ on curve No. 1.}$$

$$\text{Therefore } \frac{I\dot{\theta}^2}{2} - \frac{I\omega^2}{2} = F(\theta) = \text{the shaded area above } ab.$$

$$\text{That is, } \frac{I\omega^2}{2}\left\{\frac{\dot{\theta}^2}{\omega^2} - 1\right\} = F(\theta) = \text{the shaded area above } ab \quad (1)$$

From figure 102

$$\theta = \omega t + \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{Therefore} \qquad \frac{d\theta}{dt} = \dot{\theta} = \omega + \dot{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting this value of $\dot{\theta}$ in (1) and writing E_0 for the energy $\frac{I\omega^2}{2}$ stored at the mean speed, the expression reduces to

$$E_0\left\{\frac{2\omega\dot{\phi} + \dot{\phi}^2}{\omega^2}\right\} = F(\theta) = \text{the shaded area above } ab.$$

But $\dot{\phi}^2$ being the square of a small quantity may be neglected without serious error, so that

$$\frac{2E_0\dot{\phi}}{\omega} = F(\theta) = \text{the shaded area above } ab \quad . \quad . \quad (4)$$

$\dot{\phi}$ is the difference of the angular velocity of the wheel and the mean velocity.

When the integration is taken between the limits θ_1 and θ_2, $F(\theta)$ corresponds to ΔE, and $\dot{\phi}$ corresponds to $\omega_2 - \omega_1$ of equation (3), page 351, and the results obtained in this general way lead to the same relation as that given in equation (4), page 352.

Again, from equation (1), page 355,

$$\dot{\theta}^2 = \omega^2\left\{\frac{F(\theta)}{E_0} + 1\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\dot{\theta} = \frac{d\theta}{dt} = \omega\sqrt{\left\{1 + \frac{F(\theta)}{E_0}\right\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{d\theta}{\sqrt{\left(1 + \frac{F(\theta)}{E_0}\right)}} = \omega dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and the variables are separated.

By the Binomial theorem

$$\left\{1 + \frac{F(\theta)}{E_0}\right\}^{-\frac{1}{2}} = 1 - \frac{1}{2}\frac{F(\theta)}{E_0} + \frac{1 \cdot 3}{1 \cdot 2}\left(\frac{F(\theta)}{2E_0}\right)^2 \quad . \quad . \quad (8)$$

But the third term is by (4) of the order $\left(\frac{\dot{\phi}}{\omega}\right)^2$, which is negligibly small.

Therefore $$\int\left\{1 - \frac{1}{2}\frac{F(\theta)}{E_0}\right\}d\theta = \int\omega dt \quad . \quad . \quad . \quad . \quad . \quad (9)$$

i.e. $$\theta - \frac{1}{2E_0}\int F(\theta)d\theta = \omega t + C \quad . \quad . \quad . \quad . \quad . \quad (10)$$

But $\theta = \omega t + \phi$, and the constant $C = 0$ if $F\theta.(d\theta)$ is measured from its mean value, because mean $\phi = 0$.

Therefore $$\phi = \frac{1}{2E_0}\int F(\theta)d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Summarizing these results :—

When the accelerating couple is given as a function of the crank angle θ, the cyclical variation of angular velocity of the flywheel and

the cyclical variation of angular displacement can be found if the original function $f(\theta)$ can be integrated twice. That is to say, if the operations

$$\int f(\theta)d\theta = \mathrm{F}(\theta)$$

and

$$\int \mathrm{F}(\theta)d\theta = \psi(\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

can be performed.

Given a pair of indicator diagrams, the mean speed, and the dimensions of the engine, the crank-effort curve can be drawn as shown above. The curve representing the resisting couple against which the crank shaft is turned cannot be easily found, but it is usually sufficiently near to take the resisting couple constant and equal to the mean crank effort.

The difference between the ordinates of these curves plotted against the crank angle, gives $f(\theta)$, the accelerating couple as a function of the crank angle, as shown by curve No. 1, Fig. 102, which is derived from the crank-effort diagram of Fig. 101.

The integration of this curve can be done graphically or with the integraph, giving the curve represented by $y = \mathrm{F}(\theta)$.

And this second curve can be integrated in a similar way, giving a curve $y = \psi(\theta)$.

The problem is thus completely solved for any practical case.

Referring to Fig. 102, curve No. 1 is, as mentioned above, the curve showing the accelerating couple plotted against the crank corresponding to the crank-effort diagram Fig. 101.

Curve No. 2 is the integral curve of curve No. 1, and curve No. 3 is the integral curve of curve No. 2.

Curve No. 2 has this property, that, reckoning from the dotted axis, the ordinate CD represents to scale the shaded area in curve No. 1, and represents, therefore, the flux of energy into the flywheel as the crank turns from the angular position θ_1 to θ. It also represents to another scale, the scale on the right of the diagram, the change of angular velocity corresponding to the change of angle from θ_1 to θ, since from equation (4) it is clear that

$$\dot{\phi} = \omega - \omega_1 = \frac{\mathrm{F}(\theta)\omega}{2\mathrm{E}_0} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and that by hypothesis E_0 the mean energy and ω the mean speed are constant.

Regarding these scales, 1 in. horizontally along the axis xx = 1·395 radians; 1 in. vertically represents a couple of 50 ton-ft.

Therefore 1 sq. in. of area under curve 1 represents the energy 69·8 ft.-tons.

The vertical energy scale chosen for curve No. 2 is 1 in. represents 50 ft.-tons. But from equation (13) the change of angular velocity corresponding to a change of energy of 50 ft.-tons is $\dfrac{50 \times \omega}{2\mathrm{E}_0}$

radians per second. Therefore, reducing this to revolutions per minute, the scale on the right is

$$1 \text{ in.} = \frac{50\omega 60}{2E_0 2\pi} \text{ revolutions per minute}$$

With $I = 185$ and $\omega = 6{\cdot}28$, $E_0 = 3650$ ft.-tons

$$1 \text{ in.} = 0{\cdot}41 \text{ revolution per minute}$$

Reading from this scale, it will be seen that the maximum speed is 0·25 revolution per minute above the speed corresponding to the mean energy, and that the minimum speed is 0·22 revolution per minute below the speed corresponding to the mean energy, so that the total cyclical variation is 0·47 revolution per minute, as found on page 251.

It should be noted that the mean speed is not the same as the speed corresponding to the mean energy. From the diagram it is seen that if the speed corresponding to the mean energy is represented by ω_e,

$$\omega_e + 0{\cdot}25 = \omega + \frac{0{\cdot}47}{2}$$

so that $$\omega_e = \omega - 0{\cdot}015 \text{ radian per second}$$

Curve No. 3 has the property that, reckoning from the dotted axis, any ordinate as EF represents to scale the area shaded horizontally in curve 2. The curve is therefore the integral curve of No. 2, so that $EF = \psi(\theta)$. It also represents to another scale the angular displacement of the wheel, since reckoning from the mean axis *dd*, the angular displacement of the actual wheel, measured from the radius of reference revolving with uniform angular velocity ω, is from equation (11), page 356,

$$\phi = \frac{\psi(\theta)}{2E_0} \text{ radians}$$

and E_0 is constant.

Regarding the scales—

1 in. horizontally along the axis *ee* of curve 2 as before represents 1·395 radians.

1 in. vertically represents 50 ft.-tons.

Therefore 1 sq. in. of area under curve 2 represents 69·8 units.

The scale to the left chosen for the vertical ordinates of curve No. 3 is 1 in. represents 25 units. And the scale of angular displacement on the right is therefore

$$1 \text{ in.} = \frac{25}{2E_0} \cdot \frac{180}{\pi} \text{ degrees}$$
$$= 0{\cdot}196 \text{ degree.}$$

Reading from the curve, it will be seen that the greatest deviation of the wheel from its mean position is about 0·15 degree in advance, whilst the greatest lag behind is about 0·14 degree.

105. Representation of a Crank-Effort Curve by a Fourier Series.—The crank-effort curve does not exhibit any sensible change of form when the engine is running steadily at constant mean speed. The curve is periodic in character, the periodic time being the time of a revolution of the crank shaft. In these circumstances the curve may be represented analytically by a Fourier Series.

Referring to Fig. 103, and assuming for simplicity that the curve is identical during the two strokes, and of the form depicted in the

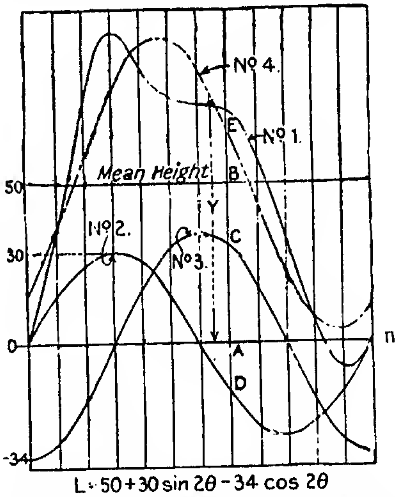


FIG. 103.—Harmonic components of crank-effort curve.

stroke from 0 to 12, this series represents the curve with considerable accuracy;

$$L = 50 + 30 \sin 2\theta - 34 \cos 2\theta + 23 \sin 4\theta - 25 \cos 4\theta - 9 \sin 6\theta - 4 \cos 6\theta \quad . \quad . \quad (1)$$

The coefficients in this series were found by means of a Henrici analyser. The coefficients can, however, be found by a graphical process without the use of an instrument, but the process is tedious.[1]

It will be noticed that the angles in the series are all even multiples of the crank angle θ; this, in fact, follows because the couple vanishes at the dead points, that is, when $\theta = 0$ and when $\theta = \pi$, with which condition all the odd multiples of θ vanish.

It will also be seen that the curve is fairly well represented if all terms involving angles greater than 2θ are neglected. The extent to which the curve corresponding to equation (1) with these terms neglected, differs from the real curve may be judged by an inspection of Fig. 103, where the full line marked No. 1 represents the actual

[1] See "Valves and Valve Gear Mechanisms," by W. E. Dalby, for an explanation of some of the well-known methods of graphical analysis.

crank-effort curve, and the dotted line marked No. 4 is plotted from the expression

$$L = 50 + 30 \sin 2\theta - 34 \cos 2\theta \quad . \quad . \quad . \quad . \quad (2)$$

The first term of the series represents the average turning couple; the second term is shown in the figure by curve No. 2, and the third term by curve No. 3. At any angle, as OA, the ordinate to the dotted curve is found by adding algebraically the ordinates of curves 2 and 3 to the mean ordinate. That is,

$$L = AB + AC + AD = AE$$

In the position lettered AD is negative.

There is sufficient correspondence between the curves No. 1 and No. 4 to show that the simple series of three terms may be used in many cases as a good analytical representation of the actual curve.

In particular, the calculation of the cyclical variation of speed and the cyclical variation of angular velocity of a flywheel can easily be made if the crank effort curve is represented by a three-term series.

By way of illustration assume that equation (2) is the curve of turning effort of a particular engine, and that I, the moment of inertia of the flywheel, is 185 $\left(\frac{\text{ton-ft.}}{g}\right)^2$ units: that the mean speed is 1 revolution per second, so that $\omega = 6{\cdot}28$, and therefore E_0, the energy stored at the mean speed, is 3650 ft.-tons.

Assume also that the resisting couple is constant, and therefore equal to the mean couple, that is, 50 tons-ft.

Then the accelerating couple, which is the difference between the couple L and the resisting couple, is

$$\text{Accelerating couple} = 30 \sin 2\theta - 34 \cos 2\theta = f(\theta) \quad . \quad (3)$$

$$\text{Integrating this,} \quad - 15 \cos 2\theta - 17 \sin 2\theta = F(\theta) \quad . \quad (4)$$

$$\text{Integrating again,} \quad - 7{\cdot}5 \sin 2\theta + 8{\cdot}5 \cos 2\theta = \psi(\theta) \quad . \quad (5)$$

To find the cyclical variation of speed it is necessary to know the angles corresponding to a minimum and a maximum value of $F(\theta)$, and this is done by equating its differential coefficient to zero. But its differential coefficient is given by equation (3), therefore equating $f(\theta)$ to zero

$$\tan 2\theta = \tfrac{34}{30} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

showing that there is a minimum value when

$$2\theta = 48\tfrac{1}{2}^\circ$$

and a maximum value when

$$2\theta = 180 + 48\tfrac{1}{2}^\circ$$

corresponding to values of θ, $24\tfrac{1}{4}$, and $114\tfrac{1}{4}^\circ$.

The value of $F(\theta)$ when 2θ is $48\tfrac{1}{2}^\circ$, is

$$- 15 \cos 48\tfrac{1}{2} - 17 \sin 48\tfrac{1}{2} = - 22{\cdot}7$$

The value of $F(\theta)$ when $2\theta = (180 + 48\tfrac{1}{2})$ is

$$15 \cos 48\tfrac{1}{2} + 17 . \sin 48\tfrac{1}{2} = + 22{\cdot}7$$

Taking the inferior from the superior limit shows that the increase in the function $F(\theta)$ as the crank turns from the angle $24\frac{1}{4}^\circ$ to $14\frac{1}{4}^\circ$ is $22 \cdot 7 - (-22 \cdot 7) = 45 \cdot 4$ ft.-tons. This quantity is the fluctuation of energy, ΔE.

Substituting this and the data given above, in equation (4), page 356,

$$\dot{\phi} = \omega_2 - \omega_1 = \frac{45 \times 6 \cdot 28}{7300} = 0 \cdot 039 \text{ radian per second,}$$

and this is the cyclical variation of angular velocity.

Again, to calculate the cyclical variation of the angular displacement, the angles for which $\psi(\theta)$ is a minimum and a maximum must be found, and then the change of $\psi(\theta)$, corresponding to the change of angle between these limits is proportional to the cyclical angular displacement.

Equating the differential coefficient of $\psi(\theta)$, that is, $F(\theta)$ to zero,

$$\tan 2\theta = -\tfrac{15}{17} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

showing that there is a minimum value when

$$2\theta = (180 - 41\tfrac{1}{2})^\circ$$

and a maximum value when

$$2\theta = (360 - 41\tfrac{1}{2})^\circ$$

The value of $\psi(\theta)$, when 2θ is $(180 - 41\frac{1}{2})^\circ$ is

$$-7 \cdot 5 \sin 41\tfrac{1}{2} - 8 \cdot 5 \cos 41\tfrac{1}{2} = -11 \cdot 32$$

The value of $\psi(\theta)$ when 2θ is $(360 - 41\frac{1}{2})^\circ$ is

$$7 \cdot 5 \sin 41\tfrac{1}{2} + 8 \cdot 5 \cos 41\tfrac{1}{2} = 11 \cdot 32$$

The difference is $11 \cdot 32 - (-11 \cdot 32) = 22 \cdot 64$.

Using this in equation (11), page 356, the cyclical variation of angular displacement is

$$\phi = \frac{22 \cdot 64}{2E_0} = \frac{22 \cdot 64}{7300} = 0 \cdot 0031 \text{ radian}$$

$$= 0 \cdot 177^\circ$$

From this example it will be understood how easily the cyclical variation can be found, both of the angular velocity and the angular displacement, when a three-term series is used to represent approximately the crank-effort curve.

Particulars of the analysis of 28 crank-effort curves into their corresponding Fourier Series, each carried to 17 terms, will be found in the *Bulletin de la Société Internationale des Electriciens*, Tome 1 (2 Serie), No. 9, November, 1901.

106. Combination of Crank-Effort Curves and the Fluctuation of Energy.—A curve of total crank effort is drawn by adding the ordinates of the crank-effort curves corresponding to each cylinder, after the several curves have been drawn on one diagram in the positions relatively to one another determined by the angle between the cranks.

In order to illustrate the method of construction, suppose that two cylinders are coupled to a crank shaft, and that the cranks are at 90° with each other. And further, let it be assumed that the crank-effort diagram is the same in form for each cylinder, and identical with the diagram given in Fig. 101, page 347. First plot the curves as indicated in Fig. 104, displaced relatively to each other through 90°, and then add the ordinates. The curve of total crank effort is shown by a thick line.

The result of the combination is that, although the mean crank effort is doubled, the fluctuation of energy, estimated from the largest

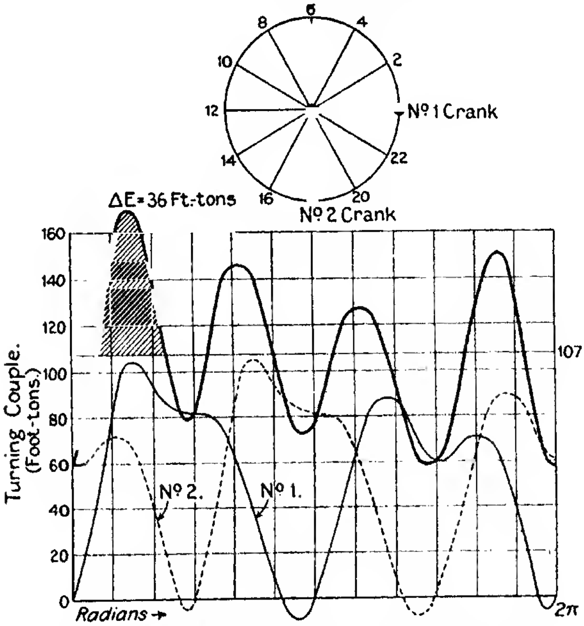


FIG. 104.—Crank-effort diagram. Two cranks at right angles.

area cut off by the mean line, is reduced to 36 ft.-tons. Comparing engines of equal power, therefore, a comparison which can be made by assuming that everything remains the same except that the joint area of the two cylinders is equal to the area of the single cylinder, the fluctuation of energy is reduced to 18 ft.-tons, which is only about one-third of the fluctuation found for the single-cylinder engine of equal power.

The fluctuation of energy is still further reduced if the power is equally distributed between three cylinders with cranks at 120°.

The fluctuation of energy depends on several variable quantities, and therefore each case must be considered separately.

The distribution of power between two cylinders reduces the ratio between the maximum and the minimum turning effort, as will be seen from the diagrams. In the single-cylinder engine (Fig. 101), page 347, the turning effort varies from 105 ft.-tons as the crank passes through

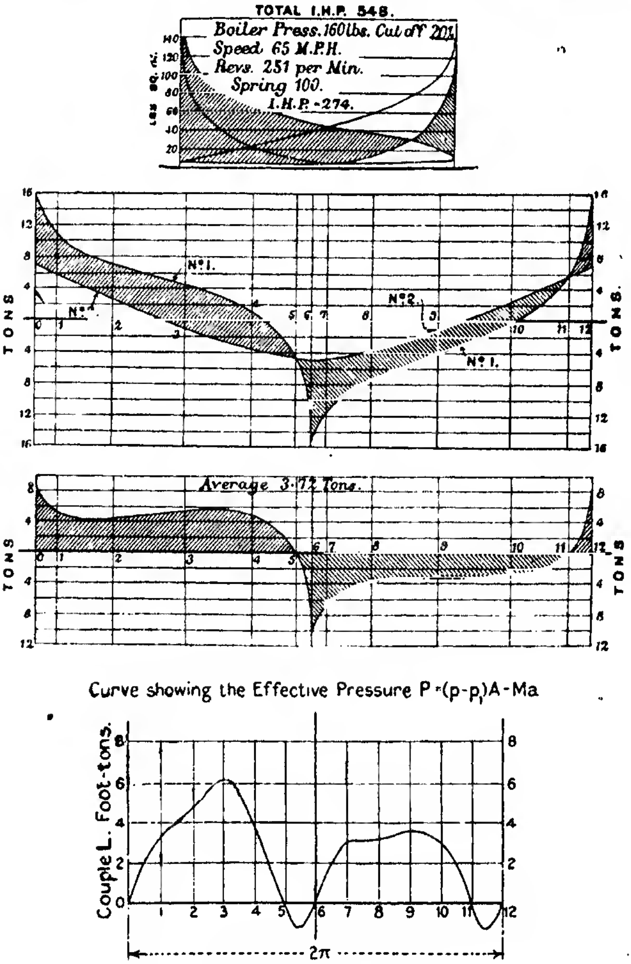


FIG. 105.—Crank-effort diagram. Locomotive.

crank position 3, to – 10 ft.-tons as it passes through position 11, a range of 115 ft.-tons. When the power is equally distributed between two cylinders with cranks at right angles, the range is from 85 ft.-tons to 40 ft.-tons, namely, 45 ft.-tons as against 115 ft.-tons.

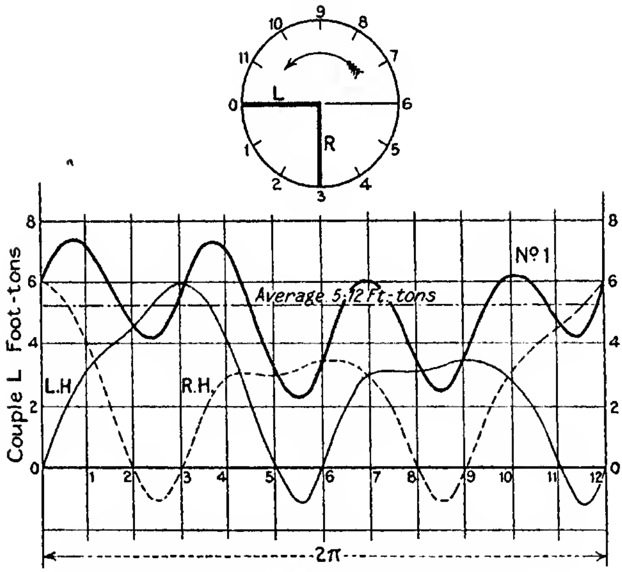


FIG. 106.—Crank-effort diagram of a locomotive at high speed.

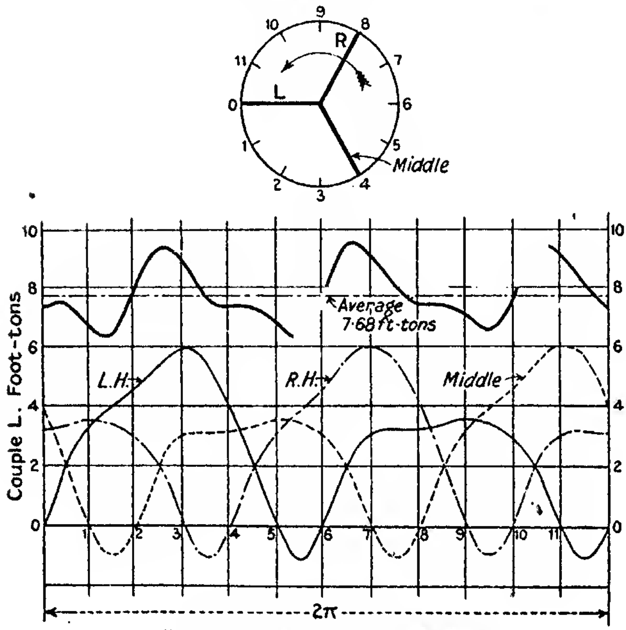


FIG. 107.—Three cranks at 120 degrees.

Figs. 106, 107, and 108, page 364, illustrate the combination of crank-effort diagrams in the cases of a two-cylinder locomotive with cranks at right angles; a three-cylinder locomotive with cranks at 120°; and a four-cylinder locomotive with two 180° pairs of cranks at right angles. It is assumed in all of these combinations that the power developed in each cylinder is the same, and that the crank-effort curve from each cylinder is identical with that shown in Fig. 105, page 363, which is derived, as shown by the curves and indicator

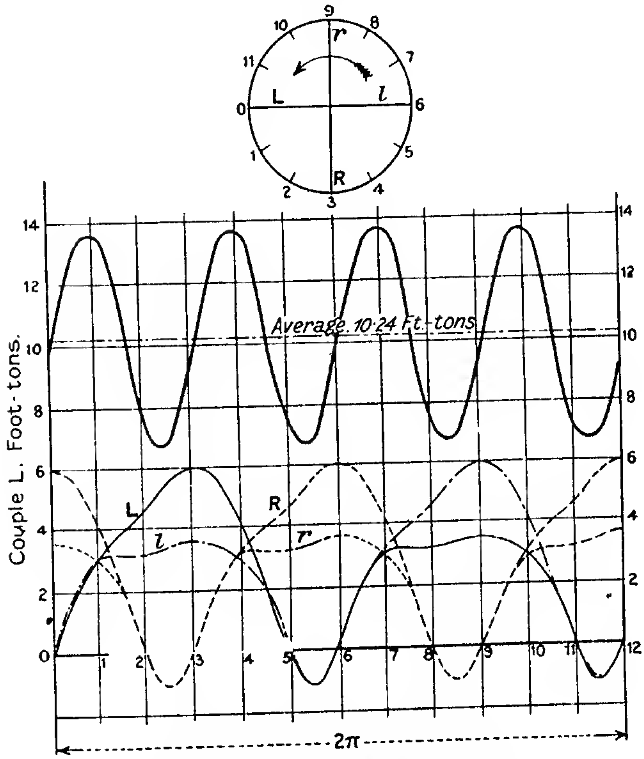


FIG. 108.—Two 180 degree pairs at right angles.

diagrams above it from a pair of diagrams taken from an express passenger locomotive when running in actual service on the road at 65 miles per hour. In Fig. 105 curve No. 1 gives the difference between the steam pressure and the back pressure; curve No. 2 shows the inertia correction. The curve below shows the net pressure acting to turn the crank. Fig. 106 shows the curve of total crank effort drawn for the engine from which the diagrams were taken. The curves of total crank effort in Figs. 107 and 108 do not correspond to

any particular engine, but are drawn for the purpose of comparison. It will be seen that the fluctuation of energy is small in the 120° three-crank combination, and that the ratio of the minimum to the maximum turning couple is small also. In the case of the four-cylinder engine the result of the combination is to correct the effect of the connecting rod, and give four symmetrical waves of effort, but that there is little change in the fluctuation of energy or in the ratio between the minimum and maximum turning couples in a four-cylinder engine of this type compared with a two-cylinder engine of equal power with cranks at right angles.

Examples of diagrams of total crank effort for typical crank arrangements of marine engines will be found in a paper by the author entitled, "A Comparison of Five Types of Engines with respect to their Inertia Forces and Couples," published in the *Transactions of the Institution of Naval Architects*, 1902.

A problem of great practical interest arises in connection with the diagram of total crank effort when in a multi-cylinder engine the crank angles are unequal, as in the case of a four-crank balanced engine where the crank angles are specially determined to suit the conditions of balancing.

In such a case the best approach to uniformity of crank effort is secured by an unequal distribution of power amongst the cylinders, found by the following rule due to Prof. Lorenz.[1] Draw an end view of the crank shaft, and then, starting from any one of the cranks as an initial line, draw another end view, but with all the crank angles doubled as measured from the initial line. Then draw a parallelogram whose sides are parallel to the double-angled cranks. If the power is then distributed between the cylinders in the proportion of the sides of the parallelogram, the resulting curve of total crank effort will be a good one.

Briefly the method of obtaining the rule is to analyse a crank-effort curve into a Fourier Series of the form

$$L = A_0 + A_2 \cos 2\theta + B_2 \sin 2\theta$$

discarding all terms in the series higher than 2θ. Then, assuming that the crank-effort curves from the several cylinders are similar in form, the 2θ components vanish on addition when the power is distributed by the rule above, and the resulting crank effort is a straight line. The conditions involved in these assumptions cannot, of course, be exactly realized in practice, because, as shown in the previous section, higher terms appear in the series when an actual curve is analysed, and also it is not possible to get similar crank-effort curves from each of the cylinders.

The curve of total crank effort is therefore variable, but the variation is less than would be the case with an equal distribution of power between the cylinders.

[1] "On the Uniformity of Turning Moments in Marine Engines," by Dr. Lorenz, *Trans. Inst. Naval Arch.*, 1900. Also "Dynamik der Kurbelgetriebe," by Dr. Lorenz. See also a complete demonstration of the principles underlying the rule in "The Balancing of Engines," by W. E. Dalby, 2nd ed., p. 225.

As will be understood from the previous sections, the "Fluctuation of Energy" ΔE depends upon many variable quantities, and the line of mean resistance cuts off a number of areas above it which depend upon the number of separate crank-effort curves used to obtain the combined curve. The particular area of the several areas belonging to each revolution which is to be used in designing a flywheel is the greatest of them.

The cut-off in the cylinder, the back pressure, the speed as affecting the inertia correction, the ratio of the length of the connecting rod to the length of the crank, the angles between the cranks, the resistance against which the engine works, all influence the magnitudes of these successive areas of fluctuation in a revolution.

Nevertheless, the order of the fluctuation expressed as a fraction of the whole work spent during a revolution may be usefully calculated for different types of engines on the assumption that the crank is turned against a constant resistance by a force which, after being corrected for back pressure and inertia of the moving parts, is constant and equal in each cylinder throughout the stroke. Let this force be P, and let r be the crank radius. Then the curve of crank effort belonging to each cylinder is given by $Pr \sin \theta$, in which θ is the crank angle made by each particular crank with its line of stroke. The ordinates to this curve will always be positive, because, although $\sin \theta$ changes from positive to negative during a revolution, P changes sign when the sine changes sign, and therefore the ordinate representing the turning effort is always positive.

The work done per revolution is $4Pr$ ft.-lbs. per cylinder; and the mean turning couple is $Pr\frac{2}{\pi}$ lbs.-ft. per cylinder. The turning couple will be equal to the mean turning couple when

$$\sin \theta = \frac{2}{\pi} = 0{\cdot}637$$

corresponding to angles of $39\frac{1}{2}°$ and $140\frac{1}{2}°$. Then the fluctuation of energy ΔE is given by

$$Pr\int_{39\frac{1}{2}}^{140\frac{1}{2}} \sin \theta d\theta - Pr\frac{2}{\pi}(\theta_2 - \theta_1) = 0{\cdot}4Pr$$

The circular measure of θ_1 and θ_2 must be substituted in the second term when reducing the result. Therefore

$$\frac{\Delta E}{\text{work per revolution}} = \frac{0{\cdot}4Pr}{4Pr} = 0{\cdot}1$$

That is to say, that with the assumption of a sine curve for the crank-effort curve of a single crank engine the fluctuation of energy is 10 per cent. of the work done per revolution. In a similar way it can be shown that with two cranks at right angles the fluctuation is reduced to 1 per cent. of the total work done per revolution, and this is reduced to 0·3 per cent. with three cranks at 120°. If three

cranks are arranged so that two of them are at right angles, and the third bisects the exterior angle, the fluctuation is about 3·5 per cent. of the total work per revolution. In each case it is to be understood that the crank effort corresponding to each crank is a sine curve, and that the component curves have equal maximum ordinates.

107. Governors.—A governor is that part of an engine which automatically adjusts the steam supply so that only small variations of mean speed follow large variations of load.

A governor of the kind invented by Watt is shown in Fig. 109, together with the connections to the engine by which the steam

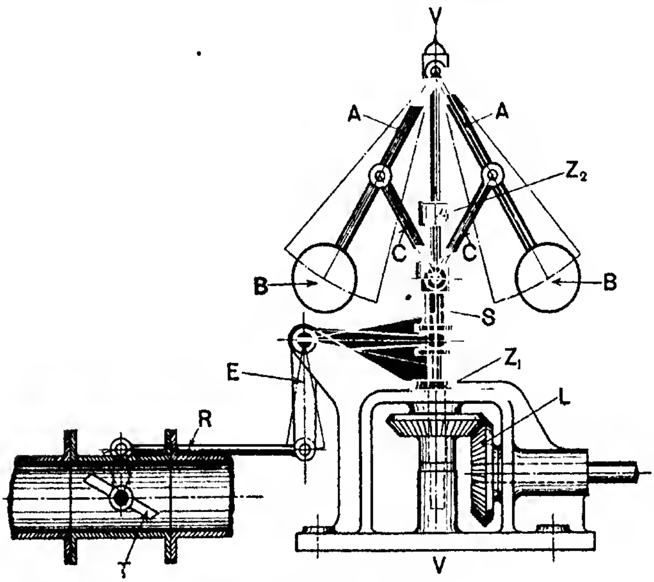


Fig. 109.—Governor and throttle valve.

supply is regulated. A vertical spindle VV is connected through bevel wheels L to the engine shaft. Two balls B, B, are secured to arms A, A. The arms are each jointed to the top of the spindle, and to links C, C. The links carry the sliding sleeve S. A bell crank lever E, jointed with the main casting, engages at the horizontal end with the sleeve S, a groove being turned in the sleeve to allow of this engagement when the sleeve is rotating, and the vertical arm is coupled to the throttle valve T in the main steam pipe by the rod R. Stops Z_1 and Z_2 limit the motion of the sleeve along the spindle. When the sleeve is against the upper stop the throttle valve is almost closed; and when it is against the lower stop the valve is wide open.

When the governor is at rest the weight of the balls B, B, acting through the links, together with the weight of the sleeve and any additional load acting upon the sleeve, either by springs or by dead weights, push the sleeve down to the bottom stop.

When the governor spindle rotates, the balls are compelled to move in a circular path, and the mechanism is of necessity compelled to apply the accelerating force required to determine the circular motion of the balls. As the speed increases, this accelerating force increases until a speed is reached at which the static load pushing the sleeve against the bottom stop is no longer able to hold it there against the reaction through the mechanism due to the rotation of the balls, a reaction, or dynamical load as it may be called, which is always equal and opposite to the force accelerating the motion of the balls in their circular path.

When the dynamical pull upwards on the sleeve is equal to the static push downwards on the sleeve, there is equilibrium; and then if the speed increases so that the dynamical pull increases, the sleeve rises and moves the throttle valve, and if the speed continues to increase, the sleeve continues to rise, still further closing the throttle valve, until it may finally reach the top stop, where the steam supply is so reduced that it is only just sufficient to keep the engine moving against its own friction. If now a load comes on to the engine the steam supply is insufficient and the speed falls, and with it the dynamical pull on the sleeve; and the static push downward on the sleeve then being greater than the dynamical pull upward, the sleeve moves downward, and in doing so opens the throttle valve and admits more steam.

There is, in fact, a continuous struggle going on at the sleeve between the dynamical pull upwards due to the mere rotation of the balls, and the static push downwards due to the mere weight of the masses or to the forces applied by the springs connected to the sleeve.

When the governor is rotating at constant speed and the sleeve is clear of the stops, the dynamical pull upwards is equal to the static push downwards if frictional resistance of the governor and its connected gear is neglected. Friction always acts to oppose any change in the configuration of the governor mechanism, and therefore, if the sleeve is in a position where there is exact equilibrium between the dynamical pull upwards and the static push downwards, the speed may increase or decrease slightly before the configuration will begin to change; generally, therefore, when the configuration of a governor is just on the point of changing, owing to a change of speed,

$$\left.\begin{array}{c}\text{Dynamical pull upward}\\ \text{on the sleeve}\end{array}\right\} = \left\{\begin{array}{c}\text{static push downward}\\ \text{on the sleeve}\end{array}\right\} \pm f$$

where f is the frictional resistance of the whole of the governor mechanism reduced to the sleeve. The *plus* sign is prefixed to f if the speed is increasing, and the *minus* sign if the speed is decreasing.

The regulation of the steam supply by means of a throttle valve

as introduced by Watt is still largely used, though the throttle valve is more usually of the double beat or equilibrium type. Another way of working is to control the cut-off in the cylinder by connecting the governor to some mechanism associated with the valves distributing the steam to the cylinder. A mechanism of this kind, as used on the Sulzer engine, is shown in detail in Fig. 170. In this case the rod Q is connected to the sleeve of the governor, and R is the end of an eccentric rod driven from a lay shaft geared to the main shaft of the engine. The movement of the end of the eccentric rod is constrained by two links, one of which, PA, is seen behind the other gear. It will be seen from the figure that a movement downwards of the end of the rod P opens the valve by means of the lever centred at A, and will continue to open it until the lever disengages from the trigger T. The fraction of the stroke at which this disengagement occurs is determined by the position in which the rod Q holds the trigger T relatively to the eccentric rod R. When the lever A disengages, the dashpot pushes the valve on to its seating and so cuts off the admission of steam. In the case of slide-valve engines fitted with separate cut-off valves, the governor sleeve is often connected with the cut-off valve through a link, so that a movement of the sleeve moves the block in the link and so alters the cut-off. In general it will be found that the steam supply to the engine is regulated either by means of a throttle valve which reduces the pressure of the steam supply without greatly reducing the quantity per stroke, or by changing the cut-off in the cylinder which alters the quantity of steam admitted per stroke without sensibly reducing the pressure.

The problem of automatically adjusting the power in the cylinder to the work required of the engine by regulating the cut-off of steam in the cylinder, has led to the invention of many ingenious mechanisms set in action by the movement of the sleeve of some form of governor.

The original Watt governor has developed into many forms—forms which, however, merely change the relation between the dynamical pull and the static push at the sleeve by modified forms of mechanism connecting the balls to the sleeve, and by adding to the sleeve additional loading above that applied by the weight of the governor balls.

The essential elements of a governor are—

1. A spindle connected to the engine crank shaft by belts or by gearing so that the angular velocity ratio between it and the crank shaft is constant. In shaft governors this spindle may be the shaft itself.

2. Masses symmetrically disposed about the spindle so that in all positions their common mass centre lies in the axis of revolution of the spindle, but so connected that each mass is free to move within limits in a radial plane containing its mass centre and the axis of revolution of the spindle.

3. A sleeve sliding along the governor spindle.

4. Mechanism connecting the balls or rotating masses with the governor spindle and with the sleeve so that for any assigned position of the balls there is one and only one corresponding position of the sleeve on the spindle.

5. Weights or springs producing the static load on the sleeve.

There are two data involved in the design of a governor of special importance, namely, the distance moved through by the sleeve from the bottom to the top stop, Δx say, and the change in speed, $\Delta\omega$, required to effect the displacement. The range of the displacement depends upon the kind of mechanism which the movement of the sleeve is required to set in motion. If the sleeve is connected to a throttle valve, a movement of $\frac{1}{8}$ of an inch may be sufficient to move the valve from the no-load to the full-load opening. On the other hand, a range of $1\frac{1}{2}$ inches may be required if the sleeve is connected to the motion block of a link. The design of a governor is conditioned by the specification of the ratio $\Delta\omega : \Delta x$, that is, the ratio of the change of speed corresponding to the displacement of the sleeve from one stop to the other stop. With a Watt governor, in which the static push is due to the weight of the balls alone, possible values of the ratio $\frac{\Delta\omega}{\Delta x}$ are very much restricted. If, however, the sleeve is loaded with additional weight or with springs, a governor can be designed to fulfil any given value of the ratio.

There is also another point to be borne in mind. The governor must be powerful in the sense that there is sufficient difference of energy in two positions of equilibrium to overcome all the resistance against which the movement from one position to the other is made, including all static and dynamical resistances. The greater the speed, the greater the difference of energy in two positions of equilibrium, other things being equal, because the energy stored varies as the square of the speed. The force which must be applied to the sleeve in a direction parallel to the spindle to hold the governor in a particular configuration when the governor is at rest is a convenient measure of the power of the governor in that configuration.

108. Speed of Equilibrium.—The accelerating force required to produce uniform rotation of a mass M in a circular path of radius r at an angular speed of ω radians per second is

$$M\omega^2 r$$

acting at the mass centre in a direction along the radius from the mass centre to the centre of the path.

The equilibrium of the forces acting on a governor mechanism can be reduced to a problem in statics by reversing the accelerating forces in direction and then considering the governor to be at r st.

Thus, referring to Fig. 110 of the Watt governor, the forces acting to produce equilibrium are, neglecting friction,

$M\omega^2 r$ reversed, and therefore acting outwards along the radius at the mass centre of each ball;

W, the weight of each ball acting downwards;
S, the weight of the sleeve and any load it carries.

The reversed accelerating force is usually known as the centrifugal force. The centrifugal force is, in fact, the reaction on the mechanism caused by the rotation of the balls in a circular path. The centrifugal force due to the rotation of the other members of the governor are negligibly small or are balanced.

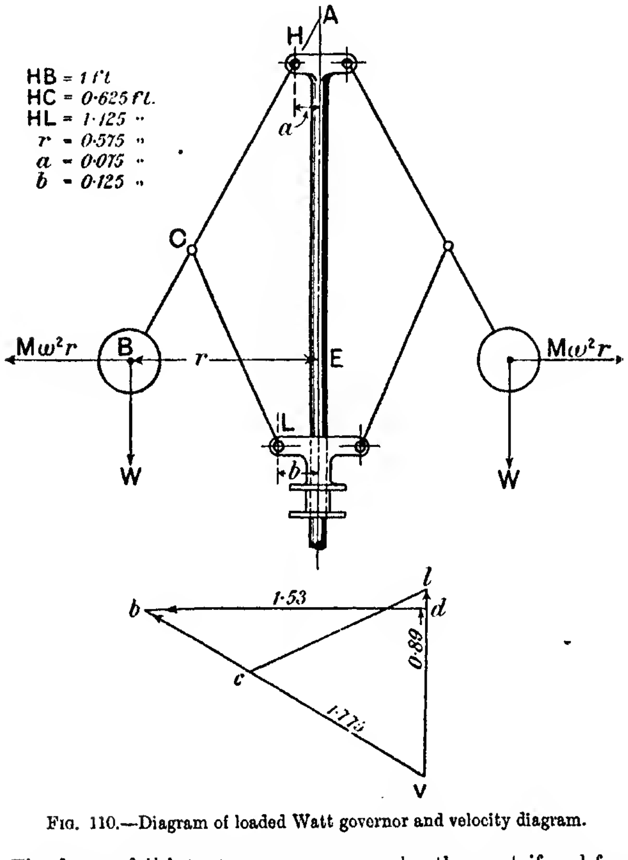


FIG. 110.—Diagram of loaded Watt governor and velocity diagram.

The forces fall into two groups, namely, the centrifugal forces, which are brought into existence by the mere rotation of the governor, and which rise and fall in magnitude as the speed of the governor rises and falls; and the static forces due to springs and dead weights.

It is convenient to reduce all these forces to the sleeve. The centrifugal forces from all the balls then combine into an equivalent

dynamical pull of the sleeve upwards towards the top stop, and the static forces combine into a **static push** of the sleeve downwards towards the bottom stop.

The reduction of the centrifugal forces to the sleeve is easily made by means of the principle of work.

Thus, assume a small change in the configuration of the governor from a position of equilibrium. Then the rate at which work is done by the centrifugal force acting at a ball is equal to that force multiplied by the velocity of its point of application in the direction of the force, which in this case is along the radius of the path of the mass centre of the ball and is horizontal.

Again, the rate at which work is done on the sleeve by the corresponding dynamical pull upwards is equal to the equivalent force P acting in the direction of motion of the sleeve, multiplied by the velocity of the sleeve.

But these rates of working must be equal, neglecting friction, therefore,

$$M\omega^2 r \times \text{radial velocity of mass centre} = P \times \text{velocity of sleeve.}$$

So that

$P = M\omega^2 r \times$ the velocity ratio of the radial velocity of the mass centre of the ball to the velocity of the sleeve along the spindle.

Similarly, it may be shown that the static load S_1 at the sleeve, equivalent to the weight W of a ball, is given by

$S_1 = W \times$ the velocity ratio of the vertical velocity of the mass centre of the ball to the velocity of the sleeve along the spindle.

These velocity ratios can be found most expeditiously for any mechanism between the ball and the sleeve by means of a velocity diagram.

Consider the equilibrium of the forces acting on the **Watt governor** in Fig. 110. The configuration of the governor is given by assigning the radius r. The velocity diagram for the mechanism in this configuration is drawn as follows :—

Choose any pole V, and from it set out Vb to represent to any convenient scale the velocity of displacement of the mass centre B of the ball in the plane of the arm and the spindle from the position defined by the radius r. The magnitude of the velocity is quite arbitrarily assumed, but the direction must be at right angles to the arm HB. Take c such that Vc : Vb = HC : HB. Then Vc is the corresponding velocity of the point C. The velocity of the point L as a point in the link CL is equal to the velocity of C plus the velocity of the point L about C. But the direction of motion of L about C can only be at right angles to the link CL. Therefore, from c in the velocity diagram draw cl at right angles to CL. Then the velocity of L is equal to the resultant velocity

of Vc and the velocity of unknown magnitude cl. But the direction of this resultant velocity is fixed by the mechanism, since the point L can move only parallel to the spindle. Therefore from V draw Vl parallel to the direction of motion of the sleeve, cutting cl in l. Then Vl is the velocity of the sleeve. Draw bd at right angles to LH. Then db is the radial velocity of the mass centre of the ball, and Vd is its vertical velocity. Then the velocity ratio which by multiplication reduces the centrifugal force acting radially at the mass centre of the ball to an equivalent dynamical pull upwards on the sleeve is $\frac{bd}{Vl}$, and the velocity ratio which by multiplication reduces the weight of the ball to an equivalent static weight on the sleeve is $\frac{Vd}{Vl}$. The equation of equilibrium is then

$$M\omega^2 r \frac{db}{Vl} = W\frac{Vd}{Vl} + S \quad . \quad . \quad . \quad . \quad . \quad (1)$$

from which ω can be found when W, r, S, and the configuration of the governor are given.

In this equation M may be taken equal to the mass of one ball, in which case W is the weight of one ball, and S is the static load at the sleeve per ball; or, if M is the mass of all the balls, then W is the total weight of the balls, and S is the actual load on the sleeve.

By way of example, suppose that the problem is to find what load must be carried by the sleeve, so that the configuration corresponding to the following dimensions may be maintained at a speed of two revolutions per second: $a = 0{\cdot}075$ ft., HB = 1 ft., $r = 0{\cdot}575$ ft., HC = 0·625 ft., HL = 1·125 ft., $b = 0{\cdot}125$ ft. The weight of each ball is 4 lbs.

The figures written against the lines in the velocity diagram correspond with a sleeve velocity of unity; they give, therefore, the velocity ratios of the various points in relation to the sleeve. Thus the ratio of the radial velocity of the mass centre of a ball to the velocity of the sleeve is 1·53, and the dynamical pull of each ball on the sleeve is $1{\cdot}53 M\omega^2 r$.

At two revolutions per second, $\omega^2 = 12{\cdot}56^2 = 158$; $M = \frac{4}{32} = \frac{1}{8}$; and $r = 0{\cdot}575$ ft.

The dynamical pull per ball is

$$\frac{1{\cdot}53 \times 158 \times 0{\cdot}575}{8} = 17{\cdot}4 \text{ lbs.}$$

Also the ratio of the vertical velocity of the ball to the velocity of the sleeve is from the diagram 0·89.

The weight of the ball reduced to the sleeve is then 3·56 lbs. And the equation of equilibrium is

$$17{\cdot}4 = 3{\cdot}56 + S$$

from which the load required on the sleeve per ball is 13·84 lbs.

A relation of similitude between the triangle Vdb in the velocity diagram and the triangle BEA in the governor mechanism itself

connects the velocity ratios with definite dimensions in the configuration of the governor itself.

Produce BH to meet the vertical axis in A. Then, since Vb is at right angles to AB, bd at right angles to the governor spindle, and Vd at right angles to the radius r, the triangles Vdb and BEA are similar. Therefore

$$\mathrm{BE} : \mathrm{EA} = \mathrm{V}d : db$$

The dimension EA is called the height h of the governor, and BE is the radius of the path in which the balls move; so that

$$\frac{\mathrm{V}d}{db} = \frac{r}{h} \quad \text{. . . . . (2)}$$

This relation gives simple expressions for the speed of equilibrium in particular cases.

Case 1.—The equilibrium of a Watt governor neglecting the load on the sleeve.

When the load S on the sleeve is neglected, S = 0, and (1), the equation of equilibrium, becomes

$$\frac{\mathrm{W}}{g}\omega^2 r \frac{db}{\mathrm{V}l} = \mathrm{W}\frac{\mathrm{V}d}{\mathrm{V}l}$$

that is,
$$\frac{\omega^2 r}{g} = \frac{\mathrm{V}d}{db} = \frac{r}{h}$$

from which
$$\omega^2 = \frac{g}{h} \quad \text{. . . . . (3)}$$

a result which shows that when the static load is due only to the weight of the governor balls, the height of the governor h is independent of the weight of the balls, and depends only upon the speed of rotation.

Case 2.—The joint H may be arranged on the axis of rotation, in which case h is measured along the spindle always from a fixed point on the axis as the configuration changes, instead of from a point A, which changes its position as the configuration changes. The relation in (3) remains unchanged.

The peculiarity of the **Porter Governor** is that a weight is carried by the sleeve which is much greater than the weight of the balls, and the mechanism connecting the sleeve with the balls and the balls with the vertical spindle consists usually of equal links jointed with the ball, the sleeve, and the spindle, as shown in the diagram, Fig. 111.

The velocity diagram for the configuration shown is drawn below the governor. From this it will be readily seen that Vl in all configurations is equal to 2Vd.

Using, this relation in equation (1) the equation of equilibrium becomes

$$\frac{\mathrm{W}}{g}\omega^2 r \frac{db}{2\mathrm{V}d} = \frac{\mathrm{W}}{2} + \mathrm{S}$$

But the triangle Vdb is similar to the triangle BEA. Therefore

$$\frac{Vd}{db} = \frac{BE}{EA} = \frac{r}{h}$$

So that

$$\frac{db}{2Vd} = \frac{h}{2r}$$

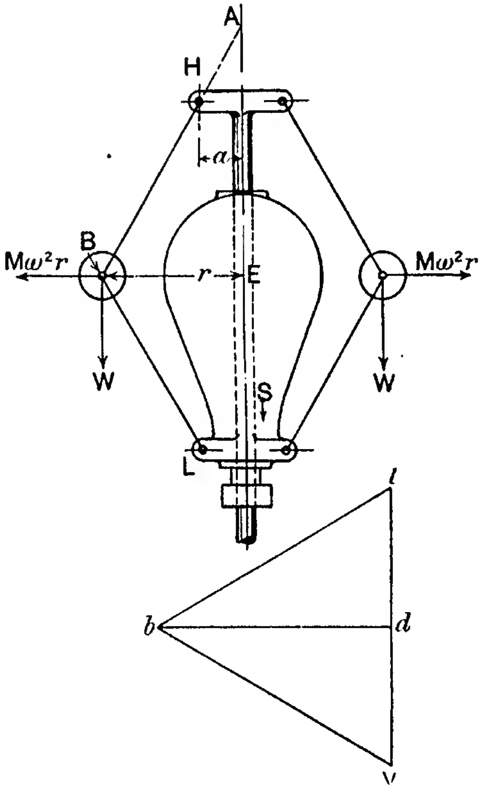


FIG. 111.—Diagram of Porter governor with velocity diagram.

Finally, therefore, the equation of equilibrium for this type of governor is

$$\frac{W}{g}\omega^2\frac{h}{2} = \frac{W}{2} + S$$

and

$$\omega^2 = \frac{(W + 2S)g}{hW} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

A **Proell governor** is shown in Fig. 112. The balls in this case are carried on upward prolongations of the bottom link.

The velocity diagram for the configuration shown is drawn as follows:—

The directions of the velocities of two points in the lower link BCL are given, and therefore the velocity image of the link can be drawn. Thus the point C must move in a direction at right angles

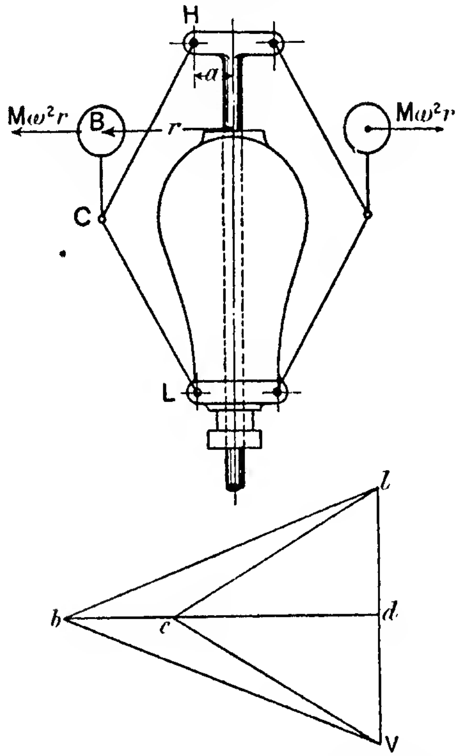


FIG. 112.—Diagram of Proell governor and velocity diagram.

to CH, and the point L must move in a direction parallel to the spindle. Hence, choose any origin V, and set out Vc at right angles to CH, Vl parallel to the spindle and equal to unity, and draw cl at right angles to CL, since the velocity of C relative to L must be at right angles to CL. The line cl is the velocity image of the part CL of the link BCL. To complete the image set out the angle lcb equal to the angle LCB, and draw lb at right angles to LB. If CB is parallel to the spindle as in the figure, then dcb is a straight line, and lcb is the complete velocity image of the link LCB, and therefore,

joining b to the origin, Vb is the velocity of the ball B. The component velocities along and at right angles to the spindle are Vd and db.

The velocity ratio which reduces the centrifugal force at the balls to the dynamical pull at the sleeve is then $\frac{db}{\mathrm{V}l}$, and the velocity ratio which reduces the weight of the balls to a static push down on the sleeve is $\frac{\mathrm{V}d}{\mathrm{V}l}$.

The equation of equilibrium is

$$\frac{\mathrm{W}}{g}\omega^2 r\frac{db}{\mathrm{V}l} = \mathrm{W}\frac{\mathrm{V}d}{\mathrm{V}l} + \mathrm{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The Spring Governor.—The static push down on the sleeve due to the weight of the balls and to the load on the sleeve may be supplemented by the addition of a spring; in many cases the static push due to the weight of the balls and to the weight of the sleeve is so small relatively to the force applied to the sleeve by spring loading that the whole static push may be regarded as due to spring loading alone.

An outline sketch of a **Hartnell**[1] **spring governor** is shown in Fig. 113. The balls are carried on the vertical ends of a pair of bell crank levers pivoted at a distance from the axis approximately equal to the mean radius at which the balls are required to run. It will be understood from the sketch that the weight of the balls reduced to the sleeve is zero when the radius of the path in which they rotate is equal to the common radius of the axes of the bell cranks, and that the weight contributes a small force which is positive if the radius is smaller, and negative if greater than this.

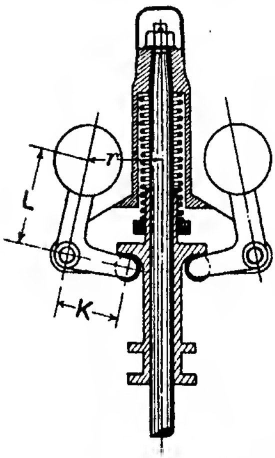


FIG. 113.—Diagram of Hartnell governor.

The static push is exerted by a spring acting directly on the sleeve. In this case S, instead of being nearly constant as in the Porter and Proell governors, varies as the displacement of the sleeve varies.

The spring necessary to determine given conditions of equilibrium is specified by two constants, namely—

1. The force a which the spring must exert on the sleeve for a given configuration, generally the configuration corresponding to the sleeve on the lower stop.

2. The force b required to compress (or extend) the spring through unit distance.

[1] *Proc. Inst. Mech. Eng.*, 1882.

Then if x is the displacement of the sleeve along the spindle measured from the point where the force exerted on the sleeve is a, the force acting on the sleeve in any position is $a + bx$ lbs.

If the effect of the weight of the balls is neglected, the equation of equilibrium is

$$M\omega^2 r\frac{dr}{dx} = a + bx \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But the velocity ratio $\frac{dr}{dx}$ is equal to the ratio of the arms $\frac{L}{K}$, where L is the distance from the axis to the mass centre of the ball, and K is the distance from the axis to the point where the arm engages with the sleeve. This ratio is in most designs practically constant over the range of action of the governor.

Making this substitution, the equation of equilibrium becomes

$$M\omega^2 r\frac{L}{K} = a + bx \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

To find a and b data must be given from which a pair of simultaneous equations can be formed.

EXAMPLE.—Find the spring required to control the governor Fig. 113, so that the sleeve just comes into contact with the top stop when the speed reaches 320 revolutions per minute, and with the bottom stop when the speed falls to 280 revolutions per minute. The mean radius is 4 ins.; the radius at the top stop, $4\frac{3}{4}$ ins.; the radius at the bottom stop, $3\frac{1}{4}$ ins.; each ball weighs 4 lbs.; $\frac{L}{K} = \frac{3}{2}$. The range of movement of the sleeve is therefore $\frac{1}{12}$ ft.

The two necessary equations of equilibrium are—

At the top stop, $$M\omega_1^2 r_1\frac{L}{K} = a + bx_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

At the bottom stop, $$M\omega_2^2 r_2\frac{L}{K} = a + bx_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Subtracting,

$$b = \frac{ML}{K}\left(\frac{\omega_1^2 r_1 - \omega_2^2 r_2}{x_1 - x_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Which from the given data becomes, since $x_1 - x_2 = \frac{1}{12}$ ft.

$$b = \frac{4}{32}\cdot\frac{3}{2}\left(\frac{443 - 233}{\frac{1}{12}}\right) = 472 \text{ lbs. per ball}$$

For two balls the spring must be of such elastic strength that a force of 944 lbs. is required to compress it one foot. This is equivalent to 78·7 lbs. per inch.

The constant a corresponds to a particular configuration denoted by x. If a is the force exerted on the sleeve at the lower stop, then this position may be conveniently chosen for the origin from which to measure x. Then $x_2 = 0$. And from equation (9) with the

given data, a works out to 43·7 lbs. per ball; that is 87·4 lbs. for two balls. This means that the spring must be so adjusted in length that when forced into its position in the governor, it will press the sleeve on to the lower stop with a force of 87·4 lbs. Adjusting nuts and washers are provided to enable a final regulation to be made.

A second case of spring loading may be considered, since it will serve both to illustrate what is called a **Shaft governor** and also to show how a supplementary controlling spring may be applied to regulate the speed by hand whilst the engine is running.

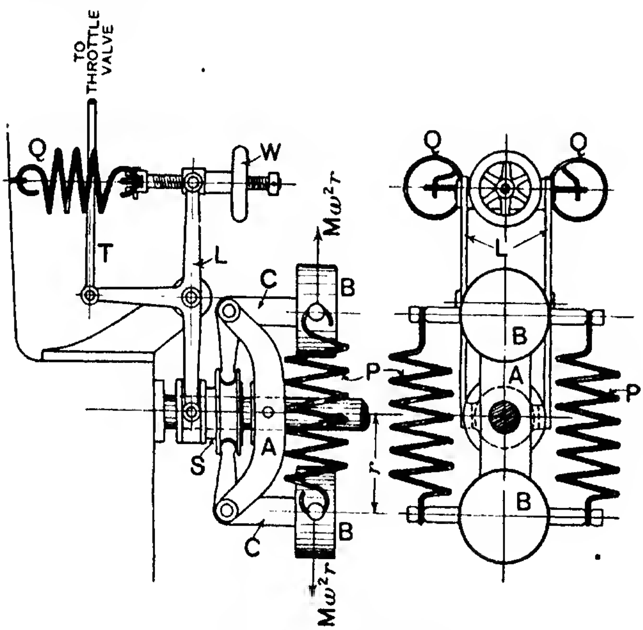


FIG. 114.—Diagram of Shaft governor.

The term **Shaft governor** is applied in general to any form of governor which is constructed on the crank shaft itself. A diagrammatic sketch of a shaft governor connected to a throttle valve together with a supplementary controlling spring is shown in Fig. 114. The crank shaft is prolonged through the end bearing, and is turned down to take the sleeve S. A cross piece A is fixed rigidly to the shaft, and its forked ends carry two bell crank levers C, C, engaging at their inner ends with the grooved sleeve, and carrying at their outer ends cylindrical masses B, B. A double bell crank lever L, pivoted to a bracket on the engine frame, engages at the lower end with the sleeve S, and at the horizontal end with

the vertical rod T, which is connected directly to the throttle valve. Springs connect the upper end with the engine frame through the bolt and adjusting hand wheel W.

When the masses B, B, move outwards, the sleeve is moved towards the right stop, and the throttle valve rod T is moved downwards, and thereby reduces the quantity of steam flowing through the throttle valve. There are four springs controlling the outwards motion of the masses B, B, namely, P, P, connecting the masses directly, and Q, Q, attached to the upper end of the lever L.

The springs P, P, act to push the sleeve towards the left stop, whilst the dynamical pull acts towards the right stop. The springs Q, Q, act against the springs P, P; so that by turning the handwheel W to increase the tension of the springs Q, Q, the statical push on the sleeve towards the left stop is reduced, and the equilibrium speed for an assigned configuration is reduced.

The regulating spring is used to bring the speed back to the mean speed after a change in the load has caused a change in the configuration. For example, if the engine is running steadily at half-load with the throttle valve open $\frac{1}{4}$ in., and an increase of load necessitates an increased opening to $\frac{5}{16}$ in., the governor must change its configuration so that the radius of the path in which the masses revolve is reduced, say from 4 ins. to 3·9 ins. But this reduction of the radius cannot take place without a decrease of speed. If now in this configuration the static load on the sleeve is increased, the configuration can only be maintained by an increase in the dynamical pull, and this increase can only be obtained by an increase of speed. Thus, by the aid of a supplementary controlling spring, the mean speed of the engine can be maintained by hand regulation through variations of load from nothing to the maximum load.

In a position of equilibrium, the static push on the sleeve is due to the sum of the forces exerted by all the springs connected to the sleeve either directly or through levers. Let there be two springs, and let the static push of one of them be $a + bx$, x being reckoned from the bottom stop, and let the supplementary spring exert the force $a' + b'x$, both springs for the moment being assumed to act directly on the sleeve. Then the equations of equilibrium are—

$$\text{Top stop,} \qquad M\omega_1^2 r_1 \frac{dr_1}{dx} = a + bx_1 + a' + b'x_1 \quad . \quad . \quad . \quad (11)$$

$$\text{Bottom stop,} \qquad M\omega_2^2 r_2 \frac{dr_2}{dx} = a + a' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Let the supplementary spring be such that $a' = qa$ and $b' = pb$. Then from (12)—

$$a = \frac{M\omega_2^2 r_2 \dfrac{dr_2}{dx}}{1 + q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

And subtracting (12) from (11)—

$$b = M\frac{\left(\omega_1^2 r_1\frac{dr_1}{dx} - \omega_2^2 r_2\frac{dr_2}{dx}\right)}{(1+p)x_1} \quad . \quad . \quad . \quad . \quad (14)$$

where x_1 is the range of movement from the bottom to the top stop. Values of a and b can be calculated when values of p and q are assigned.

The supplementary spring, hitherto assumed to be acting directly on the sleeve, is now to be replaced by a spring acting indirectly through levers, and it must be of such strength that the force which it exerts on the sleeve through the levers is equal to the force which would have been exerted by the direct-acting spring which it replaces. The advantage of this new position is that the tension can be adjusted whilst the engine is running. In this new position let its extension now be measured by the variable y, chosen so that $y = 0$ when $x = 0$. And, further, for the moment assume that the mechanism between the spring in its new position and the sleeve is such that $y = x$.

With this assumption it is easy to calculate the change in length which must be made with the hand wheel in order to bring the speed back to the mean speed without changing the configuration into which the governor passes in consequence of a change of load. All that is necessary is to calculate the magnitudes of the dynamical pull in the new configuration for the speed it assumes with the normal loads on the spring, and then again for a speed equal to the mean speed. The difference is the additional force which must be applied to the sleeve to maintain the equilibrium in the new configuration at the mean speed, and this difference is to be applied by changing the tension of the supplementary spring. Of course, after this adjustment the speeds at all configurations are changed, but when the load changes again a further adjustment of the wheel is all that is necessary to bring the engine up to the mean speed again.

The difference between the dynamic pulls at constant configuration but at two speeds, one being the mean speed ω_m and the other the speed proper to the configuration, is—

$$Mr\frac{dr}{dx}\left(\omega_m^2 - \omega^2\right) = \Delta y \, . \, b' \quad . \quad . \quad . \quad . \quad (15)$$

from which the amount by which the length of y must be changed can be calculated.

When a spring designed for direct action on the sleeve is replaced by a spring acting indirectly through levers, the forces exerted by each spring on the sleeve will be alike when the constants a, b, of the spring acting directly are related to the constants A, B, of the spring acting indirectly in the following manner—

$$A = a\rho$$
$$B = b\rho^2$$

where ρ is the ratio between the velocity of the sleeve and the velocity of extension of the substituted spring.

This is shown as follows:—

Let the law of the spring for direct action on the sleeve be $a + bx$.

Let the law of the spring which replaces it be $A + By$.

Suppose a small displacement of the sleeve Δx, and let Δy be the corresponding extension of the spring which is acting indirectly. Then the work done by a spring designed for direct action, namely $(a + bx)\Delta x$, must be equal to the work done by the spring acting indirectly, namely $(A + By)\Delta y$, therefore

$$(a + bx)\Delta x = (A + By)\Delta y$$

From which

$$A = \frac{a\Delta x}{\Delta y}$$

$$B = \frac{bx\Delta x}{y\Delta y}$$

But $\frac{x}{y} = \rho$ is the velocity ratio of the movement of the sleeve and the extension of the spring which is acting through leverage; and $\frac{\Delta x}{\Delta y}$ is also the same velocity ratio, therefore

$$A = a\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$B = b\rho^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Example.—Determine the constants of the springs required for a governor of the kind shown in Fig. 114 from the following data. The movement of the throttle valve corresponding to a change from no load to full load is $\frac{1}{10}$ in. The valve is connected to the sleeve without change of velocity ratio, so that the movement of the sleeve from no load to full load is also $\frac{1}{16}$ in. The change of speed from no load to full load is to be from 455 to 435 revolutions per minute. The radius of the path of the balls is to be 4 ins. at the lower speed, and in this configuration the ball arm of the bell crank is parallel to the shaft. The ball arm of the bell crank is 4 ins. long, and the sleeve arm $2\frac{1}{2}$ ins. long. Each of the rotating masses weighs 8 lbs. Further, let $p = -\frac{1}{5}$, and let $q = -\frac{1}{10}$, a negative sign being prefixed because the regulating spring acts not with but against the springs at the balls.

The velocity ratio $\frac{dr}{dx}$ is constant throughout the range of action of the governor, and is equal to $\frac{4}{2\cdot5} = 1\cdot6$. The radius of the path of the mass centres of the balls corresponding to the higher speed is 4 ins. plus an increase corresponding to $\frac{1}{16}$ in. movement of the sleeve, namely, 0·1 in. The conditions of the problem may now be put in Schedule form.

Higher speed, no-load conditions.	Lower speed, full-load conditions.
$r = 4\cdot1$ ins. $= 0\cdot341$ ft.	$r = 4$ ins. $= \frac{1}{3}$ ft.
$n = 455$ rev. per min.	$n = 435$ rev. per min.
$\omega^2 = 2270$	$\omega^2 = 2070$

$$x_1 = \frac{1}{16}$$

Using these data, find first the constants a, b, for a main spring, and the constants a', b', for a regulating spring, both for direct action on the sleeve, per ball. From equation (13), substituting therein the data for the lower speed, $a = 307$ lbs. per ball; from (14) $b = 672$ lbs. per inch per ball.

And since from the data $a' = -0 \cdot 1a$ and $b' = -0 \cdot 2b$, the constants of the regulating springs are $a' = 31$ lbs. per ball and $b' = 134$ lbs. per inch per ball.

A pair of regulating springs, each with the constants $a' = 31$ and $b' = 134$, arranged to act against a pair of main controlling springs, each with the constants $a = 307$ and $b = 672$, and both pairs acting directly on the sleeve, together exert a resultant static push which fulfils the conditions of the problem.

But, as will be seen from the figure, the pair of main controlling springs is to be replaced by a pair of springs acting at the balls.

The ratio ρ, between the velocity of the sleeve and the velocity of extension of the spring, remembering that the change in length of the spring is measured by the change in length of the diameter of the path of the balls is, $\frac{dx}{2dr}$, 0·313; and the square of this ratio is 0·098. The constants A and B of each of the pair of springs connected across to the balls are then

$$A = 0 \cdot 313 \times 307 = 96 \text{ lbs.}$$
$$B = 0 \cdot 098 \times 672 = 65 \cdot 2 \text{ lbs. per inch}$$

The springs are to be so adjusted in length that when the diameter of the path of the balls is 8 ins. the tension in each spring is 96 lbs.

The pair of regulating springs are to be replaced by a pair acting at the end of a lever, at a point 7 ins. from the axis of the lever, while the sleeve is connected at $5\frac{1}{2}$ ins. from the axis. The velocity ratio between the sleeve and the extension of the spring is therefore 0·786, and the square of this is 0·618. The constants A′, B′, of each of the pair of regulating springs are therefore

$$A' = 0 \cdot 786 \times 31 = 24 \text{ lbs.}$$
$$B' = 0 \cdot 618 \times 134 = 83 \text{ lbs. per inch}$$

The springs must be adjusted in length, so that when the radius of the path of the balls is 4 ins. the tension in each of the springs is 24 lbs.

It will be understood that the speed will be increased if the regulating springs are slackened, and decreased if the tension is increased.

Suppose now that the engine is running at a speed of 445 revolutions per minute, corresponding to a configuration in which the radius of the path of the balls is 4·05 ins. approximately. This speed may be regarded as the mean speed at which the engine is required to run. With the springs set as above this would determine a throttle valve opening for about half-load conditions. Suppose, further, that the load is increased, and that the increase of load

requires just as much more steam as can pass the throttle valve when the configuration changes to $r = 4$ ins., corresponding to a drop from the mean speed of 445 to the speed of 435 revolutions per minute. How much must the regulating spring be slacked to bring the speed back to 445 revolutions per minute without changing the position of the throttle valve?

The answer to this is found by solving equation (15), page 382, for Δy, remembering to use for b', the constant of the spring, the value 134; also $r = 0{\cdot}333$ ft., $\omega^2_m = 2170$, and $\omega^2 = 2070$. With these data $\Delta y = 0{\cdot}1$ in. at the sleeve. This corresponds to $\Delta y = 0{\cdot}1 \times \dfrac{7}{5{\cdot}5} = 0{\cdot}12$ in. slacking of the actual regulating springs.

This calculation sufficiently illustrates the process of designing a governor of this kind. The final adjustment of the speed is always, of course, made by hand, since the regulating spring enables the speed to be varied between wide limits.

Regarding the springs themselves, the approximate relations between the constant B and the dimensions of the spring are as follows:—

B is the load in pounds required to produce an extension of one inch in a helical spring coiled from a piece of round steel rod L inches long and d inches diameter into a regular helix, whose diameter is D inches from centre to centre of the coils.

$$B = \frac{d^4}{LD^2} \times 45 \times 10^5 \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If the diameter of the rod is reckoned in tenths of an inch, the expression reduces to the slightly more convenient form

$$B = \frac{450d^4}{LD^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Thus if a spring is coiled from a piece of round steel rod $0{\cdot}25$ in. diameter and 40 ins. long into a regular helix 3 ins. diameter,

$$B = \frac{450 \times 2{\cdot}5^4}{40 \times 3^2} = 48{\cdot}7 \text{ lbs. per inch.}$$

If T is the maximum torque in inch-lbs. brought upon the coils of the spring when compressed (or extended) to its maximum amount, the diameter in inches corresponding to a shearing stress of 25 tons per square inch may be calculated from

$$d = \tfrac{1}{22}\sqrt[3]{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

These rules only apply to springs in which the diameter of the rod is small compared with the diameter of the coils.

109. Stability of Equilibrium.—A governor will run in the configuration of equilibrium determined by equality between the dynamical pull P and the static push S only if the equilibrium is stable.

The equilibrium is stable if when displaced from a configuration

of equilibrium by the momentary application of an external force the governor comes back to the configuration of equilibrium, either by a gradual approach to it, or by an oscillation about it which gradually subsides into the configuration of equilibrium.

When the governor is displaced from a configuration of equilibrium without change of speed, the equality between the dynamical pull on the sleeve P and the static push on the sleeve S is destroyed, and the condition of stability is, that the difference between these two forces produced by the displacement should be a force whose direction of action is always towards the equilibrium position of the sleeve.

Let O (Fig. 115) be an equilibrium position of the sleeve.

Set out OP equal to the dynamic pull, and OS equal to the static push. In this position the point P will coincide with the point S, since OP = OS for equilibrium.

Draw the curve pPp, representing for various positions of the sleeve the dynamic pull calculated for a constant ω, the speed of equilibrium

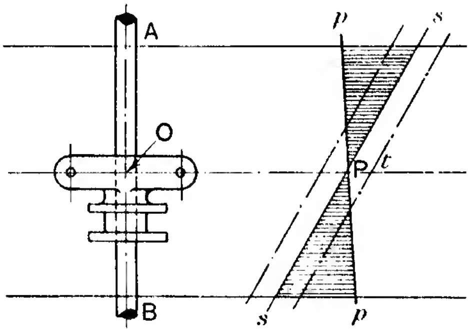


FIG. 115.—Stability and restoring force.

at O. The curve pPp is the dynamic pull plotted against the position of the sleeve. Also draw sPs, representing the static push plotted against the position of the sleeve.

Then it will be evident that if the sleeve is displaced up into the position A, there will be a force $As - Ap$ tending to push it down again. And if it is displaced into the position B, there will be a force $Bp - Bs$ tending to push it up again.

The width of the shaded areas represents the magnitude of the restoring force corresponding to displacements of the sleeve about the position of equilibrium O.

If the governor is required to run in equilibrium in the configuration corresponding to the sleeve position A, then the speed must be increased until the dynamic pull is equal to As, thus converting the position into one of equilibrium. If, on the other hand, the position B is to be one of equilibrium, then the speed must be reduced until $Bp = Bs$.

If in the position A the difference representing the restoring force

is small, the return to the position of equilibrium is slow. If there is no difference, that is to say, if the curve *pp* coincides with the curve *ss*, there will be no restoring force, and the governor will continue to run in equilibrium in the new position.

If these curves coincide along the whole range of movement of the sleeve, the governor will run in equilibrium in any position of the sleeve. It is then said to be an **Isochronous governor**

The frictional resistance, however, prevents isochronism being realized in practice even if it were desirable to do so. For suppose that the design is such that, if friction were neglected, the curves *pp* and *ss* coincide through the whole range of action, displacement from any one position of equilibrium upward could only occur when the dynamic pull P is increased by increase of speed to a magnitude greater than the static push there by an amount equal to the frictional resistance. When this speed is reached, the sleeve will move up and continue moving until it reaches the top stop, because the speed is now higher than the speed of isochronism corresponding to equality between P and S.

Similarly for a movement down. Movement will not begin until the speed has dropped sufficiently to produce a difference between the static push and the dynamic pull equal to the frictional resistance. When the speed has fallen to this, the sleeve will fall down on to the bottom stop.

An isochronous governor is in fact unstable, and it is rendered unstable by the frictional resistance.

The effect of friction on the equilibrium of a stable governor can be illustrated by means of the diagram Fig. 115. Suppose that the frictional resistance to change of configuration is reduced to a force f at the sleeve, and assume further that the force is constant. Set out on either side of the curve *ss* the dotted lines shown, each at a distance from the curve equal to f.

Then if the governor is running in equilibrium and at the equilibrium speed in the position O, the speed must be increased at constant radius until the dynamical pull has increased to Ot before the sleeve will begin to move up, and it will continue to move up, but against the much smaller kinetic friction, until it finds a position of equilibrium between the dynamical pull and the static push. The quantitative study of the effect of friction is more easily made in connection with the diagram explained in the next article.

110. The Rith Diagram.—Diagrams are a great aid in the study of the action of a governor. The diagram given by L. Rith[1] is particularly useful. The essential feature of the Rith diagram is that the forces at the sleeve are plotted against the square of the angular velocity of the governor spindle.

The advantage of this lies in the fact that the graph of the

[1] "Les Regulateurs à force Centrifugal. Remarques générales sur leur Stabilité et sur leur reglage par L. Rith." *Memoirs et Compte Rendu de Travaux de la Société des Ingenieurs Civils de France*, Bulletin de Septembre 1905, No. 9.

dynamical pull plotted against ω^2 is, for a given configuration, a straight line. Thus

$$\text{Dynamical pull} = P = M\omega^2 r\frac{dr}{dx}, \text{ as explained above.}$$

And this plotted against ω^2 gives a curve whose slope is

$$Mr\frac{dr}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a given configuration r and $\frac{dr}{dx}$ are both constant; therefore the curve becomes a straight line, and the line passes through the origin.

Further $\qquad r\frac{dr}{dx} = \frac{1}{2}\frac{dr^2}{dx}$

so that the slope is given generally by

$$\frac{1}{2}M\frac{dr^2}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Rith calls this the angular coefficient for a given configuration. The dynamical pull for any speed is therefore found by multiplying the square of the angular velocity by the angular coefficient.

To apply Rith's method, the square of the radius of the path in which the mass centres of the balls move must first be found as a function of the sleeve displacement x. This square is then differentiated with regard to x. The resulting differential coefficient multiplied by $\frac{1}{2}M$ gives a general expression for the slope.

Consider as an illustration the case of an equal-arm Porter governor, of the type shown in Fig. 111, page 376, but so arranged that the arms are pivoted on the axis of rotation. Let a be the length of an arm. Take the origin of displacement at A, and taking x as the distance from this origin to the sleeve, and a as the length of each link, the geometry of the figure gives

$$r^2 = a^2 - \frac{x^2}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{dr^2}{dx} = -\frac{x}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substituting this in (2), the slope of a "Line of Configuration" is

$$\frac{1}{4}Mx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If, for example, $M = \frac{4}{32}$ and $x = 1{\cdot}732$, the slope of the line of configuration is 0·054.

It is generally more convenient to assume a value for ω^2 and then to plot the corresponding value of P, in this way getting a point on the line; the origin is another point; therefore the line is determined.

Thus in Fig. 116, set out the angular velocity squared downwards from the origin O. When $\omega^2 = 1000$, P, the dynamical pull on the sleeve, is for $x = 1{\cdot}732$; $1000 \times 0{\cdot}054 = 54$ lbs. Choose, therefore, any convenient force scale and set out this value horizontally to

the right of the speed axis, fixing thereby a point q. Then the line through O and q is the line of configuration for $x = 1{\cdot}732$.

The diagram is completed by plotting the curve of static push also to the right of the speed axis. For the present assume this to be constant in all configurations, as in an equal-armed governor of the Porter type. Then ss, drawn parallel to the speed axis at a distance from it equal to the load on the sleeve, including the weight of the balls themselves reduced to the sleeve, will be the graph of the static push.

Properties of the Diagrams.

(1) *To find the Speed of Equilibrium corresponding to a Given Radius.*—There will be equilibrium when the dynamical pull is equal to the static push; therefore, a horizontal line drawn through a (Fig. 116), the point at which ss, the graph of the static push, intersects Oq, the graph of the dynamical pull for the given radius, will cut the vertical speed axis at the speed of equilibrium. Then the speed of equilibrium for the loading assumed is $\sqrt{On}$.

Alternatively, the load required to determine equilibrium at a stated speed can be found by reversing the construction.

Let OB (Fig. 116) be the line of configuration corresponding to the bottom stop, and OT the line for the top stop. Then the speed at which the sleeve will begin to leave the bottom stop is $\sqrt{On_1}$. The speed at which it will just come in contact with the top stop is $\sqrt{On_2}$. The change of speed which will take the governor through the whole range of its action is $\sqrt{On_2} - \sqrt{On_1}$.

The speeds corresponding to the points n, n_1, and n_2 are those for which the governor runs in its three principal positions.

The ratio between the mean speed and the change of speed required to carry the sleeve from the bottom to the top stop is called by Rith the **Isochronous Ratio.**

The isochronous ratio is given by

$$\frac{\sqrt{On}}{\sqrt{On_2} - \sqrt{On_1}} = \frac{\sqrt{On}(\sqrt{On_2} + \sqrt{On_1})}{On_2 - On_1}$$

and since in practice the speeds are close together,

$$\sqrt{On_2} + \sqrt{On_1} = 2\sqrt{On}$$

with sufficient accuracy, so that the isochronous ratio becomes

$$\frac{2 \times On}{n_1 n_2}$$

The smaller the speed required to carry the sleeve through the range of its action, the greater the isochronous ratio. In an isochronous governor the distance $n_1 n_2$ is equal to zero. Therefore, in this particular case the isochronous ratio is infinity.

(2) *To Examine the Stability of a Governor.*—As explained in the last section, the condition of stability is, that when the sleeve is displaced from its position of equilibrium without changing the speed of rotation, the difference between the dynamical pull and the static

push produced by the displacement shall have a direction towards the position of equilibrium.

To examine on the diagram if this condition is fulfilled, set out configuration lines, one corresponding to a sleeve position above the position of equilibrium, and the other to a position below the position of equilibrium. For example, in Fig. 116 let Ot be the configuration line for the top stop, and Ob for the bottom stop.

First assume that the graph of the static push is a vertical line as for loaded governors.

A change at constant speed is in the diagram represented by a horizontal line.

Let na be a horizontal line corresponding to the speed of equilibrium in the configuration Oa.

If the sleeve is displaced to the top stop, the dynamical pull decreases to nt, whilst the static push remains constant at na. There is, therefore, a restoring force, ta, tending to push the sleeve down to the equilibrium position.

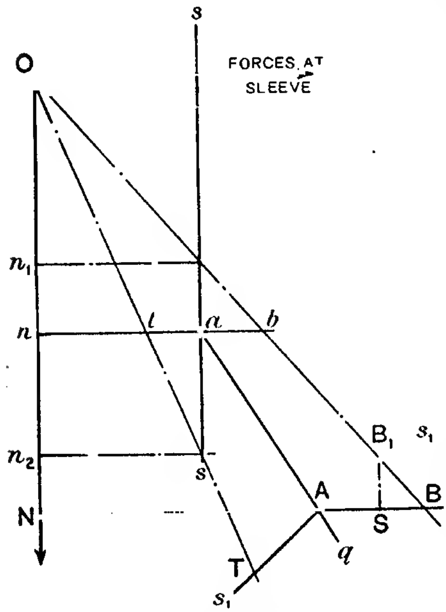


Fig. 116.—Rith diagram.

If the sleeve is displaced to the bottom stop, the dynamical pull increases to nb, whilst the static push remains constant at na. There is, therefore, a restoring force, ba, tending to pull the sleeve up to the position of equilibrium.

The equilibrium is therefore stable. It is clear that the condition of stability when the graph of the static push is a vertical line is that the configuration line for a higher sleeve position should fall below the configuration line for the position of equilibrium.

In the example of the Porter governor considered above, the slope varies as x; and as the sleeve moves upwards x diminishes, and therefore the slope of the lines of configuration diminish, and hence for the upper stop the configuration line falls below the line for the mean configuration. There is always a restoring force acting towards the position of equilibrium, and therefore the equilibrium of a governor of this type is always stable.

The closer the configuration lines lie together the smaller the restoring force. When they merge into one another there is no restoring force, and the governor is isochronous.

In the more general case where the graph of the static push is inclined or is a curve, the test for stability can be applied just as easily as for the simple case of constant gravity loading.

Thus, in Fig. 116, let s_1s_1 be the graph of the static push instead of the vertical line ss. That is to say, imagine the dead weights to be replaced by some form of spring loading which gives the curve s_1s_1. Then the equilibrium point is A; a horizontal through A gives the equilibrium speed $\sqrt{ON}$. When the sleeve is displaced to the bottom stop NB is the dynamic pull, and the horizontal distance from B_1 to the axis ON, equal to NS, is the static push on the sleeve. The dynamic pull is greater, and the sleeve will tend to return to its position of equilibrium. In this position the equilibrium is stable.

It is easily seen that the following general conditions hold:—

The equilibrium will be stable if the graph of the static push cuts a line of configuration corresponding to an increased radius of the path in which the balls revolve, in a point below the horizontal line of constant speed drawn through the equilibrium point A.

The equilibrium will be unstable if the point of intersection is above this horizontal line.

The equilibrium will be neutral if the graph is horizontal, and the governor will be isochronous.

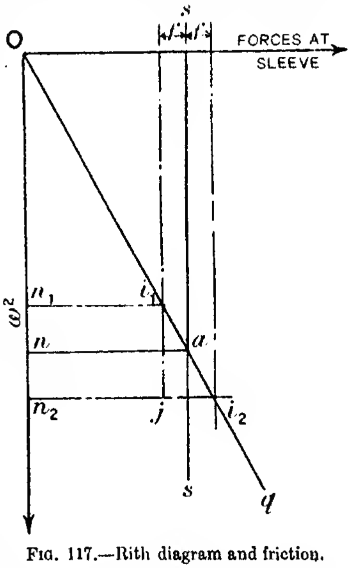


Fig. 117.—Rith diagram and friction.

(3) *The Effect of Friction on the Equilibrium.*—Let f be the frictional resistance of the whole of the governor mechanism reduced to the sleeve. Draw parallels on either side of ss at distances equal to f, as in Fig. 117. Then the sleeve will not begin to move upwards from its position of equilibrium until the dynamic pull has increased to the total magnitude n_2i_2. It will not commence to move down until the dynamic pull has decreased to n_1i_1. There can, therefore, be a total variation of dynamic pull ji_2, and a variation of speed n_1n_2 without any movement of the governor sleeve.

Rith defines the **sensibility** of a governor as the ratio of the mean speed to the change of speed required to move the sleeve from rest

against the frictional resistances; that is, referring to Fig. 117, to the ratio

$$\frac{\sqrt{On}}{\sqrt{On_2} - \sqrt{On_1}}$$

which, since the speeds differ so little, is given by

$$\frac{2 \times On}{n_1 n_2}$$

An important principle of design can be obtained from the figure.

It is clear that approximately

$$On : na = \tfrac{1}{2}\, n_1 n_2 : f$$

Now na is the static force applied to the sleeve to hold the governor in the configuration corresponding to the line Oq, and this force is defined as the **power** of the governor for that particular configuration. It follows that

$$\text{Power} = f \times \frac{2On}{n_1 n_2}$$
$$= f \times \text{sensibility}$$

From this it will be seen that the sensibility increases with the power assuming constant friction, hence the desirability of using a powerful governor.

It will also be readily understood from Fig. 116 that the restoring force increases with the power, so that the rapidity of action of the governor is greater the greater the power, another reason for using a powerful governor.

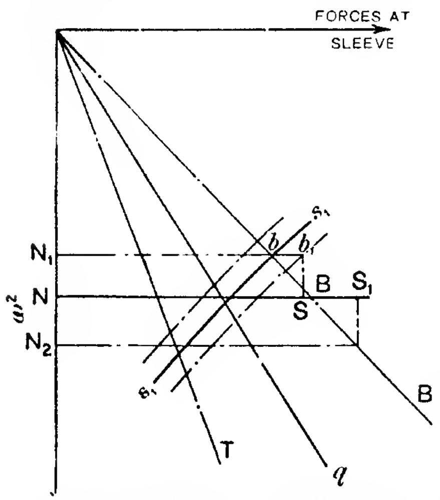


FIG. 118.—Rith diagram. Friction and stability.

(4) *The Effect of Friction on the Stability.*—Let s_1s_1, Fig. 118, be the graph of the static push. On either side of this set out curves each at a horizontal distance f from s_1s_1.

Imagine the sleeve to be pushed down to the bottom stop. Then the static force to be overcome by the dynamic pull upwards is the static force proper to the configuration, namely, N_1b plus the frictional force bb_1, making altogether a force N_1b_1, equal to NS. The restoring force is now BS.

It will be seen that the effect of friction is to reduce the magnitude of the restoring force. If the point S coincides with the point B there is no restoring force, and the governor will continue to run at the equilibrium speed $\sqrt{ON}$, but in the configuration belonging to the bottom stop.

If the point S falls to the right of B, as in the position S_1, the governor may not only continue to run at the equilibrium speed on the bottom stop, but the speed may be increased to $\sqrt{ON_2}$ before the dynamical pull is sufficient to overcome the static push plus the friction. This speed is higher than the equilibrium speed, and the sleeve will pass through the equilibrium position corresponding to the mean speed towards the top stop. The equilibrium is unstable.

These considerations apply to any governor, and in general therefore the effect of friction on the stability is to reduce it, and in certain conditions may even make the governor unstable.

Neutral equilibrium neglecting friction is converted into unstable equilibrium if friction is included.

It will thus be seen that many problems of extreme practical importance can be worked out on the Rith diagram. Although theoretically it is always possible to obtain the relation between the square of the radius and the displacement of the sleeve which by differentiation gives the angular coefficient, practical cases as a rule give rather complicated expressions to deal with. The following diagram devised by the author includes Rith's diagram in a modified form, but reduces the whole process to a graphical one, and thus avoids the necessity of differentiation.

111. Author's Development of Rith Diagram.—The dynamical pull on the sleeve for any kind of governor is given by

$$P = M\omega^2 r\frac{dr}{dx}$$

The factor $r\frac{dr}{dx}$ is the analytical expression for the subnormal of the curve found by plotting the radius of the path in which the mass centres of the balls revolve against the sleeve position.

This curve can always be plotted without difficulty for any kind of governor. Its form depends upon the mechanism connecting the balls to the sleeve, and on nothing else.

Quite generally, therefore

$$P = M\omega^2 \times \text{subnormal}$$

To estimate the dynamical pull on the sleeve in any configuration, it is therefore only necessary to construct a sufficient length of the curve $r = F(x)$ and then to construct from it the subnormal corresponding to the configuration. The length of this subnormal measured in feet multiplied by $M\omega^2$ is the dynamical pull required. Or the subnormal may be used directly to plot the corresponding line of configuration, as will appear immediately.

The explanation of the construction of the diagram will be made in connection with a Porter governor.

1. Draw two axes at right angles, Fig. 119.

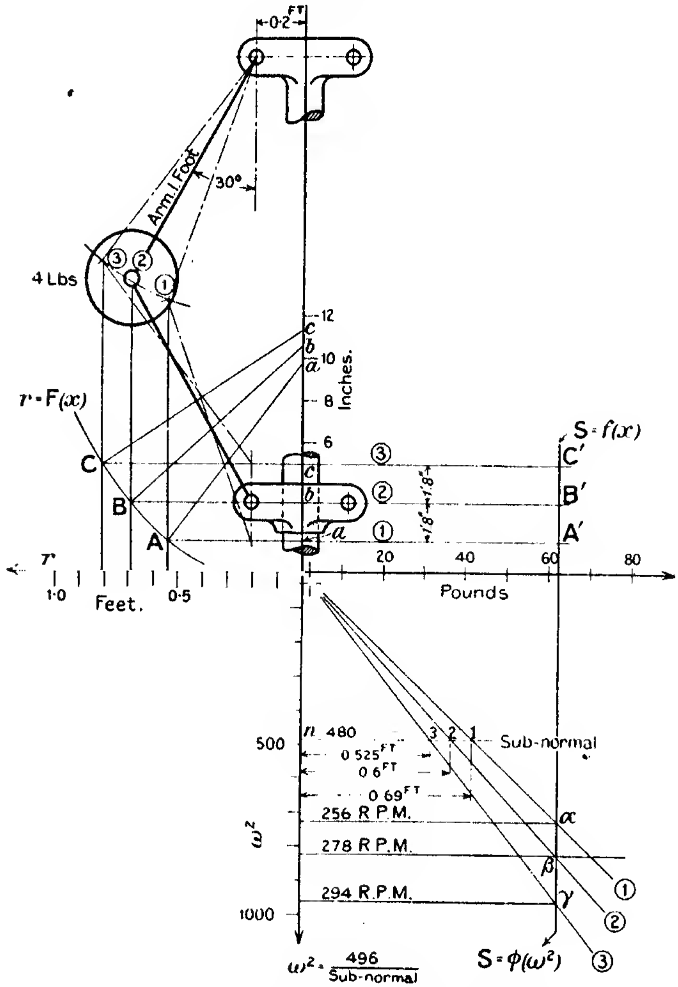


FIG. 119.—Author's diagram for loaded gravity governor.

From the origin upwards is the axis of sleeve displacement x.
From the origin to the left is the axis of the radius r.
From the origin to the right is the axis of the forces P and S.
From the origin downward is the axis of angular velocity squared, ω^2.

2. Draw the governor mechanism showing the ball and the sleeve in the three principal positions 1, 2, 3.

3. Draw horizontals through the three sleeve positions to meet verticals through the corresponding ball positions, thereby obtaining points A, B, C on the curve $r = F(x)$.

4. Construct the subnormals aa, bb, cc for the three principal positions. Choose on the ω^2 axis any point n and through it draw a horizontal line, and along this line set out $n1 = aa$, $n2 = bb$, and $n3 = cc$.

5. Through the points 1, 2, and 3 so found draw the lines of configuration O1, O2, O3.

6. On Scales. The position of the point n should be chosen so that when the subnormal corresponding to the mean position is set out, the line of configuration O2 should make an angle of about 30° with the axis of ω^2. Its position fixes the scale of angular velocity and the force scale.

When it is chosen at random the force scale and the angular velocity scale may be inconvenient.

It can be chosen to correspond with a stated force scale. The problem is then: at what angular velocity does the length of the subnormal measured on the drawing represent the corresponding dynamic pull when the force scale is given?

Let the governor mechanism be drawn $\frac{1}{n}$ full size; and let 1 in. on the paper stand for p pounds of force. Also let N be the actual length of the subnormal on the paper measured in inches.

Then on the force scale the length N stands for Np pounds. N also represents $\frac{Nn}{12}$ ft. Therefore, since the dynamical pull P is to be represented by the length N

$$M\omega^2\frac{Nn}{12} = Np \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From which

$$\omega^2 = \frac{12p}{Mn} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the figure the force scale is 20 lbs. per inch, so that $p = 20$. The mechanism is drawn $\frac{1}{4}$ full size, so that $n = 4$ and $M = \frac{1}{8}$. The particular speed at which the length of the subnormal is equal to the dynamical pull is then, from (2)

$$\omega^2 = \frac{12 \times 20 \times 8}{4} = 480$$

A convenient velocity scale is now to be selected, and 480 set out on this scale gives the point n from which the actual lengths $n1$, $n2$, $n3$ of the subnormals are to be set out. The lines of configuration are then drawn immediately. It should be noted that these lines of configuration are independent of the loading. They depend only upon the governor mechanism and upon the mass of the balls.

7. Set out the load on the sleeve as a function of the sleeve displacement. In the diagram it is a line parallel to the vertical axis, since a dead load of 62 lbs. is assumed per ball, of which 2 lbs. is due to the weight of the ball itself.

8. Project points from this curve to the corresponding lines of configuration below, thereby fixing the points α, β, γ, through which the static push curve can be drawn, thus obtaining this force as a function of ω^2. With the dead-weight loading assumed this force is constant, and α, β, γ are in a line.

9. Through the points of intersection α, β, γ, draw horizontal lines to intersect the speed axis, in this way obtaining the speeds of equilibrium in the three principal positions of the sleeve.

The subnormals actually measure 0·69 ft. on the lower stop, 0·6 ft. in the mean position, and 0·525 ft. on the top stop. Therefore, with the loading assumed, namely, 60 lbs. per ball on the sleeve and 2 lbs. as the equivalent to each ball itself, making the total loading 62 lbs., the equation of equilibrium is for this governor

$$0{\cdot}125 \times \omega^2 \times \text{subnormal} = 62$$

From which $$\omega^2 = \frac{496}{\text{subnormal}}$$

giving speeds of equilibrium of 256, 278, and 294 revolutions per minute in the three principal positions.

Without loading, the static push at the sleeve due to the weight of the balls is 2 lbs. only, and the corresponding equation of equilibrium is

$$0{\cdot}125 \times \omega^2 \times \text{subnormal} = 2$$

from which the equilibrium speed is 49 revolutions per minute. All the properties of the governor can easily be studied from this diagram.

A further example of the utility of the diagram may be given. Let the problem be, find the constant of the spring required on the sleeve of a governor of the kind shown in Fig. 114, so that the speed when the ball arm of the bell crank is vertical shall be 300 revolutions per minute, and the speed at the lower stop shall be 270 revolutions per minute, having given that the sleeve moves down ½ in. from its mean position to the lower stop. Find also the speed for the top stop determined by the spring, assuming that the sleeve moves ½ in. upward from its mean position to the top stop. The graphical solution of this problem is shown in Fig. 120.

Set out the bell crank in the positions 2, 1, corresponding to the mean position and to the bottom stop.

Draw the curve AB, connecting the radius with the sleeve displacement. In this case the curve is practically a straight line. Construct the subnormals *aa*, *bb*, for the corresponding sleeve positions.

Let the force scale be 1 in. = 40 lbs., so that $p = 40$. Also $M = \frac{1}{4}$, and $n = 1$.

Then from equation (2), above

$$\omega^2 = 12 \times 40 \times 4 = 1920$$

Choose for the speed scale 200 ω^2 units = 1 inch.

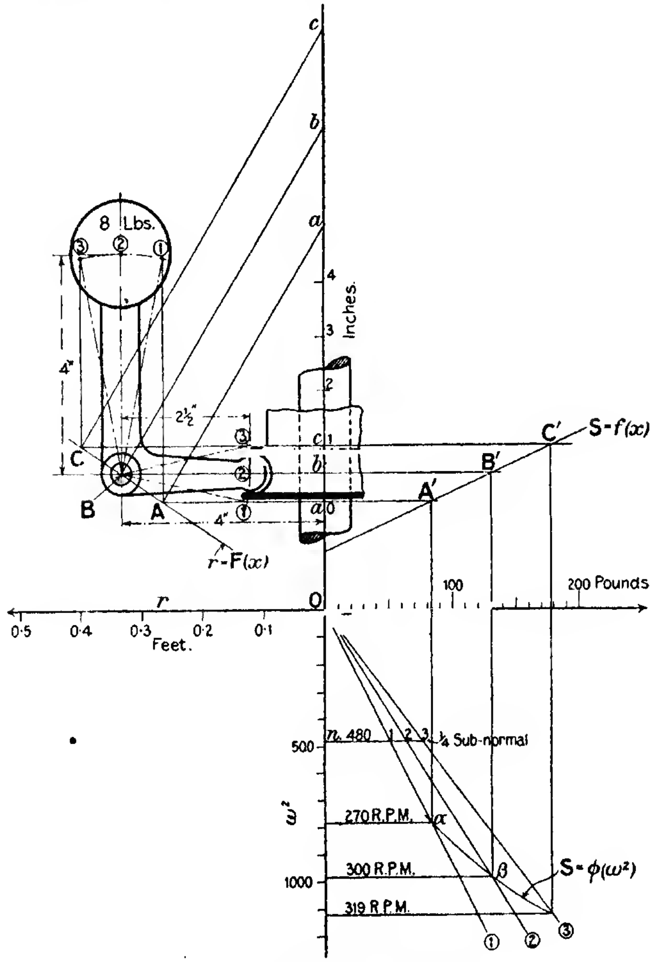


Fig. 120.—Author's diagram for spring governor.

On this scale 1920 falls outside the diagram. Retaining the scale, a point is chosen at $\omega^2 = 480$, and one-quarter of the actual length of the subnormal is set out from n. This gives the same slope as that given by setting out the whole length of the subnormal at 1920.

Set out from n, $n1 = \frac{1}{4}aa$; and $n2 = \frac{1}{4}bb$.

Then O1 is the configuration line for the bottom stop, and O2 the configuration line for the mean position.

The speed in configuration (2) is to be 300 revolutions per minute; therefore $\omega^2 = 985$. Through this point on the speed scale draw a horizontal line to cut the corresponding line of configuration (2) in β, and through β draw a vertical to cut the horizontal through the corresponding sleeve position in B′. This vertical cuts the force scale at 131 lbs., giving thus the magnitude of the dynamic pull on the sleeve in this configuration, and at the same time the equal and opposite magnitude of the force which must be exerted by the spring to maintain the configuration in equilibrium.

Similarly the given speed at the bottom stop is 270 revolutions per minute, corresponding to $\omega^2 = 789$. Points a and A′ are found by horizontal and vertical projection, and it will be seen that the dynamic pull and the equal static push in this position is 84 lbs.

The line through the points A′ and B′ is the law of the spring, from which it will be seen that the change in force for a change of sleeve displacement of $\frac{1}{2}$ in. is, reading from the force scale, 47 lbs.

Therefore the spring must be such that the constant a, the force which it exerts on the sleeve at the lower stop when the governor is at rest, is 84 lbs., and b, the compression per inch, is 94 lbs.

To find the speed at the top stop, draw the configuration line (3), obtaining the point 3 through which to draw it by setting off $n3 = \frac{1}{4}cc$. Then produce A′B′ to cut the horizontal through the corresponding sleeve position in C′. Project C′ vertically to γ, and γ horizontally to the speed axis, finding thereby that $\omega^2 = 1112$, giving the speed 319 revolutions per minute. All the properties of spring governors can be studied from this diagram.

AC is the curve $r = F(x)$; A′C′ is the curve $S = f(x)$, and is in fact the law of the spring when the spring is applied directly to the sleeve; $a\gamma$ is the curve $S = \phi(\omega^2)$, and is the relation between the static force and the speed.

The points where this curve cuts lines of configuration are equilibrium points.

The equilibrium is stable since the graph of the static push cuts a line of configuration corresponding to an increased radius in a point below the velocity horizontal belonging to the speed.

The restoring force at any displacement from a position of equilibrium can be at once found from the diagram.

If the weight of the governor ball has an appreciable effect at the sleeve, its weight must be reduced to the sleeve in each of the principal positions of the governor by means of a velocity diagram in the way illustrated above, thus giving three points of the $S = f(x)$ curve belonging to the ball.

If the spring found for direct action on the sleeve is to be replaced by another acting indirectly through levers, the constants of the substituted spring are found by multiplying a by the ratio

between the velocity of the sleeve and the velocity of extension of the substituted spring, and by multiplying b by the square of this velocity ratio. The substitution of springs acting indirectly for those acting directly on the sleeve is exemplified in the example on page 384.

If a regulating spring is to be used, then the curve A′C′ may be regarded as the algebraical sum of two springs, and the two springs may be arranged with different values of b to suit another condition of control.

In the explanation of the properties of spring governors it has been assumed that there is a linear relation between the load and the extension of the springs. When springs are used which differ sensibly from this relation, the true law of the spring is to be plotted in the first quadrant of the general diagram like that of Fig. 120, and then the conditions of running can be found. In such a case the line corresponding to A′C′ would be curved.

112. Oscillations of the Governor.—A stable governor will tend to execute oscillations about its position of equilibrium when disturbed from that position either by a small change in configuration due to a change of speed or by a rapid change of speed which brings the governor from one configuration to a new one.

For small oscillations it may be assumed that the restoring force R varies as the displacement of the sleeve from a position of equilibrium. With this assumption the oscillations of the sleeve will be simple harmonic about the position of equilibrium, and t, the time of an oscillation in seconds, is given by

$$t = 2\pi\sqrt{\frac{\mathfrak{M}}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where μ is the rate at which the restoring force R varies for a small displacement of the sleeve made at constant speed, and $\mathfrak{M}$ is the mass at the sleeve equivalent to all the governor masses.

The value of μ is found by forming the expression of equilibrium between the dynamic pull and the static push down on the sleeve, and then differentiating it with regard to x with the angular velocity ω constant.

The reduction of any mass m to an equivalent mass M at the sleeve is made from the principle that the kinetic energy stored by the actual mass, namely $\frac{mv^2}{2}$, must be equal to the kinetic energy stored by the equivalent mass at the sleeve moving with the velocity of the sleeve, namely $\frac{Mv_s^2}{2}$.

Thus,

$$M = m\left(\frac{v}{v_s}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The number of oscillations u made by the governor per revolution is given by

$$u = \frac{2\pi}{\omega t} = \frac{1}{nt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where ω is the angular velocity of rotation of the governor, and t is the time of an oscillation found from (1); n is the number of revolutions of the governor per second.

It will be remembered that there are a series of maximum and minimum speeds of the crank shaft in a revolution, the actual range of variation between a maximum and the next following minimum speed being controlled by the engine flywheel. If it should happen that the number of maximum speeds corresponds to the number of oscillations of the governor per revolution, then there will be danger of synchronism and a great probability that the governor will hunt badly.

Approximate values of u can always be found from equations 1 to 3 above, and if there is danger of synchronism the period of oscillation of the governor must be altered. This can always be done by means of a dash pot or a cataract of the kind in which a small piston connected to the governor moves in a cylinder filled with fluid, the two ends of the cylinder being connected by a pipe in which there is a small bye-pass valve. The required adjustment is easily made by regulating this valve.

Consider the case of a Porter governor with equal arms as sketched in Fig. 111, page 376, but so arranged that the arms are pivoted on the axis of rotation. Let each ball weigh W pounds, and let the weight carried by the sleeve be S pounds per ball. The dynamical pull on the sleeve is

$$P = M\omega^2 r\frac{dr}{dx} = \tfrac{1}{2}M\omega^2\frac{dr^2}{dx} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From the figure it is clear that, taking the origin at A, and denoting the length of an arm by a,

$$r^2 = a^2 - \left(\frac{x}{2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

therefore

$$\frac{dr^2}{dx} = -\frac{x}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and the dynamic pull on the sleeve is therefore $\tfrac{1}{4}M\omega^2 x$ per ball.

The actual static load on the sleeve is $S + \tfrac{1}{2}W$ per ball since the velocity ratio between the vertical movement of the ball and the movement of the sleeve is in this case constant and equal to 2. Then for a small displacement from a position of equilibrium, the restoring force R is

$$R = S + \tfrac{1}{2}W - \tfrac{1}{4}M\omega^2 x \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\frac{dR}{dx} = \mu = \tfrac{1}{4}M\omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

But ω is the speed of equilibrium found by putting R = 0 in equation (7).

Therefore $$\tfrac{1}{4}M\omega^2 = \mu = \frac{S + \frac{1}{2}W}{x} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

A particular configuration is fixed by assigning a value to x.

Again drawing a velocity diagram, it will be seen that

$$v : v_s = 1 : 2 \sin\theta$$

Therefore the mass at the sleeve equivalent to the mass of the ball is

$$\frac{M}{4 \sin^2\theta}$$

The equivalent mass at the sleeve per ball is then

$$\mathfrak{M} = \frac{1}{g}\left(S + \frac{W}{4 \sin^2\theta}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Therefore from (1)

$$t = 2\pi\sqrt{\frac{\left(S + \frac{W}{4 \sin^2\theta}\right)x}{g(S + \frac{1}{2}W)}} \quad . \quad . \quad . \quad . \quad (11)$$

The speed from the equation of equilibrium (7) with R = 0 is

$$\omega^2 = \frac{4(S + \frac{1}{2}W)g}{Wx} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Therefore from (3) $$u = \sqrt{\frac{W \sin^2\theta}{4S \sin^2\theta + W}} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If, in equations (11), (12), and (13) S is put equal to zero, the resulting equations represent the corresponding unloaded Watt governor.

If S is large compared with W, so large, in fact, that W in the denominator of (13) may be neglected without serious error,

$$u = \tfrac{1}{2}\sqrt{\frac{W}{S}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which shows that the oscillations per revolution of a heavily loaded Porter governor are approximately constant in all configurations of the governor.

Also, if W is neglected, *i.e.*, put equal to zero in the numerator of (12) and in the numerator and denominator of (11), there are obtained the approximate expressions

$$\omega = \sqrt{\frac{4gS}{Wx}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$t = 2\pi\sqrt{\frac{x}{g}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

These expressions may be applied to the particular case of a Porter governor with equal arms each 1 foot long, centred on the axis

of revolution and carrying a load on the sleeve of 50 pounds per ball, each ball weighing 5 pounds, the angle θ being 30°.

With these data $x = \sqrt{3}$.

From (11) it will be found that the time of an oscillation is 1·495 seconds, and this does not differ greatly from 1·46 seconds computed from the approximate formula (16). The oscillations per revolution from (13) are 0·151, which compared with 0·158 found from the approximate equation (15) shows only a small difference.

The equilibrium speed from the exact equation (12) is 27·9 radians per second, equal to 266 revolutions per minute, whilst from the approximate equation (15) the result is 260 revolutions per minute.

In this case, therefore, the number of oscillations which the governor would make per revolution if disturbed from a position of equilibrium and then left to itself is nearly constant and equal to 0·15, or 1 oscillation for $6\frac{2}{3}$ revolutions of the governor spindle. There is, therefore, no danger of synchronism with the variations of speed within a revolution caused by the inequalities between the turning moment and the resisting moment.

113. Relay Governors.—The governors described above have the common property that the governing mechanism regulating the steam

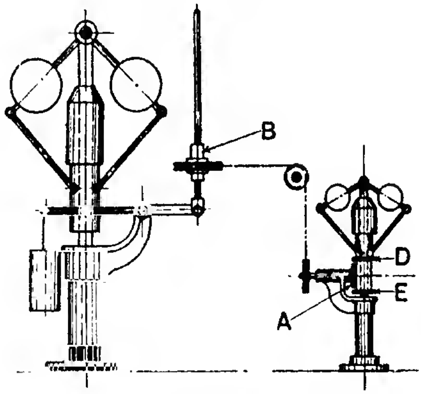


FIG. 121.—Supplementary governor.

supply, together with the mechanism of the governor itself, has one and only one configuration for a given position of the sleeve, except in so far as the configuration of the mechanism is changed by hand. Fig. 121 shows an arrangement by means of which this regulation of the governing mechanism is made automatically by what may be called a relay or a supplementary governor. The larger governor shown in the figure is designed in the usual way to keep the speed of the engine within assigned limits. The small relay governor acts as an engineman to control the speed when there is a large variation of load or a

large variation in steam pressure. The relay governor is brought into action as soon as the speed falls or increases sufficiently to bring the upper disc D or the lower disc E into contact with the wheel A. The discs D and E are formed on the governor sleeve, and are part of it. The turning of A acts to shorten or lengthen the rod connecting the sleeve of the main governor to the throttle valve by the turning of the right and left hand screwed coupling B. With an arrangement of this kind the speed can be kept within assigned and narrow limits notwithstanding a change in the boiler pressure of 25 per cent., or a change in the load of 30 per cent.[1]

Another example of a relay governor is shown in Fig. 122, which in diagrammatic form is the governor relay for a Parsons turbine.

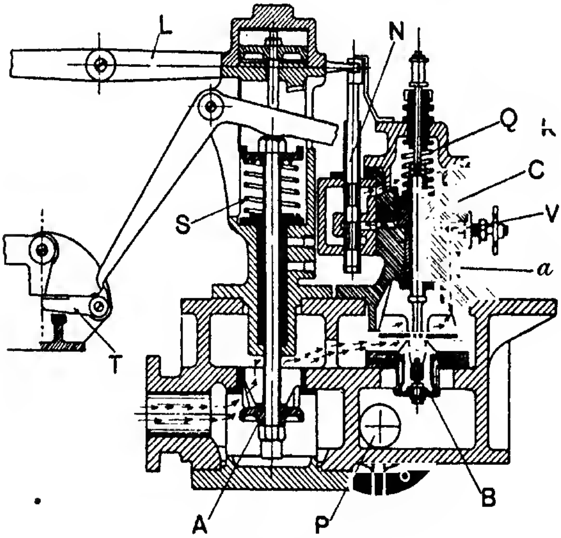


Fig. 122.—Parsons relay governor.

It will be seen that there are two valves, namely, A and B. A is an emergency valve, and only comes into action when the speed of the turbine exceeds an assigned limit; and B is the throttle valve giving access to the orifice P leading to the turbine.

The emergency valve is held open against the spring S by the trip gear indicated, which gear is connected to the sleeve of a governor. The strength and tension of the spring on the governor is so arranged that immediately the speed reaches an assigned limit, the sleeve flies to the top stop and releases the trigger T, and so allows the spring S to close the valve.

[1] "On the Moscrop Engine Recorder and Knowles Supplementary Governor," M. Longridge, *Proc. Inst. Mech. E.*, 1894.

The instability of the emergency governor is secured by so designing and adjusting the spring that the dynamical pull at the top stop, reckoned at the assigned speed, is greater than the static push exerted by the spring.

Suppose, for example, that when the speed reaches the value ω_1, the sleeve is to move rapidly from the bottom to the top stop, and is to press there with a force of F pounds. Then

$$M\omega_1^2 r_2 \frac{dr}{dx} - (a + bx) = F \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is an equation of condition in which r_2 is the radius at the top stop in feet; M is the whole mass of the balls; a is the static push exerted at the lower stop, found from

$$M\omega_1^2 r_1 \frac{dr}{dx} = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and x is the range of the sleeve from the bottom to the top stop.

The throttle valve B is connected by a piston rod to a piston C, on the top of which is a spring Q, which when free to act, closes the throttle valve.

Steam is admitted under the piston through the narrow passage a and past the regulating valve V. So that, when steam is first turned on, the pressure immediately lifts the piston C against the spring Q, and so opens the throttle valve wide. There is, however, at the side a small piston valve, N, which is really an exhaust valve. It is connected to the sleeve of a second and stable governor through the lever L, so that a movement of the sleeve causes a corresponding movement of the piston valve over its ports, and so regulates the opening of the exhaust port, through which the steam escapes from beneath the piston C into the chamber K, from which it is led through a distributing valve to the main bearings of the turbine in order to pack them with steam. To avoid the sticking of the piston valve, the groove in the governor sleeve engaging the end of the lever L is cut into a cam, the result of which is that the piston valve is kept in a state of oscillation along its line of stroke. The opening of the exhaust port is thus due to the actual position of the sleeve plus or minus the effect of the oscillation of the piston valve along its stroke produced by the rotation of the sleeve.

The two governors used are carried on one spindle, the one, as mentioned above, being an unstable governor, and the other a stable governor. The governor spindle is geared to the rotor spindle by worm gearing, so that its speed is considerably lower than the speed of the turbine rotor. The steam pressure is by this arrangement regulated not for each revolution, but for a cycle of revolutions, and in consequence it passes through the turbine in a series of gusts or waves of pressure.

CHAPTER VIII

The Motion of a Train and the Rate at which Energy must be spent to Produce, to Maintain, and to Destroy it

114. Introduction.—Marine engines, locomotives and tractors generally work against loads which increase automatically as the speed increases, because the medium through which the ship is propelled or through which the train is drawn resists the motion.

In the case of fixed installations the load against which the engine is to work is fixed arbitrarily, and the speed is fixed arbitrarily, and an engine can be designed to meet any load or speed requirement. But in the case of a ship or a train the most difficult problem of design is to find what the load really is for assigned speed conditions. In the particular case of the marine engine, the horse-power required to drive a ship of given shape and displacement at a given speed is a complicated function of the speed, and until the ship is built and tried it is always a matter of uncertainty whether the power provided is sufficient to get the speed. The uncertainty has, however, been brought within narrow limits by the extraordinarily beautiful researches of Froude[1] and Mr. R. E. Froude. A model[2] of the ship is made, and the model is towed in a tank by a dynamometric apparatus which accurately measures the force required to pull the model at definite speeds through the water. Applying the law of comparison to the experimental results, the powers required to propel the actual ship at corresponding speeds can be estimated. The law of comparison is founded on a general principle of similitude applicable to all dynamical systems. Froude applied this general principle and deduced the law, which may be enunciated as follows:—

If the linear dimensions of a ship be n times those of the model, and the resistance of the model be R at the speed V, then the resistance of the actual ship at the corresponding speed $V\sqrt{n}$ will be Rn^3, and the horse-power required to drive the ship is to the horse-power required to drive the model as $n^{\frac{7}{2}} : 1$. If, for example, the linear dimensions of the ship are sixteen times those of the model, $n = 16$. The corresponding speed would then be four times the speed of the model, and the ship's resistance would be 4096 times the observed resistance of the model. The power required to drive the actual ship

[1] *Reports of the British Association*, 1872, 1874. *Trans. Inst. Naval Arch.*, 1874, 1876, 1877, 1881, 1892.

[2] "Ship Model Apparatus," R. E. Froude, *Proc. Inst. Mech. E.*, 1893, p. 32.

would then be $4 \times 4096 = 16,384$ times the power required to drive the model at the corresponding speed. In passing from the model to the ship a correction must be made for the difference between the skin friction of the ship and the model; the complete study of ship resistance is, however, outside the limits of the present work.

The horse-power required to maintain the motion of a train against the many variable resistances opposing the motion is difficult to estimate. In the case of the locomotive, however, the load can easily be reduced by a coach or two until the desired speed is reached, providing the speed is within the limiting speed of the engine itself. The study of the motion of a train is valuable, not only because of its direct applications to railway problems, but as showing how the general principles of dynamics may be applied through experimental data to obtain approximate solutions of practical problems which are outside the possiblities of solution by exact mathematical methods.

The earlier part of this chapter is devoted to the investigation of the rate at which work must be spent to overcome the various resistances against which a train is drawn. The principles applied, and many of the results obtained, apply equally to steam and electric locomotives. The scope is then narrowed to the consideration of the power which a steam locomotive can develop to overcome these resistances. The concluding sections of the chapter are devoted to the explanation of a characteristic dynamical diagram for the motion of a train which, though developed in connection with a steam locomotive, is applicable to any kind of tractor and incidentally serves to bring into comparison the capabilities of the steam and electric locomotives during the starting period. The question of "braking" is considered in the final sections.

115. General Energy Equation for the Motion of a Train.—Train resistance may be distinguished into

(1) Resistance to change of speed and to change of level.

(2) Resistance of the general nature of friction.

The energy spent in overcoming resistance to change of speed or to change of level is recoverable. The energy spent against resistance in the nature of friction is irrecoverably lost.

Energy spent in overcoming resistance to change of speed is stored in the train as kinetic energy. Energy spent in overcoming the resistance to change of level produced by a gradient is stored as potential energy.

The relation between energy spent, energy stored, and energy dissipated against friction is expressed by the general equation on page 41, which, adapted to the immediate problem, may be written—

$$\left.\begin{array}{l}\text{Rate at which}\\ \text{work is done in}\\ \text{the cylinders of}\\ \text{the engine}\end{array}\right\} = \pm \left\{\begin{array}{l}\text{Rate at which}\\ \text{energy is}\\ \text{stored in the}\\ \text{train}\end{array}\right\} + \left\{\begin{array}{l}\text{Rate at which}\\ \text{energy is spent}\\ \text{against frictional}\\ \text{resistance}\end{array}\right\} \quad (1)$$

The sign of the storage term is plus or minus according as the store of energy is increasing or diminishing.

The problem of estimating the power which must be provided for the performance of a stated service, is the problem of estimating the values of the two terms on the right side of the equation for the severest conditions during the journey.

The rate at which work is done in the cylinders is measured by the indicated horse-power. It is more convenient to express this rate in foot-pounds per second. The left side of the equation therefore becomes 550 I.H.P.

Turning now to the right side of the equation, consider the form to be given to the term representing the rate at which energy is stored in the train.

First, consider the rate at which kinetic energy is stored in the train.

Let a be the acceleration of the train measured in feet per second per second, let M be the mass, and let W be the weight of the train in tons. Then the resistance to linear acceleration is

$$Ma = \frac{2240Wa}{g} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$g = 32{\cdot}2$.

If v is the velocity of the train in feet per second, then the rate at which energy is spent in overcoming the resistance to linear acceleration, which is also equal to the rate at which kinetic energy is stored in the train, is

$$Mav = \frac{2240Wav}{g} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad (3)$$

In addition to the resistance to linear acceleration there is the resistance to the angular acceleration of the wheels and axles. It is shown in detail, page 472, that the resistance to angular acceleration of the revolving masses may be allowed for by applying the linear acceleration a to a mass which is from 7 to 12 per cent. greater than the actual mass of the train. The lower value is applicable to ordinary trains drawn by steam locomotives, whilst the higher value is reached in the case of electric locomotives where the revolving masses are increased by the armature masses.

The rate at which energy is spent in overcoming the resistance to both the linear acceleration of a train and the angular acceleration of the wheels is therefore given by

$$\frac{2240\mathfrak{W}av}{g} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad (4)$$

where $\mathfrak{W}$ now stands for a weight, greater than the actual weight of the train by an amount which allows for the resistance to angular acceleration.

Secondly, the rate at which energy is spent against a gradient is

measured by the product of the weight of the train and the change of vertical height produced by the gradient per second.

If v is the speed of the train in feet per second and the gradient rises 1 foot in G feet, measured horizontally, then the change in height produced by the gradient per second is $\frac{v}{G}$ feet, and the rate at which energy is spent, and potential energy is increased, is

$$\frac{2240Wv}{G} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The resistance due to the gradient is

$$\frac{2240W}{G} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Consequently the rate at which energy is stored in the train, measured in foot-pounds per second, is

$$2240\left(\pm\frac{\mathfrak{W}av}{g} \pm \frac{Wv}{G}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Prefix a + sign to the first term when the speed v is increasing and a − when it is decreasing.

Prefix a + sign to the second term when the gradient is rising and a − sign when it is falling.

Finally, let F be the frictional resistance to the motion of the whole train, including the frictional resistance of the engine machinery. Then v being the speed of the trian, Fv is the rate at which energy is expended to overcome the resistance. Both the indicated horse-power and F depend upon the speed in a way which will be examined in detail below.

Collecting the above results, the general equation may be written

$$550 \text{ I.H.P.} = 2240\left(\pm\frac{\mathfrak{W}av}{g} \pm \frac{Wv}{G}\right) + Fv \quad . \quad . \quad (8)$$

This is the general energy equation for the motion of a train, and applies to the motion at every instant.

For a given value of the I.H.P. there is a definite limit towards which the speed v tends. The limiting value is found by putting the acceleration a equal to zero in equation 8, giving—

$$550 \text{ I.H.P.} = \pm\frac{2240Wv}{G} + Fv \quad . \quad . \quad . \quad . \quad (9)$$

This is the energy equation for steady running at the constant speed v on a gradient, the plus sign being prefixed for a rising gradient and the minus sign for a falling gradient.

For steady running on the level at the limiting speed the conditions are

$$550 \text{ I.H.P.} = Fv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It is difficult to calculate the time occupied in reaching a speed v. The form of the equation can, however, be obtained quite easily. Write $\frac{dv}{dt}$ for a in the general equation (8) and separate the variables; then

$$dt = \frac{1}{g}\left(\frac{2240\mathfrak{W}v}{550 \text{ I.H.P.} - Fv \mp \frac{2240Wv}{G}}\right)dv \quad . \quad . \quad . \quad (11)$$

from which t may be found by integration, but only if F and the I.H.P. are expressed as functions of the velocity.

The solution of this equation is of considerable practical importance in connection with calculations regarding the timing of trains for a service with frequent stops.

A general graphical solution is given below as one step in the construction of the characteristic dynamical diagram for the motion of a train.

116. Irrecoverable Loss of Energy due to Resistances of the General Nature of Friction.—When a vehicle is moving along a straight level track at uniform speed, energy is irrecoverably spent in overcoming resistance due to

1. Slipping between the tyres and the rails;
2. Journal friction;
3. Air;
4. Miscellaneous causes.

Resistance due to Slipping between the Tyres and the Rails.

Fig. 123 shows a pair of wheels in their central position in relation to the rails. The dimensions given are those usual on English railways. The flange is shown $1\frac{1}{8}$ ins. thick, but this dimension varies. The leading wheels of engines, engine bogie wheels, engine driving and trailing wheels usually have flanges of the dimensions shown. The middle wheel in a set of six driving wheels is usually provided with a thinner flange, and there are cases where the flanges have been omitted altogether, though the practice is rare. Carriage wheels usually have thinner flanges, $\frac{7}{8}$ in. being a common thickness.

It will be seen from the figure that the dimensions given allow $\frac{1}{2}$ inch lateral movement on each side of the central position before either flange can have contact with the rails.

The tyres are turned each to a cone and the rails are rounded at the top, in consequence of which contact between the tyres and the

rails takes place at the points a and b when the pair is in the central position.

When the vehicle which is supported by the wheels is moving along a straight level track at uniform speed without oscillations, with each axle at right angles to the direction of motion and in the central position laterally with relation to the rails, the wheels move forward with a pure rolling motion about the instantaneous axis joining the points of contact ab. The characteristics of this rolling motion are that the flanges of the wheels do not touch the rails and that there is no slipping.

Any relative motion between the tyre and the rail at a point of contact constitutes a slip. The slipping may be momentary as when a driving wheel slips, or continuous as when a wheel is compelled to move under the action of constraints which do not allow the geometrical conditions of pure rolling to be fulfilled.

If P is the pressure in pounds at a point of contact between the tyre and the rail where slipping takes place, and μ is the coefficient

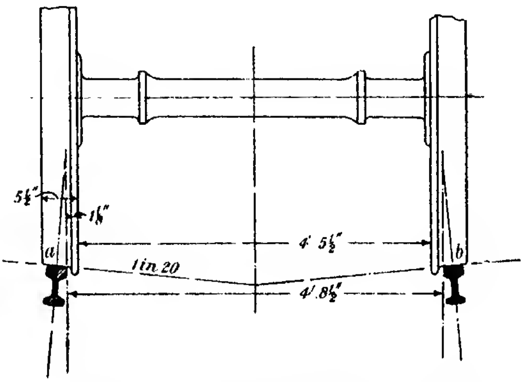


FIG. 123.—Pair of wheels in central position of the rails.

of friction, the slipping occurs against a frictional resistance $P\mu$. If u is the velocity of slipping in feet per second, that is the velocity of the tyre relatively to the rail at the point of contact, the rate at which energy is dissipated is $P\mu u$ foot-pounds per second.

The problem of estimating the losses due to slipping turns therefore upon the problem of finding values of u, the relative velocities of slip, and the corresponding pressures P. Although this cannot be done exactly, yet the following two cases may be usefully considered.

A. Slipping at the tread in the direction of rolling when the diameters of the rolling circumferences differ by the amount allowed by the side play, the axle being at right angles to the direction of motion.

B. Slipping at the tread at right angles to the direction of motion, called side slip, assuming the rolling circumferences equal.

A. *Slipping at the Tread in the Direction of Rolling on a Straight Track.*

The displacement of a pair of wheels from the mid-position introduces an inequality in the diameters of the rolling circumferences which makes pure rolling impossible unless the axle is free to turn in a horizontal plane. It is assumed that any such turning is prevented by the constraints applied by the horns, so that the wheels are compelled to roll along with the axle at right angles to the direction of motion. When the diameters of the rolling circumferences are unequal, such a motion is only possible by slipping at the points of contact. If x is the lateral displacement of the axle from the central position, and assuming the coning to be 1 in 20, then

$$\text{Radius of the larger rolling circumference} = r + \frac{x}{20}$$

$$\text{Radius of smaller rolling circumference} \quad = r - \frac{x}{20}$$

Therefore the difference in distance rolled out per second by the two circumferences $= \frac{2\pi nx}{10}$ ft., where n is the number of revolutions made per second by the axle.

If v is the speed of the train in feet per second, $n = \frac{v}{2\pi r}$. Therefore u, the relative velocity of slipping, is $\frac{vx}{10r}$. Finally, the loss of energy due to this cause is at the rate of

$$\frac{\text{P}\mu vx}{10r} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If P is the pressure between the wheel and the rail at the point where slipping occurs, the total load on the axle will be approximately 2P lbs. Let the total load be 1 ton. Then P = 1120 lbs. With $x = \frac{1}{2}$ in. $= \frac{1}{24}$ ft., the maximum possible lateral deviation, and with $\mu = \frac{1}{5}$, the expression reduces approximately to

$$\frac{v}{r} \text{ ft.-lbs. per second per ton} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This gives approximately the maximum rate at which energy is lost per ton of load on the axle due to slipping at the tread.

The movement from the central position brings the flange into contact with the rail. Every point of contact between the flange and the rail has a velocity relatively to the rail proportional to its distance from the instantaneous axis. As the wheel rolls along, the

flange and the tread polish the rail at every point of contact. The continuous polishing action due to the slipping at the tread and the flange is very apparent on the rails.

Suppose the average of all the points of contact between the flange and the rail to be $\frac{1}{2}$ in. $= \frac{1}{24}$ ft. below the instantaneous axis. Then the velocity of rubbing will be $\frac{v}{24r}$ ft. per second, where v is the speed of the train in feet per second and r is the radius of the wheel in feet. If P is the total pressure acting between the flange and the surface of contact, then the loss of energy is at the rate of

$$\frac{P\mu v}{24r} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is practically impossible to assign a value to P. However, per ton of lateral pressure and with $\mu = \frac{1}{5}$, energy is spent at the approximate rate

$$\frac{20v}{r} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Comparing this with the loss at the tread it will be seen that the flange loss is likely to be considerably greater than the tread loss.

B. *Side Slip assuming the Rolling Circumferences equal.*

Side slip occurs when a pair of wheels is rolling forward with the axle inclined to the direction of motion determined by the track.

Let a, b, Fig. 124, be the points of contact between the wheels A, B, and the rails. Rolling can only take place about the instantaneous axis ab. The wheel A tries to roll in the direction ag, a direction inclined θ to the rail: similarly the wheel B tries to roll in the direction bk, also inclined θ to the rail. Consider the wheel A. Let Of represent the actual velocity of the point of contact a along the rail: Of is equal to v the velocity of the train. Resolve this into two components, namely, Oh in the direction at right angles to ab, and hf at right angles to the rail. Then Oh represents the actual velocity of rolling and hf the velocity of slipping at right angles to the rail. The velocity of side slip is therefore $v \tan \theta$. Since, however, θ is always small, the circular measure of the angle may be substituted for the tangent, and the velocity of side slip is then $u = v\theta$ ft. per second. If P is the pressure at the point of contact a,

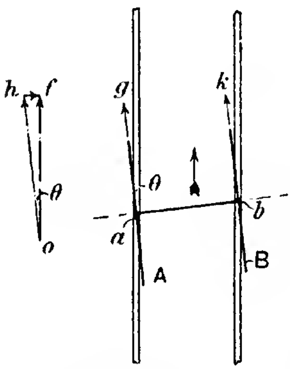


FIG. 124.—Side slip.

then the energy expended against the frictional resistance to side slip is at the rate of

$$P\mu\theta v \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The same is true for the point b. Therefore, energy is expended against the frictional resistance to side slip at the rate of

$$448\theta v \text{ ft.-lbs. per second per ton of load} \quad . \quad . \quad (6)$$

If the point b is $\frac{1}{2}$ in. $= \frac{1}{24}$ ft. in advance of a, $\theta = \frac{1}{24} \div 5 = \frac{1}{120}$ radians, and the loss of energy is approximately $4v$ ft.-lbs. per second per ton of load.

Assuming all these resistances between the tyre and the rail to take place simultaneously, the dissipation of energy between the tyre and the rail per axle is represented approximately by the general expression,

$$\left(\frac{W}{r} + \frac{20P}{r} + 448W\theta\right)v \text{ ft.-lbs. per second per axle} \quad . \quad (7)$$

where W = tons supported by the axle;
- P is the lateral pressure, now in tons, between the flanges and the rail;
- r is the radius of the wheel in feet;
- θ is the number of radians in the angle of inclination between the axle and a line at right angles to the direction of motion.

In this expression, the maximum lateral deviation is assumed to be $\frac{1}{2}$ in., and the coning of the tyres to be 1 in 20.

If $W = 1$ ton, $P = 1$ ton, $\theta = \dfrac{\frac{1}{2}\text{ in.}}{60\text{ ins.}} = \frac{1}{120}$ radians, $r = 2$ ft., the expression in the brackets reduces to 14·5. This may be regarded as the resistance per ton due to slipping between the tyre and the rail. Though inexact, it gives some idea of the order of the resistance which may be introduced from the various causes considered.

If an axle be watched in a train running at any speed it will be seen to oscillate laterally from side to side, seldom remaining in the mid-lateral position for even a few seconds. During the period of a contact between the flange and the rail on either side, there may therefore be a resistance introduced of the order 10 to 15 lbs. per ton. The whole train resistance from this cause will depend therefore upon the sum of the number of contacts per unit of time made by every axle in the train. The number of contacts and their duration will depend upon the kind of stock, the way the axles are supported, the kind of track, and upon whether the train is going up or down a gradient; an enumeration sufficiently wide to indicate that this form of resistance is likely to be a very variable one.

Journal Friction.

The surfaces of a journal and of the brass which transmits the load to the journal are kept from metallic contact by means of a film of oil or lubricating material. The oil film is formed by, and is maintained by, the rotation of the journal, and the frictional resistance at the surfaces has the characteristics of metallic friction merging into those of fluid friction according as the oil film changes from an imperfect to a perfect condition. Let W be the load supported by the bearing, μ the coefficient of friction between the surfaces of the journal and the brass, then $W\mu$ is the tangential resistance against which the journal turns, and $W\mu u$ is the rate at which energy is expended in overcoming this resistance, u being the relative velocity of the surfaces of the journal and the brass. This energy is converted into heat. Consequently the bearing increases in temperature until the rate at which heat radiates from it is the rate at which heat is produced, that is $\frac{W\mu u}{1400}$ lb.-cals. per second.

With the complete supply of oil, obtained by means of an oil bath or by forced lubrication, the film of oil is maintained by the rotation of the journal in a perfect condition. In these circumstances Mr. Beauchamp Tower[1] found that the product $W\mu$ is practically constant at constant temperature, and more recently the experiments of Lasche[2] have shown that the product $p\mu t$ is practically constant and equal approximately to 30, where

p = the quotient of the load divided by the projection of the bearing surface.
t = the temperature of the bearing Centigrade.

The expression is valid between the limits of pressure, 14 to 213 lbs. per square inch, and between limits of temperature, 30 to 100° C., and between limits of velocity 3 to 60 ft. per second.

Osborne Reynolds has shown[3] that the facts observed in connection with a journal lubricated by means of an oil bath can be explained by a theory based upon the general principles of the motion of a viscous fluid.

The lubrication of a railway axle box is not complete in the sense understood above. The pad pressed against the under surface of the journal conveys oil from the oil well to the surface by the capillary attraction of the worsted threads connecting the pad and the oil; but this is not the same as allowing the surface to run in oil, or the same as forcing oil between the journal and the brass. Consequently the coefficient of friction would be more characteristic of solid than fluid friction in the case of railway journals as at present constructed. Mr. Beauchamp Tower, in the paper quoted above, states that when the journal was lubricated by means of an

[1] "Report on Friction Experiments," *Proc. Inst. Mech. E.*, November, 1883.
[2] *Zeit. Vereins Deutsch. Ing.*, 1912, vol. 46, pp. 1881 *et seq.*
[3] *Phil. Trans.*, vol. 167, Part II.

oily pad the coefficient of friction was approximately constant at about 0·01 on the average.

Assuming this value for μ, the tangential resistance against which the journal turns is approximately 22 lbs. per ton of load carried by it. If j is the radius of the journal, r the radius of the wheel, and v the speed of the train, the relative velocity u between the surface of the journal and the brass is $\frac{vj}{r}$. And the rate at which energy is dissipated in overcoming the friction of the bearing is

$$22\frac{vj}{r} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

per ton of load on the bearing.

For a carriage wheel 43½ ins. diameter with axle journals 4 ins. diameter this reduces to $2v$ ft.-lbs. per second per ton of load. For an engine wheel 7 ft. in diameter with axle journals 8 ins. diameter it has about the same value.

This means that the effect of journal friction when running is equivalent to about 2 lbs. tractive resistance per ton of load. This must only be regarded as an approximate result, because the coefficient of friction is not accurately constant with pad lubrication, but depends to some extent upon the speed, and the ratio between the diameter of the journals and the wheel diameter $\frac{j}{r}$ varies slightly. But it furnishes an idea of the order of magnitude of the resistance due to this cause. In general, therefore, journal friction may be regarded as producing a constant resistance which is practically independent of the speed.

A system of forced lubrication for the driving axle boxes of some steam cars belonging to the Taff Vale Railway Company is described in a paper[1] in the *Proceedings of the Institution of Mechanical Engineers*. The journals are 6 ins. diameter and 9½ ins. long, and carry 7·33 tons. At the train speed of 30 miles per hour the journal makes 300 revolutions per minute. Oil is pumped between the journal and the brass at about 20 lbs. per square inch, and relief valves are fitted to prevent this pressure being exceeded.

Air Resistance.

The movement of a train throws the air in its neighbourhood into a state of commotion, the violence of which increases with the speed of the train. Every flat surface normal to the direction of motion produces air currents to the right and left of the train. Every sharp corner, ridge, or angle across the direction of motion is associated with an eddy. Air rushes backwards to fill the void caused by the motion of the train. The waves, eddies, and currents all store energy in virtue of their motion—and this energy is

[1] "Forced Lubrication for Axle Boxes." By Mr. T. Hurry Riches, and Mr. P. Reynolds, *Proc. Inst. Mech. E.*, July, 1908.

gradually frittered away in heat in consequence of the relative motion of the stream lines and the viscosity of the air.

The rate at which energy is acquired by the air from the train as the train rushes through it could be measured by observing the pull necessary to tow the train through the air at a particular speed. Such towing experiment, however, cannot be made without measuring other resistances. In practice the pull is inferred from experiments on flat planes and models. With a model the energy loss can be found from a measurement of the pull required to tow the model through air at a given speed v, or from a measurement of the force required to hold the model at rest in a current of air of velocity v. Although the pull in the first case and the force in the second case are not quite equal, yet the difference is small enough to be disregarded in practical cases. If P is the pull or the force required to tow an object through still air at a speed v, or to hold it at rest in a current of air moving with velocity v, then the rate at which energy is spent in overcoming the resistance of the air to motion is Pv ft.-lbs. per second.

A flat plate held fixed in a current of air produces a configuration of stream lines in its neighbourhood, such that there is relative motion amongst them and therefore a loss of energy due to the viscosity of the air. The change of the direction of the stream lines is greatest on the windward side of the plate, those approaching the centre being diverted towards the edges of the plate almost at right angles to their original direction of flow. The stream line configuration in the wake of the plate is determined by the shape of the plate.

Such a stream line flow causes a pressure on the windward side of the plate and a suction on the lee side. The pressure is determined by the density and the velocity of the air, and the suction by the shape of the plate. On neither side of the plate is the pressure uniformly distributed. The relation between the total pressure on a plate and the velocity of the air current which produces it has been measured by Dr. T. E. Stanton[1] in a variety of cases. In the first set of experiments the force was measured which was required to hold small circular plates at rest in a channel 2 ft. in diameter through which an air current was flowing with known velocity, and in the later experiments large planes were exposed to the action of the wind on the top of a tower specially constructed for the work at the National Physical Laboratory. The relation between the pressure in pounds per square foot and the speed V of the wind in miles per hour was found to be as follows:—

Small circular plates ½ in. to 2 ins. diameter	$P = 0{\cdot}0027V^2$
A flat plane 5 ft. × 5 ft. = 25 sq. ft.	$P = 0{\cdot}00320V^2$
A flat plane 10 ft. × 5 ft. = 50 sq. ft.	$P = 0{\cdot}00318V^2$
A flat plane 10 ft. × 10 ft. = 100 sq. ft.	$P = 0{\cdot}00322V^2$

The long axis of the 10 × 5 ft. plate was placed horizontally.

[1] Dr. T. E. Stanton, "On the Resistance of Plane Surfaces in a Uniform Current of Air," *Proc. Inst. C.E.*, 1903-1904, Part 2, page 78. "Experiments on Wind Pressure," *Collected Researches of the National Physical Laboratory*, 1909, vol. 5.

Stanton experimented with two discs, each $1\frac{1}{2}$ ins. diameter, placed on a spindle at various distances apart and placed with the axis of the spindle in the direction of the current, and found that the force required to hold the pair in the current was a minimum when the distance between the discs was one and a half times their common diameter, and that the force was in this case only 75 per cent. of that required to hold one of the plates by itself in the same current.

Stanton's early experiments on geometrically similar combinations of flat surfaces under normal impingement of the wind indicated that the pressures at corresponding points were the same for the same velocity of current, and therefore that the results of experiments on scale models of structures, such as lattice girders, would apply to the prediction of the pressure on the full-sized structure.

Recent investigations show that the experimental velocity with the model should be such that the product *velocity* × *linear dimensions* is the same for the model as for the structure.

When the plane surface on which the air impinges is the face of a prism, the configuration of the stream lines in the wake is different to the configuration in the case of a plate, and a loss of energy is incurred by skin friction along the body of the prism. The resultant force would in this case be due partly to pressure on the windward face, partly to suction on the lee face in the wake, and partly to the skin friction. This is in fact the case of the railway vehicle, except that the projections on the vehicle, the proximity of the floor to the ground, and the rotation of the wheels and axles produce eddies and vortexes superposed on the general stream line motion which increase the energy loss.

A set of elaborate experiments has been made to find the resistance of the air to the motion of prismatic models of various shapes by Frank,[1] but from a railway point of view no experiments have so much value or interest as those made in connection with the Zossen[2] trials. Single vehicles were used in these experiments each weighing about 92 tons. From measurements made at speeds ranging from 35 to 110 miles per hour it appeared that

$$P = 0{\cdot}0027V^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This figure is apparently based on the projected cross-section of the car and includes head, skin, and wake resistance.

A train of vehicles may be regarded as a long prism with transverse cuts at intervals so that the energy acquired by the air as the train passes through it would be of the same order of magnitude as if the train were a prism, but increased by the eddies and stream-line distortion produced by the spaces between the vehicles. It might be expected that the first vehicle of a train would move against the greatest resistance, the following vehicles, being shielded, would have chiefly skin friction against them which

[1] *Zeit. Vereins Deutsch. Ing.*, 1906, 593.

[2] *Berlin Zossen Electric Railway Tests*, translated from the German by Franz W. Welz. E. E. McGraw Publishing Company, New York, 1905.

would be fairly constant per vehicle, and that the last vehicle would in addition have the resistance due to the suction of the wake to overcome in addition to the skin friction.

The celebrated experiments of Professor Goss,[1] in which models singly and in train formation were exposed to the action of a current of air of known velocity in a conduit 400 sq. ins. in cross-section and 60 ft. long, show that such is in fact the case.

The relations between the velocity of the current in miles per hour and the pressure produced in pounds per square foot are as follows:—

A single model	$P = 0{\cdot}0025V^2$
The first model of a train of models . .	$P = 0{\cdot}001V^2$
The second model of a train	$P = 0{\cdot}00008V^2$
For any intermediated model in the train	$P = 0{\cdot}0001V^2$
The last model in the train	$P = 0{\cdot}00026V^2$

Each model in these experiments was 12 ins. long, $3\frac{3}{8}$ ins. wide, and $4\frac{1}{2}$ ins. high, and represented to $\frac{1}{32}$ scale a standard box car. Each was fitted with a delicate indicator by means of which the pressure exerted on it by the air current could be directly measured. The air current was regulated by a fan at the end of the conduit, and the velocity was measured by a Pitot tube.

Mr. Aspinall inferred from experiments made on a train consisting of engine, dynamometer car, and five bogie carriages that for that particular train the pressure per square foot of the equivalent cross-section of the train, which was 70 sq. ft., was represented by

$$P = 0{\cdot}003V^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

To the stream-line configuration caused by the motion of the train in still air must be added the effect due to a wind. With high speeds and moderate winds the still-air configuration would not probably be much modified. With high winds there would be a profound modification. With all winds there is a normal component acting to blow the train across the track except when the train happens to be moving in or against the direction of the wind. The effect of this is to produce flange friction against the lee rail in proportion to the magnitude of the cross component. With high winds this produces a greater resistance to the motion of the train than the stream-line configuration.

It will be understood from the foregoing remarks that it is hardly possible to devise any expression which will give the air resistance with any degree of generality, but that in any expression for the total resistance obtained by experiment a term varying as the square of the velocity would be required to allow for it. Moreover, the usual way of expressing train resistance is in pounds per ton of load. The air resistance has, however, nothing to do with the weight, but is a function of the form and volume of the train. So that in any

[1] See the *Engineer*, August 12, 1898, "Atmospheric Resistance to the Motion of Railway Trains".

application of the preceding expressions to calculate the resistance, after finding the total pressure exerted per square foot of equivalent cross-section of the vehicle, it must be reduced to resistance per ton of vehicle by means of the ratio between the cross-section and the weight of the vehicle. It is in this way that air resistance is included in the general expressions giving resistance in pounds per ton.

Miscellaneous Resistances.

The continuous deflection of the track and permanent way as the train advances, the oscillation of the spring-borne part of the train, the crowding of the carriages on one another, the fished joints, all absorb energy from the train, and therefore offer resistance to its passage. Probably the most important of these is the first-mentioned. The rail, and with it the sleepers and permanent way, deflect under every wheel that passes. The deflection is of course greatest under the engine wheels, but even then it is not the same for each wheel of a group of equally loaded wheels. The deflection at a particular point in a rail will, with a close group of wheels, hardly have time to recover before the next wheel is at the point. The energy spent in deflecting the road will therefore depend upon the load on the wheels, the pitch of the wheels, the stiffness of the rails, the pitch of the sleepers, and the degree of excellence attained in the maintenance of the permanent way. The way energy is absorbed by the track may be imagined by supposing a pair of wheels and their axle to be held in fixed bearings, and a pair of rails to be passed under the wheels, and to be supported on rollers on either side of the train wheels, the arrangement of wheels and rollers being similar in arrangement to a plate-bending mill. Then suppose the supporting rollers to be screwed up vertically until the rails press the train wheels upwards in the fixed bearings with a load equivalent to the load carried by the axle in ordinary conditions. It will then be understood that the torque which must be applied to the axle to roll the track through the combination, multiplied by the angular velocity of rolling, is the rate at which energy is absorbed to overcome the elastic resistance of the rails, and by an extension of the idea, the road bed also.

The continual packing up and ballasting of the track, which is incessantly in progress on a first-class line, is evidence of the energy which is continually abstracted by the track from passing trains.

117. Forms of the Equations used to represent the Total Frictional Resistance of a Train in Terms of the Speed.—The sum of all the resistances enumerated in the last section taken through the whole train is the total train resistance on a straight level road at uniform speed, and includes the engine and the vehicles.

Individual vehicles differ amongst themselves in the force which must be applied to haul them along according to their construction,

method of wheeling, form, and on the kind of track over which they are hauled. Again, the force required to haul a train will depend upon the length of the train, and on the dimensions of the spaces between the vehicles forming the train. In practice, however, the expressions which have been constructed to represent the results of experiments rarely distinguish individual types to a greater extent than is indicated by the terms "four-wheeled vehicles," "bogie vehicles," with "grease" or "oil lubrication". The resistance is recorded in pounds per ton, and with few exceptions no regard is given to the modifying influence of the length of the train.

The oldest form of the expression used to represent the train resistance is

$$r = A + BV^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where r stands for the resistance of the train in pounds per ton, A and B are constants determined by experiment, and V is the speed in miles per hour.

A more modern form is

$$r = A + BV + CV^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which three constants A, B, and C have to be found from experiments.

Neither of these expressions take into account the decrease of resistance per ton as the length of the train increases.

The form used by Mr. Aspinall, namely,

$$r = A + \frac{V^n}{C + DL} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which L is the length of the train in feet measured over the carriage bodies, allows for this.

Again, these equations apply only to a particular kind of stock, the constants being different for four-wheeled stock, and for bogie stock and for wagons.

Locomotives offer a greater resistance per ton than vehicles at a given speed, and the increase of resistance is at a greater rate than the increase of vehicle resistance.

Sometimes the expressions are applied to the train as a whole; in what follows, however, the locomotive resistance per ton r_e will be distiuguished from the vehicle resistance per ton r_v.

The energy dissipated in overcoming the frictional resistance of a train to motion is, therefore, represented by the expression

$$Fv = (W_e r_e + W_v r_v)v \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad (4)$$

where W_e is the weight of the engine and tender in tons;
W_v is the weight of the vehicles in tons;
r_e is the resistance of the engine in lbs. per ton;
r_v is the resistance of the vehicles in lbs. per ton;
v is the speed in feet per second.

It is generally assumed that the total frictional resistance F has the same value for a particular train on all parts of a straight road. The value of F is slightly different, however, on a gradient, and there is also a difference according as the train is going up or down a gradient. In the one case the train is strung out with all the couplings tight, and in the other case the vehicles crowd on to one another with loose couplings. In what follows F will be taken as constant for a particular train at a particular speed at all parts of a straight road.

118. Rate at which Energy is spent at the Draw-bar. Vehicle Resistance.—The pull exerted by the engine through the draw-bar is equal and opposite to the tractive resistance of the vehicles. The rate at which energy is spent in overcoming this resistance is the product of the draw-bar pull and the speed of the train. The resistance at the draw-bar is at any instant the whole resistance of the vehicles due to change of speed, change of level and friction.

Measurement of vehicle resistance is made by introducing a dynamometer[1] car between the engine and the vehicles.

The main feature of a dynamometer car is a carefully designed spring through which the pull of the engine is transmitted to the vehicles. The deflection of the spring is a measure of the pull transmitted, and it is recorded on a sheet of paper which is wound continuously on to a drum by means of mechanism operated by one of the dynamometer car axles, the wheels of which are furnished with flangeless tyres turned parallel on the tread and to an accurate diameter. This axle is specially introduced, not to carry any part of the weight of the car, but to ensure that the paper is passed under the pen at a speed proportional to the train speed.

Mr. Churchward has kindly supplied the author with the portion of the dynamometer car record of the Riviera express from Paddington shown in Fig. 125. The vehicles weighed 289 tons and were hauled by the engine "Dog Star," weighing with the tender 115 tons. The paper is drawn under the recording pens at the rate of 1 foot of paper per mile run of the train. It will be seen that there are seven lines on the diagram above the datum line.

Reckoning from the datum line, the first line shows the draw-bar pull.

The second line is traced by a pen normally at rest, but an observer in the car can produce a kick of the pen by pressing an electric button. In this way the location of the mile posts may be recorded, or the position of a station or signal-box.

Kicks of the pen tracing the third line are produced at the time an indicator diagram is taken by a member of the indicator staff on the engine.

[1] See article "Dynamometers," *Encyclopædia Britannica Supplement*, 9th ed., by W. E. Dalby.

The fourth line records particulars of the working of the engine. An observer on the footplate notes any change in the conditions of working and at the same time produces a kick of the pen tracing the line, and in order to identify the note in his book with the kick on the record, he signals one kick, two kicks, up to three kicks. Thus it will be seen that when the note was made that the boiler-pressure was 205 lbs. per square inch, and that the regulator was $\frac{3}{4}$ open with a cut-off of 19 per cent., the time of taking the note is indicated on the record by three kicks. The next signal from the engine is indicated by one kick.

The fifth line is an indication that the integrating mechanism is at work. The integrating mechanism is a roller and disc integrator, and shows the amount of energy expended from the start up to any point of the run.

The sixth line is traced by a pen connected with a clock in such a way that the pen kicks at intervals of 5 minutes.

The seventh line is also traced by a pen connected with the clock, but the pen kicks at intervals of 2 seconds. Since the paper moves under the pen at the rate of 1 foot per mile, the speed of the train is easily deduced from this line.

The record for the complete run of the train is about 300 feet long.

The complete record indicates a gradual and continuous increase of the speed from Paddington up to 70 miles per hour. The train then ran steadily at this speed until required to slow down for a junction.

From the records on the diagram the pull corresponding to the acceleration and to the gradient are computed and are then subtracted from the pull recorded on the diagram; the remainder is what the draw-bar pull would have been at a uniform speed on the level, assuming that the part of the diagram reduced was obtained on a straight road.

The vehicle resistance per ton r_v is then found by dividing the corrected draw-bar pull by the weight of the vehicles.

The reduction of the data from a dynamometer car record to obtain r_v can be made by means of the characteristic dynamical diagram explained below. An example of the method applied to data from the Riviera express, of which Fig. 125 is a part record, will be found on page 459. It is taken from a paper by the author entitled "Characteristic Dynamical Diagrams for the Motion of a Train during the Accelerating and Retarding Periods," read at the Institution of Mechanical Engineers, October 25, 1912.

The experiments[1] made by Mr. Aspinall on the Lancashire and Yorkshire Railway, furnish valuable data of the resistance of modern bogie stock on a well-maintained British railway, and are the only experiments, so far as the author knows, in which the length of the train is introduced as a variable in the general expression for the resistance.

[1] "Train Resistance," *Proc. Inst. C.E.*, 1901, 1902, vol. 147, Part 1.

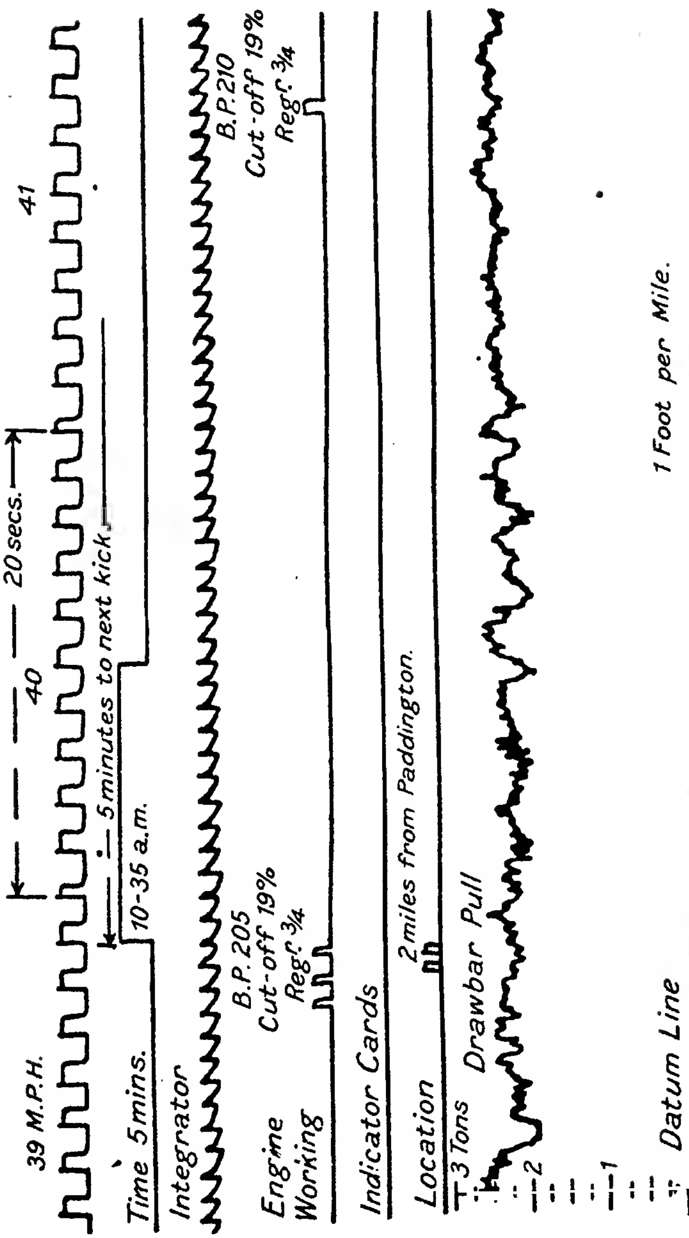


FIG. 125.—Part of a dynamometer car record of the G.W.R. Riviera express.

Mr. Aspinall's results are expressed by the following formula:—

$$r_v = 2\cdot5 + \frac{V^2}{50\cdot8 + 0\cdot0278L} \quad . \quad . \quad . \quad . \quad (1)$$

r_v = resistance in pounds per ton of vehicles;
V = speed of train in miles per hour;
L = length of the train in feet over the coach bodies.

The vehicles or coaches used in these experiments were practically of the same design throughout. Each coach was fitted with two 4-wheeled bogies, the journals of which were lubricated with oil, and each coach weighed a little over 20 tons, and measured 46 or 49 feet long. Experiments were made with trains of 5, 10, 15, and 20 bogie

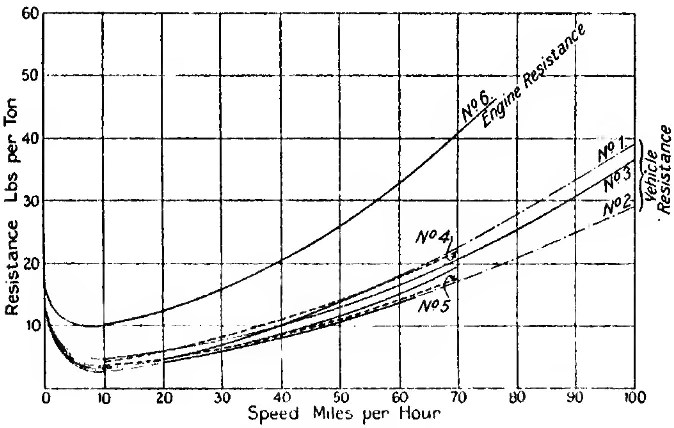


Fig. 126.—Curves of vehicle and engine resistance.

coaches behind the dynamometer car. With five coaches the total length over the coach bodies was 285 feet and the coaches weighed 115 tons. With twenty coaches the length over the coach bodies was 1057 feet and the coaches weighed 429 tons. Curve No. 1, Fig. 126, shows the resistance of the five-coach train in pounds per ton, and curve No. 2, the resistance of the twenty-coach train in pounds per ton plotted against the speed.

With the five-coach train observations are recorded up to nearly 80 miles per hour, and with the twenty-coach train up to 60 miles per hour. The curves on the diagram are continued beyond these limits by chain dotted lines up to 100 miles per hour.

If curve No. 3 is compared with curves 1 and 2, there seems some justification for supposing that this continuation will give reliable data for speeds above those at which the experiments were made, because curve No. 3 is plotted from observation of the resistance of

one of the electrically driven coaches tried in the celebrated experiments carried out at Zossen[1] in 1903, at speeds ranging from about 30 to 110 miles per hour.

The car weighed 92 tons, and the resistance per ton of load is

$$r^v = 4{\cdot}5 + 0{\cdot}022V + 0{\cdot}003V^2 \quad . \quad . \quad . \quad . \quad (2)$$

Inasmuch, however, as at high speeds the air resistance forms the greater part of the resistance on a straight level road, and that this resistance depends upon the form of the vehicle and not on the weight, the formula cannot have a general application. As it stands it represents the resistance per ton of a single coach weighing 92 tons, measured over the widest range of speed on record. The highest speed recorded in these experiments was 112·9 miles per hour.

The two following expressions, given in the *Bulletin of the International Railway Congress* for 1898, vol. 12, page 1275, by M. Barbier, represent the results of some experiments with a dynamometer car which were made on the Northern Railway of France with trains of 157 tons mean weight in the case of the 4-wheeled stock, and 202 tons mean weight with bogie stock at speeds between 37 and 77 miles per hour.

$$r_v = 3{\cdot}58 + \frac{1{\cdot}65V(1{\cdot}6V + 50)}{1000} \text{ for 4-wheel coaches oil lubrication} \quad (3)$$

$$r_c = 3{\cdot}58 + \frac{1{\cdot}65V(1{\cdot}6V + 10)}{1000} \text{ for bogie coaches oil lubrication} \quad . \quad (4)$$

These expressions are plotted in the diagram, curve No. 4, referring to 4-wheel coaches, and No. 5 to bogie coaches which weighed nearly 30 tons apiece.

A large number of expressions are given in Mr. Aspinall's paper on page 189 of vol. 147 of the *Proc. Inst. C.E.*, to which reference may be made for some of the earlier forms used by railway engineers.[2]

All the above expressions refer to carriage stock. There are no records of systematic experiments on the resistance of goods wagons. In the paper above quoted Mr. Aspinall cites a case where the resistance of a train of empty wagons 1830 feet long was 18·33 lbs. per ton at a speed of 26 miles per hour, and a train of full wagons 1045 feet long gave only 9·12 lbs. per ton at a speed of 29 miles per hour. This apparent anomaly is probably explained by the fact that the ends of the empty wagons presented a series of flat surfaces to the wind, so that the stream-line configuration produced was far more complex than in the case of the loaded wagons where the ends were not so exposed.

Mr. Aspinall deduced from experiments made to start the

[1] *Berlin Zossen Electric Railway Tests.* Translated from the German by Franz W. Welz. E. E. McGraw Publishing Co., New York, 1905.
The expression above is given in an introduction written by Louis Bell, Ph.D., in terms, however, of the American ton of 2000 pounds.

[2] See also "The Predetermination of Train Resistance," C. A. Carus-Wilson, *Proc. Inst. C.E.*, 1907-8.

5-coach train and the dynamometer car on various inclines that the resistance to starting is about 17 pounds per ton. The curves in the diagram therefore begin at 17 pounds per ton. The part between 0 and 10 miles per hour is in each case simply sketched in, the remainder of the curve being drawn through points calculated from the expressions indicated on the diagram.

119. **Engine Resistance.**—The resistance of an engine is made up of so many variable factors of an arbitrary kind that any expression for it must of necessity be very approximate. Certainly one form of expression cannot generally apply to all the experimental results obtained.

Some experiments made by the late Mr. Dugald Drummond[1] on the London and South-Western Railway are interesting in this respect.

An engine, type 4–4–0: weighing with tender 90 tons and with cylinders 18½ ins. diameter by 26 ins. stroke, and coupled wheels 6 ft. 5⅜ ins. diameter, was driven light along a short piece of straight road at various speeds and the I.H.P. was measured. These are the results—

Speed, miles per hour . . .	28·5	45	55	64	67	70
Resistance, lbs. per ton . . .	9·6	15·5	28	41	49	54·3
Indicated horse-power . . .	66	168	370	630	788	912

The author found that a formula of the following type expresses these results fairly well:—

$$r_e = AB^V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

A and B are constants and V is the speed in miles per hour. With the above data this reduces to

$$\log r_e = 0·47 + 0·018V \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This, with the aid of a four-figure logarithm table, is a very convenient form to use. The resistance when V = 10 is with this expression about 5 pounds per ton, a result which is too low for the class of engine experimented with.

A set of results from another engine of the same general type, but with cylinders 19 ins. diameter and 26 ins. stroke and driving wheels 6 ft. 10¾ ins. diameter, furnished these data:—

Speed, miles per hour	42	52	60	71½	72
Resistance, lbs. per ton	36	43·8	44	47·1	52·3
Indicated horse-power	321	487	563	719	803

The corresponding expression of form (1) above reduces to

$$\log r_e = 1·316 + 0·006V \quad . \quad . \quad . \quad . \quad . \quad (3)$$

[1] See the remarks of Mr. Vaughan Pendred and Mr. G. R. Sisterson in the discussion of "Compound Locomotives in France," *Proc. Inst. Mech. E.*, 1904, pp. 428 *et seq.*

At 10 miles per hour this gives a resistance of nearly 24 pounds per ton, which is probably too high.

Comparing this with the value obtained for the smaller wheeled engine the results appear to be anomalous. There are anomalies, however, in the data themselves, because it will be seen that an increase of speed from 52 to 60 miles per hour corresponds with an increase of resistance of only 0·2 lb. per ton.

The results as they stand probably include work done against acceleration, since on the track on which the experiments were carried out, it was not possible to obtain steady conditions of working for any length of time.

The straight level piece of line available, about $2\frac{1}{2}$ miles long, terminated at one end in buffer stops and at the other end in a curve leading into a slight gradient up. The engine therefore entered the straight portion of the track after running down the gradient and passing round the curve; and immediately afterwards the diagram had to be taken because of the necessity of shutting off steam to stop clear of the buffer stops.

The difficulty of obtaining a correct relation between the indicated horse-power and the resistance at high speeds lies in the difficulty of making a proper correction for acceleration. An acceleration of the speed may be taking place when the indicator diagram is taken which escapes observation because of the difficulty of accurately measuring small changes of speed at high speeds.

This point may be illustrated by taking a definite example. The rate at which work is done against acceleration expressed in horse-power is

$$\frac{2240Wav}{550g} \text{ H.P.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Take an engine weighing 96 tons. Then this reduces to $12{\cdot}2av$ horse-power.

At ten miles per hour v is 14·66 ft. per second. At 70 miles per hour v is 102·6 ft. per second. Using these values it will be seen that to produce an acceleration of 1 ft. per second per second (0·681 mile per hour per second) at 10 miles per hour work must be done at the rate of 179 H.P., but to produce the same acceleration when the speed is 70 miles per hour requires 1252 H.P.

Thus when the horse-power is measured at 70 miles per hour most accurate observations of speed are required, because a change in speed from 70 to 71 miles per hour in 15 seconds, a change which might easily elude observation, requires to produce it 125 H.P.

The form of expression for engine resistance which is generally used is

$$r_e = A + BV + CV^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If W is the total weight of the engine and tender in tons to which the above expression applies, then the relations between engine

resistance, draw-bar pull P, speed and indicated horse-power are expressed by

$$550 \text{ I.H.P.} - Pv = vW(A + BV + CV^2) \quad . \quad . \quad (6)$$

In this expression v is the speed in feet per second, and V is the speed in miles per hour.

The value of the observed horse-power must also be corrected for the acceleration of the engine and for the effect of the gradient on the engine.

Theoretically three experiments would furnish sufficient data with which to calculate the constants A, B, and C for a particular engine, because inserting the corrected values of the indicated horse-power and P in the equation together with the corresponding speeds, there results a set of three simultaneous equations in which A, B, and C appear as unknown quantities, and the solution of the set gives the required values.

Practically it is necessary to apply the method to several sets in order to find average values of the constants.

Engines differ so much in type that no one set of values of the coefficients A, B, and C can be obtained which have any general application. They would assume different values for different classes of engines. If, for example, the values corresponding to a particular tank engine were known, a slight structural alteration and the addition of a tender would increase the weight by 50 per cent. without sensibly changing the machinery resistance of the engine; but it would appear to be less when expressed in pounds per ton, because it would then be distributed over the whole weight of the engine and tender. Even when the constants are determined carefully for one particular engine they would be liable to large accidental variations due to the kind of lubricant employed, the condition of the machinery, the kind of permanent way over which the engine travels, and the weather.

If the constants A, B, C could be severally expanded in forms so as to include terms depending upon the type of engine an expression of wider general application would result, and a set of experiments on one type of engine could be rationally applied through these constants to engines of other types.

An endeavour is made in the next section to expand the constants in this way in order to obtain a more general type of expression for the resistance of a locomotive.

120. On the Values of the Constants A, B, C, and a General Expression for Engine Resistance.—The whole resistance of a locomotive is made up of two parts, namely, the part due to the resistance of the machinery considered merely as the resistance of a fixed steam engine, and the part due to the road resistance.

The two parts are not strictly independent, but they may be considered so without serious error.

Data obtained from experiments on testing plants show that

within reasonable limits the machinery resistance is constant at all speeds, what variation there is being due to the variation of the cut-off in the cylinders. The coefficients B and C are therefore, with this approximation, independent of the machinery resistance, and the coefficient A must be of such a form that it includes it.

The coefficient A.—The machinery resistance of an engine will increase with the number of coupled wheels. Comparing engines together, the machinery resistance may, to a first approximation, be taken as proportional to the total load W_c, carried by the coupled wheels.

The draw-bar pull equivalent to the engine resistance will therefore be proportional to the load on the coupled wheels, and it is clear that it will also be inversely proportional to the diameter D of the driving wheels. So that

$$P_m = k\frac{W_c}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant to be determined by experiment.

Prof. Goss deduced from the 97 experiments made on the engine "Schenectady" at varying boiler pressures and varying speeds that the machinery friction was practically independent of the speed and the boiler pressure, and depended only upon the cut-off in the cylinders. The mean pressure equivalent to the machinery resistance of the engine "Schenectady" is shown in column 2 of Table 20, page 430; the corresponding cut-off is shown in column 1. The draw-bar pulls calculated from these mean pressures are given in column 3. Corresponding values of the constant k, calculated from these draw-bar pulls by means of equation (1), are given in the last column. Let 120 be taken as a suitable value of k to be included in a general formula; then the resistance at the draw-bar equivalent to the machinery resistance of a locomotive is approximately expressed by

$$P_m = \frac{120W_c}{D} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

W_c is here the weight on the coupled wheels in tons, and D is the diameter of the wheels in feet.

Let W_e be the weight of the engine and tender, then the weight unsupported on the coupled wheels is $W_e - W_c$ tons. This is the weight on the carrying wheels of the engine and tender, and the constant resistance due to it may be reckoned at 4 lbs. per ton. Thus the total constant resistance of a locomotive is approximately

$$AW_e = \frac{120W_c}{D} + 4(W_e - W_c)$$

and therefore

$$A = \frac{120W_c}{DW_e} + 4\left(1 - \frac{W_c}{W_e}\right) \text{ lbs. per ton} \quad . \quad . \quad . \quad (3)$$

TABLE 20.—SHOWING THE VALUES OF THE CONSTANT k IN TERMS OF THE CUT-OFF IN THE CASE OF THE ENGINE "SCHENECTADY" FROM DATA OF 97 EXPERIMENTS RECORDED IN "HIGH STEAM PRESSURES IN LOCOMOTIVE SERVICE".

1	2	3	4
Nominal cut-off.	Mean pressure equivalent to the resistance of the engine machinery.	Corresponding draw-bar pull.	Values of k.
per cent.	lbs. per sq. in.	lbs.	
14	6·5	577	122
19	8·5	755	160
26	9·3	826	175
35	8·4	746	158
50	5·0	444	94
56	3·0	266	56

For dimensions of the engine, see Table 19, page 327.

The mean pressure in the cylinders multiplied by 88·8 is the corresponding draw-bar pull.

The weight on the coupled wheels is 27·22 tons.

The diameter of the driving wheel is 5·77 feet.

The data in columns 1 and 2 of the first four lines are taken from the group of trials in which the boiler pressure was 200 lbs. per square inch by gauge; the corresponding data in the last two lines relate to trials in which the boiler pressure was 120 lbs. per square inch by gauge.

The coefficient B.—It is more difficult to devise a correct form for the coefficient B, because it depends not only upon the engine itself but also on the kind of road on which the engine travels. The chief part of this resistance is probably due to tread resistances and miscellaneous resistance, and the resistance will certainly depend upon the number of coupled axles. Definite experiments have shown that in the case of a four-coupled engine the value of B is about 0·1. Failing direct experiments of the way B depends upon the number of coupled wheels, it may to a first approximation be assumed to vary as the number, though a more correct form would be probably $a + nb$. With the assumption that B varies directly as the number of coupled axles in the engine and that for a four-coupled engine the value is 0·1, it follows that

$$B = 0·05n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In the case of an engine with one pair of driving wheels, $n = 1$.

The coefficient C.—The part of the resistance depending upon V^2 is mainly due to wind resistance. This is indicated by a comparison of the experimental values of C found for particular engines with the values of the coefficient deduced from the experiments upon wind pressure mentioned above.

Referring to page 416 it will be seen that the pressure exerted on a plane by the air when the plane moves through the air in a direction normal to the plane at a speed of V miles per hour, may be assumed to be 0·0032 lb. per square foot of the plane.

If the plane is replaced by an engine the resistance per square foot of projected area will probably be greater because of projecting

irregularities, all of which produce eddies, and because of the proximity of the permanent way to the irregular under-surface of the engine. Allowing an increase of 25 per cent. for this and for the skin resistance of the engine, the pressure due to the wind would be 0·004 lb. per square foot of projected cross-section. If therefore J is the projected cross-section of the engine and W is the total weight of engine and tender,

$$C = \frac{0{\cdot}004J}{W_e} \text{ lbs. per ton} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The projected cross-section of modern engines of standard gauge may be taken constant at 100 sq. ft., and the weight of engine and tender may be regarded as at any approaching 100 tons, so that with these further approximation

$$C = 0{\cdot}004 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Collecting together the values of the coefficients, the resistance in pounds per ton of engine and tender working under normal conditions is given to a first approximation by the equation

$$r_e = \frac{120W_c}{DW_e} + 4\left(1 - \frac{W_c}{W_e}\right) + 0{\cdot}05nV + 0{\cdot}004V^2 \quad . \quad (7)$$

where W_e = total weight in tons of the engine, and of the tender when there is one.
W_c = weight in tons on the coupled wheels.
D = the diameter of the coupled wheels in feet.
n = the number of coupled axles.
V is the speed of the train in miles per hour.

Table 21, page 432, shows average values of $\frac{W_c}{W_e}$, and average values of D for different classes of British locomotives, together with the corresponding expression for the engine resistance r_e, calculated from equation (7).

These expressions for engine resistance are suggested as reasonable for estimating purposes failing expressions based upon actual experiments with the different classes of engines.

There are no published data sufficiently extensive to enable a computation of the coefficients A, B, and C to be made from actual experiments in connection with British locomotives. M. Barbier has determined from experimental data the values of the constants for the four-cylinder compounds used in the series of experiments on train resistance on the Northern Railway of France quoted above. The expression for engine resistance with these coefficients inserted and rounded off is

$$r_e = 8{\cdot}5 + 0{\cdot}1V + 0{\cdot}005V^2 \quad . \quad . \quad . \quad . \quad (8)$$

which is valid for speeds between 37 and 77 miles per hour. V is in miles per hour, and the resistance is in pounds per ton of engine

and tender. The locomotives were designed by the late M. du Bousquet, and were put into service in 1896. The weight of engine and tender was 83 tons.

TABLE 21.—SHOWING VALUES OF THE RATIO $\frac{W_c}{W_e}$ FOR VARIOUS CLASSES OF ENGINES, TOGETHER WITH THE CORRESPONDING RESISTANCE FORMULÆ CALCULATED FROM THE PROPOSED GENERAL EQUATION 7.

Type of engine.	Number of engines from which average values of $\frac{W_c}{W_e}$ and D were computed.	Average value of $\frac{W_c}{W_e}$.	Average value of D.	Resistance formula. Lbs. per ton, r_e.
4-2-2.				
4-4-0. Tender engine	16	0·38	6·6	$9{\cdot}5 + 0{\cdot}1V + 0{\cdot}004V^2$
4-4-2. Tender engines	10	0·35	6·8	$8{\cdot}8 + 0{\cdot}1V + 0{\cdot}004V^2$
4-6-0. Tender engine	8	0·46	6·4	$10{\cdot}8 + 0{\cdot}15V + 0{\cdot}004V^2$
0-6-0. Tender goods	13	0·56	5·0	$15{\cdot}2 + 0{\cdot}15V + 0{\cdot}004V^2$
0-8-0. Tender goods	6	0·59	4·5	$17{\cdot}2 + 0{\cdot}2V + 0{\cdot}004V^2$
0-4-4. 2-4-2. Tank engines	12 4	0·61	5·3	$15{\cdot}0 + 0{\cdot}1V + 0{\cdot}004V^2$
4-4-2. Tank engines	6	0·53	6·0	$12{\cdot}4 + 0{\cdot}1V + 0{\cdot}004V^2$
0-6-2. Tank engine	12	0·8	4·8	$20{\cdot}7 + 0{\cdot}15V + 0{\cdot}004V^2$
0-8-2. Tank engines	4	0·83	4·4	$23{\cdot}4 + 0{\cdot}2V + 0{\cdot}004V^2$

W_c is the weight on the coupled wheels.
W_e is the total weight of engine and tender.
D is the diameter of the driving wheel in feet.

An expression given by Von Borries,[1] based jointly on that of Barbier and the results of Prof. Goss on the testing plant and reduced to English units and rounded off, is

$$r_e = 9 + 0{\cdot}1V + 0{\cdot}004V^2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Frank[2] gives from an examination of recent work

$$r_e = 9 + 0{\cdot}005V^2 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Reference may be made to an article in the *Engineer*, March 26, 1909, by Mr. Fry, in which the work of continental engineers in connection with engine resistance is reviewed.

Mr. Fry, after comparing the expressions by Von Borries, Frank, Nadal, Saszin, Hefft, Coute, suggests expressions for engine resistance which, converted into resistance in pounds per ton of engine and tender and rounded off, are as follows:—

Four-coupled engines, $r_e = 8{\cdot}5 + 0{\cdot}1V + 0{\cdot}004V^2$. (11)
Six-coupled engines, $r_e = 10 + 0{\cdot}13V + 0{\cdot}004V^2$. (12)
Eight-coupled engines, $r_e = 14 + 0{\cdot}5V + 0{\cdot}004V^2$. (13)

M. Barbier's expression, equation (8), is shown in Fig. 126, page 424, by curve No. 6.

[1] *Zeit. Vereins. Deutsch. Ing.*, 1901, p. 808.
[2] *Ibid.*, 1907, p. 96.

121. The Indicated Horse-power as a Function of the Speed.—The weight of steam which flows into the cylinders of a locomotive during the period of time between admission and cut-off depends upon the boiler pressure, the extent to which the driver opens the regulator, and the fraction of the stroke at which cut-off is fixed by the position of the reversing lever. There is another cause operating to vary the flow quite outside the control of the driver, namely, the piston speed.

For a given opening of the regulator, a given cut-off, and a given and constant boiler pressure, the weight of steam which flows into the cylinder per stroke decreases as the piston speed increases.

This is illustrated by the figures in Table 22, which is compiled from data in "High Steam Pressures in Locomotive Service".

TABLE 22.—INFLUENCE OF SPEED ON THE WEIGHT OF STEAM FLOWING INTO THE CYLINDER PER REVOLUTION.

Particulars of the engine "Schenectady" No. 2, 4-4-0, are given in Table 19, p. 327.
Conditions—
Nominal boiler pressure, 200 lbs. per sq. in.
Nominal cut-off, 20 per cent. of the stroke.
Regulator, wide open.

Pressure in the supply pipe. Lbs. per sq. in. by gauge.	Actual cut-off as measured from diagram.	Piston speed in feet per minute.	I.H.P.	Weight of steam flowing into the cylinders per revolution.	Mean pressure, lbs. per sq. in.	Lb.-calories per I.H.P. min.
	per cent.			lbs.		
197	19·8	388	283	1·3	61·0	244
199	18·9	584	367	1·076	51·7	238
Not stated	19·7	780	449	0·95	47·4	229
196	20·7	975	464	0·82	39·8	239
Average 197 = 212 lbs. per sq. in. absolute						

The nominal boiler pressure in the group of experiments from which the figures in Table 22 are quoted was 200 lbs. per square inch by gauge; the nominal cut-off was at 20 per cent. of the stroke, and the regulator was wide open. It will be seen that the weight of steam flowing into the two cylinders of the engine per revolution of the driving axle was reduced from 1·3 lbs. to 0·82 lb. as the piston speed increased from 388 to 975 ft. per minute. The fall is regular, as will be seen from the dotted curve, Fig. 128, page 436.

The indicator diagrams shrink in size as the weight of steam from which they are developed is reduced. Typical diagrams are shown in Fig. 127, corresponding to piston speeds of 388, 780, and 975 ft. per minute, from which the amount of shrinkage can be seen.

In the four cases in the table the mean pressures fall to the values shown. These mean pressures are plotted against the piston

speed in Fig. 128, page 436. The points fall pretty nearly on a straight line, which is represented by the equation

$$p = 75 - 0{\cdot}037s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

p is the mean pressure in pounds per square inch when the piston speed is s feet per minute, and the nominal cut-off is constant at 20 per cent. of the stroke, with the regulator wide open and the boiler pressure constant at about 200 lbs. per square inch by gauge.

The mean pressure and the piston speed appear to be connected by a linear relation of this kind between piston speeds of 400 and 1200 ft. per minute in all the trials of simple engines which the author has examined. It is probable, therefore, that in general

$$p = c - bs \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where c and b are constants depending upon the engine, the boiler pressure, the cut-off, and the position of the regulator, and p is the mean pressure corresponding to the piston speed s.

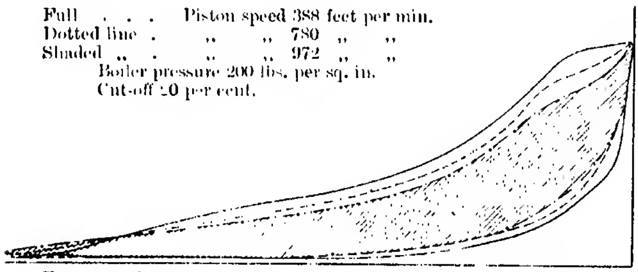


Fig. 127.—Superposed indicator diagrams to show effect of speed.

Before establishing the generality of the expression by further evidence from experiment, let us examine some of the consequences of the relation.

The usual formula for the indicated horse-power, namely

$$\text{I.H.P.} = \frac{plaN}{33000}$$

may be altered to

$$\text{I.H.P.} = \frac{psa}{33000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

a formula in which s, the average piston speed in feet per minute, replaces the product of the length of the stroke l and the number of strokes per minute N.

Introducing into (3) the value of the mean pressure given in (2), and writing n for the number of cylinders, it becomes

$$\text{I.H.P.} = \frac{na(cs - bs^2)}{33000} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This expression gives the horse-power corresponding to a given nominal cut-off, a given regulator opening, and a constant boiler pressure. This expression has a maximum value for a particular value of s. To find it, differentiate the indicated horse-power with respect to s and equate the result to zero, thus

$$\frac{d(\text{I.H.P.})}{ds} = c - 2bs = 0 \text{ for a maximum value of I.H.P.}$$

From this the piston speed corresponding to a maximum value of the indicated horse-power is

$$s_1 = \frac{c}{2b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

the corresponding mean pressure is

$$p_1 = \frac{c}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and the corresponding horse-power is

$$\text{I.H.P.}_{max} = \frac{nc^2a}{132000b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

These relations are illustrated graphically in Fig. 128, page 436. The straight line corresponding to the mean pressure is PQ. The indicated horse-power curve corresponding to equation (4) is the parabola OKQ, which passes through its maximum value at K when the piston speed is $\frac{c}{2b}$ = 1015 ft. per minute; and the mean pressure is $\frac{c}{2}$. The indicated horse-powers actually observed are shown by crosses.

Below this is shown a similar diagram, Fig. 129, drawn from data obtained by Prof. Goss from an engine tested on the plant at Purdue University and published in vol. 14 of the *Transactions of the American Society of Mechanical Engineers.* It will be seen that the linear relation is fairly true between the limits of piston speed 324 and 1184 ft. per minute.

In this case the piston speed corresponding to the maximum horse-power is 887 ft. per minute, and the expression for the mean pressure is

$$p = 55 - 0{\cdot}031s \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The value of the constants c and b should be computed from a series of actual observations of the indicated horse-power and the corresponding speeds. It is practically impossible to make the necessary experiments except on a testing plant because of the

difficulty involved in the maintenance of constant conditions of working on the road.

In the absence of a series of actual observations, the constants may be arrived at in the following manner. The intercept c on the

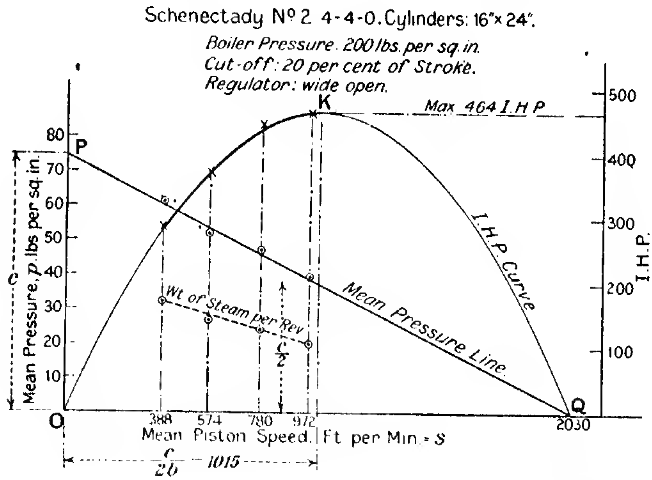


Fig. 128.—Mean pressure and I.H.P. plotted against speed. "Schenectady," No. 2.

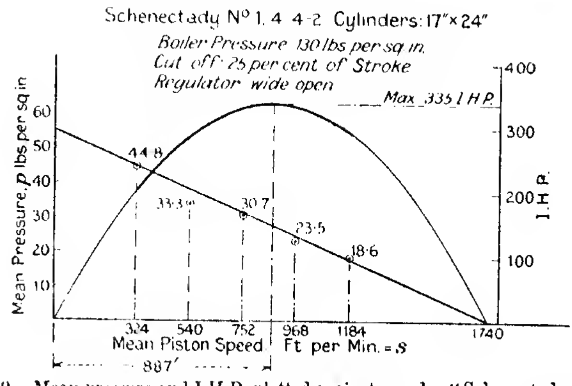


Fig. 129.—Mean pressure and I.H.P. plotted against speed. "Schenectady," No. 1.

vertical axis represents the value to which the mean pressure approaches as the speed approaches zero. This indicates that the mean pressure calculated from an indicator diagram drawn to correspond with the events of the stroke without any reduction of area for wire drawing should coincide with the value of c. The indicator diagram corresponding to the cut-off in the table is sketched

in Fig. 130 in a form corresponding with an infinitely slow piston speed. It is plotted from these data:—

Cut-off, point C, 19·7 per cent. of the stroke; volume Oc.

Release, point R, 62·0 per cent. of the stroke; volume Or.

Compression, point K, 38·0 per cent. of the stroke; volume Ok.

Admission, 1·6 per cent. of the stroke; volume Oa.

Clearance, 7·6 per cent. of the volume swept out by the pistons; so that OA is 7·6 on the scale on which AB is 100;

Back pressure, Op, 15 lbs. per square inch absolute;

Initial pressure, OP, 212 lbs. absolute. This is the average value of the absolute steam pressure in the supply pipe.

The mean pressure corresponding to the diagram is 76 lbs. per

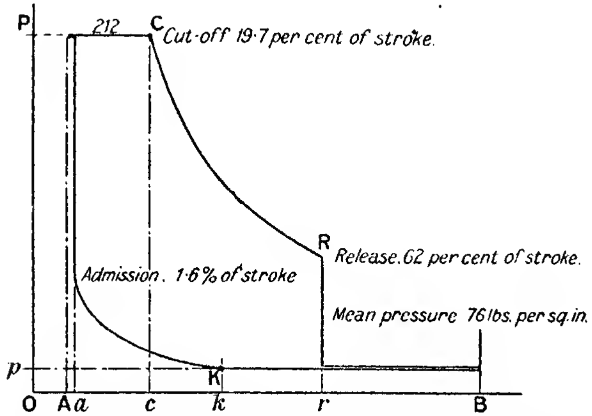


FIG. 130.—Typical indicator diagram and determination of the constant c.
The Expansion Curve through C is plotted from PV = a constant = OP × Oc.
The Compression Curve through K is plotted from PV = a constant = Op × Ok.

square inch. This practically coincides with the value of c in Fig. 128.

Comparing the values of c found, first from the mean pressure line as in Fig. 128, and secondly from the indicator diagram drawn as above, in a number of cases, the author has found that the values are sufficiently near to justify the second method of calculation when there are no data available for using the first method.

When using this second method care must be taken to find the correct fractions of the stroke at which the several events occur, and to use an initial pressure which the boiler can maintain in the supply pipe when the engine runs with the cut-off corresponding with the diagram.

Experiments on the road are required to furnish a value of b.

Theoretically a single experiment gives the data from which one point on the mean pressure line can be calculated, and this, with the point determined on the vertical axis by c, fixes the line. Practically there should not be much difficulty in making a sufficient number of observations to get a good average value of the mean pressure corresponding to one particular speed, corrected of course for acceleration and gradient.

Confirmation of the general truth of the linear relation between mean pressure and speed for a constant cut-off, a constant regulator opening, and a constant boiler pressure, is found in some experiments made with the testing plant belonging to the Chicago and North-Western Railroad on an engine with cylinders 20 ins. diameter and 26 ins. stroke, wheels 63 ins. diameter, and boiler pressure 190 lbs. per square inch. Details of these experiments are given in the *American Engineer* for June, 1901, in a valuable article by Mr. G. R. Henderson entitled "Economical Train Speed". This article and the curves which accompany it are well worth careful study. Amongst the diagrams given was one showing the indicated horse-power plotted against the speed. The analysis of these results, together with other examples, will be found in a paper by the author in the *Proceedings of the Institution of Civil Engineers*, vol. 164, p. 329, entitled "The Economical Working of Locomotives".

Further confirmation is found in the results recorded in "High Steam Pressure in Locomotive Service". The 97 separate experiments furnish data for plotting 21 mean pressure lines at various cut-offs and under six different conditions of boiler pressure. In every case the general linear relation holds good without any great error.

The piston speed corresponding to the maximum horse-power, as determined from these 21 mean pressure lines, had an average value of 900 ft. per minute, at about 14 per cent. cut-off over the whole range of boiler pressures used, whilst for all the other cut-offs and over the whole range of boiler pressures used the value averaged about 1000 ft. per minute.

These results indicate that in a particular engine the range of piston speed corresponding to the maximum indicated horse-power is not great, and for design purposes may be taken as constant.

Failing any experiments at all the probable mean pressure lines for various cut-offs may be plotted from the constant c and an assumed value of the piston speed corresponding to the maximum horse-power. From these lines, curves, like those shown in Fig. 131, showing probable indicated horse-power in terms of the speed and cut-off, may be plotted for simple engines with ordinary slide valves. Thus to estimate the probable horse-power for a simple engine—

1. Sketch a series of indicator diagrams as exemplified in Fig. 130 corresponding to a series of values of the cut-off, assuming hyperbolic expansion and compression. The mean pressures corresponding to these give a series of corresponding values of the constant c.

2. Then $b = \dfrac{c}{2s_1}$, where s_1 is an assumed value of the piston speed corresponding to the maximum horse-power. A fairly good value to take is 900 ft. per minute.

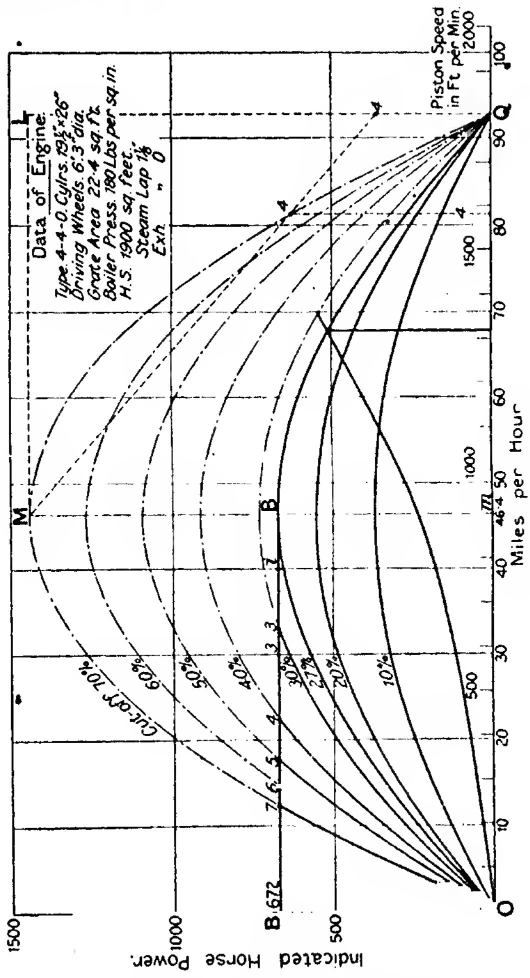


FIG. 131.—Curves of probable I.H.P. Simple engine.

3. Then the indicated horse-power $= \dfrac{na(cs - bs^2)}{33000}$; a is the area in square inches of one cylinder; n is the number of cylinders.

A group of probable indicated horse-power speed curves is shown

in Fig. 131 for cut-offs ranging from 10 to 70 per cent. of the stroke in the case of an engine with these dimensions:—

Cylinder, $19\frac{1}{2}$ ins. diameter, 26 ins. stroke.

Grate area, 22·4 sq. ft.

Diameter of driving wheels, 75 ins.

Boiler pressure, 180 lbs. per square inch by gauge = 195 lbs. per square inch absolute.

Link motion of the Stephenson type with a D slide valve. Steam lap, 1·125 ins.; exhaust lap, 0.

From a set of indicator diagrams sketched for different cut-offs the following values of c were found:—

TABLE 23.—VALUES OF c AND THE CUT-OFF.

Cut-off per cent. of stroke.	c = limit to which the mean pressure approaches as the piston speed approaches zero.
10	46
20	68
30	90
40	112
50	134
60	156
70	178

The curves are drawn assuming that the piston speed corresponding to the maximum indicated horse-power is 900 ft. per minute for all cut-offs. Therefore the value of b is $\frac{c}{1800}$, and the expression for the indicated horse-power becomes $596\left(1 - \frac{s}{1800}\right)\frac{cs}{33000}$, from which the curves may be plotted.

These curves are sketched complete in form to show what the indicated horse-power would be with the assumed conditions provided that the boiler could supply the necessary steam. The rate at which the boiler can supply steam is, however, strictly limited by its size, and therefore each curve of the group rises only to the height corresponding with the maximum indicated horse-power which the boiler can continuously maintain. This may be estimated at 30 I.H.P. per square foot of grate area. So that with the data of the problem the maximum continuous indicated horse-power would be $22·4 \times 30 = 672$. Draw the horizontal line BB across the diagram to correspond with this value: the points 7, 6, 5, 4, 3, where it cuts the curves of indicated horse-power, show the speeds up to which the corresponding cut-offs can be used.

This applies only up to the line of maximum horse-power. After that the maximum horse-power falls as the speed increases in the way shown by the particular cut-off curve, which is tangent to the straight

line of continuous boiler horse-power. The particular cut-off curve which touches this line is the one corresponding to 27 per cent. The characteristic indicated horse-power speed curve is for this engine therefore the thickened curve BBQ. As the speed increases the maximum indicated horse-power is at first approximately constant at the value determined by the size of the boiler until a speed is reached at which the power is determined, not by the rate at which the boiler can produce steam, but by the rate at which the steam can find its way into and out of the cylinders.

The assumption that the best speed is the same for all cut-offs, carries with it the condition that in the mean pressure-speed diagram all the mean pressure lines pass through the point on the horizontal axis at which the speed is twice the piston speed corresponding to the maximum horse-power. Consequently all the indicated horse-power curves pass through this point also, since the mean pressure here vanishes for all cut-offs. Also the indicated horse-power curves will all pass through the origin, because there the speed vanishes. The maximum value of the indicated horse-power is known when c is known, and can be calculated from

$$\text{I.H.P.}_{max} = \frac{\frac{1}{4}nacs_1}{33000} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

With the data of the example this is

$$\text{I.H.P.}_{max} = 8{\cdot}12c \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Therefore, three points are known on each curve, namely (1) the origin; (2) the point of maximum horse-power; (3) the point where the parabola cuts the speed axis. The co-ordinates of the point of maximum horse-power are in general $\frac{c}{2b}$ along the speed axis and the I.H.P.$_{max}$ from (13) vertically. In the present example $\frac{c}{2b}$ is assumed to be constant and equal to 900, and the I.H.P.$_{max}$ has the values calculated from (14), which are given in Table 24.

TABLE 24.—MAXIMUM HORSE-POWER AND THE CUT-OFF.

Cut-off per cent. of stroke.	Maximum horse-power assumed to take place at 900 feet per minute piston speed.
10	374
20	552
30	732
40	910
50	1089
60	1266
70	1445

The abscissa of the third point is in general $\frac{c}{b}$, and with the data of the example is 1800.

Any geometrical method may be used to draw a parabola through these three points. A convenient one is this. Let O, M, Q, Fig. 131, page 439, be three points; through Q draw a line perpendicular to OQ; divide mQ into any number of equal parts, and QL into the same number; number the dividing points in the horizontal axis from the centre m outwards, and in the vertical line from the point L downwards; draw verticals through the points in the horizontal axis and join the points in the vertical QL to the point M: lines with corresponding numbers intersect in a point on the parabola. For example, the intersection of the pair of lines 44 and 4M define the point 4 on the parabola OMQ in the figure.

The relation between the indicated horse-power and the speed with cut-off and boiler pressure constant is more difficult to ascertain with accuracy in the case of compound engines. The available experiments seem to indicate that the maximum horse-power for a given cut-off takes place at a higher piston speed than in the case of simple engines. This is probably due to the fact that the cut-offs in the high-pressure cylinder are always considerably later in the stroke than is the case with simple engines, so that there is less obstruction to the entrance of the steam into the cylinder at corresponding piston speeds.

The general trend of the horse-power speed relation is illustrated in Fig. 132, by curves plotted from the results of the trials at the St. Louis Exhibition.

It is difficult to get the indicated horse-power corresponding to a definite speed and cut-off with accuracy, because although the reversing lever may be fixed in one position in two experiments at different speeds yet the actual cut-off varies in the cylinder owing to the effect of inertia, slack at the joints, and elastic deformations of the valve gear. The actual cut-off is written against each of the dots in the diagram. In a few cases, where there was no experiment recorded at the cut-off taken for the curve, where possible a point has been found by interpolation. These points are indicated on the curve.

There are not enough data available to enable the mean pressure law to be investigated when superheated steam is supplied to the engines.

122. General Summary.—The general equation (8), page 41, is a statement based upon the principle of the Conservation of Energy, that

$$\left.\begin{array}{l}\text{the rate at which}\\ \text{mechanical energy}\\ \text{is produced in the}\\ \text{cylinders}\end{array}\right\}\text{is equal to}\left\{\begin{array}{l}\text{the rate at which energy is expended}\\ \text{in overcoming the resistance of the}\\ \text{train to motion including the resist-}\\ \text{ance to acceleration and change of}\\ \text{height.}\end{array}\right.$$

Considering the left-hand side of this equality first, the rate at which energy is produced in the cylinders is measured by the indicated horse-power. If I.H.P. stands for the indicated horse-power, the rate in foot-pounds per second is 550 I.H.P. The maximum

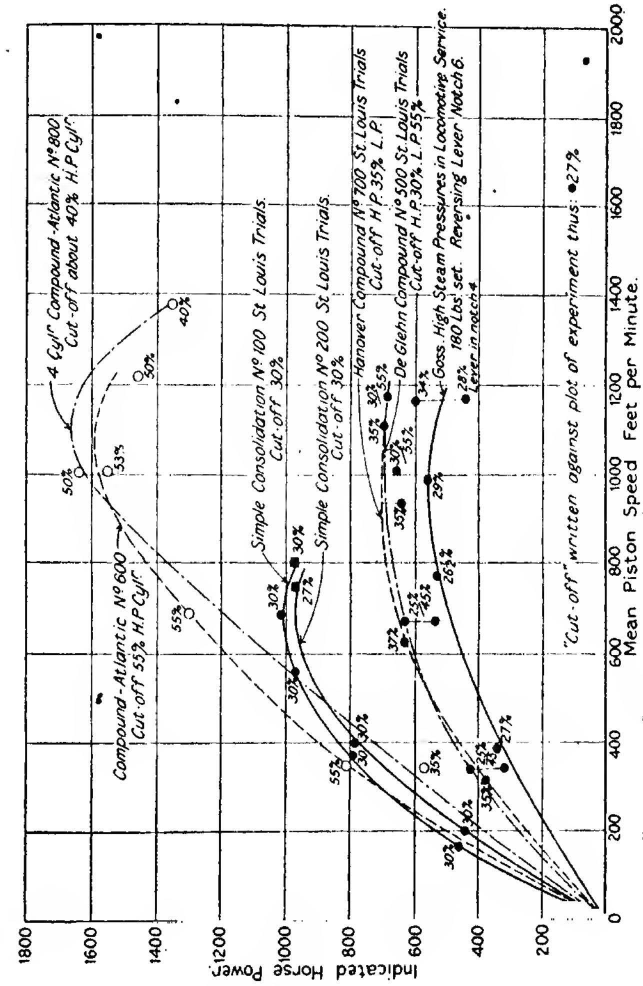


FIG. 132.—I.H.P. against piston speed. Engines tested at St. Louis Exhibition.

value which this rate can assume in a particular engine depends upon the size of the boiler up to a piston speed within the region of 700 and 1000 ft. per minute, after which it depends mainly upon the piston speed, since as the speed increases the weight of steam which

can find its way into the cylinders at constant cut-off and boiler pressure diminishes.

For given values of the boiler pressure, cut-off, and regulator opening, the mean pressure between the limits of normal working is approximately represented by the linear relation

$$p = c - bs$$

p being the mean pressure in pounds per square inch when s is the piston speed in feet per minute, c and b being constants.

The indicated horse-power curve corresponding to these conditions is therefore a parabola in form, with a maximum value at the piston speed $\frac{c}{2b}$, the corresponding mean pressure being $\frac{c}{2}$. A group of these curves is drawn in Fig. 131, page 439, for an initial boiler pressure of 180 lbs. per square inch, and cut-offs ranging from 10 to 70 per cent. The general form of the indicated horse-power curve as a function of the speed is shown in Fig. 132, page 443.

Turning now to the right side of the equation, the rate at which energy is expended may be divided into two parts, one part being the rate at which it is expended against resistance to acceleration and change of level—which is recoverable—the other part being the rate at which it is expended in overcoming resistance in the general nature of friction and which is dissipated as heat.

The rate at which energy is expended against the first kind of resistance is expressed by equation (7), page 408, as a function of the speed, and against the frictional resistances by equation (4), page 420.

Since the frictional resistance of the engine is greater per ton and increases at a greater rate than the frictional resistances of the vehicles, the engine and the vehicles must be treated separately.

In calculating the energy spent in accelerating the train the actual weight of the train must be increased by about 12 per cent. to a weight $\mathfrak{W}$, in order to make an allowance for the angular acceleration of the revolving parts of the train.

For the energy expended against change of level the actual weight of the train W is to be used.

123. Tractive Force.—The rate at which energy is expended in overcoming the resistance of the train is equal to Rv, where R stands for the sum of all the resistances opposing the motion of the train and v is the speed of the train in feet per second. And since 550 I.H.P. is the rate at which work is done on the engine pistons

$$550 \text{ I.H.P.} = Rv$$

from which

$$R = \frac{550 \text{ I.H.P.}}{v} \quad \quad (1)$$

This force R is the tractive force equivalent to the indicated horse-power developed in the cylinders.

The power developed in the cylinders is applied through the driving axle to draw the train. Let L be the average value of the turning couple acting on the driving axle, and ω its angular velocity in radians per second. Then the rate at which work is done on the driving axle is $L\omega$ ft.-lbs. per second.

The couple L is constituted by two equal and opposite forces, one of which is the tangential resistance between the tyres and the rails, and the other is the tractive pull applied at the horns of the driving wheels to haul the train along. The arm of the couple is the radius of the wheel.

Therefore the tractive pull T cannot have a greater value than the resistance to slipping at the rail. If W_c is the weight on the coupled wheels and μ is the coefficient of friction between the wheels and the rails, the wheels will begin to slip when the tractive pull at the horns reaches the value $W_c\mu$.

The weight which may be put upon a single pair of wheels is limited by the strength of the permanent way and the bridges. The weakest bridge on a line imposes the upper limit to the wheel loading. Twenty tons may be taken as an upper limit for a main line in this country.

The value of the coefficient μ for a clean dry rail is usually taken at about $\frac{1}{5}$. With greasy rails it is considerably reduced. Its value is artificially increased by sanding. With the apparatus of the usual design the sand is deposited in a small heap a few inches in front of the point of contact of the wheel and rail, so that if the engine slips at starting without moving, the sand is just a few inches away from where it is wanted. Sanding apparatus is made in which a steam jet is applied to blow the sand between the slipping wheel and the rail.

Using $\frac{1}{5}$ for the value of μ and the above limitations of wheel loading, the greatest tractive pull which can be exerted on a single pair of wheels is about 4 tons.

The tractive force T exerted against the frictional resistance of the rails is less than the tractive force equivalent to the indicated horse-power developed in the cylinders by the amount corresponding to the internal resistance of the engine. This amount has been shown, in equation (2), page 429, to be approximately equal to $\frac{120W_c}{D}$, where W_c is the total weight on coupled wheels of diameter D feet. So that

$$T = \frac{550 \text{ I.H.P.}}{v} - \frac{120W_c}{D} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For our present purpose the internal resistance is more conveniently allowed for by assuming that the value of T corresponds to a mean pressure in the cylinder of 0·8 times the boiler pressure by gauge. This is the usual practice in this respect.

Expanding the indicated horse-power into its usual factors, and

putting n for the number of cylinders and N for the revolutions of the crank axle per second,

$$T = \frac{550 \times 0 \cdot 8 p l \pi d^2 2 n N}{4 \times 550 \times v}$$

The train speed v, in feet per second divided by the circumference of the driving wheel πD ft., is equal to N, the number of revolutions of the crank axle per second. Using this relation, the above expression reduces to

$$T = \frac{0 \cdot 8 n p l d^2}{2D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this equation p is the boiler pressure by gauge, l is the length of the stroke, D is the diameter of the driving wheel, both expressed in the same units of length, and d is the cylinder diameter in inches.

The value of T calculated from this expression must not exceed the value $W_c\mu$ or the wheels will slip. Compound locomotives are usually provided with starting valves through which steam at a reduced pressure is admitted to the low pressure cylinder direct from the boiler. Assuming that the boiler steam is reduced to the pressure which when multiplied into the area of the low-pressure cylinder gives a product equal to that obtained by multiplying the boiler pressure into the area of the high-pressure cylinder, the expression for T may be applied to the low-pressure cylinders of a compound locomotive to calculate the maximum tractive force exerted at starting, remembering that p now stands for the reduced pressure.

The tractive force T given by equation (3) can only be exerted up to the speed at which it is possible to maintain a mean pressure in the cylinders equal to 0·8 times the boiler pressure. As the speed increases beyond this the cut-off must be reduced, and if this is done gradually so that steam is used in the cylinder at the maximum rate at which the boiler can supply it, the horse-power developed will be nearly constant. The tractive force T is now to be calculated from equation (2) by substituting for the indicated horse-power there the indicated horse-power corresponding to the maximum which the boiler can maintain. The second term in the expression is relatively small in these conditions and can be neglected. The tractive force T then varies inversely as the speed, and the curve representing it on a speed base is a rectangular hyperbola, and the expressions to be used are

$$T = \frac{550 \text{ I.H.P.}}{v} = \frac{375 \text{ I.H.P.}}{V} \quad . \quad . \quad . \quad . \quad (4)$$

in which v is the speed in feet per second; and V is the speed in miles per hour.

Referring to Fig. 131, page 439, it will be seen that at piston speeds beyond the piston speed corresponding to the maximum horse-power, the full boiler horse-power cannot be developed in the cylinders. The possible horse-power is then given by the ordinates of the

parabola, which is tangent to the line representing the constant boiler horse-power. Similarly, the tractive force cannot be calculated by substituting the boiler horse-power for I.H.P., in equation (4), for speeds higher than the piston speed corresponding to the maximum horse-power. The possible tractive force is then found from the horse-power given by the parabola which is tangent to the constant horse-power line.

Or the tractive force may be calculated in the following way for speeds greater than the critical piston speed.

It has been shown, page 434, that the mean pressure at constant cut-off and full regulator may be expressed by the relation

$$p = c - bs$$

where s is the piston speed in feet per minute, and c and b are constants depending upon the cut-off and the boiler pressure.

Let t be the tractive force corresponding to this. Then substitute $c - bs$ for $0{\cdot}8\,p$ in equation (3), page 446, giving

$$t = \frac{(c - bs)nld^2}{2D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In this expression n is the number of cylinders; l is the length of the stroke, D is the diameter of the driving wheel, both expressed in the same units of length, and d is the diameter of the cylinder in inches. The constants c, b are those belonging to the parabola which is tangent to the line of constant boiler horse-power.

If the speed corresponding to the maxima for each cut-off is assumed to be constant and equal to s_1, then, remembering that $b = \frac{c}{2s_1}$, the expression becomes

$$t = \frac{c\left(1 - \frac{s}{2s_1}\right)nld^2}{2D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

These considerations show that the curve of maximum tractive force which an engine can exert, plotted against the speed, is made up of three parts. The first part is a straight line parallel to the axis and at a distance from it calculated from equation (3), page 446, or alternatively from $W_e\mu$; the second part from equation (4), is a rectangular hyperbola determined by $T = \frac{550 \text{ I.H.P.}}{v}$, where I.H.P. is the maximum boiler power and v is the speed of the train in feet per second; and the third part is, as shown by equation (5), a straight line which with the proper values of c and b touches the rectangular hyperbola at the point corresponding to the maximum horse-power for the cut-off determined by c and b. An example will make these relations clear.

124. Example.—Tractive Force-Speed Diagram.—These relations are illustrated by the group of probable characteristic tractive force-speed curves shown in Fig. 133, corresponding to the group of

characteristic I.H.P.-speed curves shown in Fig. 131, page 439, for an engine with these dimensions:—

Cylinders $19\frac{1}{2}$ ins. × 26 ins. stroke.
Grate area, 22·4 sq. ft.
Diameter of driving wheels, 75 ins.
Boiler pressure, 180 lbs. per square inch by gauge.
Ordinary D slide valve.
Steam lap, 1·125. Exhaust lap, 0.
Weight on coupled wheels, 40 tons.

The tractive force corresponding to the cylinder horse-power is

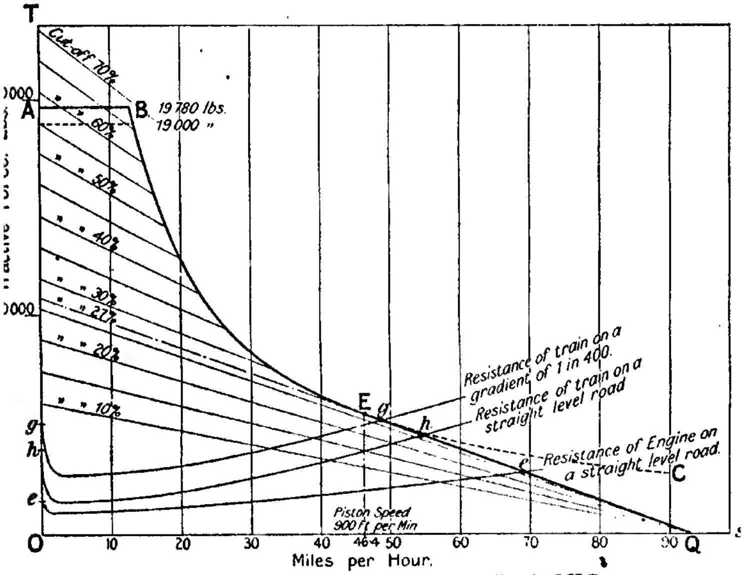


FIG. 133.—Group of tractive-force curves corresponding to I.H.P. curves, Fig. 131, page 439.

plotted vertically, against the speed horizontally, in miles per hour in Fig. 133.

The conditions assumed are—

(1) that the regulator is kept wide open;

(2) that the boiler pressure remains constant;

(3) that the maximum indicated horse-power at any cut-off is reached at a piston speed of 900 ft. per minute.

First calculate the magnitude of the maximum tractive force which can be developed by the engine from expression (3) of the last section. This is

$$T = \frac{2 \times 0{\cdot}8 \times 180 \times 19{\cdot}5^2 \times 26}{2 \times 75} = 19{,}000 \text{ lbs.} = 8{\cdot}48 \text{ tons.}$$

Therefore the weight on the coupled wheels should be

$$8{\cdot}48 \times 5 = 42{\cdot}40 \text{ tons}$$

The actual weight on the coupled wheels is 40 tons, a value sufficiently near to prevent slipping with the rails in good condition.

Before plotting this in the diagram the value 19,000 lbs. should strictly be increased by the tractive force equivalent to the internal friction of the engine, which may be calculated from

$$T_1 = \frac{120W_c}{D}$$

W_c = weight on coupled wheels in tons,
D = diameter of the driving wheels in feet,
T_1 = corresponding tractive resistance in pounds.

With the data of the example this becomes

$$\frac{120 \times 40}{6{\cdot}25} = 780 \text{ pounds},$$

which shows that compared with the maximum tractive force it is for all practical purposes negligible. The line AB, Fig. 133, is drawn to represent 19,780 lbs.

Secondly, the maximum continuous power of the boiler is estimated to be equal to thirty times the grate area, that is 672 I.H.P. Use this value of the I.H.P. in equation (4), page 446, to calculate a series of values of T and the speed V, and with these values plot the curve BC.

The tractive force T, thus calculated, cannot of course be greater in value than 19,780 lbs. The speed corresponding to this value is 12·7 miles per hour, equivalent to 57 revolutions of the driving axle per minute. Generally the maximum boiler power cannot be used by the engine until the speed has reached the region of 50 to 60 revolutions of the driving axle per minute.

Thirdly, set out the family of tractive force curves corresponding to a series of values of the cut-off from equation (6), page 447, since in this example it is assumed that the maximum horse-power at each cut-off takes place when the piston speed is 900 ft. per minute. With the data of the example, equation (6) becomes

$$t = 131{\cdot}5c\left(1 - \frac{s}{1800}\right)$$

t is in pounds, and s is the piston speed in feet per minute. From this it will be seen that for all values of c, $t = 0$ when the piston speed is 1800 ft. per minute; that is to say, all the tractive force lines produced will pass through the same point on the speed axis, namely, the point corresponding to 1800 ft. per minute, equivalent to 92·8 miles per hour. This is otherwise obvious from the power curves Fig. 131, for

since the horse-power vanishes at this point the tractive force must necessarily do so also. For a given value of c the tractive force t is represented by a straight line passing through the point $s = 1800$. The values of c for the cut-offs given in Table 23, page 440, have been used to calculate the magnitude of t when s is equal to 0, giving points on the vertical axis OT.

The family of lines in the diagram is shown by joining these points with the point Q. Amongst the family of lines which may be drawn, there is a particular line which touches the curve BC at the point E, where BC cuts the vertical line through the speed, namely, 46·4 miles per hour, at which the indicated horse-power curves with constant cut-off rise to the maximum values.

Drawing such a line through point Q tangent to the curve BC, it is found to correspond with a cut-off 27 per cent. of the stroke.

This means that at this particular cut-off the indicated horse-power rises to the maximum value of 672 I.H.P. when the piston speed is 900 ft. per minute, and that the tractive force t assumes a value which is one-half its maximum value for this cut-off. After this point the tractive force follows the line EQ.

The probable curve of maximum tractive force is therefore the curve ABEQ, with discontinuities at B and E.

The part of the curve AB is determined by the weight on the coupled wheels of the engine, and cannot have a value greater than $W_c\mu$ where μ ranges from 0·2 to 0·25. The part between the points B and E is determined by the size of the boiler, and corresponds with the maximum boiler power, which has been reckoned at 30 I.H.P. per square foot of grate, but is often higher when the boiler is forced.

The part beyond E tends to a point Q, being determined by the designs of the slide valve, cylinder ports and passages, and the fractions of the stroke at which the events of the steam distribution take place. In all cases these tractive-force lines only tend towards Q because long before they get there the engine has reached a limiting speed.

When the characteristic tractive-force speed diagram has been drawn for a locomotive, the speed which the engine will approach with a given condition of loading can be ascertained with an accuracy only limited by the accuracy with which the resistance of the engine and its load is known as a function of the speed.

Consider first the case of the engine running light. The expression for its resistance, choosing the expression for a 4–4–2 tank engine from Table 21, page 432, is

$$r_e = 12{\cdot}4 + 0{\cdot}1V + 0{\cdot}004V^2$$

With a total weight of 75 tons the engine resistance at different speeds has the values given in Table 25. A starting resistance of 20 lbs. per ton is assumed—

TABLE 25.—ENGINE RESISTANCE.

Speed in miles per hour.	r_e.	$W_e r_e$.
	lbs.	lbs.
0	20	1500
10	13·8	1035
20	16·0	1200
30	19·0	1425
40	22·8	1710
50	27·0	2025
60	32·8	2460
70	39·0	2820

The curve plotted from these figures, Fig. 133, page 448, cuts the tractive-force curve at the point *e*. At this point the tractive force is equal to the engine resistance, and there is no force available to accelerate the motion of the engine. The corresponding speed is that to which the engine approaches when it is running light on a straight level road.

Consider now the limiting speed when the engine has attached to it five bogie coaches weighing 140 tons. Using Mr. Aspinall's formula for train resistance, page 424, namely

$$r_v = 2{\cdot}5 + \frac{V^{\frac{5}{3}}}{58{\cdot}7}$$

the resistance at different speeds has the values given in Table 26. A starting resistance of 17 lbs. per ton is assumed.

TABLE 26.—VEHICLE RESISTANCE.

Speed in miles per hour.	r_v.	$W_v r_v$.
	lbs.	lbs.
0	17	2380
10	3·3	462
20	5·0	700
30	7·4	1035
40	10·5	1470
50	14·0	1960
60	18·0	2520

Plot the curve corresponding to these values, not from the horizontal axis O*s*, but from *ee*, the resistance curve of the engine. Plotted in this way, the curve cuts the tractive-force curve in point *h*. At this point the tractive force is equal to the resistance of the engine and vehicles, and there is no force available to accelerate the motion of the train. The corresponding speed is that to which the train approaches on a straight level road.

Thirdly. Suppose that the train is on a gradient of say 1 in 400. The resistance due to this is for the whole train

$$\frac{2240(W_r + W_e)}{G} = 1200 \text{ lbs.}$$

This is a constant resistance which must be added to the variable resistance due to the speed. Set it up from *hh* as a base line, giving the curve *gg*. The curve *gg* cuts the tractive force curve in the point *g*. The corresponding speed is that to which the train approaches on an up gradient of 1 in 400.

125. Characteristic Dynamical Diagram for the Motion of a Train.—The tractive-force speed diagram may be used in the way described in the previous section to find the limiting speed to which a train approaches. As will be seen immediately, the train never actually attains the limiting speed, but the time taken to reach any assigned speed within the limit, as well as the distance travelled by the train during that time, can always be found when curves of tractive force and train resistance are given. The solutions of these problems are of the greatest importance in connection with the working out of train services where stops are frequent, as in a surburban service where the distance between the stations is so small that the train has not time to accelerate to a speed at which it can run uniformly before the brakes must be applied to bring it to rest again. The problems involved in the acceleration and also in the stopping of a train can be solved with ease and rapidity by a semi-graphical method devised by the author and communicated to the Institution of Mechanical Engineers on Oct. 25, 1912, in a paper entitled "Characteristic Dynamical Diagrams for the Motion of a Train".

The characteristics of the motion of a train are defined by the following curves:—

Accelerating force-speed curve.
Time-speed curve.
Time-distance curve.
Kinetic energy-distance curve.
Speed-distance curve.

All of these curves can be plotted when curves of tractive force and train resistance on a speed base are given.

The *tractive force-speed* curve and the corresponding *resistance-speed* curve may belong to any kind of tractor, steam, electric, or petrol, so that the method is quite general in its application.

It will be seen below that the whole group of curves form a family so closely related that if any one curve is given the others may be found from it. For example, if a *time-distance* curve is given, the other curves can be found; and if in addition the actual tractive force exerted is known, the difference between it and the accelerating force derived from the curves gives at once the whole resistance against which the train is hauled. In fact, this is the way the train resistance may be found from the records of the dynamometer car.

The method is most easily explained in connection with the working out of an actual example.

Let the problem be, therefore, to draw the family of curves belonging to the following data, data which defines the *tractive force-speed* curve and the corresponding *resistance-speed* curve.

Weight of engine and tender, 115 tons = W_e.

Weight of vehicles, 290 tons = W_v.

The start to be made from rest along a gradient of 1 in 1000 up.

Maximum tractive force exerted by the engine, 12 tons.

Maximum indicated horse-power which can be maintained by the boiler during the accelerating period, 1200.

Influence of ports and passages, negligible.

The resistance of the vehicles to be calculated from Mr. Aspinall's formula for a 15-coach train, namely—

$$\text{Total resistance} = W_v\left(2{\cdot}5 + \frac{V^2}{73}\right) \text{lbs.}$$

The total resistance of the engine and tender to be taken equal to 50 per cent. of the total vehicle resistance, that is to say, it is 33 per cent. of the total resistance of the whole train.

(*a*) *Scales.*—All the quantities have to be represented in the diagram to scales which are related to one another in a definite way. The most convenient way to deal with the scale relations is to choose some unit of length on the paper and express every quantity in terms of it thus:—

1 unit of length on the paper = n units of the quantity to be represented.

(*b*) *The Diagram.*—Draw two axes at right angles. The part of the horizontal axis to the right of the origin is the *speed* axis, and the part to the left the *distance* axis.

The part of the vertical axis above the origin is the *force* axis, and the part below the *time* axis.

The first step is to construct the curves of resistance and tractive force in order to obtain the value of the force which is available for accelerating the train. At a particular speed the force available for acceleration is the difference between the tractive force corresponding to the horse-power developed in the cylinders and the whole tractive resistance including engine friction.

(*c*) *The Accelerating Force f.*—Plot the two parts of the curve of total tractive force from the data of maximum tractive force and maximum indicated horse-power given above. If R represents the total resistance corresponding to the maximum indicated horse-power given above when the speed is v feet per second, from (4), page 446

$$R = \frac{550 \times 1200}{v \times 2240} \text{ tons} = \frac{295}{v} \text{ tons} \quad . \quad . \quad . \quad . \quad (1)$$

Points on Curve 1, Fig. 134, which is the curve of total tractive force, are plotted from this expression. The curve is continued

upwards until the corresponding tractive force is 12 tons, the limit imposed by the load on the driving wheels. Draw a horizontal line, therefore, at 12 tons, rounding off the junction between it and the curve 1.

Next plot curve 2 to represent the tractive resistance of the vehicles. The ordinates are calculated from

$$R_v = 290\frac{\left(2{\cdot}5 + \frac{V^{\frac{5}{3}}}{73}\right)}{2240}\text{tons} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Plot the total engine resistance upward from curve 2 as a base from any formula suitable for the problem. In the present case it is taken equal to half the total vehicle resistance.

The ordinate bc is, therefore, equal to $\frac{1}{2}\,ab$.

The ordinate ac represents the total train resistance on the level.

From curve 3 as base, plot the constant resistance of the gradient. This is

$$\frac{(115 + 290)}{1000} = 0{\cdot}4 \text{ ton approximately}$$

Curve 4 represents the total train-resistance due to the tractive resistance and the gradient combined. If the gradient is down instead of up, the ordinates corresponding to the gradient resistance must be plotted downwards from curve 3, so that the curve of total resistance would then fall below curve 3.

Vertical intercepts between curves 1 and 4 represent accelerating forces at corresponding speeds. Thus the intercept f on the ordinate ae represents the magnitude of the accelerating force when the speed is that represented by Oa.

(*d*) *Limiting Speed.*—The point of intersection l of the tractive-force curve 1 and the total-resistance curve 4 fixes the speed at which the accelerating force vanishes. This is the limiting speed towards which the train approaches.

Draw a vertical ll through the point of intersection l. This is an asymptote which the time-speed curve approaches but never reaches. In the diagram the limiting speed for the conditions assumed is just over 60 miles per hour.

(*e*) *The Time-Speed Curve, No.* 7.—The fundamental dynamical relation between the force f, the mass $\mathbf{M}$ upon which it acts, and a the acceleration produced, is

$$f = \mathbf{M}a = \mathbf{M}\frac{dv}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Separating the variables

$$dt = \frac{\mathbf{M}}{f}dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so that

$$t = \int_0^{v_1}\frac{\mathbf{M}}{f}dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This equation could be integrated directly if $\frac{\mathsf{M}}{f}$ (containing the reciprocal of the acceleration) could be expressed as a continuous function of v reducible to one of the standard forms.

Whatever be the form of the function, it can always be integrated graphically by the following process. First plot $\frac{\mathsf{M}}{f}$. To do this draw a series of ordinates; scale off each value of f, and plot the quotient $\frac{\mathsf{M}}{f}$ (quickly found on a slide-rule) vertically downwards from the speed-axis along the corresponding ordinate. Curve 6 is obtained in this way. It is plotted to the scale: 1 unit of length = 10 units $\left(\frac{\mathsf{M}}{f}\right)$. Curve 5, the part of curve 6 from 0 to 60 ft. per second is plotted to the larger scale: 1 unit of length = 1 unit of $\left(\frac{\mathsf{M}}{f}\right)$. The mass M to be used in these calculations is that equivalent to the actual weight of the train increased by 12 per cent., a maximum allowance, to allow for the acceleration of the revolving masses; this point is discussed in detail below, page 472.

The total weight of the train is 405 tons. The mass M to be used in the calculations is therefore

$$\frac{(405 + 48)}{g} = 14$$

To integrate curve 6 graphically, imagine an ordinate to start from the origin 0 and to move to the right. At any instant the axis, the curve and the ordinate will enclose an area like the area marked A on the diagram. This area, suitably interpreted with regard to the scales, represents the value of the integral in equation (5) between the limits $v = 0$ and the value of the velocity at which the ordinate has temporarily stopped. The area therefore represents the time taken by the train to acquire the speed corresponding to the position at which the ordinate is temporarily stopped; and the number representing this area set down from the speed axis along the ordinate determines a point on the *time-speed* curve.

Curve 5 is used as long as it falls within the limits of the diagram, after which the process is applied to curve 6.

With regard to the scale on which one unit of area represents seconds—

1 unit of length horizontally to the right represents 10 units of velocity.

1 unit of length vertically downwards represents 1 unit of $\left(\frac{\mathsf{M}}{f}\right)$ on curve 5.

So that 1 unit of area represents 10 seconds.

Similarly on curve 6, 1 unit of area represents 100 seconds.

Set the times so found downwards along the proper ordinates, and

points on the time-speed curve are found. *The area A represents 130 seconds, and this is set down along the ordinate hh, thus determining the point h on curve 7.*

The time scale may be chosen arbitrarily. In the diagram it is taken so that 1 unit of length represents 50 seconds.

The most expeditious way of getting curve 7 from curves 5 or 6 is by means of the Integraph. The Integraph is an instrument with two tracing points, so connected that when one point is guided along a curve, the other point draws a curve which is the integral of the first curve. It performs, in fact, the operation $Y = \int y dx$. See Section 131, page 475.

(*f*) *The Time-Distance Curve, No.* 8.—The velocity v is equal to the time rate of change of displacement x. That is

$$v = \frac{dx}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

so that

$$dx = v\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

from which

$$x = \int_0^{t_1} v\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The velocity v as a function of the time has just been found and is represented by curve 7. Integrate this curve graphically in the way just explained, noting that the area to be integrated lies between the curve and the axis of time. The shaded area marked B represents the value of the integral between the limits $t = 0$ and $t = 50$, and it therefore represents the distance travelled in 50 seconds. This area is set out along the horizontal through the point corresponding to $t = 50$, thus obtaining the point u on the time-distance curve.

Regarding scales,

1 unit of length horizontally to the right represents 10 ft. per second.

1 unit of length vertically downward represents 50 seconds.

Therefore, 1 unit of area represents 500 ft. This is plotted to the arbitrary scale of distance: 1 unit of length represents 4000 ft.

By the aid of these two curves, the time required for the train to acquire the speed v and the distance travelled during the time can be read off. For example, the train acquires a speed of 77 ft. per second (52 miles per hour) in about 275 seconds and travels over the distance 15,200 feet (2·9 miles) during the period.

(*g*) *The Kinetic-Energy-Distance Curve, No.* 10.—The work done by the accelerating force, namely,

$$W = \int_0^{x_1} f\,dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

is equal to the kinetic energy stored in the train during its motion through the distance x_1 ft.

To find points on this curve, replot the accelerating force on the

distance axis. To find the point on the distance axis corresponding to a particular value of the accelerating force f, continue the direction of the force until it cuts the time speed curve in j; through j draw a horizontal to cut the time-distance curve in k; draw a vertical through k and set out the magnitude of f from the point p at which it cuts the distance axis. The force-distance curve 9 is drawn through points like q found in this way. Integrate this curve graphically, thus obtaining curve 10. The shaded area marked C represents the value of the integral in equation (9) between the limits $x = 0$ and $x = 15,200$ ft., and therefore represents the kinetic energy added to the train during the interval. The kinetic energy is represented by the ordinate pm.

Regarding scales,

> 1 unit of length vertically upwards represents 1 ton.
> 1 unit of length horizontally to the left represents 4000 ft.

Therefore, 1 unit of area represents 4000 ft.-tons. It is plotted to the arbitrary scale: 1 unit of length represents 5000 ft.-tons.

(*h*) *The Speed-Distance Curve No.* 11.—This curve is obtained by projecting points on the time-speed curve 7 horizontally to the time-distance curve 8, thus fixing positions on the distance axis at which corresponding values of the speed are to be set up. Thus pn is the speed corresponding to the distance 15,200 ft.

The circumstances of the train's motion are thus completely known from this group of curves, which together form the characteristic dynamical diagram for the motion of the train.

(*i*) *Checks.*—Checks should always be applied to test the accuracy of the work and the correctness of the chain of scales used in the construction of the curves. There are two checks which may be easily applied.

The first check is to calculate the energy stored at a given place in two different ways. It may be calculated from the expression $\frac{\mathsf{M}v^2}{2}$. It is also given by the ordinates of curve 10. Therefore fixing upon a particular value of the distance, measure off the speed and calculate the corresponding quantity of kinetic energy. This should agree with the quantity scaled off curve 10. The velocity corresponding to point b on curve 10 is 80 ft. per second. Therefore $\frac{\mathsf{M}v^2}{2} = 44,800$ ft.-tons. From the curve it scales 44,700, a sufficiently close agreement. Similarly at point a the velocity is 87 ft. per second, and $\frac{\mathsf{M}v^2}{2} = 52,900$ ft.-tons. From the curve it scales 52,700. The scales involved in plotting the *kinetic-energy-distance* curve are the force and distance scales. Those used in plotting the *speed-distance* curve are the M/f scale, the time-scale, and the distance scale. The agreement shows that no error has been made in the deduction of the scales.

The second check is given by the theorem that the subnormal corresponding to any point on a *speed-distance* curve represents the acceleration. Thus

$$\text{the acceleration} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

This latter expression is the subnormal to a curve of velocity on a distance base.

In the diagram *sp* is the subnormal to the velocity-curve at the point *n*, and it therefore represents the acceleration at the point and therefore the force *f* when M is known. The dimension of acceleration in terms of the velocity are $\frac{v^2}{l}$.

1 unit of length = 10 ft. per second
1 unit of length = 4000 ft.

Therefore 1 unit of length of the subnormal represents $\frac{10^2}{4000}$ units of acceleration = $\frac{1}{40}$ ft. per second per second, and since the mass is 14, $\frac{14}{40}$ = 0·35 ton.

The subnormal *sp* measures 2·8 units, and therefore represents a force of 0·98 ton. Scaling off the corresponding force *f*, it is found to be 0·97 ton, an agreement as near as can be expected.

The diagram brings out clearly how difficult it is to fix the limiting speed of a particular train. There would be no difficulty if the engine resistance and the vehicular resistance were known accurately, and if in addition the indicated horse-power developed in the cylinders of the locomotive could be predicted with accuracy. But these quantities are all difficult to determine, even with approximate accuracy, and each is subject to large accidental variations.

Another point to notice is the slowness with which the speed increases in the neighbourhood of the limiting speed. For instance, reading from the diagram, half the limiting speed—namely, 30 miles per hour—is attained in 65 seconds, but it takes about 330 seconds to reach 55 miles per hour. The diagram brings out the necessity for engines with large powers of acceleration, even for express services, so that the time required to attain the running speed may be reduced to a minimum.

The same point is illustrated by curve 11, which shows the velocity plotted on a distance base. The speed rapidly increases whilst the train is passing over the first mile, but then the curve gets flatter and flatter and the increase of speed per mile gets rapidly smaller and smaller. A speed of 55 miles per hour is acquired whilst passing over a distance of about 3½ miles, but the increase is very slow afterwards. The group of curves 7, 8, 9, 10, 11 are all related to one another, so that, given any one of them, all of the others may be deduced from it.

In order to further illustrate the utility of the diagram the characteristic curves of a train drawn by a steam locomotive and of a train of equal weight drawn by electric motors are shown in Fig. 135. The dotted lines in the diagram refer to a train drawn by a steam locomotive built with three cylinders, a large boiler, and having 10 wheels coupled; a powerful engine. The full lines show the curves corresponding to the motor-drawn train. It will at once be seen how much quicker a given speed can be reached and how much farther the electric train can travel in a given time. The reason is that with the electric motor the tractive force can be maintained at a high value for a much longer period than in the case of the steam locomotive, because the electric motor is unhampered by limitations of boiler power. It has the boiler power of the central station behind it when it wants it, whilst the steam locomotive can draw steam from its own boiler only. It should be noted that this quick gain of speed is not gained without an equivalent cost. The energy stored in the two trains is identical at the same speed, but the large boiler power enables the electric motors to store the energy in the train at a quicker rate.

To derive the curves in the reverse order, namely, 9 from 10 or 7 from 8 or the f curve from 7, the process of graphic integration must be replaced by a process of differentiation. As a graphical process differentiation is not so satisfactory as integration. But by the application of a mixed analytical and graphical process results of practical value may be obtained. These processes have a practical importance in connection with the reduction of results obtained with a dynamometer-car.

126. The Reduction of the Data from a Dynamometer-Car Record to the Curves of the Diagram.—The time-distance curve A in Fig. 136 has been plotted from data reduced from the complete record of which Fig. 125, page 433, is a part. The data relating to the diagram are given in Table 27, page 461. This curve can be plotted with accuracy, since corresponding values of the time and distance are deduced easily from the record with negligibly small error. The velocity-time curve B is derived from this curve by a process of graphical differentiation. Thus since $v = \frac{dx}{dt}$, and since this ratio also represents the slope of the curve, the velocity corresponding to the point P, say, may be found by drawing the tangent to the curve at P and then computing its slope. At P

$$v = \frac{dx}{dt} = \frac{\mathrm{PN}}{\mathrm{NQ}} = \frac{21120}{294} = 72 \text{ ft. per second}$$

This velocity is set out at NU, and a point U on the velocity-time curve B is thereby obtained.

This method is not susceptible of great accuracy since large errors in the magnitude of $\frac{dx}{dt}$ are produced by small errors in the

inclination of the tangent, and it is almost impossible to draw a tangent to a curve at a point without error in the inclination.

A better way of deriving B from A is to take a series of horizontals at a distance apart corresponding to some suitable time-interval, and then compute the ratio of corresponding steps of distance dx and time dt. The ratio found will be the average value of the velocity during the time-interval corresponding to the distance step dx. Thus referring to Fig. 136 the distance step dx during the time-interval dt from 200 to 220 seconds is 920 ft. The average velocity during the interval is then

$$\frac{dx}{dt} = \frac{920}{20} = 46 \text{ ft. per second}$$

This is set out along the horizontal passing through the point corresponding to 210 seconds, thus obtaining a point on the velocity-time curve. The corresponding steps at a particular point of time can, however, be more accurately computed from the dynamometer record itself, unless the time-distance curve is set out to a large scale. The dots on the velocity-time curve were fixed by calculation from the data on the record.

Observations were not taken before the speed reached about 40 ft. per second. The first part of the time-distance curve and consequently the time-velocity curve are therefore not known from observation. The part of the velocity-time curve between the origin and 40 ft. per second can, however, be drawn in with fair accuracy by the following method.

Two things are known about this part of the curve: first, its direction at A; and secondly, that at the end of 172 seconds the train has actually travelled 4224 ft., so that the area OAB represents the distance 4224 ft. The records show that there was no stop, so that the motion and therefore the curve are continuous. Hence draw a trial curve passing through the origin and passing smoothly into the established part of the curve ending at A. Then measure the area OAB. If it is found to be 4224, the first trial has been successful. If there is error, the shape of the curve must be altered to increase or reduce the area.

Since the acceleration is given by $\frac{dv}{dt}$, its value, corresponding to a point of the velocity-time curve B, can be found by either of the methods just explained, preference being given to the second method. By way of example calculate the acceleration at the point V. During the time interval, from 200 to 220 seconds, the velocity changes 3 ft. per second. The acceleration is then $\frac{3}{20}$ ft. per second per second. This applies equally to the vehicles and the engine. The pull on the draw-bar at this instant producing the acceleration is found by multiplying this by the mass of the vehicles. Allowing for revolving masses, the mass is 10 units, so that the corresponding draw-bar pull is 1·5 tons. This force is represented by

ab in the diagram, and b fixes a point on the curve C. Other points are found in a similar way, and the curve C is drawn through them. The curve C shows the draw-bar pull required to overcome the resistance to acceleration during the whole of the period from the start to the region of uniform speed. The uniform speed attained is, from the records, 70 miles per hour. At this point, therefore, the accelerating force vanishes.

TABLE 27.

ENGINE "DOG STAR," 4-6-0. 4-Cylinder Simple. Boiler Pressure, 225 lbs. per square inch. Weights: Engine and Tender, 115 tons; Vehicles, 289¾ tons to Westbury. Each Cylinder 14¼ inches diameter × 26 inches stroke. Coupled Wheels, 6 ft. 8½ ins. diameter.

Curve D.	Curve B.		Curve A.		
Observed D.B. Pull. Tons.	Computed speed. Ft. p. sec.	Computed speed. Miles p. hr.	Miles from Paddington.	Secs. from Paddington.	I.H.P.
3·4	39·6	27	0·8	172	—
2·75	55	37½	1·715	272	—
2·35	57·9	39½	2·0	299	—
2·50	65·2	44½	2·85	372	970. Card 2.
2·45	71·8	49	4·0	461	—
2·05	80·6	55·2	6·0	587	1046. Card 3.
2·10	88·0	60·0	8·0	711	—
1·9	91·6	62½	10½	860	—
1·7	97·9	66¾	13	996	—
1·8	100	68⅓	15	1103	—
1·75	102·6	70	17	1220	—

The observed magnitudes of the draw-bar pull at a series of speeds are given in Table 27, and curve D is plotted from these data. The part of the curve which is dotted was fitted in, allowing 17 lbs. per ton at the start and then joining in on to the established curve at 40 ft. per second.

The vertical distance between the curves C and D corresponding to a particular speed represents the total resistance of the vehicles at that speed. Thus at a speed of 65 ft. per second the total resistance is about 1·5 tons, corresponding to 11·6 lbs. per ton. At this speed the resistance to acceleration is about 1 ton. The draw-bar pull recorded is 2·5 tons. Using curve D as a base, plot curve E to represent the resistance to acceleration of the engine. The mass of the engine, allowing for revolving parts, is approximately 4 units. Thus any ordinate is $\frac{4}{10}$ of the corresponding ordinate ab.

If data were available for plotting the curve F, which represents the tractive force corresponding to the indicated horse-power, the vertical distance between curves E and F at a particular speed would represent the engine resistance at that speed. Two values of the tractive force equivalent to the indicated horse-power are plotted at

the speeds at which the diagrams were taken, namely, at 65 ft. per second and at 80 ft. per second calculated from

$$R = \frac{550 \text{ I.H.P.}}{2240v} \text{ tons}$$

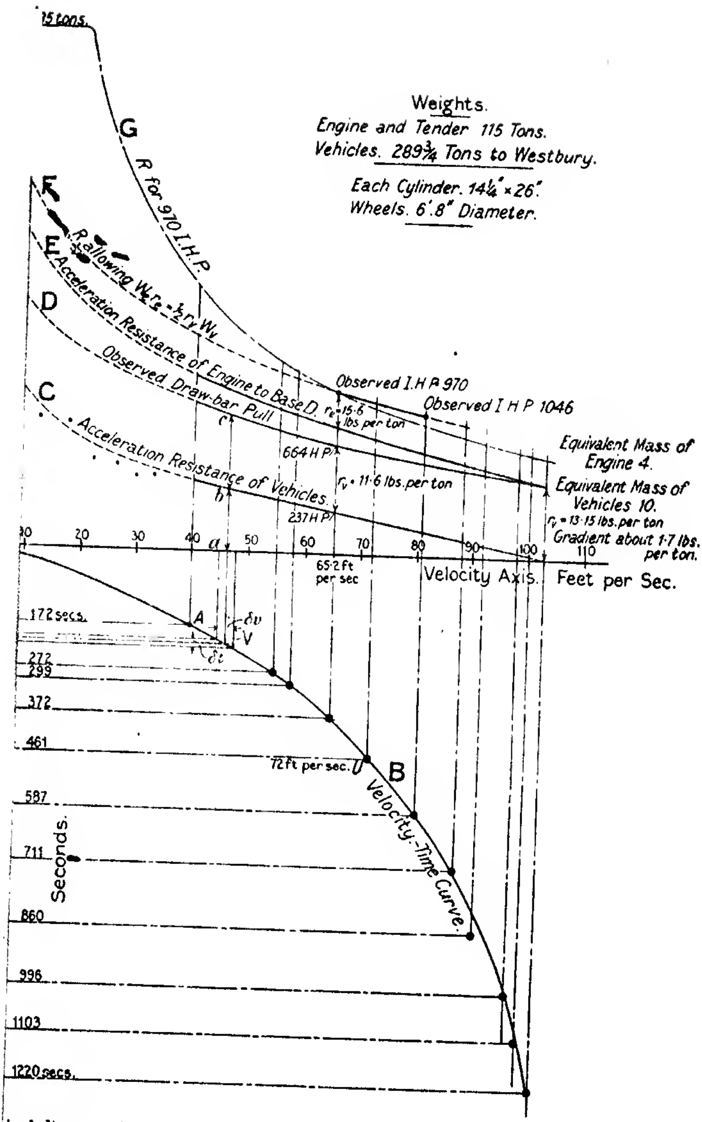
v is here in feet per second.

In the one case where the indicated horse-power was 970, R is 3·65 tons, and in the second case, where the indicated horse-power is 1046, R is 3·2 tons. The engine resistance at 65 ft. per second reduces to 15·6 lbs. per ton. The total engine resistance at this point is roughly about half the total resistance of the vehicles. The curve F is sketched on the assumption that the ratio is approximately true at all speeds. It will be understood that the dotted parts of the curves C, D, E, and F are conjectural, but that the part *OA* of B is so far correct that the corresponding area does represent the distance travelled by the train in the time 172 seconds, as it ought to do, and that its direction at *A* is correct. A curve G of tractive force corresponding to the constant horse-power 970 is drawn on the diagram, and the curve is continued to meet the maximum tractive pull which the engine can exert, namely, 11·85 tons. Comparing curve G with curve F, it will be seen that although the dotted part of F is conjectural, yet it is sufficiently correct to show that the engine had a large reserve and, if necessary, could have accelerated the train at a greater rate, or could have equally accelerated a heavier train at the same rate.

The advantage gained by reducing the dynamometer-car records in this way is that accidental errors are smoothed out, and that a comprehensive view is obtained of the working of the engine, and, moreover, the vehicle resistance can be deduced with greater accuracy from the curves than from isolated calculations. If, also, it were possible to get reliable values of the indicated horse-power at a series of speeds, the engine resistance could be deduced as well.

The resistance of the vehicles and of the engine are seen to be low at the high speeds, and the author considers that the performance of the locomotive is remarkable. To exert a draw-bar resistance of $1\frac{3}{4}$ tons at 70 miles per hour (see Table 27, p. 461), corresponding to 730 horse-power at the draw-bar, is an achievement in locomotive design which it would be difficult to surpass within the limits of the British loading-gauge.

127. Braking.—There is a characteristic diagram corresponding to the braking period of the same general type as that for the period of acceleration. The forms of the curves now depend mainly upon the pressure between the brake blocks and the tyres, that is to say upon the pull exerted by the brake rods on the brake levers. Since this pull depends largely upon the way in which the driver regulates the pressure in the air pipes of the train, the curve representing the pull as a function of the velocity is about as arbitrary in form as a curve can well be. Nevertheless there are some useful fundamental



ical diagram drawn from a time-distance curve.

[*To face p.* 462.

principles in connection with the braking of a train which are brought out by drawing a diagram from assumed data.

When steam is shut off, the motion of the train is continued against the resistances, by the gradual exhaustion of the stock of kinetic energy stored in the train. The principle of the conservation of energy applied to this problem may be stated thus:—

The rate at which the kinetic energy of the train is reduced = the rate at which it is changed into potential energy by a gradient + the rate at which it is transformed into heat by friction.

(*a*) *The Braking of a Wheel Element.*—At first it is convenient to restrict one's attention to a *wheel element* composed of a wheel carrying a load. The total weight of the wheel element is the weight which would be recorded against the wheel if the vehicle stood on a weigh bridge provided with a separate steelyard for each wheel. For practical purposes this weight may usually be regarded as equal to the total weight of the vehicle divided by the number of wheels supporting it. Let this weight be W tons. Let I be the moment of inertia of the wheel about its axis; r its radius; k its radius of gyration.

The total store of kinetic energy in the wheel element is

$$E = \frac{Mv^2}{2} + \frac{I\omega^2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this expression the first term represents the energy of translation of the mass $M = \frac{W}{g}$, and the second term represents the rotatory energy stored in the wheel.

The rate at which this stock of energy is reduced is found by differentiating the expression with regard to the time thus:—

$$\frac{dE}{dt} = M\dot{v}v + I\dot{\omega}\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is the rate at which energy is used to drive the wheel element against the resistances. In an actual train every wheel temporarily becomes a driving-wheel, just as though it formed part of a locomotive, but instead of being driven by energy derived from fuel it is now driven by energy withdrawn from the limited stock stored in the train.

The rate at which energy is transformed into heat by resistances of the nature of friction reduced to the wheel of the element may be analysed into four terms, namely:—

(1) The rate at which work is done against the natural resistance to the motion of the wheel element.
(2) The rate at which work is done against the frictional resistance of the brake blocks.
(3) The rate at which work is done against the frictional resistance of the tread if the wheel slips.
(4) The rate at which kinetic energy is transformed into potential energy by a gradient.

During the braking period (1) is negligibly small and may be

neglected; it may be assumed that no slipping takes place; thus the energy stored in the train is regarded as wholly spent in doing work against the frictional couple B caused by the application of the brake blocks to the wheel, and against the gradient.

The rate at which energy is spent against the couple B is $B\omega$, where ω is equal to the angular velocity of the wheel. The rate of working against a gradient of 1 ft. vertical to G ft. measured horizontally is—

$$\frac{Wv}{G}$$

Therefore equation (2) becomes, observing that with no slipping, $\omega = \frac{v}{r}$ and $\dot{\omega} = \frac{\dot{v}}{r}$.

$$M\dot{v} + \frac{I\dot{v}}{r^2} = \frac{B}{r} \pm \frac{W}{G} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The + sign is used before the last term if the train is running up a gradient, and the − sign if running down.

The couple B may be regarded as the sum of two couples, namely, one, represented by B_1, which produces the frictional resistance against which the linear energy of the train element is reduced; the other, B_2, which produces the frictional resistance against which the rotational energy of the wheel is reduced; and further, linear motion may be separated from the angular motion, so that

$$M\dot{v} = \frac{B_1}{r} \pm \frac{W}{G} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\frac{I\dot{v}}{r} = B_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Considering equation (4), the term $\frac{B_1}{r}$ cannot be greater than $W\mu_1$, because the motion of the wheel element being maintained by the exhaustion of the linear energy against this resistance the wheel will begin to slip immediately the resistance reduced to the rail is greater than $W\mu_1$.

Therefore $$M\dot{v} = W\mu_1 \pm \frac{W}{G} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

is an equation which gives the maximum retardation $\dot{v}$ which can be applied to the train.

Let P_1 denote the pressure applied to the brake blocks corresponding to the couple B_1; P_2 the pressure corresponding to the couple B_2; and let μ_2 be the coefficient of friction between the brake block and the tyre, so that

$$\frac{B_1}{r} = P_1\mu_2$$

and

$$\frac{B_2}{r} = P_2\mu_2$$

Then combining equations (4) and (6) with these relations

$$W\left(\mu_1 \pm \frac{1}{G}\right) = P_1\mu_2 \pm \frac{W}{G} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From which P_1, the pressure which may be applied to the brake block for the reduction of the linear energy alone, is

$$P_1 = \frac{W\mu_1}{\mu_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Similarly from equation (5)

$$P_2 = \frac{I\dot{v}}{\mu_2 r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The total pressure corresponding to the couple B is then $P = P_1 + P_2$, and this is the value of P when the wheels are on the point of slipping.

In applying these equations the maximum retardation consistent with an appropriate value of μ_1 and a given gradient is first to be calculated from (6). This value substituted in (9) together with an appropriate value of μ_2 gives the value of P_2. P_1 is calculated from (8).

If a pressure is applied to the brake block larger than P the wheel will slip. A smaller value will produce less than the possible maximum retardation, but would provide a margin against slipping.

It will be observed that the instantaneous value of P depends upon three quantities of a variable nature, namely :—

μ_1, the coefficient of friction between the tyre and the rail.

μ_2, the coefficient of friction between the brake blocks and the tyres.

G, the gradient.

With regard to μ_2 its value depends not only upon the velocity of rubbing between the block and the tyre, but upon the time. Thus, quoting from the classical experiments of Sir Douglas Galton, the observed coefficient of friction between a cast-iron brake block and a steel tyre when the brake was applied to the wheels of a train kept moving at an approximately uniform speed of 20 miles per hour was 0·18. After 5 seconds this had fallen to 0·15, and after 5 seconds more to 0·13, whilst 20 seconds after application it had fallen to 0·1. Other values are given in Table 28, page 471, which is quoted from Sir Douglas Galton's paper.[1]

The static value of μ_1 the coefficient of friction between the wheel and the rail, is variable, but for most purposes may be taken at 0·2. If the wheel is skidded the value of the coefficient of friction is a function of the velocity of sliding, but is not so markedly affected by the time as in the case of brake blocks. At 60 miles per hour Sir Douglas Galton found that the coefficient of friction of a steel tyre

[1] *Proc. Inst. Mech. E.*, 1878, 590.

skidded on a steel rail was about 0·027, increasing gradually as the speed decreased, until just before stopping it was 0·242.

(*b*) *A Vehicle composed of* n *similar Wheel Elements of which* m *are braked.*—A vehicle may be regarded as composed of a number of wheel elements, and if each one of them is braked the equations above apply to the vehicle as a whole, that is to say the maximum retardation is the value $\dot{v}$ calculated from equation (6) and equations (8) and (9) furnish the corresponding values of P_1 and P_2, remembering that in these equations W is the total weight of the vehicle divided by the number of wheels, and I is the moment of inertia of one wheel. The size of the brake levers and the brake cylinder may then be proportioned to the maximum value of P required. In some cases, however, some of the train elements forming a vehicle are not provided with brake blocks.

Consider the case of a vehicle composed of n similar wheel elements, m of which are braked, so that the retardation of the n elements is determined by the brake power on m of them. Then a mass nM is braked by the maximum frictional resistance due to a wheel load mW and a gradient resistance due to nW. Then from (6), page 464,

$$nM\dot{v} = mW\mu_1 \pm \frac{nW}{G} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In most cases this can be simplified to

$$\dot{v} = \frac{m\mu_1 g}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

since the second term is usually small in comparison with the first term. For example, suppose an 8-wheeled vehicle to consist of eight similar wheel elements, and that the total weight is 32 tons. Further, assume that four wheels only of the eight are braked, so that $n = 8$ and $m = 4$. Also let $\mu_1 = \frac{1}{2}$, and let the stop be made on a rising gradient of 1 in 300, so that $G = 300$. Then from (10)

$$\dot{v} = 3{\cdot}3 \text{ ft. per second per second}$$

Calculated from (11), $\dot{v} = 3{\cdot}2$ ft. per second per second.

This shows that in cases where the brake power applied is near the maximum, the maximum retardation can be calculated from (11) without serious error.

(*c*) *An Engine composed of* n *Dissimilar Wheel Elements.*—When a vehicle is made up of dissimilar wheel elements, equation (11) may be put in a more convenient form, a form which in practice applies mainly to locomotives.

Let W be the total weight of the engine and tender.
w be the weight on the unbraked wheels of the engine and tender.

Then
$$\dot{v} = \frac{g(W - w)\mu_1}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The instantaneous pressure P to be applied to each wheel can then be found by first calculating P_1 from equation (8), page 465, substituting for the W therein the weight on the wheel together with appropriate values of μ_1 and μ_2, and then adding to this P_2 calculated from (9), using the value of $\dot{v}$ found from (12), page 466.

The practical difficulty in connection with these calculations is that μ_2 is a function of the speed, and therefore to maintain the maximum conditions, namely, that B should be equal to $W\mu_1$, the pressure P must be reduced as the speed decreases. This is illustrated below in the characteristic diagram, Section 119, page 468.

128. Tension on a Draw-bar due to Unequal Braking of Engine and Train.—When two unequally-braked vehicles are coupled together the application of the brake produces a tension in the draw-bar (or a compression of the buffers) which in extreme cases may be serious enough to cause the draw-bar to break or to cause a derailment.

Consider two vehicles of mass A and B respectively coupled together. Let a be the retardation which would be produced by the application of the brakes to the vehicle A alone. Let b be the retardation which the application of the brakes would produce on vehicle B alone. Let c be the common retardation actually produced when the vehicles are coupled together, and let T be the tension in the draw-bar.

When the brakes are applied, the forces acting on the vehicle A to retard its motion are Aa due to the application of the brakes and T due to the draw-bar tension, so that

$$Aa + T = Ac \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly the forces acting to retard the motion of B are Bb and the draw-bar tension which acts to increase the speed of B, so that

$$Bb - T = Bc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eliminating c

$$T = \frac{AB(b - a)}{A + B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and eliminating T

$$c = \frac{Aa + Bb}{A + B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In these equations A and B are respectively the weights of the vehicles divided by g.

By way of example, calculate from the following data the tension on the draw-bar between the engine and the tender corresponding to maximum retardation on the level in the case of an engine of the 4–4–0 type with bogie-wheels unbraked, coupled to a tender and a train with all the wheels braked.

Total weight on the bogie-wheels, 16 tons. Total weight of the engine, 46 tons. Weight of the tender, 36 tons. $\mu_1 = \frac{1}{8}$. Weight of the vehicles behind tender, 284 tons. Then the maximum

retardation which the brakes on the coupled wheels can produce on the engine, calculated from equation (12), page 466, is

$$\dot{v} = a = \frac{32(46 - 16)}{46 \times 5} = 4\cdot2 \text{ ft. per second per second}$$

The corresponding retardation produced by the brakes on the tender and train calculated from equation (6), page 464, is

$$\dot{v} = g\mu_1 = 6\cdot4 \text{ ft. per second per second}$$

Therefore from (3), page 467,

$$T = \frac{46 \times 320(6\cdot4 - 4\cdot2)}{g(46 + 320)} = 2\cdot8 \text{ tons,}$$

and c, the common retardation, is from (4)

$$c = \frac{46 \times 4\cdot2 + 320 \times 6\cdot4}{366} = 6\cdot1 \text{ ft. per second per second}$$

It should be understood that in practice these maximum retardations are not produced, but it will be seen from equation (3), page 467, that since the tension on the draw-bar depends upon the difference of the retardations a and b, a large tension may be produced on the draw-bar when the actual values of the retardations are themselves quite below the possible maximum values.

129. Characteristic Dynamical Diagram for a Train stopping from a speed of 60 miles per hour.—The principles underlying the practice of braking are generally illustrated by the diagram, Fig. 137. The train is assumed to be made up of similar wheel elements similarly braked. The diagram is drawn for one wheel element, since the characteristics of the motion of one wheel element during a stop are identical with the characteristics of the motion of a train composed of a number of similar wheel elements. The weight of the element is assumed to be 4 tons, and the moment of inertia of the wheel and half axle belonging to it is taken to be 0·86 ton-foot^2 units.

Consider first the exhaustion of the translational energy of the element. Part of the energy is exhausted against the resistance due to the motion of the element along the rail and to the resistance produced by the application of the brake blocks to the wheel, and the motion of the element is maintained against these resistances by means of energy withdrawn from the kinetic energy of the element. Energy is also dissipated in overcoming the resistance of the gradient. It is assumed that the wheel will slip when the sum of the train-resistance and the brake-block resistance reduced to the rim of the wheel is equal to 1 ton. This corresponds to a value of μ_1, between the wheel and the rail, of 0·25. Curve 1 is drawn parallel to the

speed axis in Fig. 137 to represent this resistance. In doing so, the assumption is made that when the resistance at the rail reaches this value the wheel will slip whatever be the speed. Further, let it be assumed that the sum of the frictional resistances reduced to the wheel rim never exceeds three-quarters of this value in order to allow a margin against slipping. Curve 2 is drawn to represent this value, and it is assumed to increase gradually from $\frac{1}{10}$ of a ton at the beginning of the stop to $\frac{3}{4}$ of a ton, and then to decrease gradually to $\frac{1}{10}$ of a ton at the actual moment of stopping. The form of the curve is quite arbitrary and depends upon the way in which the pressure between the brake blocks and the tyre is regulated by the driver.

The chain-dotted lines near the axis show the part contributed by gradient resistance, from which it will be seen that in comparison with the resistance produced by the brake blocks, its influence on the stop is negligible, except in extreme cases, such for instance as when the gradient is steep, and the number of braked wheels in the train relatively few.

With passenger trains stopping in normal conditions of weather on gradients usually found on a main line, and with a large proportion of the wheels braked, the whole resistance against which the translational energy is exhausted may be regarded as produced by the application of the brake blocks alone.

Ordinates to curve 2 may therefore be taken to represent the product of the pressure P_1 and the coefficient of friction μ_2. Let $P_1\mu_2 = f$.

The process of finding the time-speed curve and the time-distance curve for the stop is from this point exactly the same as the process explained above in connection with Fig. 134, with the difference in detail that in this case it is the actual mass of the wheel-element which is to be used without any increase to allow for the rotatory energy in the wheel. This is considered separately below.

The mass is thus $\frac{4}{32} = 0{\cdot}125$ unit. Curve 3 is plotted from $\frac{M}{f} = \frac{0{\cdot}125}{f}$, and f is scaled from curve 2. This curve integrated with regard to the speed gives the time-speed curve 4, and curve 4 integrated with regard to the time gives the time-distance curve 5.

It will be seen from these two curves that the train is stopped from 60 miles per hour in 21 seconds, and during that period it passes over a distance of 925 feet.

Curves 1 to 5 constitute the dynamical diagram for the stop. In order to produce the frictional resistance represented by curve 2, the pressure which is applied to the brake blocks must vary as the coefficient of friction μ_2 varies.

Curve 6 shows the value of the coefficient of friction between cast-iron brake blocks and steel tyres plotted as a function of the speed from data given in Table 28. These data must not be taken as generally applicable to all cases, but rather as showing the order of variation which may be expected. The curve itself represents

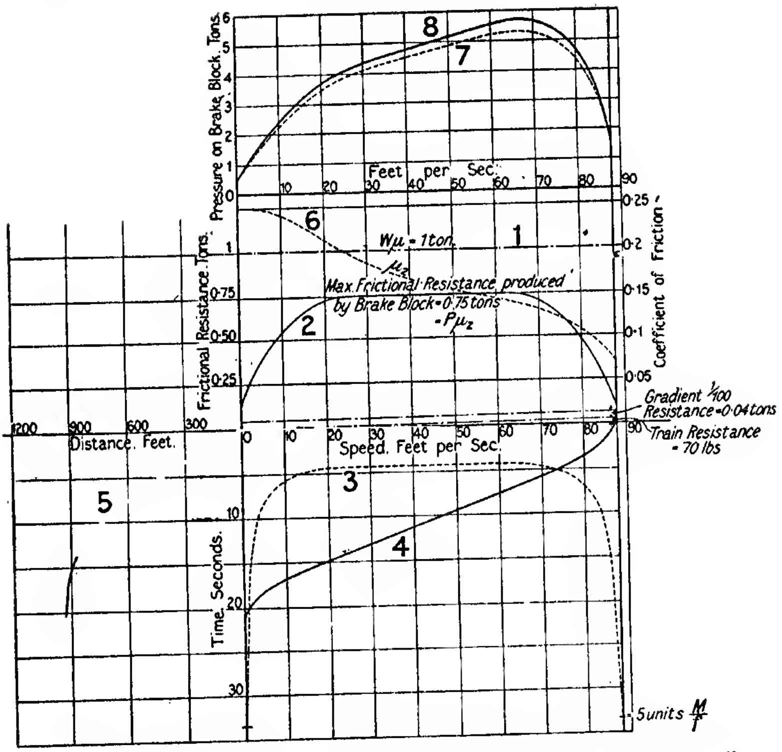


FIG. 137.—Characteristic dynamical diagram for a stop from a speed of 60 miles per hour. Every wheel similarly braked.

Train element = 4 tons.
I. of wheel = 0·86 ton-ft.^{2}.
$\mu = \frac{1}{4}$.

V ft./sec.	Curve 2. $P_1\mu_2$.	Curve 6. *P.I.M.E.*, 1878, p. 599. μ_2.	Curve 7. P_1 tons.
0	0·1	0·25	4
10	0·54	0·242	2·23
20	0·71	0·218	3·94
30	0·75	0·182	4·12
40	0·75	0·171	4·89
50	0·75	0·153	4·9
60	0·75	0·144	5·2
70	0·7	0·132	5·8
80	0·49	0·106	4·6
88	0·1	0·072	1·89

values of μ_2 actually found for cast-iron blocks on steel tyres during experiments carried out with extreme skill and care on the London, Brighton and South Coast Railway.

TABLE 28.—THE VALUE OF THE COEFFICIENT OF FRICTION BETWEEN CAST-IRON BRAKE BLOCKS AND STEEL TYRES IN TERMS OF THE SPEED AND THE TIME. QUOTED FROM CAPTAIN DOUGLAS GALTON'S PAPER ON THE "EFFECT OF RAILWAY BRAKES," *Proc. Inst. Mech. Engineers*, Oct., 1878, page 599.

Dynamic friction. Cast-iron on steel.	Speed in		Value of the coefficient of friction, μ_2.				
	Ft. per second.	Miles per hour.	At beginning of expts.	After 5 secs.	10 secs.	15 secs.	20 secs.
Just before coming to rest	1 to 3	2/3 to 2	0·25	—	—	—	—
When moving at	10	6·8	0·252	—	—	—	—
,, ,,	20	13·6	0·218	0·193	—	—	—
,, ,,	25	17·0	0·205	0·157	—	0·110	—
,, ,,	30	20·4	0·182	0·152	0 133	0·116	0·099
,, ,,	40	27·8	0·171	0·130	0·119	0·0[illegible]1	0·072
,, ,,	45	30·7	0·163	0·107	0·099	—	—
,, ,,	50	34·1	0·153	—	—	—	—
,, ,,	55	37·5	0·152	0·096	0·083	0·069	—
,, ,,	60	40·9	0·144	0·093	—	—	—
,, ,,	70	47·7	0·132	0·080	0·070	—	—
,, ,,	80	54·5	0·106	—	—	0·045	—
,, ,,	88	60·0	0·072	0·063	0·058	—	—

If any ordinate of curve 2, which gives $P_1\mu_2$, is divided by the corresponding ordinate of curve 6, which gives μ_2, the quotient is the value of the pressure P_1 which must be applied to the brake block to actually produce the resistance shown in curve 2. Curve 7 has been plotted in this way. Owing to the small value of μ_2 at 60 miles per hour, a considerable pressure may be applied without skidding the wheels at this speed. For example, at 60 miles per hour a pressure of nearly 14 tons is required to produce the skidding resistance of 1 ton on the brake blocks, but just before stopping where the coefficient of friction is about 0·25 a pressure of 4 tons would skid the wheels. In the example it is assumed that the pressure is applied to one brake block. If there are two brake blocks acting on the one wheel, the same frictional resistance is produced by half the pressure applied to each block.

The values of the pressure shown in curve 7 are those required to produce the friction necessary to abstract the translational energy from the wheel-element. The pressure is greater than this to the extent shown by curve 8, the extra pressure being required to produce a frictional couple equal and opposite to the couple corresponding to the angular retardation of the wheel. With no slipping the angular retardation is calculated from the linear retardation.

Thus, consider the point on the time-speed curve 4, corresponding to 54 feet per second. At this speed the retardation is uniform and

equal to 6 feet per second per second. The radius of the wheel is 1·89 feet (3 feet $7\frac{1}{2}$ inches diameter). The corresponding angular retardation is $\frac{6}{1\cdot 89}$ radians per second per second. The moment of inertia of the wheel is $\frac{0\cdot 86}{g}$ in dynamical units = 0·0268. The force of the couple required to produce this angular retardation is found from

$$P_2\mu_2 = \frac{6 \times 0\cdot 0268}{1\cdot 89 \times 1\cdot 89} = 0\cdot 045$$

From curve 6 the corresponding value of μ_2 is 0·15. Therefore the increase of pressure P_1 is $\frac{0\cdot 045}{0\cdot 15} = 0\cdot 3$ ton. Points on curve 8 have been calculated in this way, and its ordinate represents the force which must be applied to the brake blocks to produce a frictional couple against which both the translation and the rotatory energy of motion are dissipated; it shows also how the pressure must be varied in order to produce the stop represented by the characteristic curves. This is the couple which is represented on page 464 by B, neglecting the effect of the small resistances.

130. The Resistance due to the Angular Acceleration of the Wheels and Axles.—Let I be the moment of inertia of a pair of wheels and the axle connecting them, and let w be their weight. Let ω be the angular velocity of the pair, then the angular acceleration is $\frac{d\omega}{dt}$, and the couple required to cause the acceleration is $I\frac{d\omega}{dt}$.

The forces of this couple are, a tangential resistance at the rail, and the equal and opposite force acting from the carriage horns to the axle box and so through the axle itself, which force appears as a resistance to the motion of the train. Denote it by f. The arm of the couple is the radius of the wheel r. Also with no slipping at the tread $\omega = \frac{v}{r}$, so that $\frac{d\omega}{dt} = \frac{dv}{rdt}$. If k is the radius of gyration of the pair, $I = \frac{wk^2}{g}$. The magnitude of the couple is fr, and therefore

$$fr = I\frac{d\omega}{dt} = \frac{wk^2}{gr}\cdot\frac{dv}{dt} = \frac{wk^2}{gr}a \quad . \quad . \quad . \quad . \quad (1)$$

So that the resistance to the angular acceleration of the pair of wheels which must be overcome at the horns is

$$f = \frac{wk^2}{gr^2}a \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For the whole train the resistance is

$$\frac{ak^2}{gr^2}\Sigma w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the ratio $\frac{k^2}{r^2}$ being placed outside the summation sign, because for practical purposes it may be regarded as constant for ordinary wood-centred carriage wheels of normal design.

As will be seen from Table 29 the value of the ratio for a typical pair of wood-centred carriage wheels is 0·45. From this it will be seen that the resistance to the angular acceleration of the carriage wheels may be allowed for by applying the linear acceleration a to a mass which is greater than the actual mass of the train by 0·45 times the mass of the wheels and axles.

This addition is usually about 7 per cent. in the case of ordinary trains, but may increase to as much as 12 to 14 per cent. in the case of electric locomotives or motor coaches. If greater accuracy is required the value of I must be calculated for each pair of wheels separately. Actual values of the moment of inertia and the ratio $\frac{k^2}{r^2}$ are shown in Table 29. These were calculated by dividing the wheels into a series of concentric rings and then computing the moment of inertia of each ring. The calculations were made by Mr. Deuchar, of the Civil and Mechanical Engineering Department of the City and Guilds Engineering College, from drawings supplied by Mr. Pickersgill, locomotive engineer of the Caledonian Railway.

TABLE. 29.—MOMENTS OF INERTIA; RADII OF GYRATION; TOTAL WEIGHT; AND THE RATIO OF THE SQUARE OF THE RADIUS OF GYRATION TO THE SQUARE OF THE RADIUS OF THE WHEEL, IN FOUR TYPICAL CASES.

Type of wheel.	Total weight of 2 wheels and 1 axle.	k^2.	Moment of inertia.	$\frac{k^2}{r^2}$.
	Lbs.	Ft.^{2}	Lb.-ft.^{2}	
1 pair of driving wheels 6 ft. 1 in. diameter, connected by a crank axle	8,473	4·1	34,133	0·444
1 pair of trailing wheels 6 ft. 1 in. diameter, connected by a straight axle	6,723	4·52	30,389	0·49
1 pair of 3 ft. 9½ ins. bogie-wheels and axle	3,080	2	6,164	0·555
1 pair of 3 ft. 9½ ins. wood-centred carriage wheels and axle	2,356	1·63	3,840	0·453

The following rule for estimating the moment of inertia of an engine wheel is based on the assumptions that the complete wheel is composed of tyre, wheel-rim, mansell rings and n spokes; that each spoke is of uniform section and is ρ feet long, where ρ is the distance from the centre of gravity of a radial section through the rim to the centre of the wheel. With this assumption the spokes would be crowded out at the centre, and the material may be imagined to spread laterally, thus making some allowance for the wheel-boss.

Let A be the area in square inches of a radial section taken through the tyre, the wheel-rim, and the mansell rings. Then allowing 0·28 lb. per cubic inch for the material, the moment

of inertia about the wheel-centre of a rim corresponding to this section is with sufficient approximation

$$0{\cdot}28 \times 12 \times 2\,\pi\rho^3\mathrm{A}\ \text{lb.-ft.}^2\ \text{units} = 21\mathrm{A}\rho^3$$

And the moment of inertia about the wheel-centre of one spoke, whose mean section is a square inches, is

$$0{\cdot}28 \times 12\,a\rho\left(\frac{\rho^2}{4} + \frac{\rho^2}{12}\right) = 1{\cdot}12a\rho^3$$

The moment of inertia of one wheel with n spokes is therefore

$$\rho^3(21\mathrm{A} + 1{\cdot}12\,na)\ \text{lb.-ft.}^2\ \text{units}$$

For a pair of wheels and the axle it may be taken double this. Before using the value so obtained in dynamical calculations divide by g.

Similarly for a wood-centred carriage wheel the moment of inertia of the rim is $21\mathrm{A}\rho^3$. This is about 85 per cent. of the total, so that the moment of inertia for a pair of wheels and an axle is approximately

$$49{\cdot}4\ \mathrm{A}\rho^3\ \text{lb.-ft.}^2\ \text{units}$$

Finally, rounding the figures off—

Moment of inertia of an axle and pair of wheels with n spokes may be approximately calculated from

$$\mathrm{I} = 2\rho^3(20\mathrm{A} + na)\ \text{lb.-ft.}^2\ \text{units}$$

Moment of inertia of a pair of wood-centred carriage wheels and axle may be approximately calculated from

$$\mathrm{I} = 50\mathrm{A}\rho^3$$

By way of illustration, in the case of the trailing wheel in Table 29, page 473,

A, the area of the cross-section of the rim	= 26·6 sq. ins.
ρ, the distance of the centre of gravity of this section from the centre of the wheel	= 2·85 ft.
a, the main area of a spoke	= 5·8 sq. ins.
n, the number of spokes	= 20

Then I = 30,000. Comparing this with 30,389, the accurate figure, it will be seen that the error is about 1·3 per cent.

This expression makes no allowance for a balance-weight, but when there is one, its moment of inertia can be separately calculated and added to the sum found from the formula.

Adding, therefore, 400 to I to allow for the balance-weight, I = 30,400, and the error is less than 1 per cent.

In the case of the wood-centred carriage wheel—

A . . . . . = 15·55 sq. ins.
ρ . . . . . = 1·7 ft.
I . . . . . = 3,880, which, compared with 3,840, the accurate figure, shows an error of less than 1 per cent.

131. The Integraph.—Considerable time and labour are saved if the integral curves in the characteristic dynamical diagram

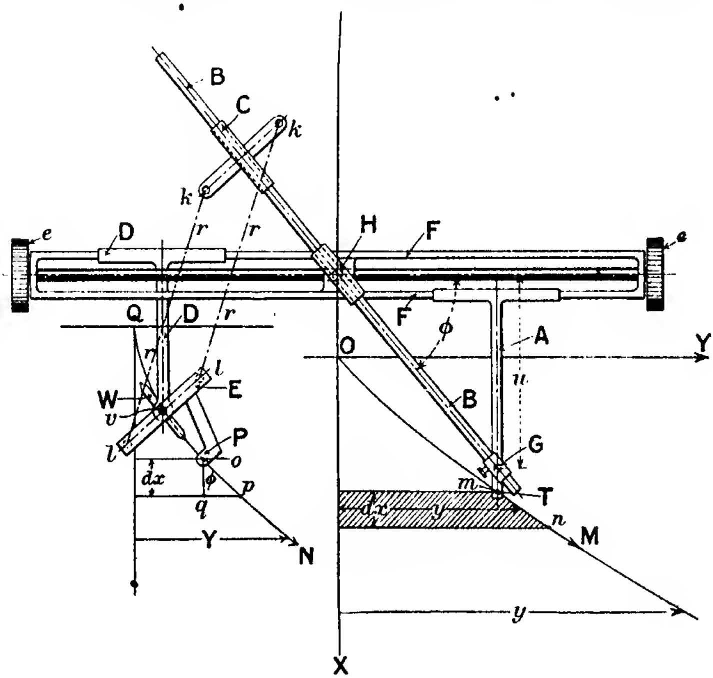


FIG. 138.—The integraph.

are drawn by means of an integraph, an instrument made by Coradi of Zurich. A diagrammatic sketch of the instrument is given in Fig. 138. The two points, m and P, touch the paper. As the tracing point m is guided by hand along the curve OM the pen P draws the integral curve QN. That is to say, the ordinate Y of the curve drawn by the instrument represents to scale the area A contained by the ordinate corresponding to the position of the tracing point m on the curve, the axis of X and the curve itself.

The main frame FF is carried on a pair of wheels e, e rigidly connected by an axle. The frame is thus free to move only in a

direction parallel to itself. The sides of the frame carry sliding pieces A and D. Each piece is carried on small pivoted wheels in order to avoid sliding friction.

A leg, with which is combined the tracing point m, is attached to the end of the arm of the piece A, so that the weight of the instrument is supported by this leg and the wheels e, e.

The arm also carries a vertical axis G, which can be clamped in any position along the arm A, thus fixing the dimension u, which, as will be seen below, fixes the relative scale between the two curves. The top of this vertical axis is enlarged into a slide through which a long bar, B, passes, and this bar can be clamped in position by the set screw shown.

At the centre of the frame FF there is a vertical axis, the top of which is enlarged to form a guide through which the bar B can pass freely. (The actual slide is formed by friction wheels.)

It will be seen that as the piece A is moved along the frame the bar B is free to follow it, partly turning at H and partly sliding through the guide at H.

Sliding on the bar is a piece C (actually on friction wheels). The lower arm, kk, of this piece is fixed rigidly at right angles to the upper part.

A frame, E, carries a knife-edged following wheel, W, mounted so that its axis is along the direction ll. The centres l, l are the same distance apart as the centres k, k, and they are connected by the parallel rods, r, r. Thus wherever the piece E is placed, the axis of the following wheel w stands parallel to kk, and therefore at right angles to the bar B.

The motion of the frame E is further constrained by its attachment to the arm D, so that its vertical axis v moves parallel to the frame FF, and always at a constant distance from it. It will be understood from this that when the instrument is rolled over the paper the following wheel, W, is free to roll in a direction determined solely by the instantaneous inclination of its axis.

To draw the integral curve corresponding to the curve OM the instrument must be placed on the drawing board so that the frame FF is at right angles to the axis of X; the slider A is in the middle position; and the bar B lies over the axis of x. The instrument is then rolled along X until the tracing point T is over the origin. With the instrument so placed, the piece D can be moved right or left of the middle position in order to place the drawing pen P at an origin, as Q, convenient for the starting of the integral curve.

The fundamental property of the mechanism is that however the tracing point m be moved, the plane in which the following wheel W is free to rotate is always parallel to the bar B. With this in mind the theory of the instrument is simple.

Let the tracing point m be guided along the curve OM. Then the pen P will draw the curve QN.

Consider a small element mn of the curve OM. As the tracing

point moves along mn the pen draws the element op of the integral curve QN.

To show that QN is the integral curve of OM it is sufficient to show that qp represents the shaded area $y\,dx$.

It is clear that in all positions of the instrument $y = u \tan \phi$, therefore the area $y\,dx$ is equal to $u \tan \phi\, dx$. But in all positions of the instrument the angle $qop = \phi$. Therefore $qp = dx \tan \phi$. Therefore the area $y\,dx$ is equal to $u\,qp$.

That is to say that qp represents the area $y\,dx$ to a scale determined by the length u.

It follows that the ordinate Y is the area of the curve OM reckoned up to the ordinate y, i.e.

$$uY = \int y\,dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It will be noticed that a movement of the tracing point T may be analysed into two steps, namely, a step parallel to the axis of y and a step parallel to the axis of x. A movement parallel to y alters the inclination of the plane of the following wheel W, the mechanism ensuring that the plane is always parallel to the bar B.

A movement parallel to X causes the whole instrument to move like a parallel ruler parallel to the axis of y, this movement causing the following wheel W to rotate on its axis and run along a line inclined to the axis of X by an amount depending upon the displacement of the slider A from its central position.

To calculate the value of the length u.—Assume that the scale along OX is such that 1 in. represents a units; and that the scale along OY is such that 1 in. represents b units; then one sq. in. represents ab of the derived units, and $\frac{1}{ab}$ = area corresponding to one unit of the derived unit.

Choose arbitrarily the scale on which the derived unit is to be represented by the ordinates of the integral curve. Thus in the integral curve let 1 in. represent c of the derived units, then one derived unit is represented by $\frac{1}{c}$ inches.

Referring to (1) it will be seen that

$$uY = \text{area.}$$

$$\text{Therefore } u = \frac{\text{area in square ins. representing one derived unit}}{\text{distance representing one derived unit}}$$

$$= \frac{c}{ab} \text{ ins.}$$

Set the instrument to the value of u calculated from this expression; and then it is ready for use, and the integral curve of OM can be drawn to a scale such that 1 in. of Y represents c derived units.

As an example, suppose that the instrument is to be used to draw

a time-distance curve from a time-velocity curve. Let these be the given scales;

1 in. along the velocity axis represents 13·33 ft. per second = a
1 in. along the time axis represents 66·66 seconds = b
1 in. along the distance axis represents 5330 feet = c

Then $$u = \frac{5330}{13{\cdot}33 \times 66{\cdot}66} = 6 \text{ ins.}$$

In particular relation to the curves of the dynamical diagram it may be convenient to put the results in this way:—

(*a*) When the time-velocity curve is derived from the auxiliary curve $\frac{M}{f}$,

$$u = \frac{\text{Number of seconds represented by 1 in. on the paper}}{\frac{M}{f} \text{ units per in. on the paper} \times \text{velocity units per in.}}$$

(*b*) When the time-distance curve is derived from the time-velocity curve,

$$u = \frac{\text{Number of ft. per in.}}{\text{Vel. units per in.} \times \text{time units per in.}}$$

(*c*) When the energy curve is deduced from the force distance curve,

$$u = \frac{\text{Number of energy units per in.}}{\text{Force units per in.} \times \text{distance units per in.}}$$

CHAPTER IX

THE BALANCING OF ENGINES

132. Application of Newton's Laws to the Balancing of Engines.—"Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by force to change that state." This, Newton's first law of motion, defines the natural mode of motion of a body. If a body is moving in any other manner than in a straight line with uniform velocity the inference is that a force is acting on the body. The instant that force ceases to act the motion of the body changes to uniform velocity in a straight line.

The application of a force to a body cannot take place without the simultaneous action of an equal and opposite force on some other body. Newton expresses this principle in the third law, namely, "To every action there is always an equal and contrary reaction; or the mutual action of any two bodies are always equal and oppositely directed". A stone in a sling is compelled to move in a circle by the force applied to it by the sling string: the equal and opposite force appears as a pull at the hand, and the two aspects of the force, the action and the reaction, constitute a tension in the string.

The inward pressure acting between the tyres and the outer rail of a railway curve is the force constraining the train to pass round the curve, turning it from its natural path in a straight line, whilst the equal force from the tyres to the rail, outwards, tries to displace the track radially outwards.

The pressure produced by the explosion of a charge in a gun accelerates the shot from rest and accelerates the gun itself in the opposite direction. The flying shot and the slower recoil of the gun together illustrate the action and reaction of the pressure in the gun.

The explosion in the cylinder of a gas engine accelerates the piston masses and would fire them out of the cylinder were they not constrained by their connection to the crank shaft to move in a regular manner, and usually it does not require very close observation to see the recoil of the engine frame which follows the explosion.

In a steam engine the steam pressure accelerates the piston masses and tries to blow them out, but here again their connection to the crank shaft constrains them to move in a regular way with periodic motion and the recoil of the frame is always there though it cannot always be seen. Briefly, the balancing of an engine may be described

as the design of the moving parts so that the recoils on the frame which result from their accelerations, neutralize one another.

In general the velocity of every particle in the moving parts of an engine or machine is changing either in magnitude or in direction, or both in magnitude and in direction, and in consequence there is at any instant, acting on the engine frame, a system of forces which are the reactions to the forces actually producing the accelerations of the particles in motion.

At any given instant the system of frame reactions reduce to a force and a couple, each varying from instant to instant but passing periodically through the same values. The periodic change in the force and in the couple produce, or tend to produce, periodic displacements or recoils of the frame of the mechanism which if the periodic time is sufficiently short become vibrations.

These vibrations may be transmitted to the foundation to which the machine is bolted, and from these foundations to the surrounding ground, and where the ground is resonant to the periodic time of the engine a small vibration of the frame may become a nuisance to an entire neighbourhood.

The problem of balancing a machine or engine in a technical sense is the problem of so designing the moving parts, both as regards mass and disposition, that during every instant of the motion the resultant of the forces and the resultant of the couples acting on the bed plate or frame of the engine or machine are separately zero.

133. Forces produced on the Frame by the Rotation of a Single Mass.—Consider the mass M, Fig. 139. It is attached to a disc which is keyed to the shaft OX. Let the shaft rotate with angular velocity ω. Then the mass M is compelled to move in a circular path, and if at any instant it were freed from its connection with the disc it would continue to move in a straight path tangential to the circular path. The force required to compel the mass to move in its circular path is

$$\frac{M\omega^2 r}{g} \text{ lbs.-wt.}$$

where M is the weight of the mass in pounds, r is the radius of its mass centre in feet, and ω is the angular velocity of rotation in radians per second.

This force must act on the mass in a direction along the radius towards the axis.

The force is applied in the example above by means of tensions in the plate connecting it to the shaft. In consequence of this there is an equal and opposite force acting on the shaft itself.

If a mass M_1 equal to M were attached to the plate on the prolongation of the line joining the mass centre of M with the axis, and at a distance r on the other side of it, it would require for its acceleration in the circular path a force exactly equal, but opposite to the force accelerating M. Therefore what may be called the

dynamical load on the shaft due to the rotation of M is equilibrated by the dynamical load caused by the rotation of M_1.

Consequently the shaft, and through the shaft the frame, are not required to supply any reactions to the dynamical loading, and there is no tendency to vibrate. The mass M is balanced by the mass M_1; M_1 is therefore called the **balance weight**.

The balance weight is applied to produce an equal and opposite dynamical load on the shaft to that caused by the rotation of the mass M.

The dynamical load due to the rotation of M is $M\omega^2 r$ poundals. The dynamical load due to the rotation of M_1 is $M_1\omega^2 r_1$ poundals. Equality between the dynamical loading is secured if

$$M\omega^2 r = M_1\omega^2 r_1$$

or since ω is the same on each side of the equation,

if $$Mr = M_1 r_1$$

The balance weight M_1 may therefore have any magnitude, providing it is placed at the radius

$$r_1 = \frac{Mr}{M_1}$$

or it may be placed at any radius r_1 providing its magnitude is

$$M_1 = \frac{Mr}{r_1}$$

Thus a mass of 12 lbs. at 2 ft. radius is balanced by 24 lbs. at 1 ft. or 6 lbs. at 4 ft. or 3 lbs. at 8 ft. or 4 lbs. at 6 ft. The only limitation placed on the choice of the mass and the radius is that their product shall be 24, and that the radius shall be a continuation of the radius of the mass M.

If no balance weight is applied, the dynamical load due to the rotation of M must be equilibrated by straining actions in the shaft itself, and again these actions must be equilibrated by reactions at the bearings which support the shaft. The shaft is thus in the condition of a beam carrying a load F, and the bearings which hold the shaft must supply the reactions to this load. There is, however, this important difference, namely, that a beam is at rest, so that the reactions to the loads act in fixed directions, whilst in the rotating shaft the load and the reactions are rotating, and the constant change of direction tends to set up vibrations of the frame.

When there are several unbalanced masses attached to a shaft the solution of the problem of finding the magnitude and position of the masses which must be added to equilibrate them is reduced to a practicable and simple form, if it is assumed that the shaft is supported at one point only, the point O in Fig. 139 for example.

This is in fact the fundamental assumption in the method of finding balance weights about to be explained. The method was first communicated by the author to the Institution of Naval Architects

on the 24th March, 1899, in a paper entitled "The Balancing of Engines with special reference to Marine Work".

Consider the mass M (Fig. 139) attached to a truly turned disc D, carried by the shaft OX which is supported only at the fixed point O, and is distant a feet from M's plane of revolution.

Rotation of the system at angular velocity ω causes a force, $M\omega^2r$, to act on the shaft in the plane of the disc D. This is equivalent to—

(1) an equal and parallel force, $M\omega^2r = F$, acting at the fixed point O, and shown by a dotted line;

(2) a couple whose moment is $M\omega^2ra$, tending to cause rotation about an axis through O, at right angles to the plane of the couple, in the positive direction.

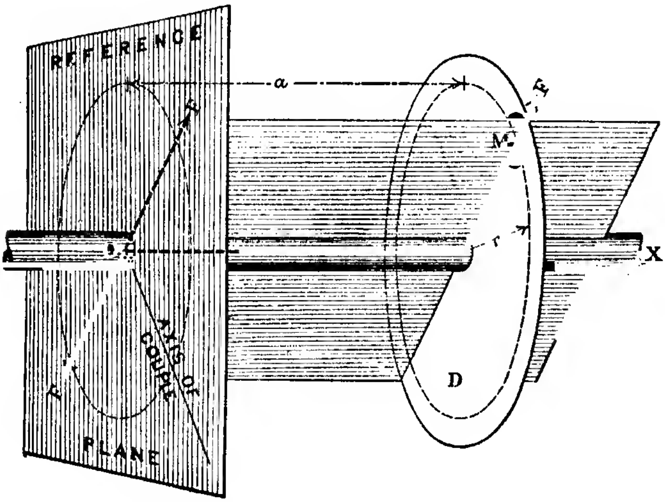


FIG. 139.—Reference plane.

A plane through the fixed point O, at right angles to the axis of rotation, and revolving with it, will be called the **reference plane.** It contains both the force at the fixed point O, and the axis about which the system is assumed to be free to turn under the action of the centrifugal couple. The **reference plane** may be thought of as a drawing-board keyed to and rotating with the shaft. It is on this drawing-board that all the vector summation which is required in a problem may be imagined carried out.

Example.—The rotation of a mass of 10 lbs., 4 times per second, at 5 ft. radius, in a plane distant 5 ft. from the reference, causes

(1) a force $\dfrac{M4\pi^2n^2r}{g} = 980{\cdot}47$ lbs. weight, to act at the fixed point O, in the plane of reference;

(2) a couple of moment $980{\cdot}47 \times 5 = 4902$ ft.-lbs., tending to turn the system about the axis shown in Fig. 139.

If a balancing mass or masses be applied to the system, giving rise to an equal and opposite centrifugal couple, there will be no tendency to turn about the fixed point. If at the same time the balancing masses have a resultant centrifugal force, equal and opposite to the resultant centrifugal force at the fixed point, there will be no pressure acting on it. In these circumstances, the constraint applied to the fixed point may be removed, and the system will continue to rotate without trying to change the direction of the main shaft. It would be a balanced rotating system, and held in bearings, would put no dynamical load upon them.

134. The Balancing of any Number of Given Masses by the addition of Masses placed in Two Given Planes.

The artifice used by the author to obtain a solution of the problem consists in placing the reference plane in co-incidence with the plane of revolution of one of the two necessary balancing masses. The balancing mass in the reference plane has then no moment about the origin O, and it may therefore be added to or taken away from the system without altering the system of couples due to the other masses. It follows that the couples may be balanced by the addition of a mass placed at a suitable radius and at a suitable distance from the reference plane, and then that the forces may be balanced by the addition of a mass at a suitable radius in the reference plane.

Let a_4 be the distance between the two given planes in which the balancing masses are to be placed.

Choose one of these planes for the reference plane, thus fixing the point O. Let M_1, M_2, M_3, etc., be the given masses, at radii r_1, r_2, r_3, etc., respectively, revolving in planes a_1, a_2, a_3, etc., feet from the reference plane.

Let M_5 be the balancing mass in the plane of reference at radius r_5, and let M_4 be the balancing mass at radius r_4 in the plane, which is by the terms of the problem a_4 feet from the reference plane.

When the system rotates, the centrifugal force corresponding to each mass acts upon the axis, which in turn causes an equal and parallel force to act at the fixed point O, and a couple. The condition that there may be no couple is expressed by—

$$\text{Vector sum } (M_1r_1a_1 + M_2r_2a_2 + \ldots + M_4r_4a_4)\omega^2 = 0 \quad . \quad (1)$$

and the condition for no force on the fixed point O by—

$$\text{Vector sum } (M_1r_1 + M_2r_2 + \ldots + M_4r_4 + M_5r_5)\omega^2 = 0 \quad . \quad (2)$$

Whatever be the value of ω, these two conditions are separately fulfilled if the vector sums of the terms in the brackets are in each case zero—that is, if the lines representing them, when set out to scale, form a pair of closed polygons. Consider equation (1). All the terms are given but $M_4r_4a_4$, of which, however, the factor a_4 is given. Calculate their arithmetical values and set them out to scale,

their relative directions being specified by a drawing. The axes of the couples which the terms represent are of course at right angles to the axial planes in which they respectively act. A little consideration will show, however, that the directions of the cranks themselves may be used in actually drawing the couple polygon, if the following rules are observed :—

Rules for the Way of drawing Couple Vectors.—If the masses are all on the same side of the reference plane, the direction of drawing is *from* the axis, *outwards,* to the mass, in a direction parallel to the respective crank directions. If the masses are, some on one side of the reference plane, and some on the other side, the direction of drawing is *from* the axis, *outwards,* towards the mass, for all masses on one side; and *from* the mass, *inwards,* towards the axis for all masses on the opposition side of the reference plane, drawing always parallel to the respective crank directions.

The line closing the polygon, called by the author **a closure,** represents $M_4r_4a_4$. Scaling this off and dividing by a_4, M_4r_4 is known.

Again, calculate the arithmetical values of the terms in equation (2), and set them out to scale, the relative directions being given by the drawing, and include of course the value of M_4r_4 just found from the couple polygon, observing the following rule :—

Rule for the Way of drawing Force Vectors.—Draw always *from* the axis *outwards* towards the mass parallel to the respective crank directions.

The line closing the polygon, that is **the closure,** represents M_5r_5.

Choosing the radii, r_4 and r_5, the magnitude of the balancing masses may be calculated at once. These added to the given system, so that their radii are placed in the relative positions specified by the closing sides of the two polygons respectively, completely balance it for all speeds of rotation.

Checking the Accuracy of the Work.—Having found the balancing masses, add them to the drawing in their proper positions relatively to the given masses; choose a new reference plane anywhere along the shaft, and draw a new couple polygon relatively to it. If it close, it is safe to infer that no mistake has been made in the work. The force polygon is the same for all positions of the reference plane.

135. Nomenclature.—It will be noticed that each term in the brackets of equation (2), page 483, is a mass moment, and that each term in the brackets of equation (1) is the moment of a mass moment with reference to the reference plane. The term, "product of inertia," is also used to denote terms of this form. The conditions of balance may evidently be concisely stated as follows :—

(1) The sum of the products of inertia about the axis of rotation must vanish;

(2) The sum of the mass moments about the axis of rotation must vanish.

When these conditions are fulfilled, the axis is called a **principal**

axis. To avoid using these somewhat unfamiliar terms, the angular velocity may be supposed equal to unity, since it may have any value, in the problem under discussion, without affecting the balance in any way. Then a term of the form Mr may be referred to as a centrifugal force, and a term of the form Mra as a centrifugal couple.

136. Typical Example.—Three masses, page 486 (shown black, Fig. 140), rigidly connected to a shaft, are specified in the following list, the distances, a, being measured from a given plane of reference:—

$$M_1 = 1\cdot0 \text{ lb.} \qquad r_1 = 1\cdot5 \text{ ft.} \qquad a_1 = 7\cdot0 \text{ ft.}$$
$$M_2 = 2\cdot0 \text{ ,,} \qquad r_2 = 1\cdot0 \text{ ,,} \qquad a_2 = 3\cdot5 \text{ ,,}$$
$$M_3 = 1\cdot8 \text{ ,,} \qquad r_3 = 1\cdot25 \text{ ,,} \qquad a_3 = 1\cdot8 \text{ ,,}$$

The angles between the mass radii are specified by the dotted lines (Fig. 141).

Find the magnitude and position of the two balancing masses, which are to be placed, one in the plane of reference at unity radius, the other in the plane of M_1, also at unity radius.

It will be found convenient to arrange the data in a schedule of the following kind, in order to calculate the arithmetical values of the different terms.

SCHEDULE I.

Number of the plane of revolution.	Magnitude of the mass in pounds (M).	Mass radius measured in feet (r).	Distance from the reference plane in feet (a).	$\omega = 1$. The products Mr, the centrifugal forces.	$\omega = 1$. The products Mra, the centrifugal couples.
No. 1	1	1·5	7·0	1·5	10·5
No. 2	2	1·0	3·5	2·0	7·0
No. 3	1·8	1·25	1·8	2·25	4·05
No. 4	unknown	1·0	7·0	0·63	4·4 (closure)
No. 5	unknown	1·0	0·0	1·7 (closure)	

Draw the couple polygon first (Fig. 142), setting out AB, BC, CD parallel respectively to the crank directions given by Fig. 141, and representing to scale 10·5, 7·0, and 4·05, the respective magnitudes of the couples given in the last column of the schedule. The closure DA measures 4·4, and this is the value of $M_4r_4a_4$, in which a_4 is given equal to 7 ft.; therefore—

$$\therefore \qquad M_4r_4 = 0\cdot63$$

The angular position of the radius in the plane of M_4, which in this case is given coincident with the plane of M_1, is determined

by drawing a line parallel to DA, *from* the axis. Further, if $r_4 = 1$ ft., 0·63 lb. placed at this radius, shown in Fig. 140, will balance the centrifugal couples.

$M_4r_4 = 0·63$ may now be written in the schedule.

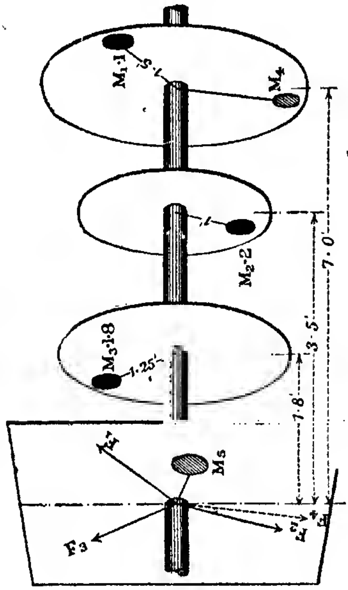


Fig. 140.

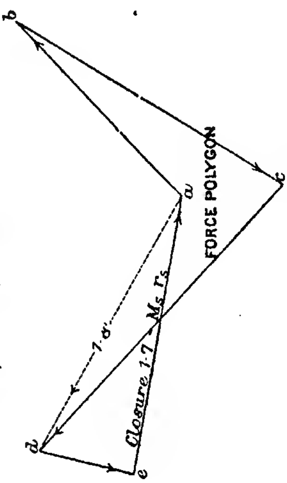


Fig. 143.

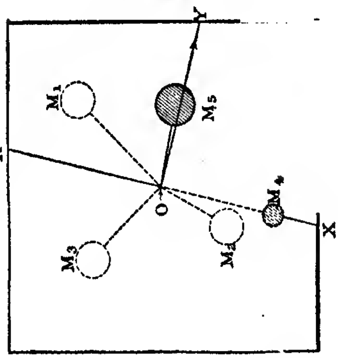


Fig. 141.

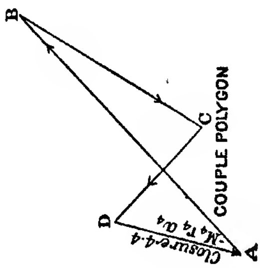


Fig. 142.

The balancing of a system of four revolving masses.

Again, set out the force polygon (Fig. 143) *abcde*, the sides being parallel to the radii given by Fig. 141, and proportional to 1·5, 2·0, 2·25, and 0·63 respectively, the forces acting at the point O. The closure *ea* measures 1·7, and this is the value of M_5r_5. Taking $r_5 = 1$, M_5 is equal to 1·7 lbs. The angular position of the

radius in the reference plane is determined by drawing a line parallel to *ea*, *from* the axis, as indicated in Fig. 141.

These two masses completely balance the given system; in mathematical language, they convert the axis of rotation into a principal axis. Bearings may be applied to the shaft in any position along it and in any number. There will be no dynamical load on the shaft at any ordinary speed of rotation, and therefore no vibration will be produced at any ordinary speed of rotation. At exceptionally high speeds dynamical loading due to slight variations of the density of the materials may become troublesome.

137. To find the Unbalanced Force and the Unbalanced Couple, with respect to a Given Reference Plane due to a System of Masses Rotating at a Given Speed.—The unbalanced force is the vector sum of the forces at O, equal and parallel to the centrifugal forces. The unbalanced couple is the vector sum of the centrifugal couples.

Consider the example of Section 136. The unbalanced couple is represented by AD (Fig. 142), the magnitude of which is $4{\cdot}4\,\frac{\omega^2}{g}$ ft.-lbs. The axis of the couple is at right angles to AD, and is shown by OY (Fig. 141). The magnitude of the resultant force is represented by *ad* (Fig. 143), which measures 1·8 feet to scale; the magnitude of the force is therefore $1{\cdot}8\,\frac{\omega^2}{g}$ lbs. weight acting at O, parallel to AD.

If the shaft rotates at 10 revolutions per second, these values are 543 ft.-lbs., and 221 lbs. weight respectively.

The force and the couple may now be balanced in a more general way than that given in Section 136. To balance the force, a mass, M, must be placed in the reference plane in a direction, *da*, at such a radius, r, that—

$$Mr = 1{\cdot}8$$

To balance the couple, masses M_a, M_b, at radii r_a, r_b, respectively may be placed anywhere in the axial plane, of which XX (Fig. 141) is the trace, so that if a_c be the axial distance between their radii—

$$M_a r_a a_c = 4{\cdot}4 = M_b r_b a_c$$

their disposition being such that they give rise to a couple opposite in sign to the unbalanced couple. This method gives in general three balancing masses, which of course may be combined into the two of Section 136. The first method is by far the most convenient for practical use, because it gives the balancing masses without any necessity of thinking of their ways of action, these being determined automatically by the closures of the polygons.

138. Reduction of the Masses to a Common Radius.—It is often convenient to make a preliminary reduction of the masses to a common radius, the crank radius in an engine problem, or unity in a

general problem. The centrifugal force $M\omega^2 r$ is proportio[illegible] product Mr, and the individual factors, M and r, may h[illegible] to the value, providing that their product remains constant. If R repr[illegible] any the common radius, the reduced masses $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, etc., a[illegible] obtained from the several equations—

$$\mathbf{M}_1 R = M_1 r_1,\ \mathbf{M}_2 R = M_2 r_2,\ \mathbf{M}_3 R = M_3 r_3, \text{ etc.}$$

Substituting these equivalent products in equations (1) and (2), page 483, they become—

$$\text{Vector sum } (\mathbf{M}_1 a_1 + \mathbf{M}_2 a_2 + \ldots + \mathbf{M}_4 a_4)\omega^2 R = 0 \quad . \quad . \quad (3)$$

$$\text{Vector sum } (\mathbf{M}_1 + \mathbf{M}_2 + \ldots + \mathbf{M}_5)\omega^2 R = 0 \quad . \quad . \quad (4)$$

If the vector sums in the brackets separately vanish, the two conditions of balance are fulfilled for all speeds. $\mathbf{M}$ is really a mass moment, and $\mathbf{M}a$ a moment of a mass moment, but, to avoid using unfamiliar terms, the factor $\omega^2 R$ may be supposed equal to unity; then $\mathbf{M}$ may be referred to as a centrifugal force, and $\mathbf{M}a$ as a centrifugal couple.

To calculate the value of the centrifugal force and couple when the vector sum in the brackets is not zero, the quantities $\mathbf{M}$ and $\mathbf{M}a$ representing the respective vector sums must be multiplied by $\frac{\omega^2}{g}R$, the g being introduced to obtain the result in lbs: weight units of force. In what follows the ordinary capital M denotes mass at crank radius unless otherwise stated.

139. Conditions which must be satisfied by a Given System of Masses so that they may be in Balance amongst themselves.—Suppose all the masses to be first reduced to equivalent masses at a common radius so that the terms "equivalent mass" and "equivalent mass moment" may be used instead of "mass moment" and "moment of mass moment" respectively. Then—

(1) It must be possible to draw a closed polygon whose sides are proportional to the equivalent masses, and parallel in direction to the corresponding mass radii;

(2) It must be possible to draw a closed polygon whose sides are proportional to the equivalent mass moments taken with respect to any reference plane.

If condition (1) is satisfied and not (2), there is no unbalanced force, but there is an unbalanced couple.

Condition (2) cannot be satisfied unless (1) is satisfied, for although the couple polygon may be closed for any reference plane, yet if the plane is moved into a new position, the couple polygon for the new position will close, only if there is no force in the old reference plane—that is, only if condition (1) is satisfied.

Balancing problems are conditioned, therefore, by the geometrical properties of two polygons, whose sides are parallel, but of different lengths, the sides of the one being obtained from the sides of the

other by multiplication, the multiplier in each case being the distance of the equivalent mass from the reference plane.

140. Experimental Apparatus.—These principles may easily be verified experimentally by means of the apparatus shown in Fig. 144. A wood frame, slung by chains from a supporting frame, carries an accurately turned and well-mounted steel shaft, on which are arranged four carefully turned and balanced discs. One disc, shown to the front in the figure, is fixed to the shaft and carries a protractor, the others are capable of angular and longitudinal adjustment relatively to the shaft. The radius defined by a disc is fixed by a small hole, drilled near the periphery. The system is driven by a carefully balanced motor, also carried on the frame, so that the frame is free from the action of any external driving force. The only unbalanced forces acting on the system are therefore those due to the rotation of the discs. The four discs serve to carry any assigned set of crank-pin masses. These are bolted to the discs at the crank-pin holes (two such masses are shown in the figure), the discs are set to the proper crank angles by means of the protractor on the front disc, and the proper distance apart by means of the longitudinal adjustment. Any want of balance is at once shown by the oscillations set up when the system is driven. The first apparatus of this kind was designed by Professor Ewing for the Engineering Laboratories at Cambridge.

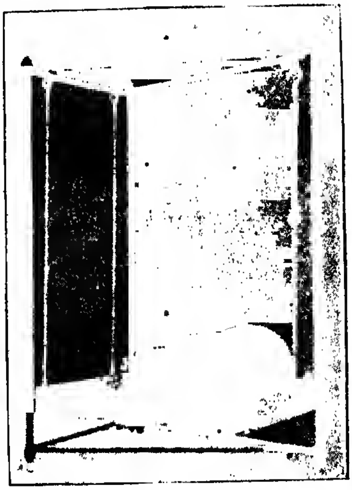


Fig. 144.—Experimental apparatus.

141. The Balancing of Reciprocating Masses.—The reciprocating masses of a steam engine move in the natural straight path, but both the speed and the direction of motion are changed in a periodic manner. In general if M is the mass in pounds of the moving body and A the acceleration produced by the force F acting in the direction of motion at the mass centre

$$F = MA \text{ poundals}$$

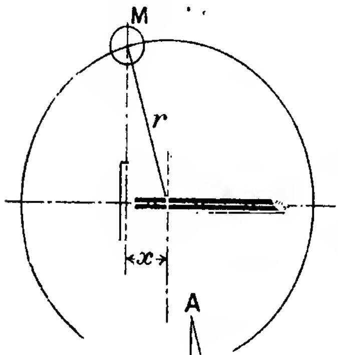
or

$$F = \frac{MA}{g} \text{ lbs.-wt.}$$

In balancing problems the magnitudes of the forces are not generally concerned; it is therefore more convenient to use the first expression thus avoiding the introduction of g into the work.

The exact expression for the piston acceleration is given by equation (8), page 338. In many cases in practice the difference between the true accelerating force acting on the piston and the force calculated on the assumption that the connecting rod is infinitely long is small enough to be negligible. The error due to this assumption increases as the length of the connecting rod in relation to the length of the crank decreases. The errors are considerably reduced if the balancing is based on the approximate expression (5), page 338. When an engine is balanced on the assumption that the reciprocating masses move as if they were operated by infinitely long connecting rods, the balancing is said to be for Primary Forces and Primary Couples. If the balancing is based on the motion corresponding to the acceleration given in expression (5), page 338, then balancing is said to be for Primary and Secondary forces and for Primary and Secondary Couples.

Fig. 145.



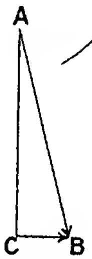


Fig. 146.

Relations in simple harmonic motion.

When the connecting rod coupled to a reciprocating mass is assumed to be infinitely long, the motion of the mass may be regarded as the projection on the line of stroke of the circular motion of the crank pin. It is in fact the kind of motion called Simple Harmonic. The force constraining the motion of the reciprocating mass is then the projection on a line parallel to the line of stroke of the force required to constrain the uniform circular motion at crank-pin radius, of a mass equal in magnitude to the reciprocating mass. Let M (Fig. 145) be a mass at the crank pin equal to the reciprocating masses indicated by the piston. Then when the piston is in the position shown, the instantaneous magnitude of the force acting to accelerate its motion is the projection, on a line parallel to the line of stroke, of the force required to constrain the mass M to move with uniform velocity in a circular path, the diameter of which is equal to the stroke.

This force is $M\omega^2 r$, and it is represented by AB (Fig. 146) drawn parallel to the radius r. CB is the projection of AB on a line parallel to the line of stroke, and is the instantaneous magnitude of the force causing the acceleration (or the retardation) of the motion

of the piston. The equal and opposite force BC is the action on the frame which results from the acceleration of the reciprocating mass.

The general application of this principle to the problems relating to the balancing of reciprocating masses assumed to be moving with simple harmonic motion will be easily understood by reference to a balanced system of revolving masses.

Let ABCD (Fig. 147) be a closed force polygon in the reference plane, which is supposed to be keyed to, and to revolve with, the shaft, whose end is shown at O. The shaft carries four masses whose respective centrifugal forces are represented by the sides of the polygon. Let ZZ be any fixed line. It is clear that in the position shown in Fig. 147, the sum of the projections of the sides of the polygon on the line ZZ is zero. But these projections are the components of the centrifugal forces represented by the sides of the force polygon, and, therefore, if the masses concerned in drawing the

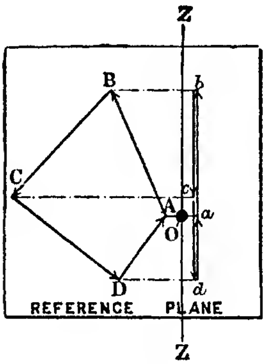


Fig. 147.—Force polygon in the reference plane.

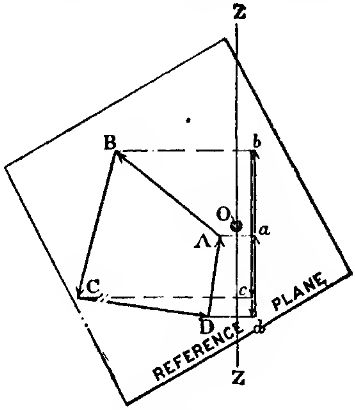
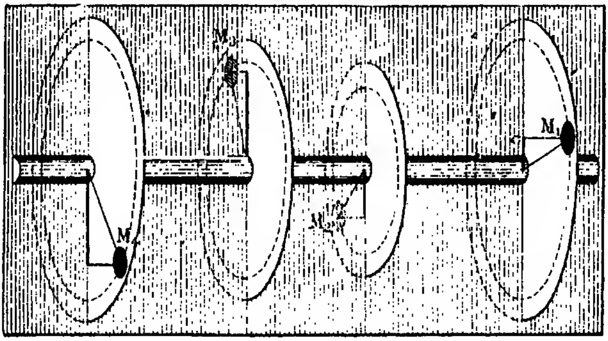


Fig. 148.—Force polygon in the reference plane.

polygon are reciprocated in the plane of which ZZ is the trace, the sum of the disturbing forces due to their reciprocation is, for the position shown, zero, since these forces are instantaneously represented by the several projections (drawn to the right of ZZ for clearness) shown in Fig. 147. Again, consider the system when the shaft, and therefore the reference plane and the polygon on it, has turned into the new angular position with regard to ZZ shown in Fig. 148. The sum of the projections of the centrifugal forces, viz. ab, bc, cd, da, is still zero, and therefore there is still balance amongst the reciprocating forces. In fact, always providing that the force polygon is closed, the sum of the projections of its sides on ZZ is continuously zero during the rotation of the shaft, though the individual magnitudes of the projections are continually varying. Similar reasoning applies to a closed couple polygon. During rotation the sum of the projections of its several sides on ZZ is

always zero, and, therefore, the reciprocation of a corresponding set of masses in the plane, of which ZZ is a trace, gives rise to no tilting action on the engine frame. From this it follows that to

FIG. 149.—Revolving system of four masses.



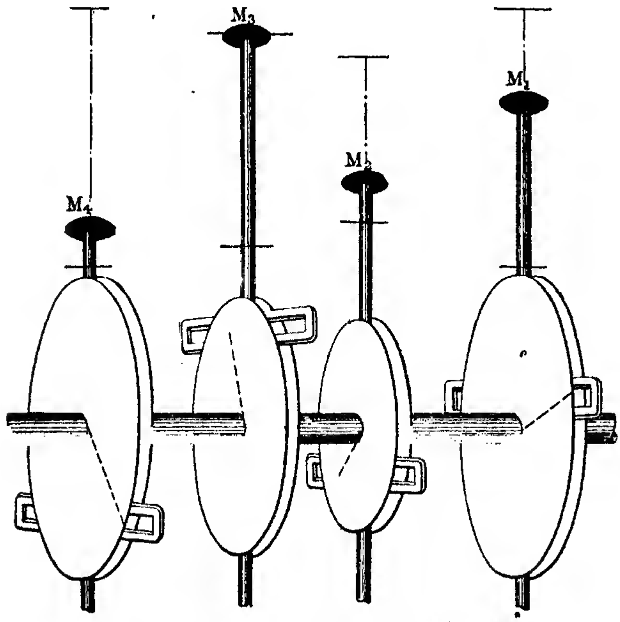


FIG. 150.—Corresponding system of reciprocating masses.

investigate the balancing conditions amongst a given system of reciprocating masses, it is only necessary to imagine them transferred to their respective crank pins, and then to proceed by the rules of page 484.

To fix these principles in the mind, assume the system of revolving masses M_1, M_2, M_3, M_4, shown in Fig. 149, to be in balance amongst themselves. The corresponding force and couple polygons will therefore be closed on a reference plane taken in any position along the shaft. The sum of the projections of these two polygons on the plane indicated by shading will, therefore, be continuously zero during the rotation of the system. If, therefore, the revolving system is changed into the reciprocating system shown in Fig. 150, where the masses have been taken from their respective crank pins and placed as pistons on the slotted bars (whose mass is here neglected), the system of reciprocating masses so formed will be balanced.

If the force polygon is unclosed, the resultant is the unbalanced force in the revolving system, and the projection of this resultant on the plane of reciprocation is the instantaneous value of the unbalanced force in the corresponding reciprocating system.

For example, let ABCD (Fig. 151) be the unclosed force polygon corresponding to a system of three revolving masses carried by the shaft, whose end is indicated by O. Let ZZ be a fixed line, and let OX be a line drawn in the reference plane by means of which the angular position of the plane may be specified relatively to the fixed line ZZ. When the angle between OX and ZZ is known, the angles between all the crank directions and ZZ are known. The unbalanced centrifugal force is represented by the vector AD. The projection of this on ZZ is the instantaneous value of the unbalanced force due to a corresponding system of reciprocating masses.

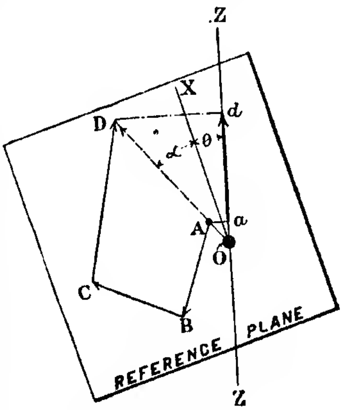


FIG. 151.—Unbalanced forces and couples due to a reciprocating system.

It will be noticed that the value of the unbalanced force is a maximum at the instant AD is parallel to ZZ. And remembering the rules for drawing a couple polygon, the tilting couple is a maximum also when AD, taken now to represent the unbalanced couple, is parallel to ZZ.

EXAMPLE.—Find the unbalanced force and the unbalanced couple due to the reciprocating masses of a three-crank engine arranged with cranks mutually at 120°, and running at 88 revolutions per minute, having given—

Mass of each set of reciprocating parts . .	5 tons.
Crank radius	2 feet.
Cylinders	16 feet pitch.

Take a reference plane at the central crank. Imagine the reciprocating parts concentrated at and moving with their respective crank pins, and apply the method of page 484.

The force polygon is a closed equilateral triangle, so that there is no unbalanced force.

The couple polygon is open, requiring to close it a side 138·5 units long, inclined 90° to the direction of the central crank. The couple this represents at 88 revolutions per minute is $138 \cdot 5 \frac{\omega^2 r}{g} = 728$ ft.-tons. This is the maximum value of the tilting couple for the reciprocating masses, and the value in terms of the angle which the central crank makes with the line of stroke is given by—

$$728 \cos (\theta + 90°)$$

142. Elimination of the Connecting Rod.—Not only does the connecting rod disturb the simple harmonic motion of the reciprocating masses, but the motion of the rod itself, partly reciprocating, partly turning, and in each case changing from instant to instant, requires the action of varying accelerating forces to constrain its motion, and the equal and opposite aspects of these forces disturb the frame. Therefore, in addition to the assumption that the connecting rod is so long that the difference between the acceleration of the mass it reciprocates, and the acceleration it would give if it were infinitely long, is negligible, the forces due to its own acceleration must be reckoned with before it can be finally discarded from the problem. A full discussion of this subject will be found in Chapter VIII. of "The Balancing of Engines".[1] For the present purpose it will be sufficient to state that its effect on the frame in the line of stroke is imitated by two separate masses, one being concentrated at the crank pin, the other at the crosshead. These two masses are then included, the one with the revolving masses of the engine, the other with the reciprocating masses. The magnitudes of these masses are inversely as the mass centre of the rod divides the line joining the crank-pin centre to the centre of the crosshead-pin, and their sum is equal to the mass of the rod.

To find the mass centre of the rod, take it in its finished state, complete in every detail, and balance it on a knife-edge. Let c be the distance from the crank-pin centre to the knife-edge, and l the length of the rod centre to centre. Weigh the rod, and let M be its mass in pounds or tons as the case may be. Then—

$$\left.\begin{array}{l}\text{Mass supposed attached to, and moving}\\ \quad\text{with the crosshead}\end{array}\right\} = \frac{Mc}{l} = m$$

$$\left.\begin{array}{l}\text{Mass supposed attached to, and revolving}\\ \quad\text{with the crank pin}\end{array}\right\} = M - m$$

[1] "The Balancing of Engines," W. E. Dalby. (Edward Arnold, London, 1906.)

Another way of arriving at the proper division of the mass is to place the rod with its centres on knife-edges, supported on the platforms of two separate weighing-machines. The reading given by the scale supporting the crank-pin end gives the mass to be included with the revolving masses. The reading given by the other scale gives the mass to be included with the masses at the crosshead. It is obviously only really necessary to support one knife-edge on a scale, the pressure on the other edge being found by difference when the mass of the rod is known.

EXAMPLE.—A connecting rod, 6 ft. centre to centre, weighs 500 lbs. It is found to balance about a knife-edge 1 ft. 6 ins. from the crank-pin centre. The mass to be included with the reciprocating masses is 125 lbs., the rest, 375 lbs., is reckoned with the revolving masses.

143. General Method of Procedure for Balancing an Engine, when the Motion of the Reciprocating Parts may be considered Simple Harmonic.

(1) Reduce each mass to an equivalent mass at a common crank radius (page 488), distinguishing between revolving and reciprocating masses.

(2) Distribute the mass of each connecting rod between the revolving and reciprocating parts which it connects by the method given on page 494.

(3) Fill in a schedule of the following type for the reciprocating masses, choosing the reference plane to coincide with a plane of revolution for which the reciprocating mass is unknown.

SCHEDULE II.

RECIPROCATING MASSES. Plane of reference at.............

Number of crank.	Distance of centre line of cylinder from plane of reference.	Column I. Equivalent mass at crank pin or centrifugal force, when $\omega^2R = 1$.	Column II. Equivalent mass moment or centrifugal couple, when $\omega^2R = 1$.

(4) Treat the quantities in the schedule exactly as though they formed a revolving system, and find the balancing masses by Section 134, page 483. These masses when reciprocated will be the two balancing masses for the system of reciprocating masses under consideration.

(5) Choose a new reference plane. Fill up a new schedule, including in it the balancing masses found in (4). Draw the couple

polygon corresponding to it. If the work has been correctly done this polygon will close. This should always be done to check the accuracy of the work. Notice, however, that this only checks (3) and (4). (1) and (2) must be checked independently.

The result of these five operations is to fix the crank angles and the reciprocating masses so that the reciprocating system is in balance. There now remains the revolving masses connected with the crank shaft to deal with. Since in this revolving system the crank angles are fixed, masses must be added to the system to balance it.

It will be noticed that the problem of balancing reciprocating masses presents itself in a slightly different form from problems on revolving masses. In the latter case, the problem is usually—

Given a system of revolving masses to find the masses which added to the given masses will produce a balanced system.

The least number of masses in the general case which must be added is 2, so that if there are n masses given, there results a system of $(n + 2)$ masses in balance. There is little difficulty in adding revolving balancing masses to a system. With reciprocating masses it is more difficult. A balancing mass in this case means a new crank, connecting rod, guides, etc., to operate a mass which in every other respect but the enclosing cylinder may be looked upon as a piston. Such a mass has been called a "bob-weight". Mr. Yarrow, in a paper[1] at the Institution of Naval Architects in 1892, described how the engines of a torpedo boat had been balanced by the addition of two bob-weights. This paper of Mr. Yarrow's is full of interest, as in it are given the details of the calculations used to determine the bob-weights for balancing the reciprocating masses of a three-cylinder engine. It will be perceived that, providing there are a sufficient number of cranks, the reciprocating parts operated by any two cranks may be considered as the bob-weights balancing the rest of the reciprocating parts.

The general problem of balancing a reciprocating system, therefore, presents itself in this way—

Given a system of n reciprocating masses to find how the masses must be arranged so that they mutually balance.

It has been shown, in Section 35 of "The Balancing of Engines," that the number of independent variables concerned in a revolving system of n cranks is in general $3(n - 1)$. This applies equally to a system of reciprocating masses. Consequently, if an engine is to be built with n cylinders, there will be $3(n - 1)$ variables to be considered in the balancing of the reciprocating masses, which have to satisfy four conditions; consequently, $3n - 7$ of these variables must have values assigned to them, but no more. The remaining four variables can then be found by the foregoing methods to balance the system.

Thus, in a four-crank engine, there are nine independent variables;

[1] On "Balancing Marine Engines and the Vibration of Vessels". By Mr. A. F. Yarrow, *Trans. Inst. Naval Architects*, London, 1892.

these are, or, rather, may be (for any one of the M's or a's may be used for the common divisor)—

Variables of magnitude . .	$\frac{M_2}{M_1}, \frac{M_3}{M_1}, \frac{M_4}{M_1}$	= 3
Variables in a	$\frac{a_2}{a_1}, \frac{a_3}{a_1}, \frac{a_4}{a_1}$	= 3
Variables of direction . . .	$\theta_{12}, \theta_{13}, \theta_{14}$	= 3
	Total	= 9

The double subscript to the θ's indicate the particular numbers of the cranks between which θ is measured. If the magnitude of one of the masses is put equal to unity, and if one of the distances from the reference plane is put equal to unity, the letters left in then represent independent variables. These letters are now proportional numbers, and the solution must be interpreted in terms of the M and the a which were put equal to unity.

If, for instance, M_1 and a_1 be considered each equal to unity, the variables are—

$$M_2, M_3, M_4, = 3$$
$$a_2, a_3, a_4 = 3$$
$$\theta_{12}, \theta_{13}, \theta_{14}, = 3$$
$$\text{Total} = 9$$

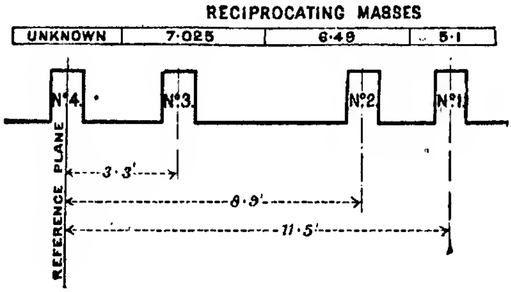
Of these $3 \times 4 - 7 = 5$ must be fixed. Fixing the pitch of the cylinders is equivalent to fixing all the variables in a, that is, three. The remaining two variables may then be chosen at will from the above set. For instance, any two of the M's may be fixed, or any two of the angles, or one M and one angle. It is evident that if all the masses are fixed as well as the centre lines of the cylinders, there are too many data chosen, and no solution of the problem is possible. It is also clear that there are as many solutions possible as there are ways of choosing the two quantities, supposing the centre lines to be fixed.

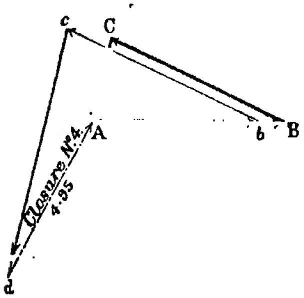
144. Example.—Given the stroke, the cylinder centre lines, and the masses corresponding to three cylinders in a four-cylinder engine find the crank angles, and the mass of the reciprocating parts belong ing to the fourth cylinder so that the reciprocating masses may be in balance amongst themselves.

The first step is to examine the data. There are nine variables concerned in the problem (Section 143), and of these five must be fixed. The fixing of the cylinder centre lines accounts for three, and the fixing of three masses for the remaining two, because although three masses are given, this only corresponds to two ratios. It is the same as putting one of the given masses equal to unity.

Take the reference plane so that it contains the centre line of the gear whose mass is to be determined as shown in Fig. 152, the figured data there shown being given. From it fill up Schedule III.

Fig. 152.





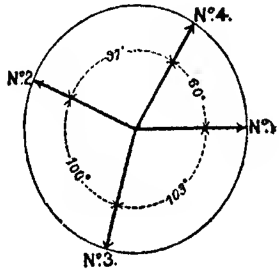


Fig. 153. Fig. 154.

The balancing of a four-crank engine.

Schedule III.

RECIPROCATING MASSES. Plane of reference at No. 4 cylinder.

Number of crank.	Distance of centre line of gear from plane of reference.	Column I. Equivalent mass at crank pin or centrifugal force, when $\omega^2 R = 1$.	Column II. Equivalent mass moment or centrifugal couple, when $\omega^2 R = 1$.
	Feet.	Tons.	
No. 4	0·0	Unknown (4·95)	0·0
No. 3	3·3	7·025	23·2
No. 2	8·9	6·49	57·8
No. 1	11·5	5·1	58·6

By supposition there is to be balance, therefore the couple polygon must close. In this case the polygon becomes a triangle.

Choosing a convenient scale, draw a triangle, as in Fig. 153, in which—

$$AB = 58{\cdot}6\,; \quad BC = 57{\cdot}8\,; \quad CA = 23{\cdot}2$$

Then—

AB is the direction of No. 1 crank
BC ,, ,, of No. 2 ,,
CA ,, ,, of No. 3 ,,

These directions are transferred to the end view of the crank-shaft centre lines (Fig. 154).

One condition of balance is fulfilled, *viz.* that the couple polygon close. The second condition, *viz.* that the force polygon close, is easily satisfied by taking advantage of the adjustment which may be made by crank No. 4, since, in whatever direction it is placed, the mass it operates has no moment about the reference plane, and consequently it may be fixed in any position without disturbing the balance amongst the couples.

Choosing a convenient scale, make Ab (Fig. 153) = to 5·1, bc = 6·49 and parallel to crank No. 2, cd = 7·025 and parallel to crank No. 3. The polygon fails to close by the side dA.

Close it by means of the 4th crank. Thus, dA is the direction of crank No. 4 relatively to the others, and its length represents the equivalent mass of the reciprocating parts attached to the crank—dA scales 4·95 tons.

Check the work in this way—

Suppose the reference plane to be at No. 1 crank. Make a new schedule for the masses with reference to this plane, including, of course, No. 4 crank. Draw the couple polygon. If it closes, the work is correct.

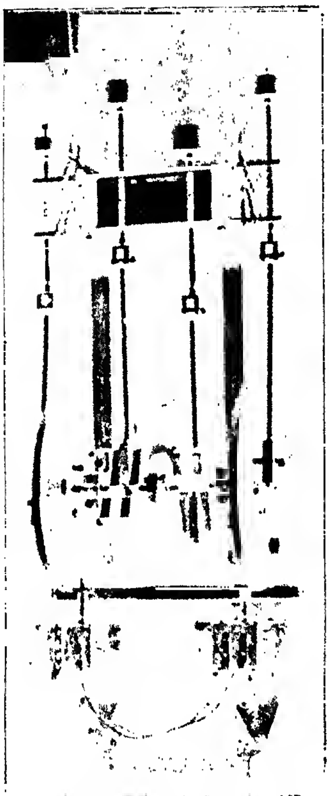


FIG. 155.—Model with four lines of parts, illustrating the Balancing of Marine Engines or Locomotives.

145. Experimental Apparatus.—Fig. 155 shows an apparatus which has been designed by the author to illustrate the principles of the balancing of reciprocating parts. There are four cranks, all mutually

adjustable, three of the flanges of the crank shaft being divided into degrees for this purpose. Adjustable masses may be fixed to the tails of the respective piston rods. Of the nine variables concerned in the balancing of a four-crank engine six are susceptible of variation in this apparatus. Suspended from a frame as shown, its motions when running unbalanced exhibit the way a marine engine tries to wobble when running under similar conditions. Properly balanced, it hangs motionless at all speeds, showing only a little uneasiness when the speed is passing through the natural period of oscillation of the supporting springs. Placed on rollers in the way shown in Fig. 164, page 517, it shows the effect on the tractive force due to the unbalanced parts of a four-cylinder locomotive. In this model the revolving parts are so arranged that their balance is not disturbed by any adjustment that may be made in the reciprocating parts or crank angles. If provided with short connecting rods, it serves also to illustrate the principles of secondary balancing.

Additional examples will be found in the author's "Balancing of Engines," and in particular an example relating to the balancing of the engines of a torpedo-boat destroyer, in which the effect of the valve gear is included. In this case there are twelve lines of parts in the system corresponding respectively to the four main cranks and to the eight eccentric sheaves; there is also the condition that the angles are not all independent, since the angles of the eccentric sheaves are functions of the main crank angles.

146. The Balancing of Two-cylinder Locomotives.—The method described in Section 134, page 483, may be applied to find the revolving balance weights required to equilibrate the revolving masses. The general method, page 495, may be applied to find separately the reciprocating balance weights required to equilibrate the reciprocating masses. An engine with two cylinders requires two reciprocating balance weights to equilibrate the piston masses. In practice, reciprocating balance weights have not yet been applied to a two-cylinder locomotive. Instead, revolving masses are placed in the driving wheel. In this case the components of the forces constraining the circular motion of the balance weights form with the unbalanced reciprocating forces acting on the frame a system in equilibrium in the plane of reciprocation. These revolving balance weights have a vertical component which acts to vary the pressure between the wheels and the rails, and if the magnitude of this vertical component in a wheel is greater than the static weight on the wheel, the wheel will leave the rail for an instant, returning to contact with a blow. This contingency is avoided in practice by adding masses sufficient to produce only partial balance in the plane of reciprocation. In a two-cylinder locomotive of normal proportions it is usual to balance two-thirds only of the reciprocating masses. This minimises the variation of rail pressure, and yet at the same time gives sufficient balance to secure smooth and safe running at high speeds.

The balance weights required for the revolving system are different in magnitude and are differently situated in the wheel to those required to equilibrate the reciprocating masses. The balance weight actually seen in a wheel is the resultant of the weight belonging to the revolving system and the weight belonging to the reciprocating system. There is, however, no necessity to find the component weights separately. The resultant balance weight can be found directly by a single application of the method of Section 143, page 483, to a system of unbalanced masses formed by the unbalanced revolving masses, together with the unbalanced reciprocating masses, including in the system only the proportion of each unbalanced reciprocating mass which is to be balanced.

In the next Section the general method is applied in detail to find the balance weights for an inside single engine.

147. A Standard Set of Reciprocating Parts, Lancashire and Yorkshire Railway. Cylinders, 18 ins. diameter × 26 ins. stroke.

1 piston, 18 ins. diameter	146	lbs.
2 piston rings	13	„
1 piston rod and 1 crosshead	151	„
1 nut	6	„
1 crosshead pin	17½	„
2 side blocks	66	„
Total	399½	„

The connecting rod weighs 444 lbs., and this mass is to be divided between the reciprocating masses and the revolving masses by the method of Section 142, page 494. The position of mass centre is 0·659 the length, measured from the small end, therefore—

$$0{\cdot}659 \times 444 = 292\tfrac{1}{2} \text{ lbs.}$$

is to be included with the revolving masses, the rest 151½ lbs., being included with the reciprocating masses.

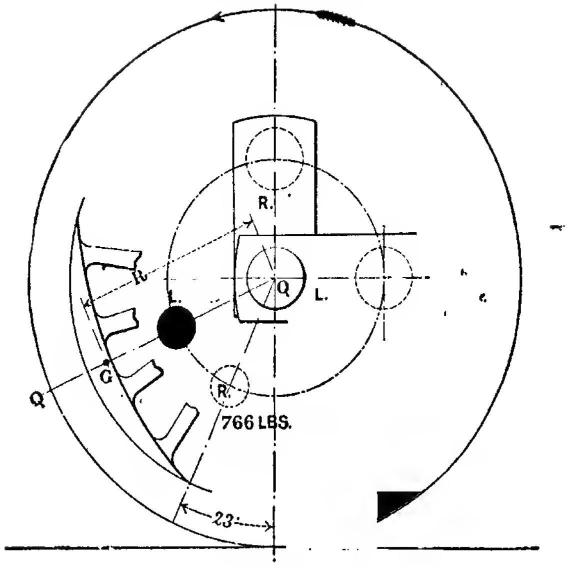
This gives finally—

Mass reciprocated by the connecting rod	399½	lbs.
Proportion of mass of connecting rod	151½	„
Total reciprocating mass per cylinder	551	„

148. The corresponding **Revolving Parts of the Crank Axle** are—

1 pair of crank arms	296 lbs.	at 13 ins.
1 crank journal	56 „	„
Proportion of connecting rod	292½ „	„
Total revolving mass per crank pin	644½ „	„

Fig. 156.



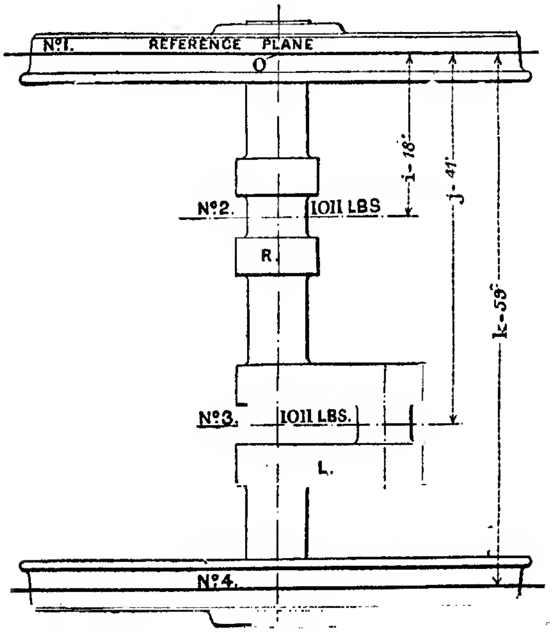


Fig. 157.

Plan and elevation of driving axle. Two-cylinder inside single locomotive.

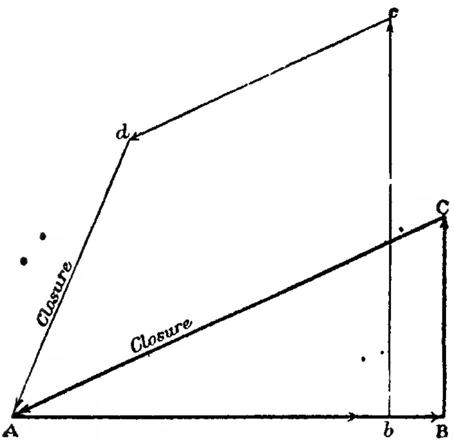


FIG. 158.—Force and couple polygons. Two-cylinder inside single locomotive.

149. **Scales.**—In the following examples the distances from the reference plane are expressed in inches. As a consequence, the mass moments, or couples, are given by numbers involving sometimes five figures. It will be sufficiently exact for all practical purposes if the scale to which the couple polygons are drawn is chosen so that three significant figures can be read, a fourth being estimated, the fifth being considered zero. The scale of the force polygon should allow three significant figures to be read.

150. The Balancing of an Inside Cylinder Single Engine.—

DATA.

Stroke	26 ins.
Distance centre to centre of cylinders	1 ft. 11 ins.
Distance between the planes containing the mass centres of the balance weights	4 „ 11 „
Mass of unbalanced revolving parts per crank pin reduced to 13 ins. radius	644 lbs.
Mass of reciprocating parts per cylinder at crank radius	551 „
Proportion of reciprocating parts to be balanced,	two-thirds
Masses to be balanced are therefore—	
Revolving	644 lbs.
$\frac{2}{3}$ reciprocating	367 „
Total at each crank pin	1011 „

Draw the plan and elevation of the crank axle as shown in Figs. 156 and 157. Notice that in elevation the left-hand driving wheel shows to the front. Choose a reference plane to coincide with

the plane containing the mass centre of the right-hand balance weight, and mark on the drawing the three dimensions i, j, and k. The balance weights are found by the general method. The quantities concerned are shown in Schedule IV.

SCHEDULE IV.

Two-cylinder inside single engine. Reference plane at No. 1 (Fig. 157).

Number of crank.	Distance from reference plane.	Equivalent mass at crank radius, or centrifugal force, when ω^2R = unity.	Equivalent mass moment, or centrifugal couple, when ω^2R = unity.
	Ins.	Lbs.	
No. 1. R.H. balance weight . .	0	766	0
No. 2. R.H. crank	18	1011	18198
No. 3. L.H. crank	41	1011	41451
No. 4. L.H. balance weight . .	59	766	45220

ABC (Fig. 158) is the couple polygon, and the closure CA measures 45,220.

This represents the product of the mass of the left-hand balance weight and its distance from the reference plane, which distance is 59 ins. The mass of the balance weight is therefore 766 lbs. Its angular position in relation to the cranks is at once given by drawing QQ (Fig. 156) parallel to CA (Fig. 158), remembering to draw from the centre of the axle in the direction from C to A. The balance weight is shown in black.

The force polygon is A*bcd* (Fig. 158). Its closure measures 766 lbs., and this is the mass of the balance weight in the right-hand wheel, and its angular position is defined by the direction *d*A (Fig. 158). The balance weight is shown dotted in Fig. 156.

It is unnecessary to take a new reference plane to check the work, since the polygons check one another when the masses at each crank pin are equal, and their planes of revolution and the planes in which the balance weights are placed are symmetrically disposed with reference to the central plane of the engine. Under these conditions the balance weights are equal in magnitude, and their angular positions are symmetrical with respect to the cranks. One balance weight is found from the couple triangle ABC (Fig. 158), the other is therefore known, and the drawing of the force polygon A*bcd* is therefore only necessary to check the accuracy of the work.

The actual mass M_0 of the balance weight depends upon the distance R of its mass centre G from the axis. If r is the crank radius, M_0 is found from—

$$M_0R = M_1r = 766r \text{ for the present example.}$$

Taking $r = 13$ ins. and $R = 36$ ins., which would be about the practicable distance for a 7 ft. 3 ins. wheel—

$$M_0 = 376 \text{ lbs.}$$

This should be arranged in crescent form between the spokes, as shown in Fig. 156.

No difficulty will be found in applying this method to the case of an outside single engine or to coupled engines. In the case of a coupled engine each coupled axle may be treated as a separate balanced system in which is included its proper share of the coupling rod. The driving axle then carries balance weights sufficient to balance the assigned proportion of the reciprocating masses as well as its share of the coupling rod. Another way is to divide the balance weights actually required for the balancing of the reciprocating masses between the coupled wheels either equally or in a stated proportion, and so reduce the hammer blow from the driving wheel. Completely worked examples of this method will be found in the author's "Balancing of Engines".

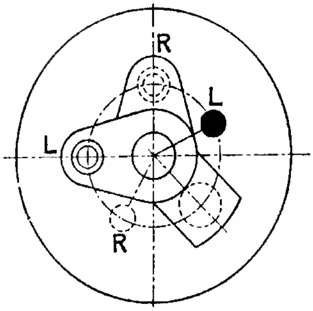
151. The Balancing of Three-cylinder Locomotives.—Although comparatively few three-cylinder locomotives have been built yet it is worth while to consider in detail the balancing of an engine of this type, because a three-cylinder engine offers advantages both in mechanical arrangement and in balancing which may yet determine its more general adoption.

In fact when the crank angles are arranged as in the example immediately to be considered, that is with the outside cranks at right angles and the middle crank bisecting the obtuse angle between them, as in the Smith compound engines running on the Midland railway, the engine may be better balanced than a four-cylinder engine of the type now so much used in which the cranks are placed mutually at right angles. The example relates to the balancing of the driving wheels.

The data assumed for the example are that the engine is of the four-coupled type with outside cylinders, and that in addition a third cylinder is placed centrally inside the frames, the three cylinders driving on to the leading axle. Assume also the following data:—

Proportion of the reciprocating masses to be balanced	$\frac{2}{3}$
Stroke	26 ins.
Distance centre to centre of the outside cylinders	6 ft. 3 "
Distance from centre to centre of the coupling rods	7 " 0 "
Distance between the mass centres of the wheel cranks	5 " $1\frac{3}{4}$ "
Distance between the planes containing the mass centres of the balance weights	4 " 11 "
Inside cylinder placed midway between the outside cylinders.	
Unbalanced mass at each outside crank pin due to a part of the coupling rod and its crank pin, in planes 1, 9	290 lbs.

Fig. 159.



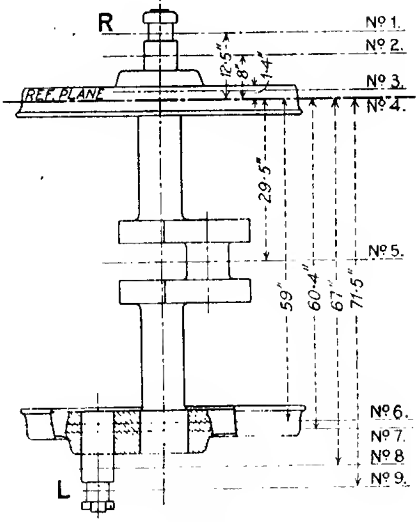


Fig. 160.

Plan and elevation of driving wheel. Three-cylinder locomotive.

Unbalanced mass at each outside crank pin due to $\frac{2}{3}$ of the mass of a set of reciprocating parts (367 lbs.) plus the revolving part of the connecting rod (292 lbs.), and crank-pin journal (21 lbs.) in planes 2 and 8 . . . . 680 lbs.

Unbalanced mass of each wheel crank and the part of the pin in it, in planes 3 and 7 reduced to crank radius . . . . 100 „

Unbalanced mass at central crank pin made up of 644 lbs. due to crank webs and journal, and $\frac{2}{3}$ of the reciprocating masses taken at 400 lbs. 1044 „

Schedule number V. corresponds to these data, and the crank axle is shown in Figs. 159 and 160.

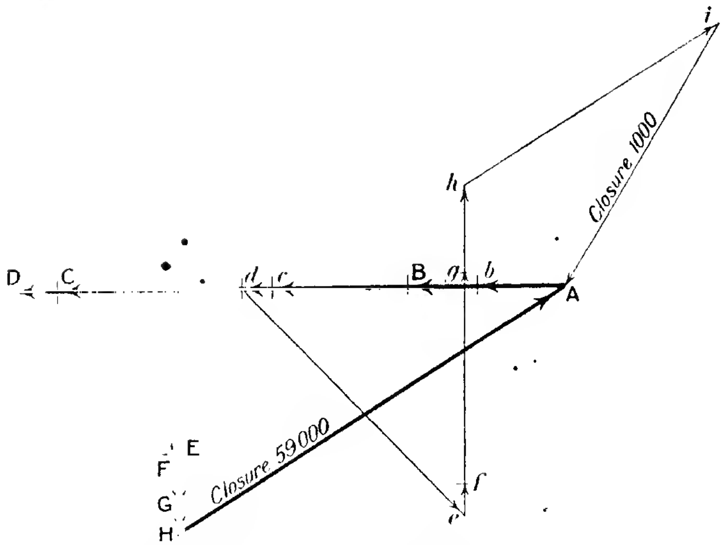


FIG. 161.—Force and couple polygons. Three-cylinder locomotive.

SCHEDULE V.

4-coupled 3-cylinder engine. DRIVING WHEEL. Crank radius 13 ins. Reference plane at No. 4.

Number of crank.	Distance from reference plane.	Equivalent mass at crank radius or centrifugal force, when ω^2R = unity.	Equivalent mass moment, or centrifugal couple, when ω^2R = unity.
No. 1	– 12·5	290	– 3625
No. 2	– 8·0	680	– 5440
No. 3	– 1·4	100	– 140
No. 4	0·0	1000 bal. wt.	0
No. 5	29·5	1044	30798
No. 6	59·0	1000 bal. wt.	59000
No. 7	60·4	100	6040
No. 8	67	680	45560
No. 9	71·5	290	20735

ABCDEFGH (Fig. 161) is the couple polygon, and the closure HA measures 59,000; dividing this by 59 the quotient 1000 gives the balance weight for the left-hand wheel.

A*bcdefghi* is the force polygon. The closure *i*A measures 1000, and this is the magnitude of the balance weight in the right-hand wheel. Since the parts are symmetrically disposed about the centre line the balance weights should be equal in magnitude, and that they

are found to be so from the construction is a check on the accuracy of the work.

Notice that the couple vectors EF, FG, and GH, are drawn oppositely to the corresponding crank directions because the cranks are on the opposite side of the reference plane to the rest of the cranks.

The balance weights for the trailing wheel can be found in a similar way.

The hammer blow due to the part of the revolving balance weight required to balance the reciprocating masses may be reduced if it is divided between the coupled wheel in the way mentioned on page 505.

If the reciprocating masses are proportioned so that the two outside masses are each 0·71 times the middle mass the force polygon in the reciprocating system closes, but the couple polygon does not close. The balance weights are then reduced in magnitude with a corresponding reduction in the maximum magnitude of the hammer blow.

The balance weights are also smaller if the middle crank is balanced by extending the crank webs to form balance weights, but there is no reduction in the hammer blow.

The reciprocating masses of a three-cylinder engine with cranks at 120 degrees also form a system in balance so far as the forces are concerned providing the reciprocating masses are equal, but there remains an unbalanced horizontal couple to be equilibrated by balance weights. The balance weights required are found by the general method, and the case presents no difficulty.

152. The Balancing of Four-cylinder Locomotives.—The type of four-cylinder locomotives which has been generally adopted is that in which the cranks are arranged in two 180 degree pairs at right angles.

In some cases the four cranks are all on one axle, in other cases the two inside cylinders drive on to one axle and two outside cylinders on to the trailing axle, the coupling rod keeping the phase relation between the cranks.

Consider the balancing of the driving wheel of an engine of the type where the four cranks are all on one axle, and let the data be as follows:—

Stroke of each cylinder		26	ins.
Distance centre to centre of the outside cylinder	6 ft.	3	,,
,, ,, ,, ,, inside cylinders .	2 ,,	0	,,
,, ,, ,, ,, coupling rods .	7 ,,	0	,,
Distance between the planes containing the mass centre of the wheel cranks	5 ,,	$1\frac{3}{4}$	,,
Distance between the planes containing the mass centre of the balance weights	4 ,,	11	,,

Unbalanced mass at each outside crank pin due to a part of the coupling rod and its crank pin, in planes 1, 9	290 lbs.
Unbalanced mass at each outside crank pin due to $\frac{2}{3}$ of the mass of a set of reciprocating parts (367 lbs.) plus the revolving part of the connecting rod (292 lbs.), and the crank-pin journal (21 lbs.) in planes 2 and 8	680 ,,
Unbalanced mass of each wheel crank and the part of the pin in it	100 ,,
Unbalanced mass at each inside crank pin . .	1044 ,,

SCHEDULE VI.

4-cylinder engine. DRIVING WHEEL. Cranks arranged in two 180-degree pairs at right angles. Crank radius 13 ins. Reference plane at No. 4.

Number of crank.	Distance from reference plane.	Equivalent mass at crank radius, or centrifugal force, when ω^2R = unity.	Equivalent mass moment, or centrifugal couple, when ω^2R = unity.
No. 1.	– 12·5	290	– 3625
No. 2.	– 8·0	680	– 5440
No. 3.	– 1·4	100	– 140
No. 4.	0·0	678 bal. wt.	0
No. 5.	17·5	1044	18270
No. 6.	41·5	1044	43326
No. 7.	59·0	678 bal. wt.	40000
No. 8.	60·4	100	6040
No. 9.	67·0	680	45560
No. 10.	71·5	290	20735

Fig. 126 shows the diagrammatic elevation of the left side of the driving wheel and the directions of the four cranks. The closure of the couple polygon corresponding to Schedule VI. is 40,000, and this divided by •59 gives a balance weight of 678 lbs. in the left-hand wheel in the direction shown in Fig. 162. The closure of the force polygon is 678 in the direction indicated by dotted lines in Fig. 162. The agreement in the magnitude of the balance weights checks the work.

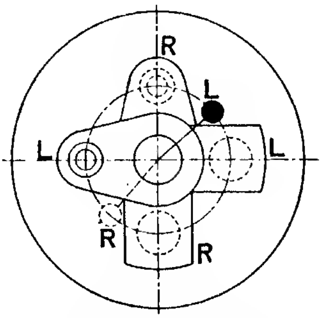


FIG. 162.—Elevation of driving axle. Four-cylinder locomotive.

In the case of a four-cylinder engine in which the inside cylinders are connected to one axle and the outside cylinders to a second axle coupled to the first, the balance weights for each wheel are found by dealing with the revolving system and the reciprocating system separately, and afterwards in

each wheel combining into one resultant weight the balance weight belonging to each system.

The first step is to apply the general method described in Section 143, page 495, to each of the axles separately, and so find the balance weights required to equilibrate the revolving masses belonging to each axle.

The simplest way to find the balance weights for the reciprocating system is to imagine that all the cylinders are connected to one axle on which the cranks are arranged in the same relative angular positions that the actual cranks occupy with regard to one another on the separate axle positions determined and maintained by the coupling rod. The balance weights found may then be concentrated in one wheel, or they may be distributed between the coupled wheels.

The actual balance weight in each wheel is then the resultant of that already found for the revolving system, and the component belonging to the reciprocating system.

153. Completely Balanced Four-cylinder Locomotive.—A locomotive with four cylinders may be completely balanced so that there is no variation of pull on the train due to the motion of the reciprocating masses, no horizontal swaying couple and no hammer blow on the rail. This complete balancing is accomplished by adding balance weights to the wheels to equilibrate the unbalanced revolving masses only, and then by so arranging the magnitudes of the reciprocating masses and the crank angles that the reciprocating masses form a balanced system of themselves. It will easily be understood from a consideration of the force and couple polygons that it is not possible to arrange a reciprocating system in balance unless there are at least four lines of parts except in the special case where the cranks are mutually at 180°, when it may be done with three lines of parts. A little consideration will also show that with four lines of parts a balanced system cannot be formed with cranks arranged in two 180° pairs at right angles, since with these angles it is impossible to close the couple polygon.

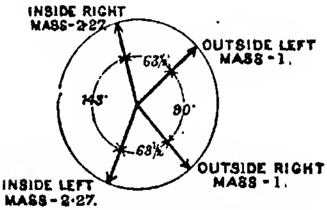


Fig. 163.—End elevation of cranks. Four-cylinder locomotive.

One of the reasons which has led to the adoption of a four-cylinder locomotive with two 180° pairs at right angles is that one valve gear may be used to operate the two valves of the 180° pair. This mechanical simplicity must be sacrificed if the locomotive is to be completely balanced, and a set of valve gear must be fitted to each cylinder.

Fig. 163 shows the crank angles of the driving axle for a completely balanced four-cylinder engine in which the cross pitch of the

cylinders outside is 6·7 ft. and that of the cylinders inside is 2·2 ft. and the angle between the outside cranks to which the coupling rods are connected is 90°. With these angles the reciprocating masses must be adjusted by thickening the pistons so that each inner mass is 2·27 times each outside reciprocating mass. In these circumstances there would be no unbalanced force, no variation of rail pressure, and no swaying couple, except to the extent introduced by the obliquity of the connecting rods, which, in the case of locomotives where the ratio of the rod to the crank is always relatively great, is negligible. Up to the present, no locomotive in this country has been balanced so perfectly as this, and whether the gain in smoothness of running and absence of vibration is worth the extra mechanical complication, or whether an engine with such crank angles would work well on the road, are points which can only properly be decided by a practical trial. It should be noticed that an engine with the crank angles of Fig. 163 has only one dead centre at a time. To obtain a good crank-effort curve the work may be distributed amongst the cylinders by the method described on page 366. The revolving parts must be balanced independently as a separate system.

154. Comparative Schedule.—Schedule VII. has been prepared so that the different types of engine may be compared with one another with regard to their possibilities of balance. In making this comparison it is assumed that the revolving masses are completely balanced, as they always can and always should be, so that they form a system in equilibrium at all speeds. Their motion then disturbs neither the tractive pull nor the rail pressure. Comparing the various types with regard to the balancing of the reciprocating parts three things require consideration:—

(1) The magnitude of the maximum variation in the pressure between the driving wheel and the rail caused by the revolving mass added to balance the reciprocating masses horizontally.

(2) The maximum value of the unbalanced horizontal periodic force in the direction of traction.

(3) The maximum value of the horizontal periodic couple, a couple tending to produce an oscillation of the engine about a vertical axis through its mass centre, and which results in a swaying of the engine from side to side as it travels along the road.

The results given in the schedule for the magnitude of these three quantities are severally obtained by applying the method of Section 143, page 495, to each case, the reference plane being taken at the centre of the engine.

Consider the case of a two-cylinder engine with cranks at right angles, in which the fraction q of the reciprocating mass M per cylinder is balanced, leaving the mass $(1 - q)$M unbalanced. Drawing force and couple polygons, assuming that a mass equal to this

quantity is concentrated at each crank pin, it will be found that the length of the closure of the force polygon is $\sqrt{2}(1-q)M$, and the length of the closure of the couple polygon $\sqrt{2}(1-q)Ma$, where a is the distance from the reference plane to each of the two cylinders; that is, it is equal to half the cross pitch of the cylinders.

The corresponding unbalanced force is $\dfrac{\sqrt{2}(1-q)M\omega^2 r}{g}$ lbs.-wt.,

and this reduces to $$1 \cdot 7(1-q)A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
where A is written for Mn^2r.

Similarly the couple corresponding to the closure of the couple polygon is $\dfrac{\sqrt{2}(1-q)M\omega^2 ra}{g}$ lbs.-wt.,

and this reduces to $$1 \cdot 7(1-q)Aa \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In both these expressions M is the mass of one set of reciprocating parts; n is the number of revolutions of the driving axle per second; r is the crank radius in feet; A is the product Mn^2r; q is the fraction of the reciprocating mass which is balanced; and in (2) a is half the cross pitch of the cylinders in feet.

In order to find the hammer blow it is necessary to calculate separately the magnitude of the balance weight which is placed in the driving wheels to balance the fraction q of the reciprocating masses. The actual balance weight seen is the combination of this weight with the balance weight required to balance the revolving masses. In the case of a two-cylinder engine with cranks at right angles the calculation is easily made, because the couple polygon from which the magnitude of the balance weight is found is a right-angled triangle, such as is shown by ABC, Fig. 158, page 503. Referring to Fig. 157 for the distances represented by the symbols i, j, and k, it is easily shown that if the mass qM is to be balanced the length of the closure in the couple polygon ABC is equal to $qM\sqrt{i^2+j^2}$; consequently the balance weight corresponding to this, which is represented by m, say, is given by

$$m = qM\sqrt{\frac{i^2+j^2}{k}} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equations (1), (2), and (3) apply only to two-cylinder engines with cranks at right angles.

After m has been calculated, the variation of rail pressure which it produces is $\dfrac{m\omega^2 r}{g}$ lbs.-wt.; and this reduces to

$$\text{Maximum variation of rail pressure} = 1 \cdot 22\frac{m}{M} \cdot A \quad . \quad (4)$$

Referring to Schedule VII., page 514, each type is represented by a single-wheel engine. Two conditions of balancing are shown opposite each type, namely, the conditions which result when none of the reciprocating masses are balanced, corresponding to $q = 0$; and the conditions which result when the whole of the reciprocating masses

are balanced by weights placed in the driving wheels, corresponding to $q = 1$. The conditions corresponding to a coupled engine of the same type are deduced directly from the schedule. Let q, as above, be the fraction of the reciprocating mass per cylinder which is balanced by revolving balance weights placed in the driving wheels. Then the hammer blow is found by multiplying the quantities in column 4 by q (q in practice is often $\frac{2}{3}$); the maximum unbalanced periodic force is found by multiplying the quantities in column 5 by $(1 - q)$; and the maximum unbalanced periodic couple is found by multiplying the quantities in column 6 by $(1 - q)$.

If there are c pairs of coupled wheels, and the revolving balance weight required to balance the fraction q of the reciprocating masses is equally divided between the coupled wheels, the hammer blow per wheel is found by multiplying the quantities in column 4 by q and then dividing the product by c. The corresponding unbalanced force and couple are found by multiplying the quantities in columns 5 and 6 by $(1 - q)$ as in the case of a single engine.

Example.—Find the balance weight in the driving wheel required to balance the reciprocating masses; the maximum and the minimum values of the rail load at 60 miles per hour; the speed at which the driving wheel lifts from the rail, in the case of a two-cylinder locomotive having given the following data: $q = \frac{2}{3}$; $M = 551$ lbs. per cylinder; $i = 18$ ins.; $j = 41$ ins.; $k = 59$ ins.; $r = 1$ ft.; diameter of driving wheel, 7 ft.; static load on the driving wheel = 7 tons.

At 60 miles per hour a 7-ft. driving wheel makes 4 revolutions per second. Therefore $A = Mn^2r = 551 \times 16$.

The magnitude m of the mass required to balance two-thirds of 551 lbs. with the dimensions given above, found from equation (3), is

$$m = \frac{\frac{2}{3} \times 551\sqrt{18^2 + 41^2}}{59} = 279 \text{ lbs.}$$

The maximum variation of the rail pressure which this produces is from (4)

$$1{\cdot}22\frac{m}{M}A = \frac{1{\cdot}22 \times 279 \times 551 \times 16}{551} = 5450 \text{ lbs.-wt.} = 2{\cdot}4 \text{ tons}$$

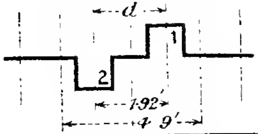
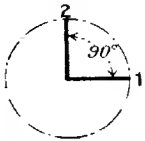
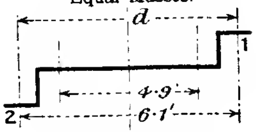
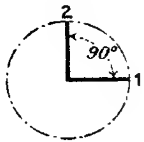
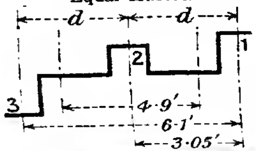
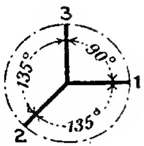
With a static load on the driving wheel of 7 tons (the static load is the load recorded on the weigh-bridge), at 60 miles per hour the maximum load on the rail will be $7 + 2{\cdot}4 = 9{\cdot}4$ tons, and the minimum load will be $7 - 2{\cdot}4 = 4{\cdot}6$ tons.

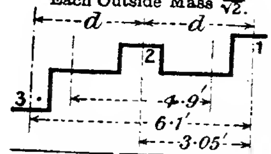
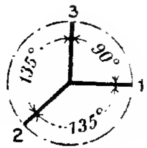
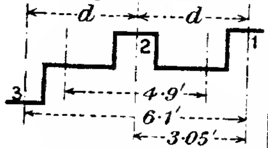
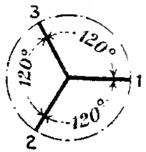
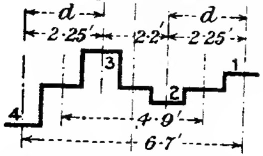
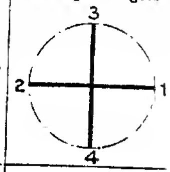
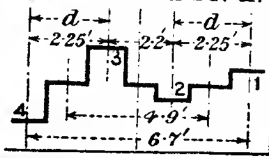
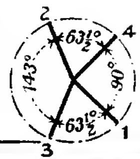
When the variation of rail load calculated from equation (4) is equal to the static load, the maximum load is double the static load and the minimum load is zero. At speeds greater than the speed which produces this condition the wheel lifts away from the rail, and returns to it with a true hammer blow. The speed at which the wheel lifts from the rail is calculated from $\frac{m\omega^2 r}{g} = W$, the static load on the wheel in pounds. The number of the revolutions of the crank axle per second is therefore $n = \sqrt{\frac{W}{1{\cdot}22mr}}$ . . . . . (5)

COMPARATIVE SCHEDULE—No. VII.

$A = Mn^2r.$

M = the mass in pounds of the reciprocating parts per cylinder.
n = the number of revolutions per second of the crank axle.
r = the radius in feet of the crank.
d = the distance in feet between cylinder centre lines.

Col. 1.	Col. 2.	Col. 3.	Col. 4.	Reference Plane at Centre. Col. 5.	Reference Plane at Centre. Col. 6.
Type of Engine.	Crank Angles.	Proportion of reciprocating masses balanced by revolving masses placed in the driving wheels.	Maximum value of the variation of rail load per wheel, i.e., the hammer blow.	Maximum value of the unbalanced periodic force in the line of traction.	Maximum value of the unbalanced periodic couple causing horizontal oscillations.
			Lbs.-wt.	Lbs.-wt.	Ft.-Lbs.
2 Cylinder Inside Single Equal Masses. 		NONE ALL	0 0·93A	1·7A 0	1·65A (0·86Ad) 0
2 Cylinder Outside Single. Equal Masses. 		NONE ALL	0 1·38A	1·7A 0	5·25A (0·86Ad) 0
3 Cylinder Single. Equal Masses. 		NONE ALL	0 1·1A	0·51A 0	5·25A (1·73Ad) 0

3 Cylinder Single Middle Mass M. Each Outside Mass $\frac{M}{\sqrt{2}}$. 		NONE	0	0	3·72A (1·22Ad)
		ALL	0·76A	0	0
3 Cylinder Single. Equal Masses. 		NONE	0	0	6·45A (2·12Ad)
		ALL	1·32A	0	0
4 Cylinder Single. Equal Masses. 	Two 180° Pairs at Right Angles. 	NONE	0	0	3·88A (1·72Ad)
		ALL	0·79A	0	0
4 Cylinder Balanced Single. Each Outside Mass M Each Middle Mass 2·27 M. 		0	0	0	0

155. On the Bending Moment produced on the Crank Shaft by the Unbalanced Masses.—In the preceding sections the balance weights have been applied between the spokes of the wheels, following the usual practice, both for the equilibration of the revolving and the reciprocating masses. Although when balanced in this way the crank axle as a whole is in equilibrium under the action of the system of forces corresponding to the dynamical loading, yet the parts of the axle between the loads are subject to bending moment just as if the axle supported a system of static loads. This bending action on the axle may be avoided if a balance weight is added to equilibrate each unbalanced revolving mass in its plane of motion. This may be done for inside cranks by extending the crank webs and forming the extension into a balance weight, and for outside cranks by merely extending the crank into the spokes on the other side of the boss. If, for example, the revolving masses of a four-crank axle were balanced in this way, there would be six balance weights attached to the axle instead of the two which could be used for the purpose. The six balance weights comprise the extension of the four webs of the inside cranks and the extension of the two bosses of the outside cranks. The total weight of metal added to the axle as balance weights would be considerably greater if six weights were used instead of two, but the axle would be entirely freed from dynamical bending moment, and its life would undoubtedly be considerably prolonged.

The bending moment on the axle due to the crank webs and crank pins of an inside cylinder engine is as great at high speeds as the bending moment produced by the maximum steam pressure at starting. The crank axle suffers severe stresses at starting, due to the steam pressures; these diminish as the speed increases, but if the increase is continued to speeds in the region of 60 and 70 miles per hour the stresses may again become as great as at starting, owing simply to the bending moment produced by the rotation of the crank webs, crank pins, and attached revolving masses.

156. Experimental Apparatus.—It has been tacitly assumed that the balancing of an engine is unaffected by its linear motion along the rails. This fundamental assumption is easily seen to be true because whatever forces act on the engine frame to produce the linear motion of the engine they cannot disturb the equilibrium existing between the moving parts of the machinery which has been produced by the addition of balance weights. If there is much variation in the turning couple the corresponding variation in the tractive force may be sufficiently pronounced at low speeds to be felt in the train—but such oscillation is due to the variable torque and is not due to any disturbance of equilibrium amongst the moving parts caused by the linear motion of the train.

Consider for a moment the motion of any one revolving mass. With the frame fixed, the path of its mass centre is a circle, its

velocity is uniform in the circle, and the force acting to cause the circular motion is $M\omega^2r$, acting always along the radius towards the centre. When the engine is running along the rails with uniform velocity, the path of the mass centre is now trochoidal, the velocity of the mass is variable in the trochoidal path, but the force producing the motion is still $M\omega^2r$ acting along the radius of the mass, since no force acts to produce the uniform linear motion. This point is examined in detail in an article by the author in *Engineering*, June 13, 1902.

The balancing of a locomotive is sometimes tested by hanging it in chains or supporting it on live wheels and then running it light.

Fig. 164 shows a model, designed by the author, of an inside four-coupled engine, by means of which the various problems of locomotive balancing may be studied. It is shown resting on

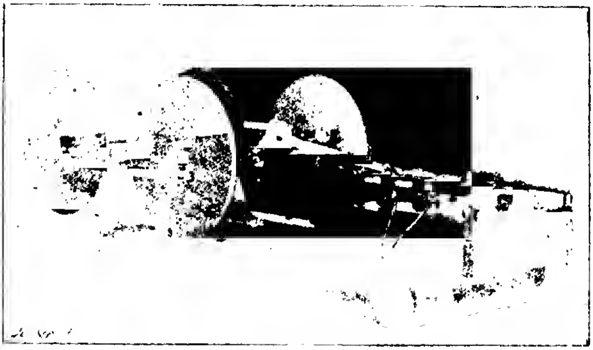


FIG. 164.—Model illustrating the balancing of two-cylinder locomotives.

rollers. Supported in this way, the effect of the unbalanced masses on the tractive force is separated from the other effects. Unbalanced, the model rolls backwards and forwards when the gear is driven. When the proper masses are added to balance the whole of the reciprocating and the revolving parts, the model stands quite still on the rollers at all speeds of rotation. If the model is suspended by three chains after the manner of the apparatus shown in Fig. 144, page 489, the effect of the swaying couple would be seen; if one of these chains is replaced by an elastic link, or a spring, the vertical oscillations indicate the hammer blow.

157. Secondary Balancing. The Yarrow-Schlick-Tweedy Engine.—In the preceding articles and illustrations it has been assumed that the motion of each of the reciprocating masses in a multi-crank engine is simple harmonic, so that the force acting to accelerate the motion of a mass M, reciprocated in the line of stroke by a crank, radius r, is $M\omega^2r \cos \theta$ at the instant when the crank

passes through the angle θ, θ being the angle between the crank and the line of stroke, and ω the angular velocity of the crank.

It has also been shown that the expression

$$F = M\omega^2 r\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)$$

gives a much nearer approximation to the actual force because it allows for the disturbance produced in the simple harmonic motion by the obliquity of the connecting rod. The proof of this equation is given in Section 98, page 338.

This expression may be analysed into two terms, the second being both multiplied and divided by 4, giving

$$M\omega^2 r\cos\theta \text{ and } M(2\omega)^2\frac{r^2}{4l}\cos 2\theta$$

The second term is the projection on the line of stroke of the force required to constrain the mass M to move in a circle whose radius is $\frac{r^2}{4l}$, with an angular velocity 2ω. The mass M may therefore be conceived as concentrated at the crank pin of an imaginary crank $\frac{r^2}{4l}$ long, which is associated with, but revolves twice as fast as the main crank.

The force is in this way separated into a primary element and a secondary element each producing its effect independently of the other.

This conception is illustrated in Fig. 165. Two reference planes Z, Z, are now required, one to which the primary forces are to be transferred, and a second to which the secondary forces are to be transferred. Both planes are, however, coincident.

In a multi-crank engine in which the ratio between the crank and the connecting rod of each line of parts is the same, the balance weights required to equilibrate the effects of the primary forces are found in No. 1 reference plane; and the balance weights required to equilibrate the effects of the set of imaginary cranks revolving at twice the speed of the main crank are found in No. 2 reference plane. When found these balance weights must be separately applied to the engine, mechanism being designed to reciprocate secondary balance weights twice as fast as the speed of the pistons.

In Fig. 165 OX_1 is a line in the primary reference plane from which the crank angles are severally measured by angles a, a, etc.; and $Oabc$ shows a closed polygon for a four-crank engine and OABC the corresponding couple polygon, also closed.

Again, O_1X_2 is a line in the secondary reference plane from which the angles of the imaginary cranks are measured by angles $2a$, $2a_1$, etc.; and O_1stu and O_1STU are respectively the secondary-force and the secondary couple polygons, both shown closed.

In each case the lines OX_1 and OX_2 are fixed in the reference

planes and rotate with them, so that the angle θ measured from a fixed vertical line gives the actual angular displacement of the

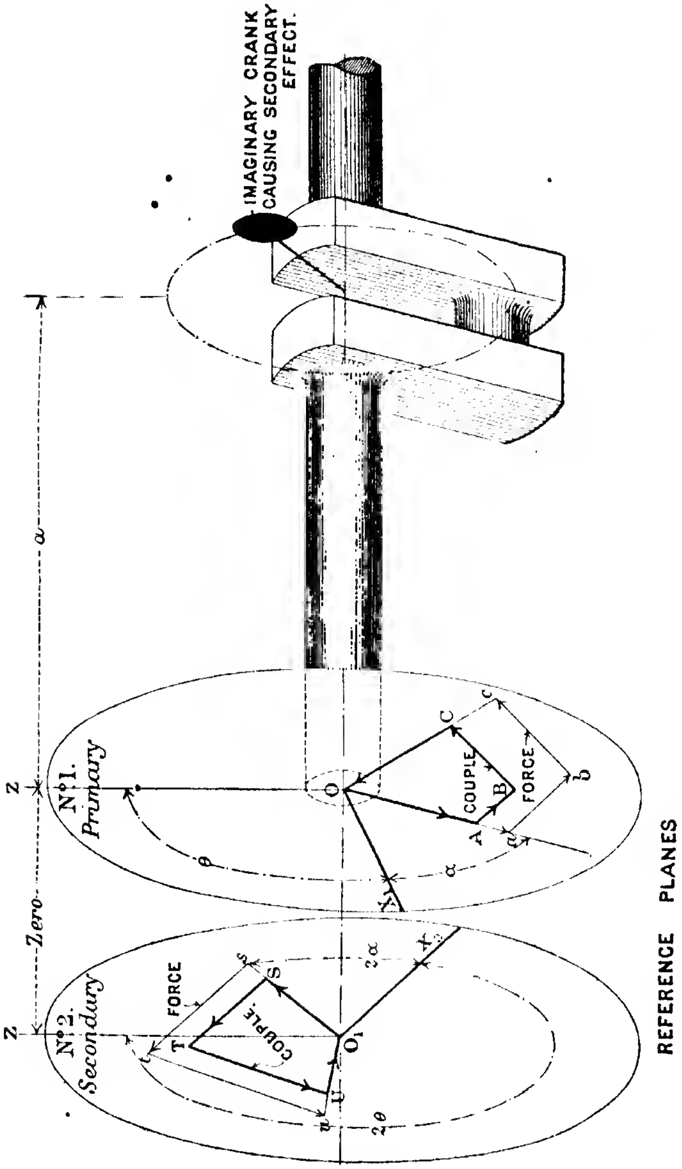


FIG. 165.—Reference planes. Primary and secondary balancing.

primary plane, and the angle 2θ, the corresponding angular displacement of the secondary plane, which plane it will be understood revolves at just double the speed of the primary plane.

Since the four polygons are closed, two in each plane, the engine is balanced both for primary forces and couples and for secondary forces and couples.

It will be seen from this illustration that when the effect of the connecting rod is taken into consideration by the addition of a 2θ term, the conditions of balance are, that the primary force polygon; the primary couple polygon; the secondary force polygon and the secondary couple polygon; shall separately close.

The practical question which emerges from these considerations is, can a multi-crank engine be designed in which these conditions of balance are satisfied by the reciprocating parts amongst themselves, without the necessity of adding either primary balance weights or secondary balance weights, or bob-weights as they would be called, since each set would of necessity be reciprocated, the secondary set twice as fast as the primary set? The answer to this is Yes, providing that there are a sufficient number of cranks. The minimum number of cranks is 5. If the two outer cranks of a group of five are placed parallel with one another, and the two inner pair parallel with one another but at 120° with the outer pair, and then if a central crank is placed at 120° with either pair, masses can be found which satisfy the conditions of primary and secondary balance. If the cranks are equally spaced along the shaft, then the four outer masses must be equal and the mass on the central crank must be twice the mass carried by each of the others. A six-crank engine is immediately obtained from this by splitting the central crank into two cranks and moving them apart along the shaft dividing the mass carried by the single crank between them. Five and six crank engines set in this way satisfy the conditions of balance even when the accelerating force is expanded into terms involving higher harmonics.

Although the geometrical method is easily applied when primary balancing alone is considered, and is a useful method to test the primary and secondary balance of an existing engine, and is valuable also to illustrate the principles involved in the conception of primary and secondary balancing, yet the general relations are better obtained by analysis. The application of vector analysis to these problems of balancing will be found in the author's book on the "Balancing of Engines," chapter 5, and some of the results obtained are quoted below.

The conditions for primary and secondary balance can only be partially satisfied amongst the reciprocating masses themselves in a four-crank engine. All the polygons can be made to close except the secondary couple polygon.

Four cranks symmetrically disposed with regard to the centre line are used in the Yarrow-Schlick-Tweedy balanced engine, a type which has been extensively used in marine practice. The general

disposition of the cranks of a Yarrow-Schlick-Tweedy engine are shown in Fig. 166. The reference plane is placed at the centre of the crank shaft. Let a_1 be the distance of each outer crank from it, and a_2 the distance from it of each inner crank. The end view of the cranks show that they also divide the crank circle symmetrically. Let x_1 and y_1 be the co-ordinates of the end of crank No. 1; and x_2

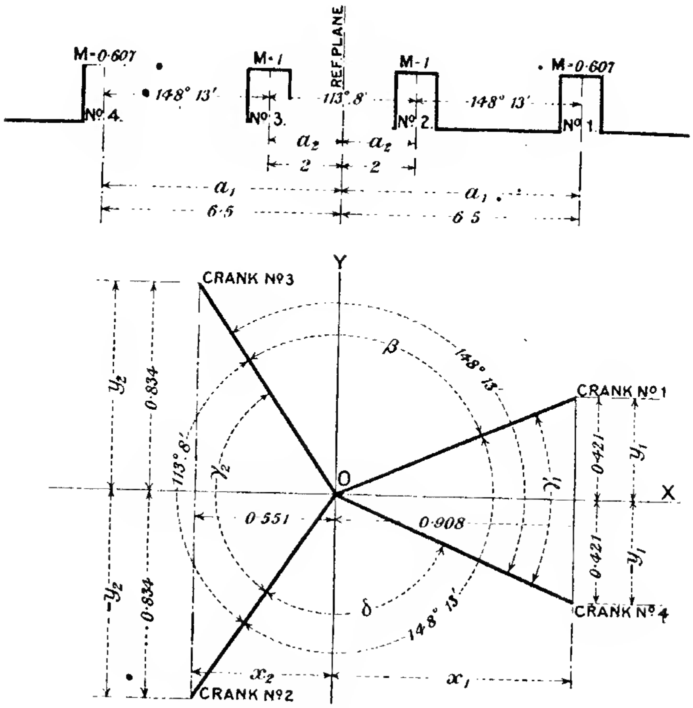


FIG. 166.—Four-crank symmetrical engine.

and y_2 the co-ordinates of crank No. 3; then the co-ordinates of crank No. 2 are x_2 and $-y_2$, and of No. 4, x_1 and $-y_1$.

The relations between these quantities to secure primary and secondary balance of the forces, and primary balance of the couples are:—

$$x_1 x_2 = -\tfrac{1}{2} \quad \dots \quad (1)$$

$$x_1^2 = -P \pm \sqrt{P^2 + Q^2} \quad \dots \quad (2)$$

$$x_1^2 = \frac{M_2}{2M_1} \quad \dots \quad (3)$$

$$y_1^2 = 1 - x_1^2 \quad \dots \quad (4)$$

$$y_2^2 = 1 - x_2^2 \quad \dots \quad (5)$$

$$P = \frac{a_1^2 - a_2^2}{8a_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$Q = \frac{a_1^2}{4a_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Therefore, given the values of a_1 and a_2, the spacing of the cylinder centre lines,

(a) calculate P and Q from (6) and (7)
(b) substitute P and Q in (2) and find x_1
(c) substitute x_1 in (1) and find x_2
(d) substitute x_1 in (4) to find y_1 and substitute x_2 in (5) to find y_2.

All the data are then found for setting out a balanced engine of the symmetrical type. A numerical example may be taken.

Given that $a_1 : a_2 = 6{\cdot}5 : 2$, find the crank angles and the ratio of the inner and the outer piston masses so that the engine may be balanced for primary and secondary forces and for primary couples.

With these data

$$P = \frac{6{\cdot}5^2 - 2^2}{8 \times 2^2} = 1{\cdot}195$$

therefore—

$$P^2 = 1{\cdot}428$$

$$Q^2 = \frac{6{\cdot}5^2}{4 \times 2^2} = 2{\cdot}643$$

$$x_1^2 = -1{\cdot}195 \pm \sqrt{1{\cdot}428 + 2{\cdot}643}$$

$$x_1 = 0{\cdot}908$$

$$x_2 = -\frac{1}{2 \times 0{\cdot}908} = -0{\cdot}551$$

$$y_1 = \pm\sqrt{1 - 0{\cdot}908^2} = \pm 0{\cdot}421$$

$$y_2 = \pm\sqrt{1 - 0{\cdot}551^2} = \pm 0{\cdot}834$$

$$\frac{M_1}{M_2} = \frac{1}{2x_1^2} = 0{\cdot}607$$

Curves are plotted in Fig. 167, showing the various quantities against the ratio $\frac{a_1}{a_2} = b$.

Curves Nos. 1 and 2 give the values of x_1 and x_2 respectively. Curve No. 3 shows the ratio of the inner and outer masses. Curves 4 and 5 give respectively the crank angles and curve No. 6 shows the secondary couple left unbalanced. To find its magnitude in a particular case in which the ratio b is given, measure the value of the ordinate to the scale on the left of the diagram and multiply by $2M_1a_2B$, where $B = \frac{\omega^2 r^2}{gl}$.

For example, when $b = 2$ the ordinate measures 3·2. If $2a_2 = 16$ and $M_1 = 5$ and $B = 1{\cdot}5$, which is the case when $r = 2$ ft., $l = 7$ ft.,

and the revolutions per minute are 88, the maximum magnitude of the unbalanced secondary couple is $1{\cdot}5 \times 5 \times 16 \times 3{\cdot}2 = 384$ ft.-tons.

Equations for the unsymmetrical four-crank engine are complicated and are not quoted. Mr. Inglis[1] has solved them for various relations between the spacing of the cranks along the shaft and has drawn families of curves to express the results.

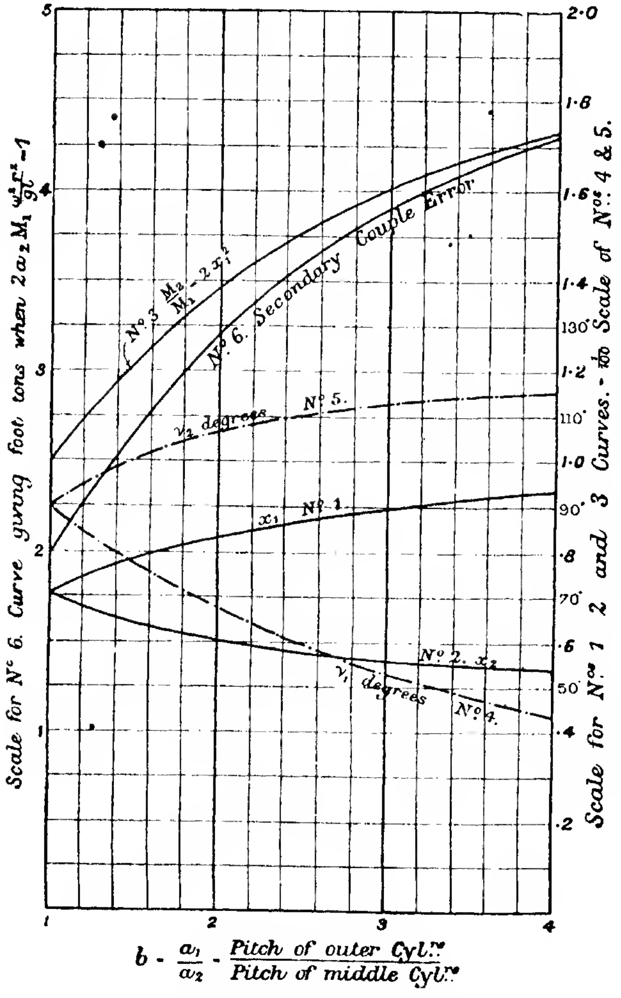


FIG. 167.—Curves for the balancing of a four-crank symmetrical engine.

[1] C. E. Inglis, "The Balancing of the Four-crank Marine Engine," *Trans. Inst. Naval Arch.*, 1911, Part 1.

From these curves data from which to construct an unsymmetrical partially balanced engine are obtained.

These results both for the symmetrical and unsymmetrical engine are obtained, however, without considering the effect of the valve gear. It introduces primary unbalanced forces and couples, and in any practical case must be taken into consideration.

The method which the author has found most convenient and which gives a well-balanced engine is, after fixing the spacing of the cranks along the shaft, first find the angles and the masses for primary and secondary balance in the way illustrated, or read off the quantities involved from a set of curves like those shown in Fig. 167, or for the unsymmetrical engine, like those drawn by Mr. Inglis; and then start afresh, assuming the data found for three cranks, but supposing the fourth crank to be as yet undetermined, and proceed by the general method illustrated in Section 50 of the author's book on the "Balancing of Engines". The final result will be an engine balanced accurately for primary forces and couples including the effect of the valve gear, and very nearly balanced for secondary forces.

The method is equivalent to upsetting the balance first found to the small extent required to give a force and couple equal and opposite to the unbalanced force and couple from the valve gear.

The revolving masses are dealt with separately.

CHAPTER X

VALVES AND VALVE-GEAR MECHANISMS

158. The Distribution of Steam to the Cylinder.—After the stop valve on the steam supply pipe is opened, steam finds its way into the cylinder through an **admission valve**; and after having done work on the piston it escapes through the **exhaust valve** either to the atmosphere or to the condenser. The opening and closing of these valves is done automatically by the valve-gear mechanism. The fractions of the stroke at which these valves open or close can be measured from an indicator diagram. The opening or closing of a valve constitutes an **event** in the pressure-volume cycle through which the steam passes in the cylinder, and an event is shown on the indicator diagram by a change in the general direction of the curve.

Admission and **cut-off** are the events of the cycle determined by the admission valve; and **release** and **compression** are the events determined by the exhaust valve.

A double-acting engine requires an admission valve and an exhaust valve at each end of the cylinder, so that in general there are four valves concerned in the distribution of steam to a double-acting engine.

The mechanism which operates the distributing valves is usually arranged to open and close the release valves at fixed fractions of the stroke, and to open the admission valve at a fixed fraction of the stroke, but its closing is accelerated or delayed by the action of a governor so that the quantity of steam admitted to the cylinder is adjusted from stroke to stroke according to the load on the engine. In many cases the steam supply is cut off at a fixed fraction of the stroke and then the steam supply is controlled by a throttling governor.

One form of mechanism in which governing is done by varying the cut-off, a form which was introduced by Corliss in 1849, is shown in Fig. 168. S, S_1 are admission valves, and E, E_1 are exhaust valves. There are two circular plates one behind the other free to oscillate about a fixed axis A. Each plate is connected by an eccentric rod to an eccentric sheave on the crank shaft, and receives therefrom an angular oscillation. The front disc is connected to its eccentric rod at Z, and to the exhaust valves by the rods BD and B_1D_1. By these means the exhaust valves receive angular oscillations from the discs,

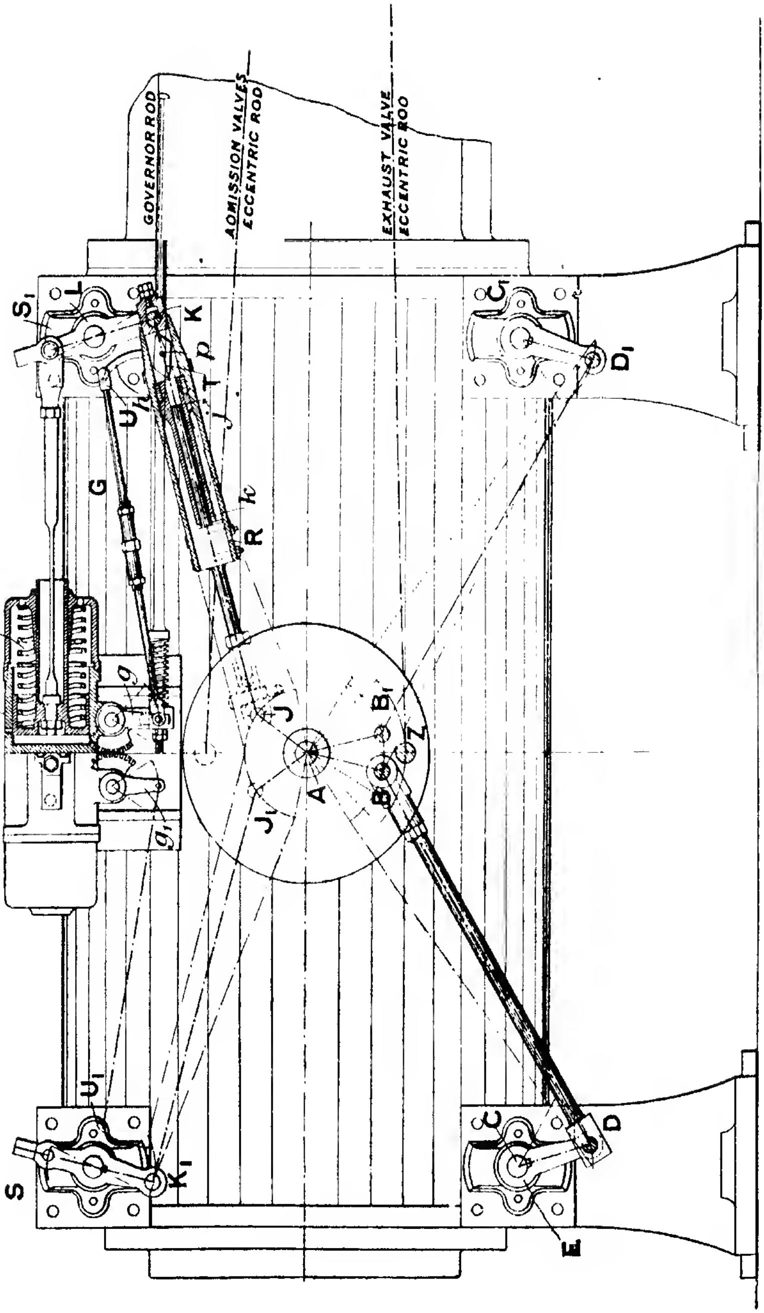


FIG. 168.—Corliss valve gear.

which open and close them at the specified points of the respective cycles to which they belong.

The admission valves are connected to the back plate by the rods KJ and K_1J_1, but these rods each contain a trigger T, which when turned about the pin p forces apart two springs which hold together the two parts of which the coupling rod is constructed. Immediately the springs are opened, the upper part of the rod is free, and the valve is closed by the spring Q acting through the link connecting it to the valve. The spring acts through a dashpot P in order to prevent shock. The trigger is coupled at U to a joint g by the rod G. The motion of the rod KJ swings the trigger about the joint U, and it is the position of this joint which determines at what fraction of the stroke the springs are released and the valve is closed. The position of this joint is determined by the position of the joint g, and this is coupled through a short shaft to the governor rod. The disengaged parts of the rod, JR and Kk, automatically re-engage as the lower part swings back again to the right, and then as it swings to the left again the complete rod opens the valve at a definite fraction of the stroke, and continues to open it wider and wider until the trigger comes into action again and breaks the connection, and so leaving the valve free to close.

Valves of this kind are termed rocking valves, and a section of a steam valve at right angles to the axis of oscillation is shown in Fig. 169. The valve engages with the valve spindle by slots cut across the ends, but it is free to find its seat on the curved port surface under the action of the steam pressure.

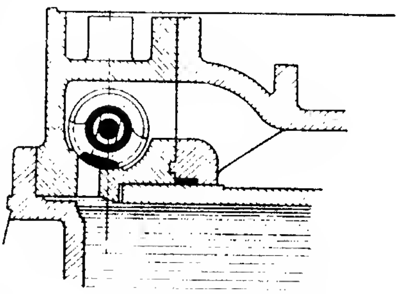


FIG. 169.—Section of Corliss valve.

A distributing valve of another kind is shown in Fig. 170. It is of the equilibrium type, and has four seatings, and is specially designed to work with moderately superheated steam. Valves of this type are fitted to many of the Sulzer engines, and the drawing serves to indicate another type of trigger mechanism. The valve itself is lifted off its seat against a spring contained in a dashpot, by the downward movement of the rod R acting through the lever A. A trigger T is carried on the joint P and this trigger is separately connected to the rod Q, which is actuated by the governor through simple mechanism. The downward movement of the rod R continues to lift the valve until a point is reached where the trigger, turning about the end of the rod Q, which is motionless unless the governor sleeve is moving, breaks the connection between the trigger and the arm A. The spring then closes the valve, and at the same time forces the air beneath the piston in the dashpot through a small adjustable

orifice. By suitably adjusting the size of this orifice the valve may be forced back into its seating almost without a sound. The load

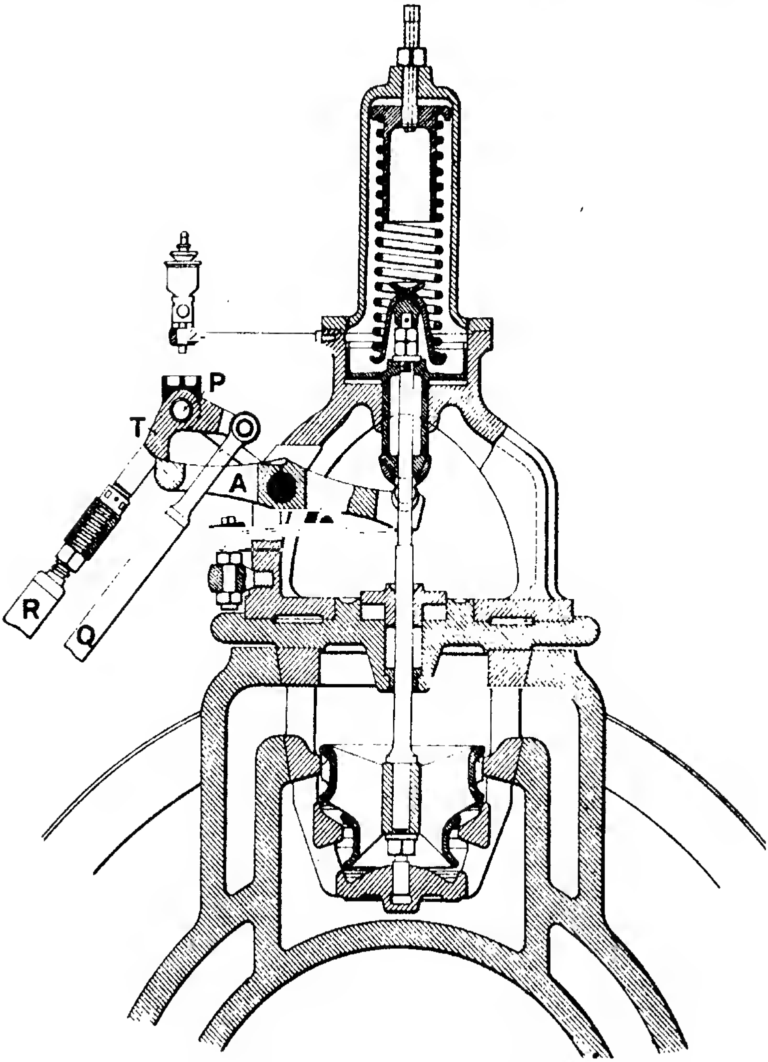


FIG. 170.—Four-seated valve and trigger. Sulzer engine.

against which the valve is lifted is found by multiplying the steam pressure by the projected area of the four seatings. The rod R is connected to an eccentric sheave on a lay shaft running parallel to

the engine and driven by bevel gearing from the crank shaft. The exhaust valves are similarly constructed and similarly operated from the lay shaft, but there is no trigger mechanism in the connections, so that the events of release and compression occur at fixed fractions of the stroke.

Reference has already been made on pages 290, 302, to the Unaflow Cylinder, and some particulars have been given of the performance of engines fitted with cylinders of this type. A longitudinal section and a sectional end view of a Unaflow Cylinder are shown in Fig. 171. There are two admission valves, A and A_1, built respectively into each of the cylinder covers, and exhaust valves are dispensed with. The steam escapes from the cylinder through an exhaust port E at the centre of the barrel, and this port is closed and opened by the piston itself. The ports PP_1, for the admission of steam, are formed in the cylinder covers, and the clearance can, therefore, be reduced to the smallest possible amount. The effect of small clearance

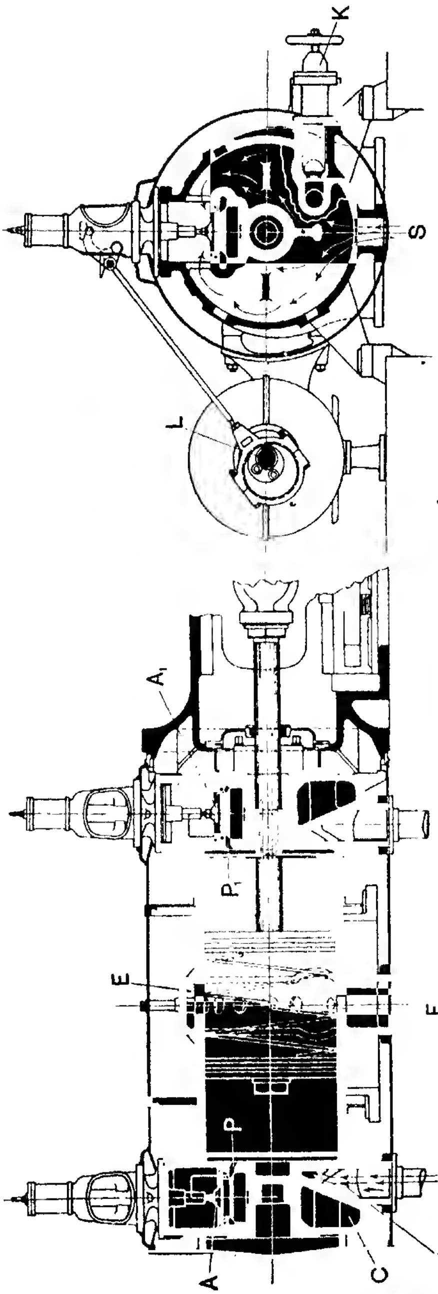


FIG. 171.—Sulzer Unaflow cylinder.

on the cylinder size has been discussed in Section 77, page 289. Each cylinder cover is a complex casting, and besides the admission valve and passages leading thereto it contains a steam jacket J, and a supplementary jacket space C in addition. The steam entering the engine flows through a jacket on its way to an admission valve entering respectively at S and at S_1. Each steam jacket J, J_1 may, in fact, be regarded as an enlargement of the steam pipe. The second chamber C may be regarded as a second jacket, which jackets the steam jacket with air when the engine is working normally as a condensing engine, but this second jacket C is required primarily for another purpose. Compression begins soon after the piston begins the return stroke. Consequently, if the clearance is small, the pressure at the end of compression is a large multiple of the pressure at the beginning. This leads to no difficulty when the engine is working condensing, and the pressure at the beginning of compression is merely a few pounds per square inch, but when the engine is starting with no vacuum, and the pressure at the commencement of compression is about 15 pounds per square inch, the final pressure becomes impossibly high. An inordinate pressure at the end of compression is prevented by increasing the volume of the clearance. The primary function of the jacket C is to provide the additional clearance necessary. This space may be connected to the cylinder through the valve K, and in this way the normally small clearance in the cylinder itself may, at will, be increased by the volume C. The valve K, and a second valve like it at the right end of the cylinder, are opened at starting, and they are kept open until the full vacuum is obtained, and then they are closed, and normal working with small clearance is established. The drawing from which the diagram was prepared was kindly supplied to the author by Messrs. Sulzer Brothers. The drop valves are worked from a lay shaft L. Oil is forced into the cylinder at the centre where the temperature is relatively low, and flows right and left along the barrel. This particular type of engine is especially designed for working with highly superheated steam, and the economy achieved is remarkable, as will be seen from the results exhibited by the curves in Fig. 87, page 302.

159. The Slide Valve.—Steam may be distributed to a double acting engine by a single valve sliding across ports formed in the steam chest to which the main stop valve admits the steam. A diagrammatic view of a cylinder designed for slide valve distribution, together with the slide valve, is shown in Figs. 172 and 173. S_2, S_1 are steam passages; EE_1 is the exhaust passage shown curving round the cylinder for diagrammatic purposes only; H is the steam chest cover; B is the valve buckle or valve spindle, turned at D, and coned and slotted at the end for connection to the valve rod, or the eccentric rod; V is the slide valve. These parts are lifted above their real position, but are shown assembled in the cross-section, Fig. 173.

When the valve is placed centrally over the ports, as in Fig. 173, its edges overlap the ports giving access to the cylinder, and steam can neither flow into nor escape from the cylinder. A slight

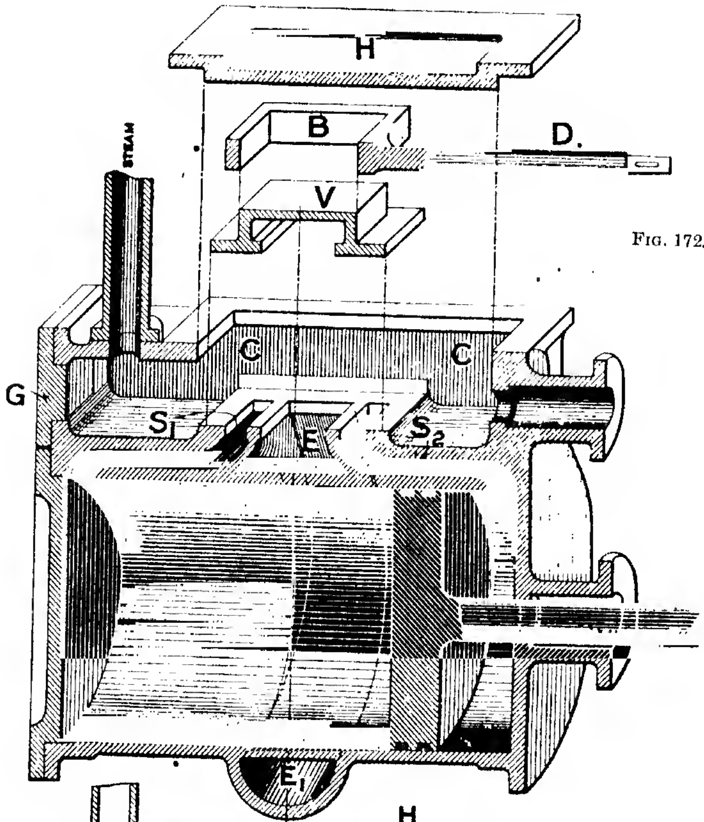


FIG. 172.

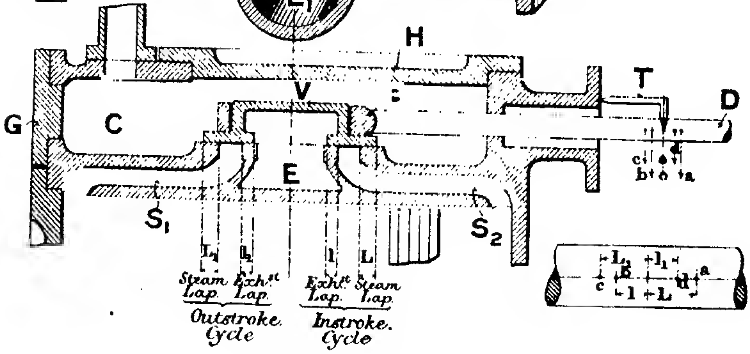


FIG. 173.—The slide valve.

displacement of the valve to the right places the left end of the cylinder into communication with the steam chest, and the right end in communication with the exhaust pipe through the inside of the valve. A movement to the left of the central position reverses these connections. The reciprocation of the valve across the ports thus admits and releases steam alternately at each end of the cylinder.

When the valve is in its central position over the ports, the amount of overlap on the admission side of a port is called the **steam lap,** and the amount of overlap on the exhaust side is called the **exhaust lap.** These quantities are indicated by L and l for the right channel S_2 and by L_1 and l_1 for the left channel S_1.

In some cases the valve in its mid-position does not overlap the steam ports, and the ports may even be open. The width of the port opening is in these circumstances called **negative lap.** Negative lap on the steam side rarely occurs; on the exhaust side it is more common. For example, it is often found in the case of locomotives intended for high speeds, and it is arranged for in order to prolong the period between release and compression.

The positions of the slide valve which determine respectively admission, cut-off, release, and compression, for one pressure-volume cycle are called the **critical positions** of the valve for that cycle. Considering the cycle at the left end of the cylinder it is easily seen that the slide valve is either just opening or just closing the port to steam when the valve is at a distance from the central position equal to the steam lap. And that it is just opening or just closing the communication between the cylinder and the exhaust pipe when it is at a distance from its central position equal to the exhaust lap. The direction in which the valve is moving discriminates between admission and cut-off, and again between release and compression. There are therefore two critical positions of the valve during a cycle, namely, the position where the valve is at a distance from its central position equal to the steam lap, and the position where it is at a distance equal to the exhaust lap.

But there are two cycles in a double acting engine so that there are altogether four critical positions of the valve, two for each cycle.

The cycles may be distinguished from one another by naming them the **instroke** and the **outstroke cycles** respectively. The action of the steam in the outstroke cycle is to push the piston rod out of the cylinder, and the pressure-volume changes of the cycle therefore take place at the end of the cylinder remote from the crosshead. The opposite is the case for the instroke cycle.

Steam can be admitted to the cylinder from the inside of the valve and released to the outside. In this case the pipe E becomes the steam pipe and the pipe S becomes the exhaust pipe. The steam pressure now acts to lift the valve off its seat, and provision must be made to prevent this. A piston valve is generally used where steam is admitted on the inside of the valve. The steam lap is now placed on the inside of the valve, and the exhaust lap on the outside.

160. The Single Eccentric Valve Gear.—The simplest way to produce reciprocating motion of the slide valve is to connect it by an eccentric rod to an eccentric sheave keyed to the crank shaft. An eccentric sheave and an eccentric rod are equivalent to a crank and a connecting rod, but the eccentric sheave is employed instead of a crank when the stroke is short. It has the advantage that it can be put on the crank shaft in two halves. An eccentric sheave and eccentric rod are shown in Fig. 174. E is the eccentric sheave, surrounded by the eccentric strap S, which is bolted to the eccentric rod R. The sheave is grooved with the strap as shown in cross-section to prevent lateral motion. The valve spindle is coupled to the rod at the joint P. The stroke of the point P is twice the crank radius OK. This radius is generally referred to as the **eccentric radius** or the **eccentricity** of the sheave.

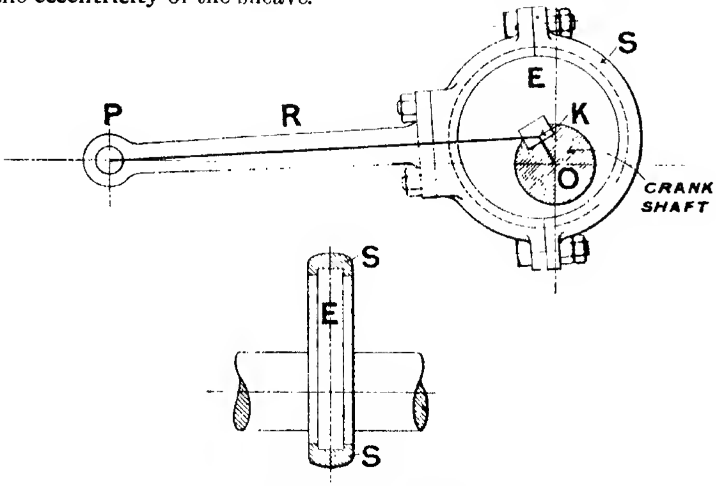


FIG. 174.—Single eccentric gear.

Let a slide valve V, Fig 175, be connected to a single eccentric sheave whose radius is r, by the eccentric rod EK. Let ϕ be the angle between the eccentric radius and the crank. This angle is called the **angular advance** of the eccentric sheave. The direction of rotation is in the direction in which this angle is measured from the crank.

We now proceed to investigate the distribution of steam which can be produced by the reciprocation of a slide valve.

The first step in the investigation is to plot two curves, one showing the displacement of the valve from its central position, the other showing the displacement of the piston from its central position, both curves being plotted on a common crank-angle base.

Points on the piston displacement curve are found thus :—

From a centre on the line of stroke, Fig. 175, and with a radius equal to the length of the connecting rod AL, draw an arc through

the crank pin A to cut the line of stroke in a. Then, $Oa = X$ is the displacement of the piston from its central position when the crank angle is θ.

Again, from a centre on the line of stroke of the valve and with a radius equal to the length of the eccentric rod, draw an arc of a circle through the point K to cut the line of stroke of the valve in the point k. Then $Ok = x$ is the corresponding displacement of the valve from its central position. If it should happen that x is numerically equal to the steam lap, then X would be the corresponding position of the piston at cut-off.

A repetition of this construction for a number of equidistant crank positions enable a sufficient number of points to be found through which the **valve displacement curve** and the corresponding **piston displacement curve** can be drawn. A diagram obtained in this way is shown in Fig. 176. TT is the crank angle base set out to any convenient scale. Usually the

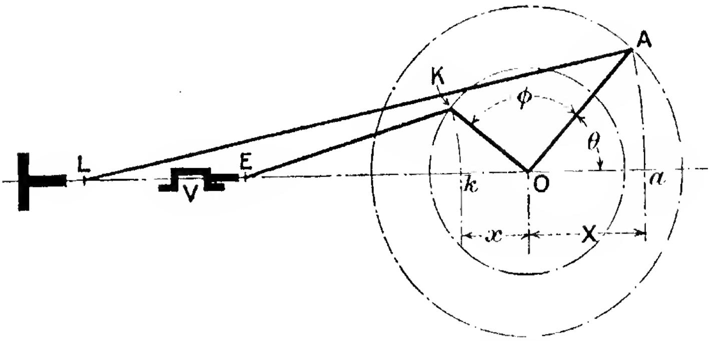


FIG. 175.—Construction to give piston and valve displacement.

eccentric rod is so long relatively to the eccentric radius that its obliquity may be neglected and the position of the points on the valve displacement curve are then projected vertically from the position of the end of the eccentric radius.

If now any horizontal as XX is drawn across the diagram, the intercepts on this line between the curves and the axis TT are simultaneous values of the displacements of the valve and the piston when the crank is at the angle corresponding to the position of the horizontal XX.

Draw LL and EE parallel to the axis TT and at distances therefrom respectively equal to the steam lap and the exhaust lap. The intersection of these lines with the valve displacement curve in the points c, r, k, and a, identify the points on the curve corresponding to the critical positions of the valve for the instroke cycle. Horizontals through these points cut the piston displacement curve in points which fix the corresponding displacements of the pistons; and verticals through these points on the piston curve to cut the

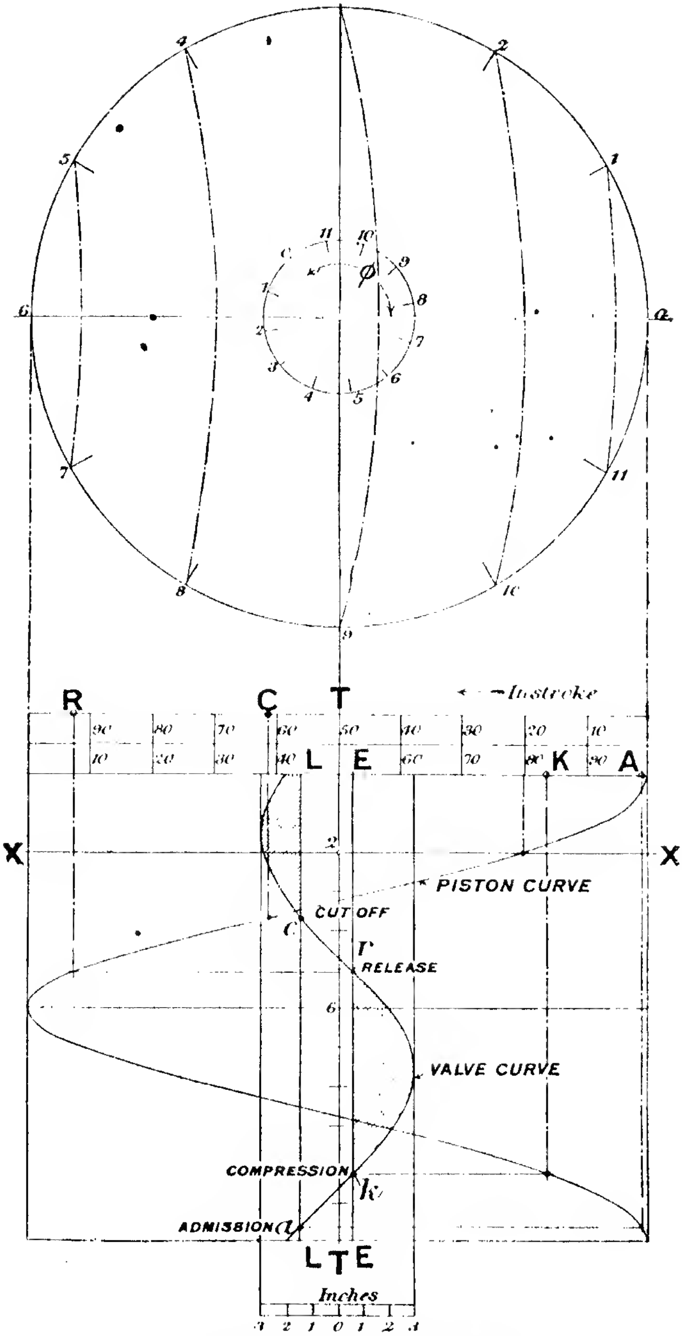


FIG. 176.—The rectangular valve diagram.

percentage scale constructed at the top of the diagram, in A, C, R, and K, show these piston displacements as a percentage of the stroke, the way in which the piston displacement is usually expressed. At the point *a* the valve is just opening and admission is just beginning. At the point *c* the valve is just closing so that this marks the event of cut-off. At the point *r* the valve is just opening on the inside so that this corresponds to the event of release; and at *k* the valve is just closing on the inside so that the event of compression is just beginning. The whole distribution during the pressure-volume changes of the instroke cycle are thus clearly exhibited by the diagram.

For an assigned piston position the opening of the port for steam is the corresponding valve displacement minus the steam lap, so that the cross-hatched area to the left of LL shows by its horizontal width the gradual opening and closing of the steam port during the admission period. Similarly the gradual opening and closing of the exhaust port is shown by the width of the dotted cross-hatched area during the exhaust period. The width of the opening of the port for steam when the piston is on a dead point is called the **lead of the valve** and this is given in the diagram by the intercept between the valve curve and the lap line LL on the horizontal line through the crank angle 0.

The events of the outstroke cycle are found in a similar way; but now the lap line LL must be drawn to the right of the axis TT and the exhaust lap line EE to the left. These distances are not necessarily equal in the respective cycles.

If there is no exhaust lap then the line EE is in both cycles identical with the axis TT.

If there is negative exhaust lap then the line EE is drawn on the same side of the axis as the corresponding steam lap line.

This method of exhibiting the distribution of steam was taught by Professor Reynolds. It may be described as the **rectangular valve diagram** because the displacement curves are set out by rectangular co-ordinates.

The distribution of steam may also be exhibited in a diagram obtained by plotting the valve displacement against the piston displacement, and then adding lap lines. With these co-ordinates the valve displacement is represented by a closed curve, hence the name **oval diagram,** by which it is usually called.

It may be noted here that if steam is admitted to the cylinder from the inside of the valve, the angular advance must be increased by 180° beyond what its value would be for a corresponding distribution with outside steam admission. This increase brings the eccentric radius in the fourth quadrant so that it appears to lag behind the crank. Strictly the eccentric radius is always to be thought of as in advance of the crank, so that the angular advance may have any value measured out from and in advance of the crank. The interposition of a rocking shaft between the end of the eccentric rod and the valve spindle may again alter the phase 180°, and so bring the angular advance for inside admission back to the second quadrant.

161. Example. Rectangular Valve Diagram.—It is clearly unnecessary to draw the piston displacement curve full size, since only the percentage position of the piston is required; but it is necessary to draw the valve displacement curve full size, because the displacements are relatively small. It is therefore usual to choose the piston scale so that the percentage scale is reduced to a convenient length.

A complete valve diagram, set out with suitable scales, is shown in Fig. 177 (p. 538). The data are:—

Ratio of length of the connecting rod to the crank, 5·5 to 1.

Eccentric radius, 1·5 ins.

Angular advance, 130 degrees.

Steam lap, 0·8 in. for both cycles.

Exhaust lap, 0·28 in. for both cycles.

In this diagram the percentage scale for the outstroke cycle is placed at the bottom to avoid confusion.

LL, EE, are the respective steam and exhaust lap lines for the instroke cycle; L_1L_1 and E_1E_1, those for the outstroke cycle.

The distribution points for the instroke cycle are c, r, k, a; those for the outstroke cycle are c_1, r_1, k_1, and a_1.

These points, referred first to the piston displacement curve and then to the percentage scales, give A, C, R, K, for the fractions of the stroke at which the events occur in the instroke cycle; and A_1, C_1, R_1, K_1, for the fractions of the stroke at which the events occur in the outstroke cycle.

Reading from these scales and from the diagram it will be found that the complete distribution is as follows:—

Instroke Cycle:

Admission begins at 99½ per cent. of the return stroke.
Cut-off takes place at 61 per cent. of the stroke.
Release takes place at 92½ per cent. of the stroke.
Compression begins at 84½ per cent. of the return stroke.
The maximum port opening for steam is 0·7 in.
The maximum port opening for exhaust is 1·22 ins.
The lead is 0·17 in.

Outstroke Cycle:

Admission, 99½ per cent. of the return stroke.
Cut-off, 69½ per cent. of the stroke.
Release, 94½ per cent. of the stroke.
Compression, 79 per cent. of the return stroke.
Maximum port opening for steam, 0·7 in.
Maximum port opening for exhaust, 1·22 ins.
Lead, 0·17 in.

The velocity of the valve is represented at any point by the slope of the displacement curve. At q, on the curve, the ratio $\frac{qx_1}{x_1x_2}$ suitably interpreted as regards scales is the velocity at the crank angle x_1. A velocity curve may be constructed by measuring the slope at a sufficient number of points. Again the slope of this velocity curve

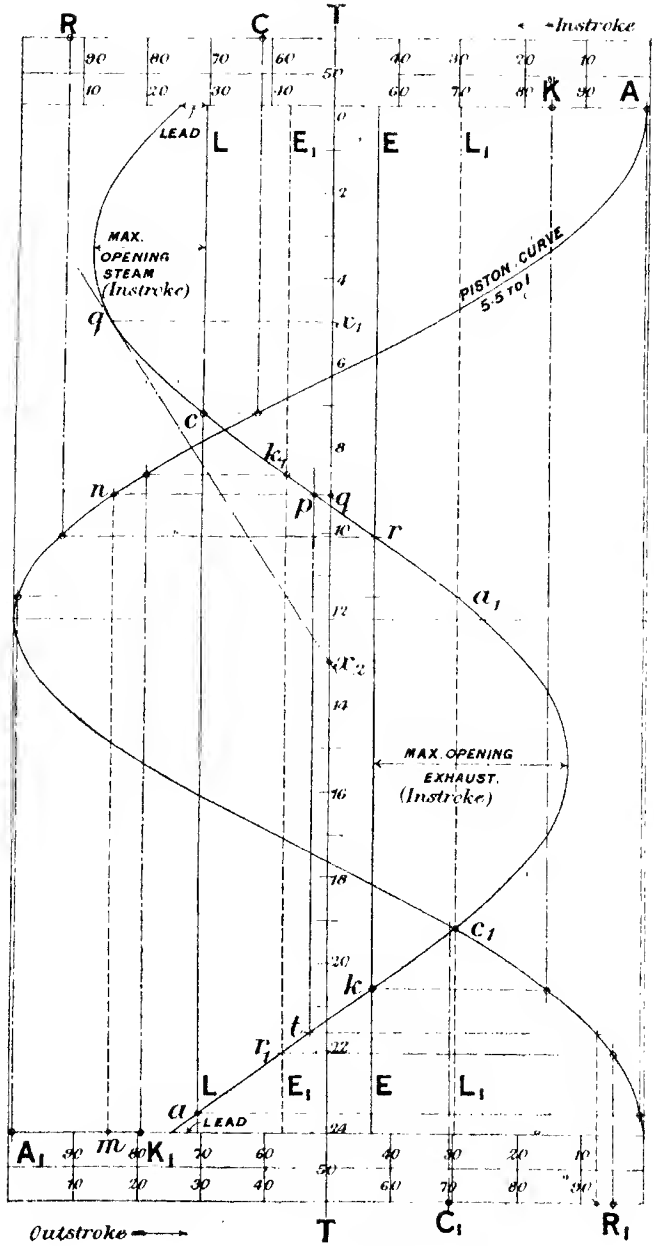


Fig. 177.—The rectangular valve diagram.

at any point gives the acceleration there, suitably interpreted as regards scales, and if the mass of the valve and of the reciprocating parts connected with it is known, then the force required to accelerate the valve at the dead points and at any other point in its stroke can be calculated by methods already illustrated in connection with the piston masses, pages 338 *et seq.* Usually the calculation can be made assuming the motion of the valve to be simple harmonic.

Comparing the two cycles it will be seen that corresponding events do not occur at quite the same fractions of the stroke. There is 7 per cent. difference at cut-off; 2 per cent. difference at release; and $5\frac{1}{2}$ per cent. difference at compression. These inequalities are chiefly due to the obliquity of the connecting rod, and would not exist if the piston were reciprocated by a rod, long in relation to the crank.

These inequalities may be adjusted to some extent by giving slightly different laps to the valve. For example, take the point m on the lower scale at $84\frac{1}{2}$, the percentage at which compression occurs in the instroke cycle; project m vertically to n, and then project n horizontally to p. If the exhaust lap is made equal to qp on the right inner side of the valve, the events of compression will be equalized, and as will be seen from the diagram, where t marks the new intersection of the lap line which determines release, the events of release are brought near to equality also.

The events of cut-off can be brought nearer to equality in a similar way, but in this case the movement of a steam lap line in the diagram reduces the lead, and it will be found that if the events of cut-off are equalized at 61 per cent. the lap must be increased to nearly 1 in. and this increase reduces the lead in the outstroke cycle to nothing. All that can be done by this method of unequal laps is to reduce the inequality of the events of cut-off to as great an extent as the maintenance of adequate lead will allow.

Another way of correcting these inequalities is to introduce a rocking lever between the valve rod and the valve spindle. The angle between the arms of the rocking lever can be chosen so that the valve displacement curve has the proper shape to determine equality between a pair of events without the necessity of introducing inequality in the laps.

162. The Zeuner Diagram.—This is the polar form of the rectangular valve diagram. The valve displacement curve takes the form of a figure 8 formed round an axis inclined to the horizontal at an angle equal to the angular advance, ϕ. Points on this curve are found by plotting the valve displacement radially along a radius vector which may be regarded as the engine crank. The convenience of the diagram lies in the fact that if the obliquity of the eccentric rod is neglected, the figure of 8 becomes two circles touching at the origin, the diameter of each circle being equal to the radius of the eccentric sheave, as shown in Fig. 178, where x is the displacement of the valve corresponding to the crank angle θ.

The proof of this is simple. Neglecting the obliquity of the eccentric rod the valve displacement, x, is given by

$$x = r \cos (\theta + \phi)$$

Produce the direction of the crank arm through the centre O, when it will be seen that the angle $(\theta + \phi)$ is equal to the angle AOC. The cosine of this angle is $\frac{x}{r}$, since the angle in the semicircle is always a right angle, so that $x = r \cos (\theta + \phi)$. This is true for all positions of the crank, the upper circle cutting off x for all positions of the crank above the common tangent at O, the lower circle serving to cut off x for all positions below the common tangent.

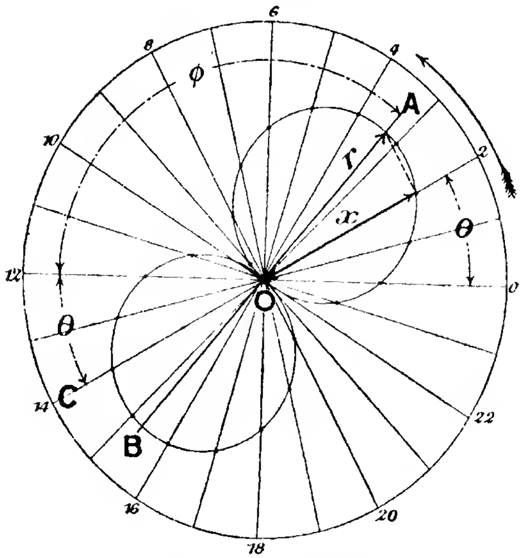


FIG. 178.—Polar valve displacement curves.

The position of the crank for the various events can be found by fixing the points on the circles where x is equal to the respective steam and exhaust laps. This is most conveniently done by drawing arcs from the centre O, with radii equal to the steam and exhaust laps respectively, Fig. 179 cutting the circles in the points

$$a, c, k, r, a_1, c_1, k_1, r_1$$

The several radii drawn from O through these points fix the angular positions of the crank corresponding to the eight events of the two cycles. Consider, for instance, the position of the crank Oc; the displacement x is given by Oc. But this is equal to the steam lap, and the displacement is decreasing, therefore cut-off is just taking place. Therefore Oc is the crank position for " cut-off".

The port opening for steam on the instroke cycle is the intercept on the crank arm between the displacement curve and the arc *ac*. The port opening for exhaust is the intercept on the crank arm between the displacement cycle and the arc *rk*. The outstroke cycle can be similarly treated. The leads for the two cycles are the respective intercepts when the crank is in the 0° and the 180° positions.

The corresponding piston positions are projected from the crank-pin positions, by the curved arcs shown in dotted lines, on to the diameter of the crank-pin circle which is taken to represent the stroke.

The ease with which this diagram can be constructed has led to

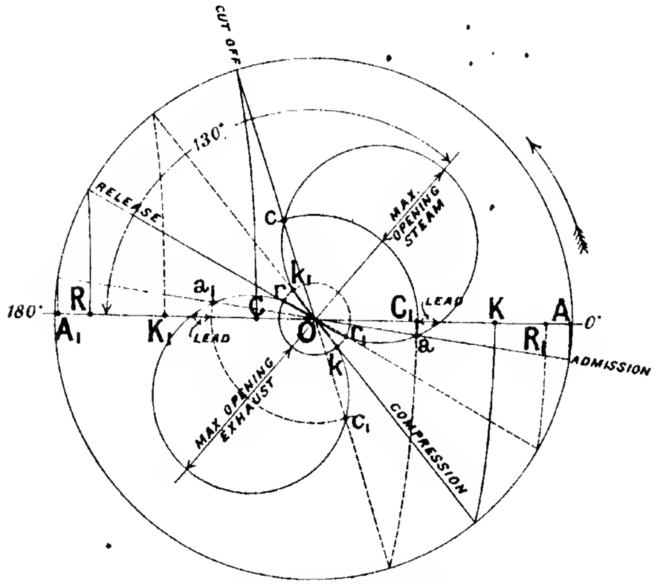


FIG. 179.—Zeuner valve diagram.

its extensive use. It must not be overlooked that the displacement curves are only circles when the obliquity of the eccentric rod is neglected. If this is great, and requires to be taken into account, the curves then take as long to draw as in the rectangular form of the diagram, since they must be set out by actually finding values of x corresponding to a series of crank angles, and then plotting these several values along the crank arm in its various angular positions instead of plotting them on a crank base.

Fig. 179 is drawn with the data on page 537.

163. The Reuleaux Diagram.—In the explanation of this and the following diagram it is convenient to assume that the eccentric

rod is infinitely long, and then to use the sine form for the valve displacement. Thus, since $\phi = 90 + \delta$,

$$x = r \cos (\theta + \delta + 90) = - r \sin (\theta + \delta)$$

In the Reuleaux diagram, Fig. 180, the angle δ is set out below the zero position of the crank, so that when the crank is in any position OQ, the angle QOD is equal to $(\theta + \delta)$. The radius of the circle OQ is equal to r, the eccentricity. The perpendicular QS, from Q on to OD, is therefore equal to

$$r \sin (\theta + \delta)$$

The positions of the crank corresponding to the various events of the stroke are those positions in which the perpendicular QS is equal to the respective laps. Draw Q_1Q_2 parallel to OD at a distance from it equal to the steam lap, and Q_3Q_4 at a distance on the

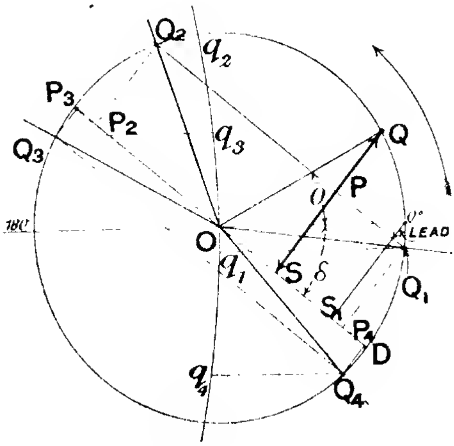


FIG. 180.—Reuleaux diagram.

other side equal to the exhaust lap. These lines cut the circle in the points Q_1, Q_2, Q_3, Q_4, and these points fix the crank positions for the corresponding events, admission, cut-off, release and compression. The positions for the instroke cycle only are shown. If the circle represents the path of the crank pin to scale, Q_1, Q_2, Q_3, Q_4, represent the positions of the crank pin. Considering the crank position Q_2, for instance, $Q_2P_2 = r \sin (\theta + \delta)$ is equal to the displacement for that crank position, but this distance is equal to the steam lap, and the displacement is decreasing; therefore cut-off is just taking place.

The opening for steam or exhaust for an assigned crank-pin position is equal to the length of the perpendicular from the crank pin on to OD minus the appropriate lap. Therefore the lead is the length of the perpendicular from the crank-pin position 0° to the lap line Q_1Q_2.

The corresponding positions of the piston with reference to the centre of the stroke are exhibited by the horizontal distances, Q_1q_1, Q_2q_2, Q_3q_3, Q_4q_4, between the crank-pin positions and the arc drawn through the centre O with radius equal to the length of the connecting rod.

The diagram is drawn for the data on page 537.

164. The Bilgram Diagram.—This is a diagram of the same type as the previous one, only now the angle δ, Fig. 181, is set up from the 180° position of the crank in a direction opposite to the direction of rotation of the crank. A circle is drawn from O, with radius equal to the eccentricity, thereby fixing a point Q. Suppose

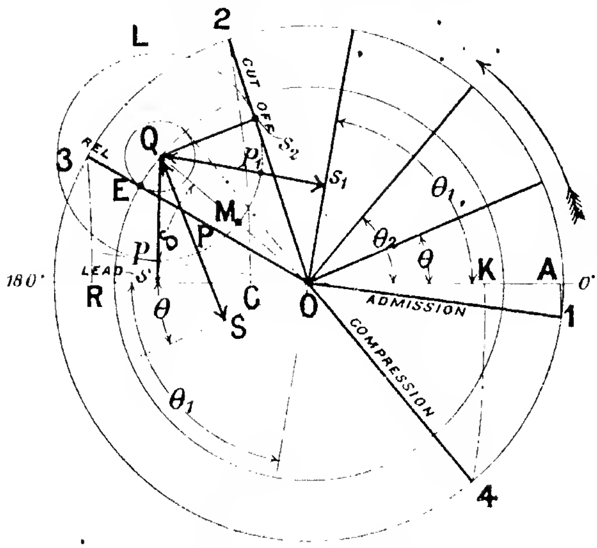


FIG. 181.—Bilgram diagram.

the crank to be placed in any position defined by the angle θ. Produce the crank through O, and it will be seen that the angle this direction makes with the line OQ is equal to $(\theta + \delta)$. Therefore, a perpendicular QS from the fixed point Q on to the crank is equal to $r \sin (\theta + \delta)$, and it therefore represents the valve displacement. In the crank position θ_2 the crank is at right angles to QO; QO is therefore the valve displacement. Between the angles θ_2 and $\theta_2 + 180°$, the foot of the perpendicular falls on the crank arm itself, as shown for the angle θ_1.

If circles be drawn from the centre Q with radii respectively equal to the steam and exhaust laps, the crank positions for the four events of the instroke cycle are fixed by the four tangents which can be drawn to these circles from the point O. Consider the event of

"cut-off," for instance. The valve displacement is given by Qs_2 and this is clearly equal to the steam lap, therefore the crank is in the position corresponding to cut-off by the valve.

In the position θ_1, Qs_1 is the displacement of the valve, and Qp_1 is the steam lap; therefore the difference p_1s_1 is the opening for steam. When the crank is at zero, Qs is the displacement, and Qp is the steam lap; therefore ps is the lead.

The figure is drawn for the instroke cycle with the data on page 537.

The piston positions A, C, R, and K are projected from the crank-pin positions 1, 2, 3, and 4 by the arcs shown.

165. Problems.—Both the Rectangular valve diagram (also called the Harmonic valve diagram, and sometimes the Wave diagram), and the Oval valve diagram give a general view of the distribution, whilst the Zeuner, the Reuleaux, and the Bilgram diagrams may be regarded as geometrical constructions which enable the critical positions of the valve, and therefore the percentage positions of the piston, to be quickly found when the assumption is made that the valve moves with simple harmonic motion.

Many problems may be devised as exercises; in practice, however, the problems requiring the aid of these constructions in connection with the design of a valve gear are few.

Two problems are of particular interest, the first one of which may be stated as follows:—

Problem 1. Given the crank angle at cut-off θ_c, the lead, and the eccentric radius; find the steam lap and the angular advance. The solution of this depends upon the facts that the angle turned through by the eccentric radius from the position corresponding to the zero position of the crank up to cut-off is equal to θ_c; that the position of the crank at zero, produced, bisects the angle between the eccentric radius at admission and at cut-off; and that the projection of the eccentric radius on the zero position of the crank, produced, is equal to the steam lap. Hence the following construction, Fig. 182:—

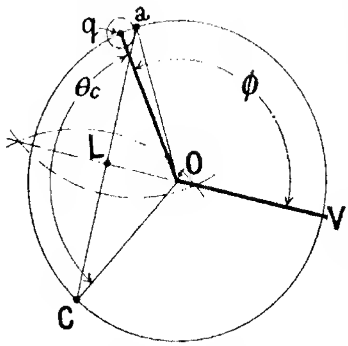


Fig. 182.—Problem.

Draw a circle with the given radius r, and set out two radii Oq, OC, to contain the angle θ_c. With radius equal to the lead, draw a circle from q as centre; and from O draw a tangent to it, cutting the eccentric circle in a. Bisect aC in L and draw LOV. OV is the zero position of the crank; VOq is the angular advance; and OL is the steam lap.

From these, and the given data, the valve displacement curve

may be set out and the various events of the stroke determined. The displacement curve being now fixed, the exhaust lap may be arranged to determine either release or compression at a stated per cent. of the stroke.

The second problem is in particular called the design problem.

Problem 2. Given the crank angle at cut-off θ_c, the lead, and the maximum opening of the port for steam, find the eccentric radius, the angular advance, and the steam lap.

The Bilgram diagram, page 543, may be used to solve this problem, and though not strictly a geometrical solution, it is a very good practical one.

The construction is as follows, Fig. 183.

Set out the position of the crank at cut-off, OC; draw a circle with radius OM equal to the given maximum opening, and draw qq parallel to OV_1 and at a distance from it equal to the lead. Find

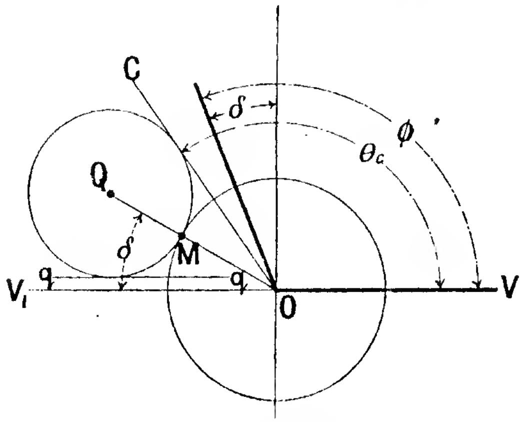


FIG. 183.—Problem.

by trial the centre Q of a circle which will touch OC, qq, and the circle OM. Then QM is the steam lap. The angle $QOV_1 = \delta = \phi - 90$ is the difference between the angular advance and 90 degrees, and OQ is the eccentric radius.

166. Valve Gears with Independent Cut-off Valves.—The simplicity of the slide valve is only gained by the sacrifice of the independent control of the events of the stroke. A slide valve cannot be set to cut-off early in the stroke, without at the same time producing an early release and an early compression. The difficulty is avoided by the use of a second valve sliding on the back of the main slide valve and driven by its own eccentric rod and sheave. The steam passes from the steam chest through ports in the main valve and into the steamways leading to the cylinder. The second valve cuts off the supply of steam to the main valve. This second valve is often called an expansion valve.

Such an arrangement is shown in Fig. 184. V is the main slide valve driven by the rod L, and V_1 and V_2, which are connected by the right- and left-handed screw K, together form the second slide valve. The steam laps at S and *s*, and the exhaust laps inside the main valve, are measured just as if the main valve were an ordinary slide valve. The construction of the valve is modified in order to form steam channels at the ends and to provide a smooth surface over which the second valve can slide. The lap of the second, or cut-off valve is measured in relation to the ports in the main valve. In the figure it will be seen that the cut-off valve is in its central position in respect of the ports of the main valve but there is no overlap. The ports in fact stand open by the amount *l*; *l* is therefore the negative lap. This can be adjusted by the turning of the screw K, and this is done from outside whilst the engine is running by the wheel W, the index nut I showing how far the blocks are separated. By this device the cut-off can be varied through a wide range, without interfering in any way with the events of release, compression, and admission, since these are controlled by the main valve.

The following example illustrates the way the Rectangular Valve Diagram may be applied to examine the distribution of steam which can be obtained by the use of a pair of valves of this kind.

The following are the data:—

Main gear.	Eccentricity of sheave, 1·5 ins.
	Angular advance of sheave, 130°.
	Steam lap, 0·8 in.
Expansion Gear.	Eccentricity of sheave, 1·5 ins.
	Angular advance, 180°.

Let the problem be to find the lap of the expansion valve so that cut-off may take place at 20 per cent. of the stroke in each cycle.

The first step is to draw the valve diagram for the main valve in the way already explained. This is shown in Fig. 185, from which it will be seen that the distribution effected by the main valve is:—

	Instroke.	Outstroke.
Lead . . .	0·17″	0·17″
Cut-off . . .	61 per cent.	69½ per cent.
Release . . .	92½ ,,	92½ ,,
Compression .	84½ ,,	84½ ,,

It will be observed from the diagram that the compression has been equalized by using different exhaust laps.

Next, add to the diagram the displacement curve for the expansion valve, using the data above. The ordinates to this curve represent the displacement of the expansion valve from the centre of its travel.

In order to solve the problem the distance of the expansion valve from the centre of the main valve must be known when the piston is at 20 per cent. of its stroke.

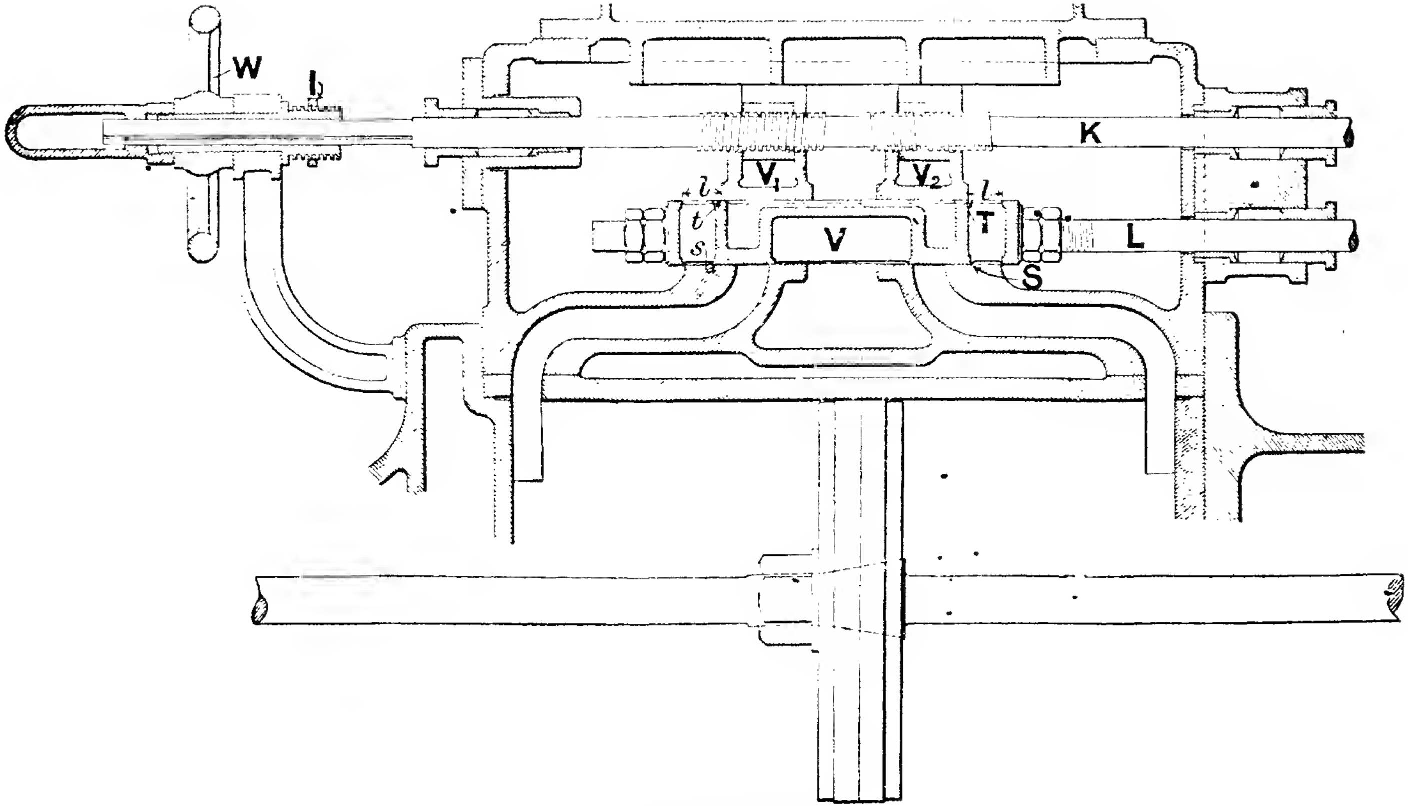


Fig. 184.—Meyer valve gear.

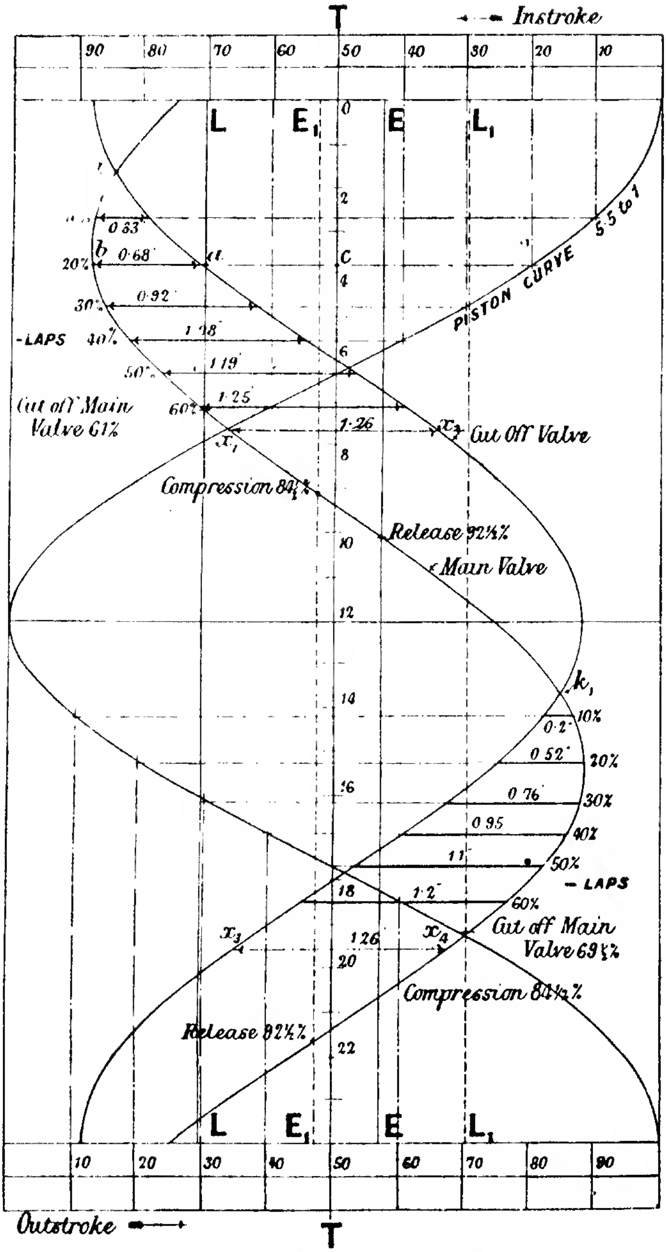


FIG. 185.—Valve diagram. Meyer gear.

Consider the instroke first. From 20 per cent. on the instroke scale at the top of the diagram draw a vertical line to cut the piston curve, and then a line at right angles to cut the displacement curves in the points a and b, and the axis of the diagram in c.

Then, when the piston is at 20 per cent. of the instroke, the main valve is the distance cb from its central position, and the expansion valve is the distance ca from its central position. The distance of the expansion valve from the centre of the main valve is therefore

$$cb - ca = ab$$

This distance therefore represents the required lap of the expansion valve with respect to the ports of the main valve. But is it to be negative or positive lap?

This matter is settled by observing whether the displacement of the expansion valve from the axis of the diagram is greater or less than the displacement of the main valve.

When the displacement of the expansion valve is less than the corresponding displacement of the main valve, the lap is negative; when greater, positive.

The distance ab on the diagram measures 0·68 in., and, so far as the instroke is concerned, this must be the negative lap of the expansion valve.

Consider the outstroke. Projecting, in a similar way to that just described, from the 20 on the outstroke scale, the intercept between the two valve displacement curves measures 0·52 in.

If therefore the length of the valve spindle is adjusted, and the blocks set the proper distance apart, so that the expansion valve has these respective negative laps, cut-off will take place at exactly 20 per cent. of the stroke for both strokes, whilst the leads remain equal at 0·17 in., and release and compression retain the values stated above.

The problem may now obviously be extended to find the laps for a series of stated values of the cut-off between zero and the cut-off of the main valve. The projecting lines from 10 to 60 per cent., increasing by intervals of 10 per cent., are drawn in the figure for both the in- and the outstroke cycles, and the value of the intercepts between the two curves marked in. Thus, to cut-off at 40 per cent. of the stroke for both strokes the negative lap for the instroke cycle must be 1·08 ins., and for the outstroke cycle 0·95 in. The difference between the laps for 20 per cent. cut-off is 0·16 in. The difference for 40 per cent. is 0·13 in.

Assuming that the screws are constant in pitch the cut-off valve can only be set to have one difference between the laps, because when once the difference is established by adjusting the valve spindle to the proper length, the turning of the wheel W changes the lap on each side by equal amounts.

Thus, after the diagram has been drawn, the differences of the laps must be written down and then that particular difference chosen which gives the best general result.

Taking the differences of the figures marked on the valve diagram,

Per cent. cut-off.	Difference of laps required to equalize the cut-off.
10 per cent.	0·13 in.
20 „	0·16 „
30 „	0·16 „
40 „	0·13 „
50 „	0·09 „
60 „	0·05 „

If the valve spindle is adjusted for a difference of 0·16 in., cut-off will be exactly equalized over the range from 20 to 30 per cent.; very nearly equalized over the whole range from 10 to 40 per cent., and there will be small errors over the range from 40 to 60 per cent.

Engines provided with this kind of gear can therefore be set to give almost perfect indicator cards over a large range of expansion, the cut-off being equalized by the expansion valve, and the compression and release by the main valve.

At the point k on the valve diagram, where the two displacement curves cut one another, the expansion valve is in its central position with regard to the main valve, and if the expansion valve has no lap, cut-off will take place. For regions above this point it will be seen that the expansion valve is at a greater distance from the axis of the diagram than the main valve, and therefore if cut-off is required to take place at the excessively early fractions of the stroke to which the horizontals crossing this region correspond, the expansion valve must have positive lap. The same remarks apply, of course, to the region included between the horizontals through k_1 and the point 12 on the vertical axis.

When considering the general design of the gear it will be found convenient to draw the displacement curve for the expansion valve on tracing paper. It may then be pinned over the valve diagram for the main valve with any required angular advance, and the effect of varying this can be studied.

Care must be taken when designing the valve to see that, when the blocks are separated to the greatest distance, the ports in the main valve are not opened by the inside edges of the blocks. The blocks should be wide enough to prevent their inner edges from ever approaching near enough to the inner edges of the ports to allow steam to leak through.

An instructive way of working problems connected with the relative motion of the expansion and the main valves is to consider the main valve at rest and then find an imaginary eccentric sheave which, keyed to the main shaft and coupled to the expansion valve, would give to it a movement over the main valve precisely the same as it has when both valves are moving in the engine. The way to do this is to combine with each valve a motion equal and opposite to the motion of the main valve, which has the effect of reducing the

main valve to rest. The imaginary eccentric which will give the expansion valve the motion resulting from the combination is found by subtracting the eccentric radius of the main valve as a vector from the eccentric radius of the expansion valve. The line giving this vector difference represents the imaginary eccentric in its proper angular position with regard to the main crank and of the proper radius. The way to take this difference is to set out the two eccentrics with their proper angular advances with regard to the main crank and then to join their ends, placing an arrow on this joining line "in circuit" with the subtracted vector. Thus, in Fig. 186, the main crank and eccentric cranks, corresponding to the example, are set out, and ab is the required difference. The dotted eccentric crank, drawn parallel to ab from the centre O in the direction of the arrow, represents the required imaginary crank, and this, if coupled to the expansion valve, would give it the proper

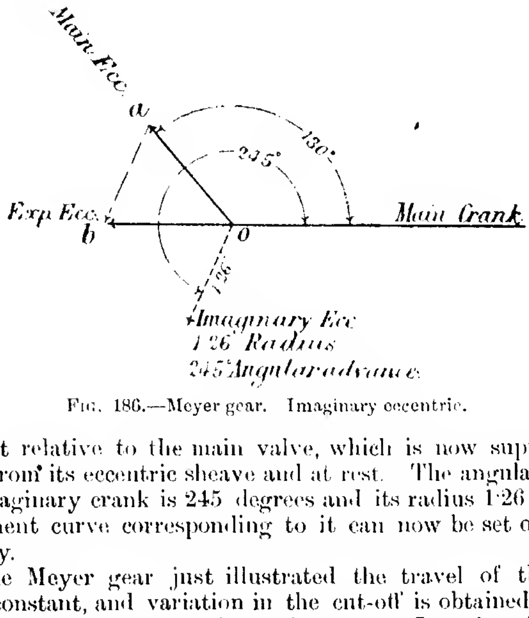


FIG. 186.—Meyer gear. Imaginary eccentric.

movement relative to the main valve, which is now supposed uncoupled from its eccentric sheave and at rest. The angular advance of the imaginary crank is 245 degrees and its radius 1·26 ins. The displacement curve corresponding to it can now be set out in the usual way.

In the Meyer gear just illustrated the travel of the cut-off valve is constant, and variation in the cut-off is obtained by varying its lap in relation to the main valve. In other forms the lap is kept constant whilst the angular advance of the eccentric sheave actuating the cut-off valve is varied, often by the action of a governor. Or, again, the travel may be varied by the action of a governor, or in some cases both travel and angular advance are varied. The principle of all these gears is however the same, namely, to give independence of action to a separate valve in order that the cut-off can be varied whilst the other events are determined at constant fractions of the stroke by the main distributing valve.

167. Forms of Slide Valve.—The slide valve shown in Fig. 172, page 531, is the form in which it is generally used when the piston speed and the steam pressures are moderate. With increase of piston speed this typical form is modified with the object of presenting a larger opening for the admission of steam in order to prevent excessive drop of pressure due to wire drawing. With increase of pressure, or where the area of the valve is large, the design is modified in order to partially relieve the valve from the pressure pressing it on to the surface over which it slides.

The Trick or Allan valve, Fig. 187, has been largely used in locomotive practice to minimize the wire drawing of the steam at high speeds. A passage P is cast in the back of the valve, so that when it is just opening for steam the usual supply at the edge is supplemented by a second supply introduced into the port through the passage P, the opening and closing of this passage being effected by the edges of the valve seat E and E_1 as the valve slides across

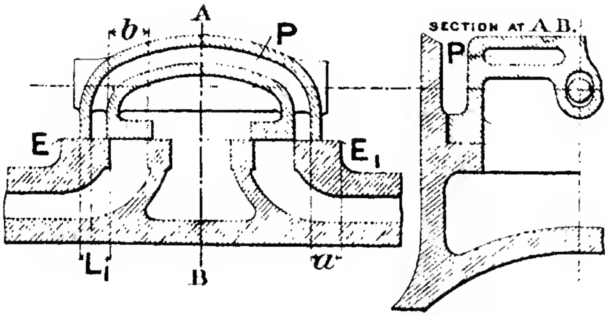


FIG. 187.—Trick valve.

them. If the valve, Fig. 187, is separately drawn and placed in the critical positions it will be apparent that, considering the admission from the left edge of the valve, the distance a from the inside edge of the passage P to the edge of the seat E_1 must be equal to the steam lap L_1 in order that the steam port and the passage P may open simultaneously. When the valve has moved a distance b to the right of its central position, a further movement to the right increases the width of the opening determined by the steam admission edge of the valve, but decreases the opening of the passage P by an equal amount, so that the opening for steam remains constant until the passage P is entirely closed by the bridge between the steam and the exhaust port. Admission at the right edge of the valve for the instroke cycle is similarly supplemented by steam passing through the passage from the left, the edge E controlling admission and cut-off so far as the passage is concerned.

In designing a valve of this kind care must be taken that sufficient width exists between the edge of a steam port and the

adjacent edge of the exhaust port to prevent the passage P opening communication between the steam chest and the exhaust passage.

A valve designed to secure sufficient port opening with a relatively short stroke, and in general use for the low pressure cylinders of marine engines and known as the double ported valve, is shown in Fig. 188. Here, two steam ports are formed on each side of the exhaust port, and the right side of the valve is provided with two edges B_1, B_2 to give admission at these two ports simultaneously; and corresponding to these are two edges C_1, C_2 which open communication with the exhaust passage E. A passage K is formed across the valve to convey steam to the edge B_2. A similar set of edges B_3, B_4, C_3, C_4 determine the distribution at the left side of the valve for the outstroke cycle. The steam and exhaust laps are indicated by L, L, l and l for the instroke cycle and L_1, L_1, l_1 and l_1 for the outstroke cycle.

Valves of this kind can be made with three, or even more, ports for each steam way into the cylinder. A section through the inter-

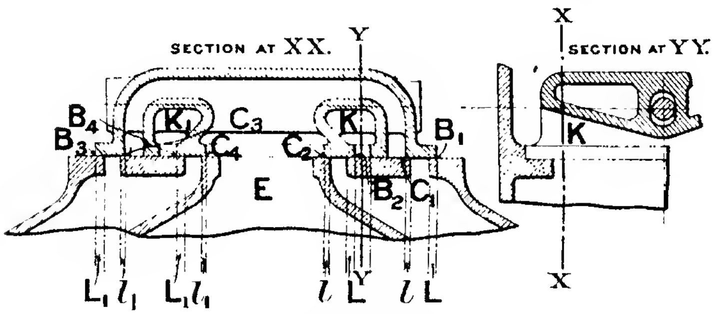


FIG. 188.—Double-ported valve.

mediate pressure and one of the low-pressure cylinders of a battleship engine of 11,000 I.H.P. is shown in Fig. 189, which illustrates, amongst other things, the general arrangement of the triple-ported slide valve used with the low-pressure cylinders of this engine. The triple ported entry to the steam ways at each end of the cylinder is clearly shown. It will be noticed that the face on which the valve slides is really one face of a plate through which the ports are cut, ports in the cylinder casting corresponding with them. The exhaust port E lies between the steam ways.

Balancing Piston.—The weight of a slide valve of a large engine is considerable. The triple ported valve just described weighs $1\frac{1}{2}$ tons, and in the largest engines the weight may be as much as 3 tons. In order to relieve the valve gear the weight of the valve is taken by a balancing piston B, Fig. 189. The bottom of the balancing piston cylinder is in free communication with the steam chest, a clear way being left round the piston rod. The upper part of the cylinder is maintained in continuous communication with the

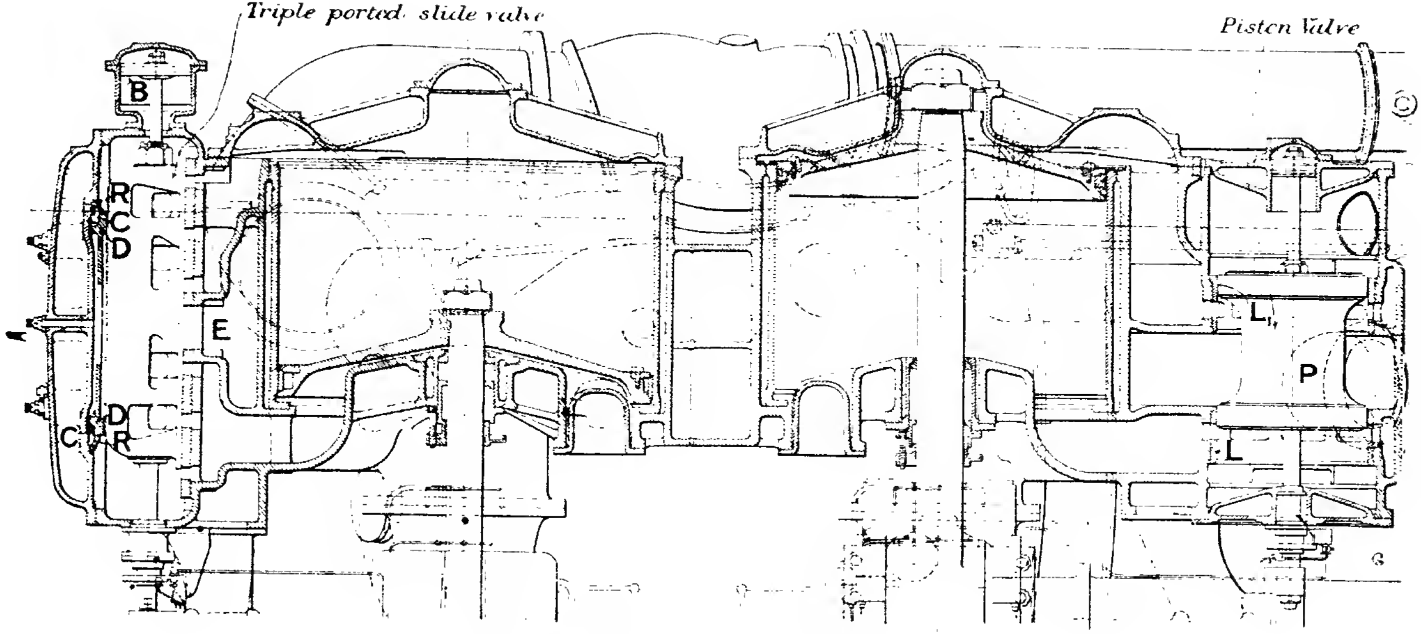


FIG. 189.—Piston valve and triple-ported balanced slide valve.

condenser. Thus the difference of pressure per square inch between the lower and the upper sides of the piston is the difference between the steam chest pressure and the condenser pressure. The area of the piston is proportioned to take the weight of the valve and valve gear, less the weight of one eccentric rod.

Balanced Valves.—In recent years the steam pressure used with all types of engines has gradually increased until at the present time 200 lbs. per square inch is not uncommon either in marine or in locomotive practice. In consequence of this, methods of relieving the pressure on the back of the valve in order to decrease the frictional resistance to sliding are used much more generally than in the days of lower pressures.

One way of reducing the total pressure on the valve is to form a rectangular groove in the back of the valve and then to fit metal strips in the groove. Springs at the bottom of the groove press the strips upwards into contact with a face prepared in the cylinder cover. In this way the area enclosed by the frame is cut off from the action of the high-pressure steam and the frictional resistance of the valve is reduced in proportion. These Richardson strips, as they are called, have been extensively employed in the United States, and on some railways of this country.[1]

The triple-ported valve shown in Fig. 189 is 4521 sq. ins. in area. A considerable portion of this is cut off from the action of the steam by a relief frame. A circular plate is pressed into contact with the back of the valve by springs. This plate is free to move in a direction perpendicular to the port face, being guided by the steel ring RR, secured to the cover by 12, $\frac{7}{8}''$ studs. Steam is prevented from getting behind the plate by means of a flexible copper ring CC in the form of a single deep corrugation. This copper corrugation is fastened by one limb to the plate by 48, $\frac{1}{2}''$ studs, and by the other limb, to the cylinder cover by 52 studs. The copper corrugation forms a flexible connection between the cylinder cover and the circular plate resting on the back of the valve, the plate is therefore free to move into close contact with the surface of the valve under the action of the springs. In this way an area of 1503 sq. ins. is relieved from steam pressure.

The Piston Valve.—Theoretically the slide valve can be relieved of all pressure on the back by changing it to the cylindrical form, obtaining what is known as a piston valve. Two essential properties of the flat slide valve are lost by this change. The valve is no longer kept up to its seat by the steam pressure, and the valve can no longer lift off its seat against the steam pressure to allow water to escape. When a piston valve is used therefore it is absolutely necessary to provide relief valves on the cylinder to allow water to escape.

If a thin section of the slide valve shown in Fig. 172, page 531, is revolved about the axis of the valve spindle it will describe a corresponding piston valve in space. The ports of the passages, both for

[1] J. A. F. Aspinall, "On the Friction of Slide Valves," *Proc. Inst. Civil Engineers*, vol. 133.

the supply and for the exhaust of steam, now encircle the valve, and provision is made for placing the ends of the steam chest or valve chamber in communication with one another. This is generally done either by casting a passage for the purpose with the valve chest or with the cylinder, or by casting a passage through the valve itself.

This change in form of the valve does not involve any change in the distribution of steam it can effect. The definitions of the steam and of the exhaust laps and of the critical positions are precisely the same in a piston valve as in the corresponding valve of the ordinary type.

A usual construction of a piston valve is shown in Fig. 189. The steam chest is bored out to receive two liners, L and L_1, in which the ports are machined to the correct size. The corresponding ports in the cylinder are cast slightly wider than those in the liner, so that when the liner is forced into its place the ports in it have a fair opening into the steam passages. In some cases the liner is made in one piece and all the ports are cut in it. The piston valve P is indicated in general elevation and it will be seen that its weight is carried by a balancing piston.

A section of the piston valve for the high pressure cylinder of the engine of which Fig. 189 is a part is shown in Fig. 190. This valve is two feet in diameter. The cylindrical body of the valve carries a boss inside it supported by three webs, the boss being connected to the valve spindle which below is continued to the valve gear, and above to a balance piston. The ends of the valve are increased in diameter and are formed into circumferential grooves carrying packing rings. One side of a groove is formed by the addition of a flat steel ring to the end of the valve. The removal of this ring enables the packing rings to be put in place, after which it is replaced and held securely in position by a ring of studs. The packing rings are split and a tongue piece is inserted as indicated, thus leaving the ring free to expand against the liner. The edges of the packing rings form the respective steam and exhaust edges of the valve. The valve shown works with outside steam admission.

Sometimes an uncut floating ring is used for packing the valve. The ring is turned a few thousandths of an inch smaller than the liner in which it works and is free to move a small amount in a radial direction relatively to the valve, so that it can accommodate its position to the small distortions of the liner due to changes of temperature without danger of sticking.

A floating ring can either be put over the end of the valve, being held in its place by a washer or junk ring, or it can be sprung over the end of the valve into a groove turned to receive it in the way discovered by Mr. Yarrow.[1] A floating ring passes a small amount of steam, but the gain in other directions in the opinion of some eminent engineers more than compensates for this. The advantage of this form of packing is that there is practically no frictional resistance to sliding, because the uncut packing ring is actually

[1] See *Engineering*, January 30, 1903, p. 122.

smaller in diameter than the liner in which it slides; there is therefore no necessity for lubrication. This is an enormous advantage in the case of condensing engines where the condensed steam is returned to the boiler, since the successful working of modern water-tube boilers is imperilled if oil gets into them with the feed water. Some makers dispense with even a floating ring and use a solid valve. In this case the design must be carefully considered with regard to the

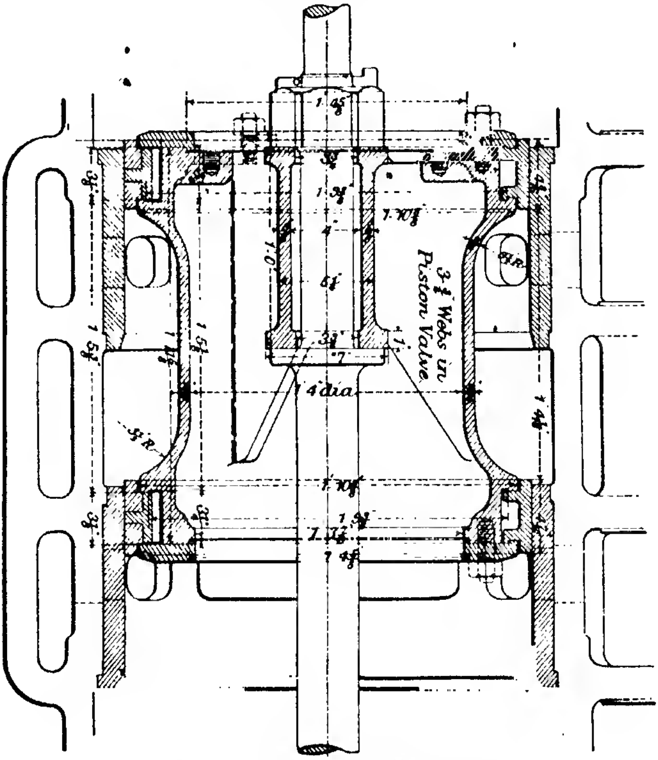


FIG. 190.—Piston valve. Marine engine.

change of form likely to be caused by the changes of temperature to which the valve and seat will be subjected.

Care is required in the design of a piston valve required to work with superheated steam. In some cases the piston valves are fitted with solid packing rings. Precision is required in the workmanship if these rings are to work satisfactorily. M. Sauvage quotes a case where on a Belgian locomotive the loss of steam by leakage through the piston valves fitted with solid rings amounted to 3300 lbs. per hour whilst the engine was standing. M. Sauvage states that on the Continent elastic rings are usually preferred.

Fig. 191 shows a piston valve specially designed for use with superheated steam. It is of the Schmidt type. The figure is in fact constructed from the drawing of the piston valve used in the four-cylinder simple express locomotives designed by Mr. Bowen Cooke for the London and North-Western Railway. These engines are fitted with superheaters of the Schmidt pattern.

The valve consists essentially of two broad piston rings held one at each end between end pieces and flanges. Each ring is cut for trick ports communicating through the centre of the valve. Steam is admitted inside the valve. The steam lap is the distance L, and the exhaust lap the distance *l*, which in this case is negative. Considering the construction of the valve in detail, two end plates, of

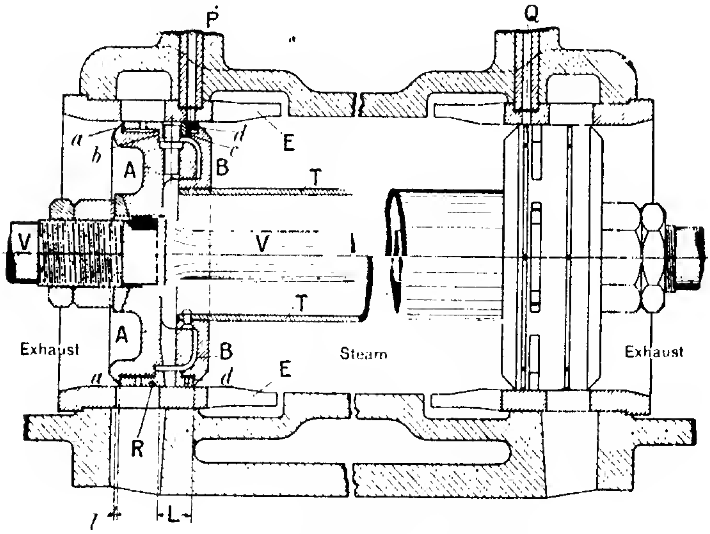


FIG. 191.—Piston valve for superheated steam.

which A is one, are clamped against shoulders turned on the valve spindle V. A centre piece consisting of a tube T terminating in flanges, of which B is one, lies between the end plates, and short bosses on the flanges engage with the end plates as shown, and thus hold the tube centrally, but without constraint endways. Six ports lying circumferentially are formed in the single piston ring R. These ports are seen in elevation at the right-hand end of the valve. Corresponding ports are formed in the end plates, and the ports at each end of the valve are in communication through the central tube T. There are four radial faces to keep steam tight, namely, *a*, *b*, *c*, and *d*. The spaces behind the piston ring are put into communication with the outer face by means of small holes drilled radially, four of which are seen in the section of the ring.

This prevents the accumulation of steam behind the ring. The faces b, c prevent the passage of steam into the centre of the valve except through the trick port. The piston ring is cut in one place, and is fitted with tongue pieces which prevent it from turning. The valve works in liners, one of which is shown at E. Oil is supplied for lubrication through pipes, the ends of which are seen at P, Q.

168. Reversing and Expansion Gears.—In locomotive, in marine, and to some extent in stationary practice, the valve-gear mechanism must be capable of operating the valve so that the crank shaft can be driven in either direction. A valve-gear mechanism which possesses this capability usually possesses also the valuable property that it enables the cut off in the cylinder to be varied through a wide range in whichever direction the crank shaft is turning.

The **link motion** is the earliest, the simplest, and the most generally used reversing gear, though on the Continent the Walschaerts gear is rapidly displacing the link motion in locomotive practice, and it has recently been fitted to engines in this country. The Joy gear has been extensively used by two of the leading railway companies, owing chiefly to its compactness and to the absence of eccentric sheaves on the driving axle. The link motion and the Joy gear stand each as representatives of a class; those mechanisms represented by the Joy gear being known generally as radial gears.

Both link and radial gears are used in connection with some form of slide valve, and all types possess the common property that they impart to the valve a periodic sliding motion which approximates to simple harmonic motion so nearly, that it is exceeding useful to assume this character of the motion at the outset of any analytical investigations relating to these gears. With this assumption the distribution of the steam can be found when the dimensions of the gear are given, and alternately, the dimensions of the gear can be found to give a stated distribution at least to an approximation which is usually sufficiently accurate for most practical purposes, and always sufficiently accurate for the purposes of a preliminary examination or design.

The assumption of simple harmonic motion of the valve carries with it the consequence that whatever be the particular type of link or radial gear under consideration, the motion which the gear imparts to the valve for a given setting is sensibly the same as the valve would receive if it were disconnected from the actual gear and then re-connected to a substituted single eccentric gear in which the eccentric rod is long in comparison with the eccentricity. The idea of this substituted single eccentric gear is so useful that its eccentric sheave has received a special name. It is called the **equivalent eccentric.** One of the main objects of the kinematic analysis of valve-gear mechanisms is the determination of this equivalent eccentric for given conditions. The equivalent eccentric for any particular

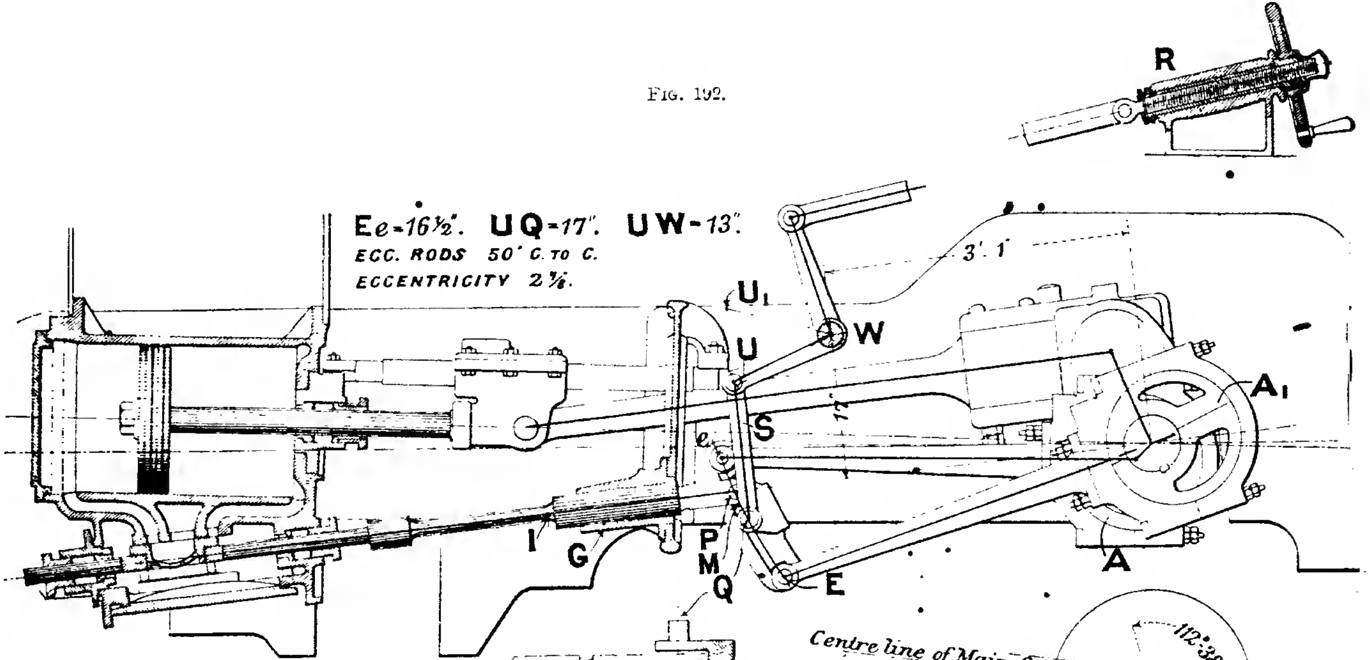
setting of the valve gear is defined by its radius and by its angular advance. In order to distinguish these quantities, in what follows the radius of an equivalent eccentric will be represented by ρ and the corresponding angular advance by ψ.

Actual displacement curves do, however, possess an individuality according to the mechanism which is associated with the slide valve. The only way to determine the actual curves is by geometry. Corresponding positions of the crank angle and valve are found by plotting the gear in a series of positions. The process is tedious, and in most places the gear is set up in model form, so that the valve displacement curves can be drawn automatically by the gear itself. Such a model is illustrated in Fig. 200, page 568.

169. The Link Motion.—In a link motion there are two eccentric sheaves, one of which is keyed to the crank shaft at an angular advance, giving a distribution of steam which determines a clockwise rotation of the crank, the other at an angular advance, giving a counter-clockwise rotation. The ends of the eccentric rods belonging to these sheaves are jointed to a link, hence the name of the gear, and the link is supported so that it is free to move in the way defined by the motion of the ends of the eccentric rods to which it is jointed. The valve itself is coupled to the link by the valve rod. The link is suspended in such a way that it can be moved up or down, so that one or other of the two eccentric rods can be brought in line, or nearly in line, with the valve rod. The crank shaft turns in the direction determined by the particular eccentric with which the valve rod is brought into line.

A Stephenson link motion is shown in Fig. 192. The two eccentric sheaves are A and A_1. E*e* is the link. The link is suspended from the end of an arm belonging to a shaft, W, by suspension links, of which UQ is one. The shaft is held in bearings fixed on the engine frame; it is called the **weigh-bar shaft.** The shaft is held in any assigned angular position by the **reversing screw** R, to which it is connected by the rod as indicated. As the crank shaft rotates, the motion of the link is defined by the motion of the ends of the two eccentric rods together with the motion of the end of the swinging link UQ, which is an arc of a circle struck from the centre U with radius UQ. This combination of two eccentric sheaves and their rods with a link suspended from a weigh-bar shaft, constitutes the special constructional element of a link motion. The valve is connected to the valve rod I, and this rod is free to slide in the guide G. It is jointed with the link at P. The block M involved in this jointing with the slotted link is called the **motion block**. When the link is moved from one extreme position to the other by the turning of the reversing screw R, the motion block, P, must pass the joint Q. Fig. 193 shows the way in which the link is constructed in order to allow this to take place. A somewhat unusual feature of this gear is that the line of stroke of the piston is inclined to the line of stroke of the valve. This arrangement is

Fig. 192.



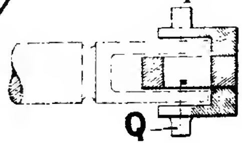


Fig. 193.

Stephenson link motion, Great Eastern Railway.

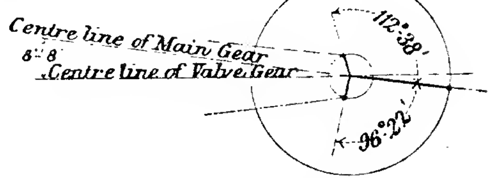


Fig. 194.

made in order to allow the valve to be placed beneath the cylinder in order to gain more room for the valve chests. The actual motion of the valve rod is the resultant of the motions of the ends of the eccentric rods. When the link is in the extreme lower position, as shown in the figure, the resultant motion is very nearly the same as the component motion of the end of the eccentric rod *e*, and the crank rotates in the positive direction. If the link is lifted into the highest position by the turning of the screw R, the lower rod is brought nearly into line with the valve rod, so that its resultant motion is very nearly the same as the component motion of the end of the rod E. In an intermediate position the cut-off is reduced and the distribution effected will cause positive or negative rotation according as the upper or lower component motion of the end of the rod predominates.

The Stephenson link motion is distinguished into the open rod type, or the crossed rod type. A characteristic feature of the open rod type is that the lead increases as the link approaches mid-gear from either extreme position; whilst the opposite is the case with the crossed rod type. If the rods are crossed when both eccentric radii are pointing towards the link, the motion belongs to the crossed rod type.

170. Link Motion Valve Diagram.—The valve diagram for the Stephenson link motion just described is shown in Fig. 195. There are seven valve displacement curves shown in the diagram together with the displacement curve of the piston, drawn for a connecting rod to crank ratio of 5·85 to 1. The piston curve is drawn as though the line of stroke of the piston and the line of stroke of the valve were parallel. The corrected angular advances after this adjustment has been made are, for the forward eccentric, $104\frac{1}{2}^\circ$ instead of $112\frac{1}{2}^\circ$ as actually set in the engine; and for the backward sheave, $104\frac{1}{2}^\circ$ instead of approximately $96\frac{1}{2}^\circ$ as set in the engine.

Each valve displacement curve corresponds to a particular position of the reversing screw, or, what is the same thing, to a definite position of the block in the link when the crank angle is zero. That is, the position of the block is defined by its distance from the centre of the link when the piston is on the dead centre of the instroke cycle. This distance, represented by the letter u, is positive when the block is above the centre and nearer to the eccentric rod which determines positive rotation, and negative when it is below the centre and nearer to the eccentric rod which determines negative rotation. The valve displacement curves in the diagram correspond to the values of u written against them.

The essential dimensions of the gear are shown in the centre line drawing above the valve diagram.

The vertical centre line TT may be regarded as the centre line through the exhaust port, or a line symmetrically situated with regard to the ports. In the diagram it is drawn to bisect the distance between the points where the $4\frac{1}{2}$ in. displacement curve cuts the horizontals through 0° and 180°. Lines are drawn parallel

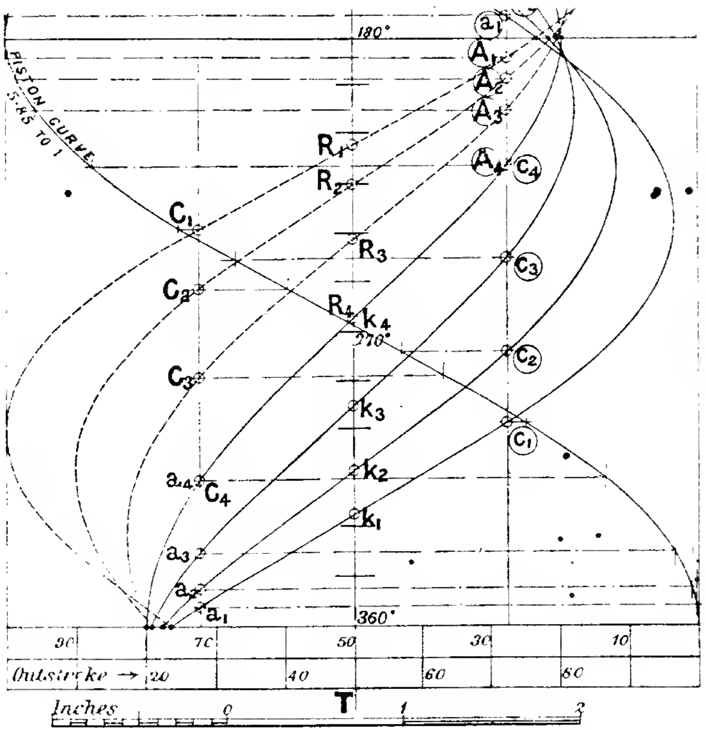


FIG. 195.—Valve diagram for link motion illustrated in Fig. 192.

[*To face p.* 562,

to TT, on either side of it, at distances equal to the respective steam laps, which in this case are each $\frac{7}{8}$ in. There is no exhaust lap, so that the centre line itself determines, by its intersections with the displacement curves, the points at which release and compression occur for the respective displacements and in the respective cycles.

Consider the displacement curve marked $u = 4\frac{1}{2}$ ins. Starting from the zero crank position it will be seen that the lead is 0·17 in., and that cut-off, release, compression, and admission are respectively determined by the points c_1, r_1, k_1, a_1, which, after reference horizontally to the piston displacement curve and then vertically to the top scale, show that the distribution is—

Cut-off . .	75 per cent. of the stroke.			
Release . .	91	,,	,,	,,
Compression	92	,,	,,	,,
Admission. .	99	,,	,,	,,

Also the maximum opening for steam is 1·12 ins., and the maximum opening for exhaust is 1·82 ins.

These values are plotted against the value of u in Figs. 196 and 197, thus obtaining points on the extreme ordinates through which the thick curves of the diagrams pass.

Similarly, the points c_2, r_2, k_2, a_2, determine the distribution when $u = 3$ ins., and the points c_3, r_3, k_3, a_3, the distribution when $u = 1\frac{1}{2}$ ins.

There are similar sets of points for the outstroke cycle, beginning when the crank angle is 180°. These points are similarly lettered to those above, but are distinguished from them in the valve diagram by being ringed.

The sequence of events in the distribution beginning with admission follow an increasing value of the crank angle. When the engine is reversed the crank passes through the angles in the direction 360, 359 . . . to 0. It will be seen from the diagram that the dotted displacement curves, corresponding to negative values of u, determine a sequence of distribution which corresponds with this upward reading of the angles. Starting from the angle 360° and going along the curve $u = -4\frac{1}{2}$ ins. the lead is 0·17 in. and the points C_1, R_1, K_1, A_1 determine a distribution which is practically the same as that determined by the displacement curve $u = +4$ ins. Points relating to backward running are all lettered with capitals and the ringed capitals belong to the outstroke cycle.

The four points in each set are plotted in the diagram, Figs. 196 and 197, together with the corresponding values of the lead and port openings, and the curves in these figures are sketched in through the points so plotted. The distribution corresponding to any assigned value of u may be found from this diagram.

When $u = 0$ the motion block is at the centre of the link, and the motion is said to be in mid-gear. The valve displacement curve corresponding to mid-gear is shown in the diagram, from which it can be found, or more directly from Figs. 196 and 197, that the lead

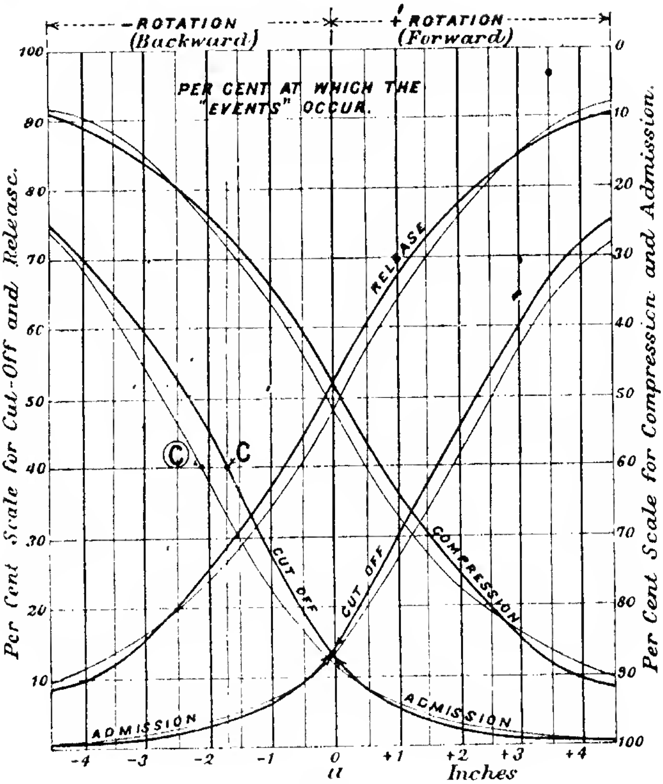


Fig. 196.

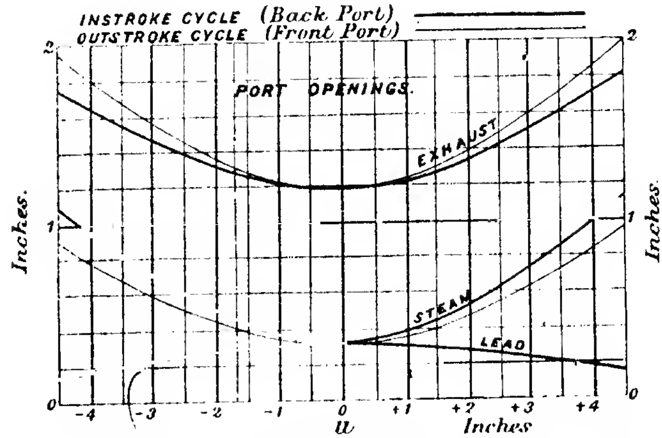


Fig. 197.—Percentage and port opening diagrams for the link motion shown in Fig. 192, page 561.

is a maximum and equal to the maximum opening of the port to steam, that cut-off takes place at 13 per cent. of the stroke, and that exhaust and release occur, each at 52 per cent. of the stroke. The sequence of events along this curve can be read either up or down, showing that if there is enough positive work done to drive the engine at all, it will run either forward or backwards when in mid-gear. For example, a locomotive running light may continue to run when the motion is put into mid-gear. If the engine is reversed it may continue to run backwards when the motion is brought back again to mid-gear. Usually, however, an engine will not run against the load even of its own friction when in mid-gear, and can never be started in mid-gear.

It will be observed that the displacement curves in the valve diagram intersect one another nearly at a common point, just below

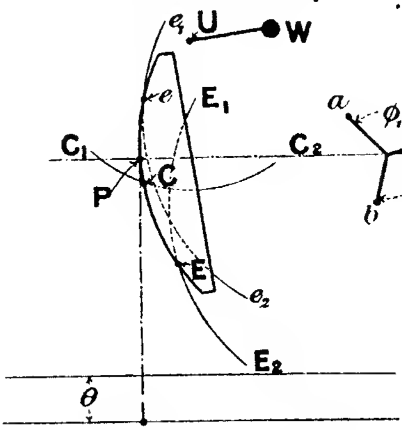


Fig. 198.—Template of link.

the horizontal through 0°. If a horizontal is drawn through this common point it will cut the vertical at about 5°, showing that if the zero of the scale is moved down 5°, the lead will be very nearly constant for all values of u. This movement of the zero is equivalent to increasing the angular advance of the forward eccentric by 5° and decreasing the angular advance of the backward eccentric by a like amount. This change also equalizes the leads at the 180° dead point, so that, as far as forward running is concerned, there is no difficulty in equalizing the lead, but it can only be done at the expense of producing a greater than the normal inequality in back gear, and the adjustment is rarely made.

The valve displacement curves shown in the valve diagram were obtained geometrically in the way illustrated generally in Fig. 198. The crank and the eccentric radii are set out for a given crank angle θ, and then the loci, e_1e_2, and E_1E_2, of the ends of the eccentric rods are drawn. The position of the suspension point U

is then plotted, corresponding to the given value of u, for which the displacement curve is to be drawn, and from it as centre the locus C_1C_2 of the end of the suspension link is drawn. A template of the link is then cut out and fitted to these three curves so that the centre line of the link can be drawn. The intersection P of the line of stroke of the valve with the centre line of the link, fixes

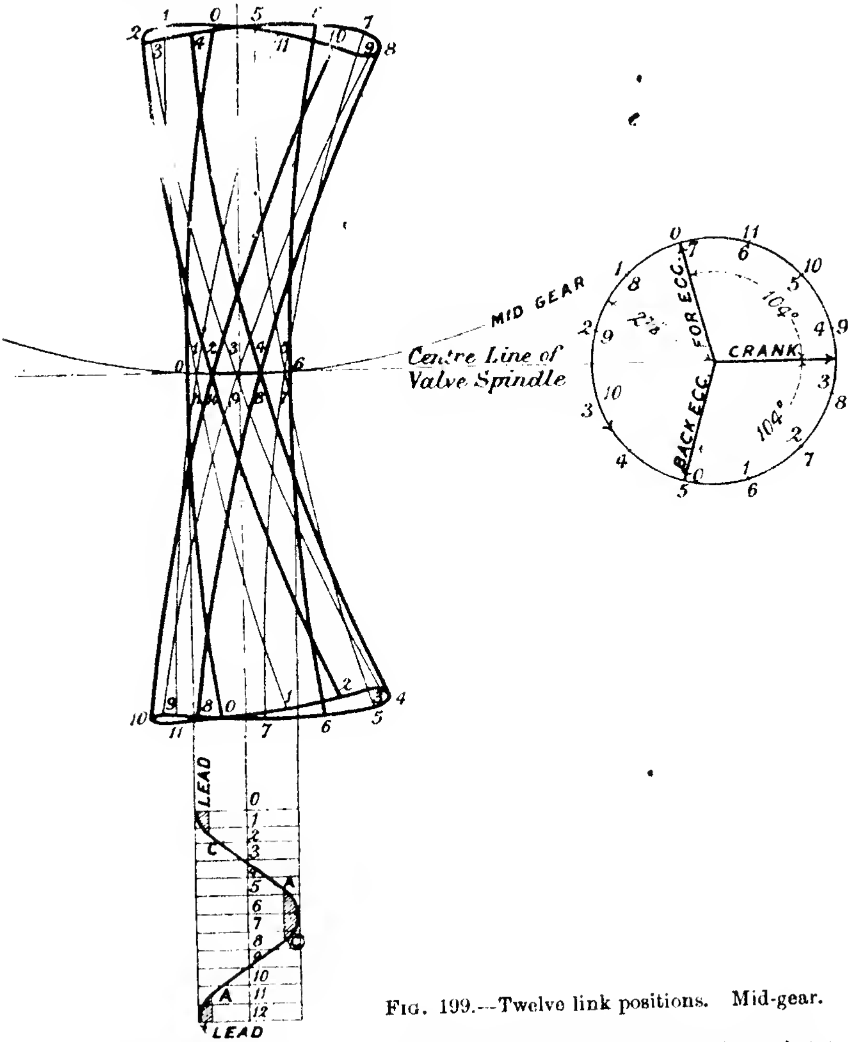


Fig. 199.—Twelve link positions. Mid-gear.

the position of the motion block, and a vertical from this point to the horizontal line in the valve diagram below representing the angle θ, fixes a point on the displacement curve. The process is repeated for twenty-four equiangular positions of the crank for each displacement curve. Twelve mid-gear positions of the link are shown in Fig. 199, together with the corresponding displacement curve

beneath them. The template of the link should always be as large as possible, and the inconvenience of drawing the circular arcs representing the loci of the ends of the connecting rod may be avoided by means of templates cut to the proper curvature. The application of these drawing office methods to the geometrical analysis of mechanisms is fully illustrated in the author's book on Valves and Valve-Gear Mechanisms.

The term **slip** is used to denote the small relative sliding motion between the motion block and the link which occurs during a revolution. In consequence of this the value of u is not quite constant during a revolution, but it passes through its stated value each time the crank passes through its zero position. The amount of slip depends upon the way in which the link is suspended. It is generally inconsiderable in a well-designed link motion.

171. Apparatus for drawing Valve Displacement Curves Mechanically.—The labour involved in drawing out a set of displacement curves is considerable. Many drawing offices are provided with a skeleton link motion, the different parts of which are adjustable, so that it may be arranged to represent any link motion in course of design. Parts representing the connecting rod and piston are combined with the valve gear, so that when the crank is placed at any angle the corresponding position of the piston and the valve may be observed, and thus all the peculiarities of the distribution effected by the gear may be studied. In some cases apparatus is added by means of which displacement curves may be drawn.

Fig. 200 represents an apparatus of a simpler kind than that usually employed for the purpose of investigating the properties of a link motion, which was designed by the author for the Mechanical Engineering Department of the City and Guilds of London Technical College, Finsbury. The eccentricities, angular advances, lengths of the eccentric rods, position of the weigh-bar shaft, and the length of the arm of the weigh-bar shaft which fixes the point of suspension, are all adjustable within reasonable limits. Simplicity is obtained by omitting the connecting rod and piston altogether, a graduated ring being used in order to fix the crank angle corresponding to a given valve position.

What is obtained from the apparatus is a family of rectangular displacement curves, like those, for example, shown in Fig. 195. These curves are drawn on the drum seen to the left of the figure. The piston displacement curve can be quickly and accurately drawn on the diagram, and then the additions of a common vertical axis and the lap lines convert the drawing into a valve diagram from which all the circumstances of the distribution can be found.

172. Theory of the Stephenson Link Motion.—Fig. 201 shows a centre line sketch of the Stephenson link motion in which Oa, Ob are the respective sheaves connected by the rods ae, bE, to

the link Ee. The link is suspended from the point U on the arm of the weigh-bar shaft whose axis is at W. G is the guide for the valve rod. As the crank rotates, the distance u between the centre of the motion block P and the centre of the link Q, remains con-

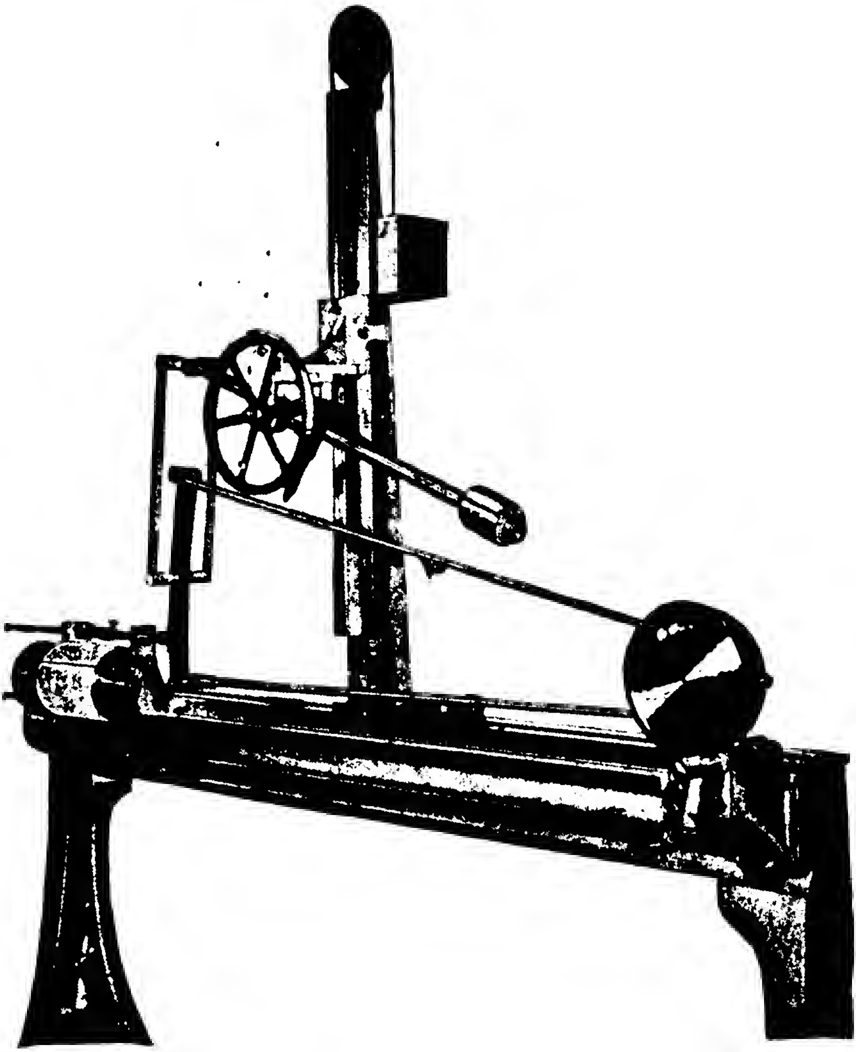


FIG. 200. Apparatus for drawing the displacement curves.

stant to a first approximation. Let l be the common length of the eccentric rods, and let $2c$ be the length of the link. The usual practice is to make the radius of the link equal to the common length l of the eccentric rods.

In the following investigation it is assumed :—

(1) that the point e moves with simple harmonic motion in the straight path e_0O; that the point E moves with simple harmonic motion in the straight path E_0O.

(2) that the inclinations, β and γ, of the two paths to the line of stroke of the valve are small, so that, l being the common length of the eccentric rods, the sines of the angles are given respectively by

$$\frac{Pe}{l} \text{ and } \frac{PE}{l},$$

that is, by

$$\frac{c-u}{l} \text{ and } \frac{c+u}{l}$$

where c represents half the length of the link; and that the cosines of the angles are so nearly unity that $\cos\beta = \cos\gamma = 1$ may be written without introducing sensible error.

(3) and that the value of u remains constant during rotation of the crank.

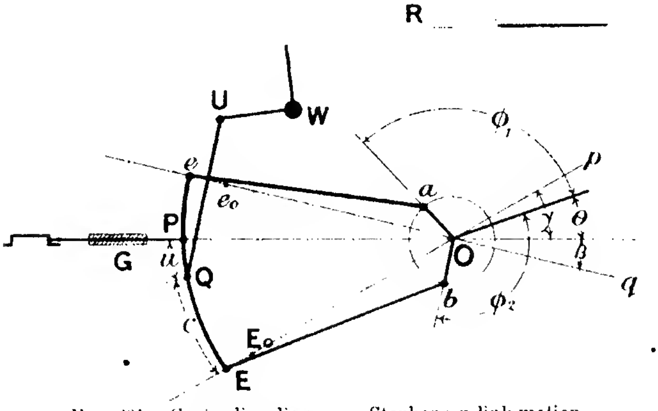


FIG. 201.—Centre line diagram. Stephenson link motion.

Let the gear have the configuration shown in Fig. 201. U is fixed, and u is assumed to have a constant value. The only variable angle is θ.

The angles made by the eccentrics Oa, Ob, with their respective lines of stroke, are aOq and bOp. Expressing these in terms of θ, the angular advances, and the inclination of the lines eO and EO,

$qOa = (\theta + \phi_1 + \beta)$ measured counterclockwise from Oq,
$pOb = 360° - (\gamma + \phi_2 - \theta)$ measured counterclockwise from Op.

The displacement of e, from its central position in its line of stroke is, writing r_1 for Oa,

$$r_1 \cos(\theta + \phi_1 + \beta),$$

and since $\cos\beta$ is taken equal to unity this also represents the displacement in the direction parallel to the direction of motion of P.

The corresponding displacement of P relative to the point E is

$$\frac{c+u}{2c} \times r_1 \cos(\theta + \phi_1 + \beta)$$

Again, the displacement of E from its central position is, writing r_2 for Ob,

$$r_2 \cos(\gamma + \phi_2 - \theta),$$

and since $\cos\gamma$ is taken equal to unity, this also represents the displacement parallel to the direction of motion of P.

The corresponding displacement of P relative to the point e is

$$\frac{c-u}{2c} \times r_2 \cos(\gamma + \phi_2 - \theta)$$

The displacement, x, of P, from its central position, is given by the sum of these separate displacements. That is

$$x = r_1 \frac{c+u}{2c} \cos\{\theta + (\phi_1 + \beta)\} + r_2 \frac{c-u}{2c} \cos\{(\gamma + \phi_2) - \theta\} \quad (1)$$

The displacement x can be calculated from this expression for any assigned value of u and θ.

Expanding this expression, dropping the suffixes for r and ϕ, since these quantities are usually made respectively equal, and writing the terms as factors of $\sin\theta$ and $\cos\theta$, (1) becomes

$$x = r\cos\theta\left\{\frac{c+u}{2c}\cos(\phi+\beta) + \frac{c-u}{2c}\cos(\gamma+\phi)\right\}$$
$$+ r\sin\theta\left\{\frac{c-u}{2c}\sin(\gamma+\phi) - \frac{c+u}{2c}\sin(\phi+\beta)\right\} \quad . \quad (2)$$

Expanding the cosines and sines in the brackets and writing

$$\cos\beta = \cos\gamma = \text{unity},$$

$$x = r\cos\theta\left[\cos\phi - \sin\phi\left\{\frac{c+u}{2c}\sin\beta + \frac{c-u}{2c}\sin\gamma\right\}\right]$$
$$+ r\sin\theta\left[\cos\phi\left\{\frac{c-u}{2c}\sin\gamma - \frac{c+u}{2c}\sin\beta\right\} - \frac{u}{c}\sin\phi\right] \quad . \quad (3)$$

Substituting the value $\frac{c-u}{l}$ for $\sin\beta$ and the value $\frac{c+u}{l}$ for $\sin\gamma$ this expression further reduces to

$$x = r\cos\theta\left\{\cos\phi - \frac{c^2-u^2}{cl}\sin\phi\right\} - r\sin\theta\left\{\frac{u}{c}\sin\phi\right\} \quad . \quad (4)^1$$

This is the simplest form to which it can be reduced.

[1] If $\cos(90+\delta) = -\sin\delta$ and $\sin(90+\delta) = \cos\delta$ are respectively introduced into this equation in place of $\cos\phi$ and $\sin\phi$, it reduces to

$$-r\cos\theta\left\{\sin\delta + \frac{c^2-u^2}{cl}\cos\delta\right\} - r\sin\theta\left\{\frac{u}{c}\cos\delta\right\}$$

Changing the signs all through, this is the form in which the equation was originally given by Zeuner.

The coefficients of $\cos\theta$ and $\sin\theta$ in expression (4) are constants for a particular value of u, neglecting the small variations of u mentioned on page 567. Therefore the displacement of the valve from its central position may be written

$$x = \mathrm{A}\cos\theta - \mathrm{B}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where A and B are constants, having the values

$$\mathrm{A} = r\left\{\cos\phi - \frac{c^2 - u^2}{cl}\sin\phi\right\} \quad . \quad . \quad . \quad (6)$$

$$\mathrm{B} = r\left\{\frac{u}{c}\sin\phi\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

No restriction was put upon the magnitude of the angle ϕ in the preceding investigation, so that the expressions are true for all values of ϕ between 0° and 360°. The expressions (5), (6), (7), apply to all arrangements of the Stephenson link motion, whether with open or crossed rods; inside or outside steam admission; or with a rocking shaft included between the link and the valve, providing that the value assigned to ϕ is the angular advance of the sheave which produces positive rotation of the crank, and that this angular advance is measured from the crank in the positive direction. For example, with open rods and outside steam admission, ϕ lies between 90° and 180°, but if a rocking shaft is interposed between the link and the valve, which changes the phase of the motion 180°, ϕ lies between 270° and 360°.

It will be found that substituting in (6) and (7) the dimensions of the gear of which the valve diagram is drawn in Fig. 195, namely, $l = 50$ ins.; $c = 8{\cdot}25$ ins.; $r = 2{\cdot}875$ ins.; and $\phi = 104\frac{1}{2}°$, so that $\cos\phi = 0{\cdot}25$ and $\sin\phi = 0{\cdot}968$, equation (5) reduces to

$$x = -1{\cdot}042\cos\theta - 1{\cdot}517\sin\theta$$

when the gear is set so that $u = 4\frac{1}{2}$ ins.

This is the equation of valve displacement in terms of the crank angle, and it may be used to plot the approximate displacement curve corresponding to the value of u taken.

Expression (5) is a general form for all types of link motion and for radial gears as well. The coefficients A and B depend upon the type of mechanism used.

In particular, the values of the coefficients A and B for the simple eccentric gear, a gear in which the valve displacement is given by the expression

$$x = r\cos(\theta + \phi) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$$

are

$$\mathrm{A} = r\cos\phi$$
$$\mathrm{B} = -r\sin\phi$$

It follows that the distribution of steam produced by any of the types of gear to which equation (5) applies is the same as the distribution which would be produced by a simple eccentric gear for

which the coefficients A and B in the general equation (5) are found, from

$$A = \rho \cos \psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$
$$B = \rho \sin \psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and $\tan \psi = \frac{A}{B}$

ρ and ψ are respectively the eccentricity and the angular advance of the equivalent eccentric sheave already mentioned on page 559. In a link motion, for example, there is an equivalent eccentric corresponding to every setting of the gear by the reversing wheel or lever. The values of ρ and ψ for each setting are separately determined from the two equations formed by equating the right side of (8) to the right side of (6), and the right side of (9) to the right side of (7). It will be seen that to every setting of the link motion there corresponds a definite value of u, and therefore definite values of ρ and ψ.

It is easy to show that the following rule, due to Mr. Macfarlane Grey, gives the value of ρ and ψ to a near approximation.

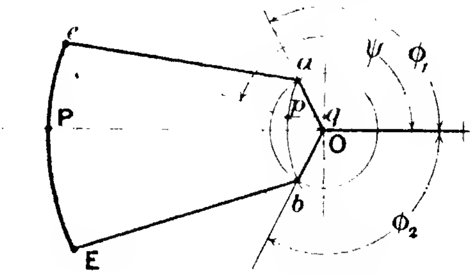


FIG. 202.—Macfarlane Grey's construction.

Let Oa, Ob (Fig. 202) represent the centre lines of the two sheaves of a link motion, and ϕ_1 and ϕ_2 their respective angular advances. Draw an arc of a circle through the points a and b, using the radius R, found from the formula

$$R = \frac{s \times l}{2k},$$

where s is the distance between the points a and b, l is the common length of the eccentric rods, and k is the length of the link eE.

Take a point p in the arc ab so that it divides the arc in the same proportion as the axis of the motion block divides the distance between the end centres of the link. That is,

$$ap : pb = e\text{P} : \text{PE}$$

Then Op is the radius ρ of the "equivalent" eccentric, and ψ is its angular advance.

173. The Design of a Link Motion.—After making a general arrangement, the quantities which remain to be settled by calculation

are the eccentric radii, and their angular advances to give an assigned distribution for the full gear value of u. First by calculation or from a Bilgram diagram find the value of ρ and ψ, giving the distribution defined. From equations (6), (7), (8), (9), pages 571 and 572, in which the eccentric radii and their angular advances are assumed to be equal,

$$\rho \cos \psi = A = r\left(\cos \phi - \frac{c^2 - u^2}{cl} \sin \phi\right) \quad . \quad . \quad (1)$$

$$\rho \sin \psi = B = r\left(\frac{u}{c} \sin \phi\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In these simultaneous equations r is the eccentricity, and ϕ is the angular advance of the eccentric which is to be coupled to the top of the link.

To facilitate the solution of these equations calculate the value of $\frac{c^2 - u^2}{cl}$, using the maximum value of u, and denote it by p. Then calculate the value of $\frac{u}{c}$, using the same value of u, and denote it by q.

Substituting these values in (1) and (2) and dividing (1) by (2) to eliminate r,

$$\cot \phi = q \cot \psi + p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which ϕ can be calculated.

Then r can be found from equation (2).

If the actual eccentrics are made with the eccentricity r and set with respective angular advances $+ \phi$ and $- \phi$ the gear will give approximately the distribution specified in full gear. The distribution for values of u other than the maximum, and, finally, the actual distribution in full gear, must be found from a drawing of the actual displacement curves.

Example.—Design a link motion so that cut-off in full gear occurs at 70 per cent. of the stroke; having given that the lead and the maximum port opening are respectively 0·1 in. and 1 in.

(1) Assume the motion to be of the general character shown in Fig. 192, page 561. The motion block cannot be brought exactly opposite the end of the eccentric rod in full gear, so that the maximum value of u will be less than c. Let this maximum value be 5 ins., let the length of the link be 16 ins., so that $c = 8$ ins., and let the length of the eccentric rod l be 48 ins.

(2) It will be found that with the data given

$\psi = 124° \, 50'$,
$\rho = 2·05$ ins.
Steam lap $= 1·1$ ins.
Crank angle at cut-off $= 113° \, 35'$, neglecting the obliquity of the connecting rod.

(3) Calculating the values of p and q,

$$p = 0{\cdot}101, \qquad q = 0{\cdot}625$$

Using these values in equation (3),

$$\cot \phi = -0{\cdot}333,$$

from which

$$\phi = 108^\circ\ 27'$$

With this value of ϕ in equation (2), $r = 2{\cdot}91$ ins. Hence the eccentricity of the actual eccentrics must be 2·91 ins., and they must be set on the shaft at the respective angular advances, 108° 27′ and − 108° 27′.

The length of the arm of the weigh-bar shaft and the position of its axis may be chosen so that the inequalities of cut-off due to the obliquities of the connecting and eccentric rods may be reduced. The method is geometrical and tentative.

174. Resolution of the Valve Displacement Curve into Two Components.—Consider the eccentric radius Oa, Fig. 203. Resolve it into two components, Oh and Od respectively, perpendicular to and in the direction of the main crank. Then the valve displacement curve produced by the eccentric Oa is the resultant of the valves displacement curves produced by the component eccentrics Oh and Od. The component furnished by Od is a cosine curve whose maximum ordinate is A, Fig. 204, so that Od = $r \cos \phi$ = A; and points on the curve may be calulated from

$$x = \mathrm{A} \cos (\theta + 180) = -\mathrm{A} \cos \theta$$

The component furnished by Oh is a sine curve whose maximum ordinate is B, Fig. 204, so that Oh = $r \sin \phi$ = B; and points on the curve may be calculated from

$$x = \mathrm{B} \sin (\theta + 90) = -\mathrm{B} \sin \theta$$

The component curves together with the resultant curve are shown in Fig. 204. Any ordinate, as O_1a_1, is equal to the sum of the ordinates to the component curves, so that $\mathrm{O}_1a_1 = \mathrm{O}_1h_1 + \mathrm{O}_1d_1$.

When the crank angle is 0 or 180°, the displacement of the valve from its central position is equal to the lap plus the lead, and this displacement is furnished entirely by the component eccentric Od, in fact the displacement is, in these circumstances, equal to the length of the eccentric itself. This condition fixes the amplitude of the A component uniquely.

Again, B is the displacement of the valve from its central position when $\theta = 90°$.

If the angular advance of Oa is changed to an equal negative value in order to reverse the direction of rotation, the new rectangular components are Od and Og. The A component furnished by Od remains unchanged, whilst the B or 90° component changes in sign. Thus the combination of the A component with a 90° component determines positive rotation; and the combination of the A component with a 270° component determines negative rotation.

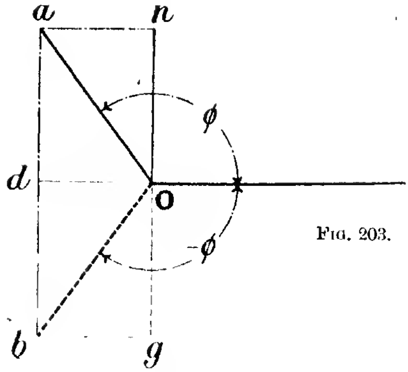


Fig. 203.

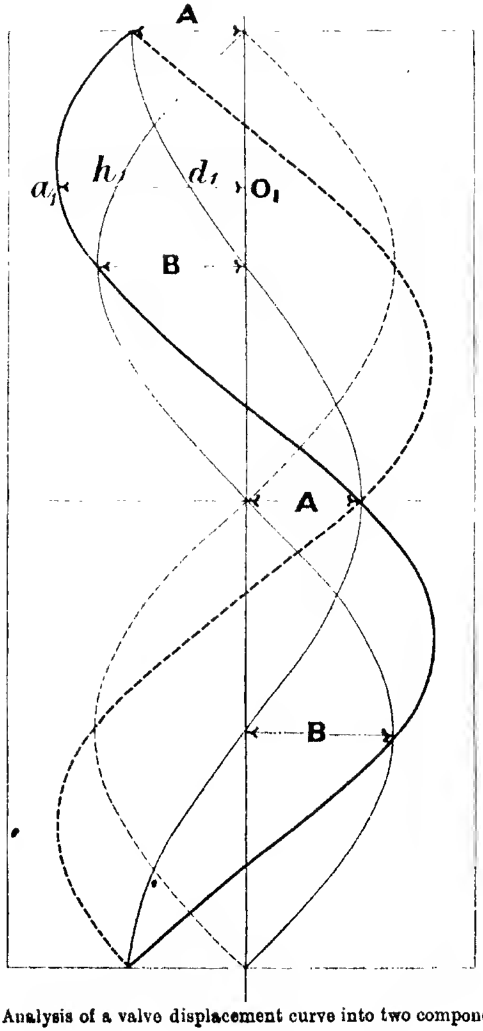


Fig. 204.—Analysis of a valve displacement curve into two component curves.

Also if the A component, constant in maximum value, is combined with either a 90° or a 270° component of smaller maximum value than Oh, the resulting displacement curve will have a smaller maximum value, and will determine cut-off at an earlier fraction of the stroke.

When B is equal to zero, the distribution is determined by the A component alone, and this corresponds to the mid-gear condition. If A is zero, then distribution is only possible if the valve has no lap and is set with no lead. The indicator diagram is then a rectangle.

From these considerations it will be understood that a family of displacement curves, like those shown in Fig. 195 for the link motion, may be described from an A component having a fixed value and a B component whose value ranges from a positive to a negative value.

Valve-gear mechanisms based on the combination of two rectangular components in the direction of, and perpendicular to, the main crank, are usually called **radial gears.** This class of mechanism originated in Hackworth's gear, patented in 1859. The Joy gear is a recent representative of the type. The Walschaerts gear is also included in this class since it is based on the combination of the rectangular components of the actual valve motion, and this is the essential feature of a radial gear.

175. The Walschaerts Reversing Gear.—The Walschaerts gear, sometimes called the Waldegg gear, is shown by a centre line sketch in Fig. 205. The A, or 180° component, is obtained from the crosshead through the link DJV and the swinging link HD. Assuming for the moment that the point J is fixed (it is actually motionless when the B component is zero), it will be seen that the motion of the point V is a copy to a reduced scale of the motion of the crosshead, reversed in phase by the placing of the pivot joint J between V and D.

And since the maximum value of the A component is to be equal to the lap plus the lead the proportions of the link can be established at once. Thus, if R is the radius of the main crank OK,

$$\frac{VJ \times R}{JD} = \text{lap} + \text{lead} = A \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the displacement curve furnished by this component is

$$x_1 = -\frac{VJ \times R}{JD} \cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The B component is furnished by the eccentric sheave Oj, which is keyed at 90° to the main crank. It is connected to a link, free to oscillate about a fixed axis Q. The motion of the eccentric is conveyed to the valve through the link, the valve rod PJ, and through the lever DJV oscillating about the joint D, which for the moment may be regarded as fixed. The valve rod PJ is supported from the end of the arm UW of the weigh-bar shaft, which is centred at W.

Neglecting the obliquity of the eccentric rod jF, the displacement of the point F is given by $Oj \cos(\theta + 90) = -Oj \sin\theta$. The amplitude of this displacement is altered before it reaches the point V, first by the link in the ratio u to c; and secondly by the link VJD, oscillating about the centre D, in the ratio VD to JD.

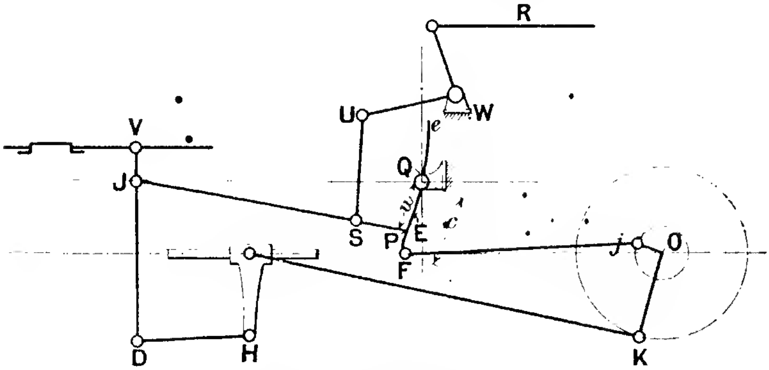


FIG. 205.—Walschaerts valve gear.

Therefore if r is written for Oj, the eccentric radius, the displacement of V is given by

$$x_2 = -\frac{ru}{c} \times \frac{\mathrm{VD}}{\mathrm{JD}} \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

The whole displacement x of the valve is then found by adding the component displacements together, giving:—

$$x = -\frac{\mathrm{VJ}}{\mathrm{JD}}\mathrm{R}\cos\theta - \frac{ru}{c} \times \frac{\mathrm{VD}}{\mathrm{JD}} \sin\theta \quad . \quad . \quad . \qquad (4)$$

The positive direction along the link from Q as origin must be chosen so that a positive value of u produces a B component of the proper sign to combine with A in giving positive rotation.

In the sketch, for example, u is positive when measured down below the centre Q, and negative when measured from Q upwards.

When the block P is brought to the middle of the link $u = 0$, and the amplitude of the B component is reduced to zero; then the A component alone appears in the motion of the valve. When the block P is in an extreme position in the link, the A component is combined with the B component of maximum amplitude.

The value of B, the maximum ordinate of the 90° component of the actual valve displacement curve, is

$$\mathrm{B} = \frac{ru}{c} \times \frac{\mathrm{VD}}{\mathrm{JD}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

These expressions are approximate because the obliquities of all the rods have been neglected, but they bring out the principles on which

the gear is based and may be used for a preliminary examination of an existing gear or for the preliminary design of a new gear.

In order to design a gear of this class a drawing is made to settle the general arrangement, after which the proportions of the lever VJD and the radius Oj of the eccentric are found, so they together determine a valve displacement which will produce a stated distribution of steam in full gear.

The equivalent eccentric corresponding to the given distribution is determined either by calculation or by means of a valve diagram, and then its rectangular components are found, giving:—

$$A = \rho \cos \psi$$
$$B = \rho \sin \psi$$

A is negative for outside admission of steam and positive for inside admission.

These values of A and B are then substituted in expressions (1) and (5).

The joints V and J should be designed first in order to fix the minimum distance between them consistent with the provision of sufficient bearing surface, and then JD is found from (1), after which r is found from (5), after substituting therein the value of u corresponding to full gear, and the length c, as determined by the general arrangement.

A drawing of the gear fitted to the high pressure cylinders of certain of the De Glehn compound locomotives of the Northern Railway of France is shown in Fig. 206. It is similarly lettered with the diagram of Fig. 205.

Special attention should be given to provide a substantial guide G for the valve spindle.

In some cases the valve rod is prolonged through the link, and the point of suspension S is placed on the other side of the link to that in which S is shown in the drawing.

This gear is specially convenient for outside cylinder engines, because the eccentric radius can be provided for by means of a return crank as shown in the drawing, Fig. 206, so that eccentric sheaves are entirely dispensed with.

176. The Hackworth Gear.—A general arrangement of this gear is indicated in Fig. 207. An eccentric OD is placed at 180° with the crank and drives a crosshead or slider through the eccentric rod DJ. The line of stroke of this slider can be fixed at small inclinations on either side of a vertical to the line of stroke of the piston OQ by mechanism which enables the path PP to be turned and clamped. The motion of any point V on the eccentric rod may be analysed into two components. First a component differing in phase with the crank by 180°, the A component; and secondly a component due to the angular oscillation of the eccentric rod about D caused by the horizontal projection of the motion of the slider in its path PP. This component differs in phase from the crank by

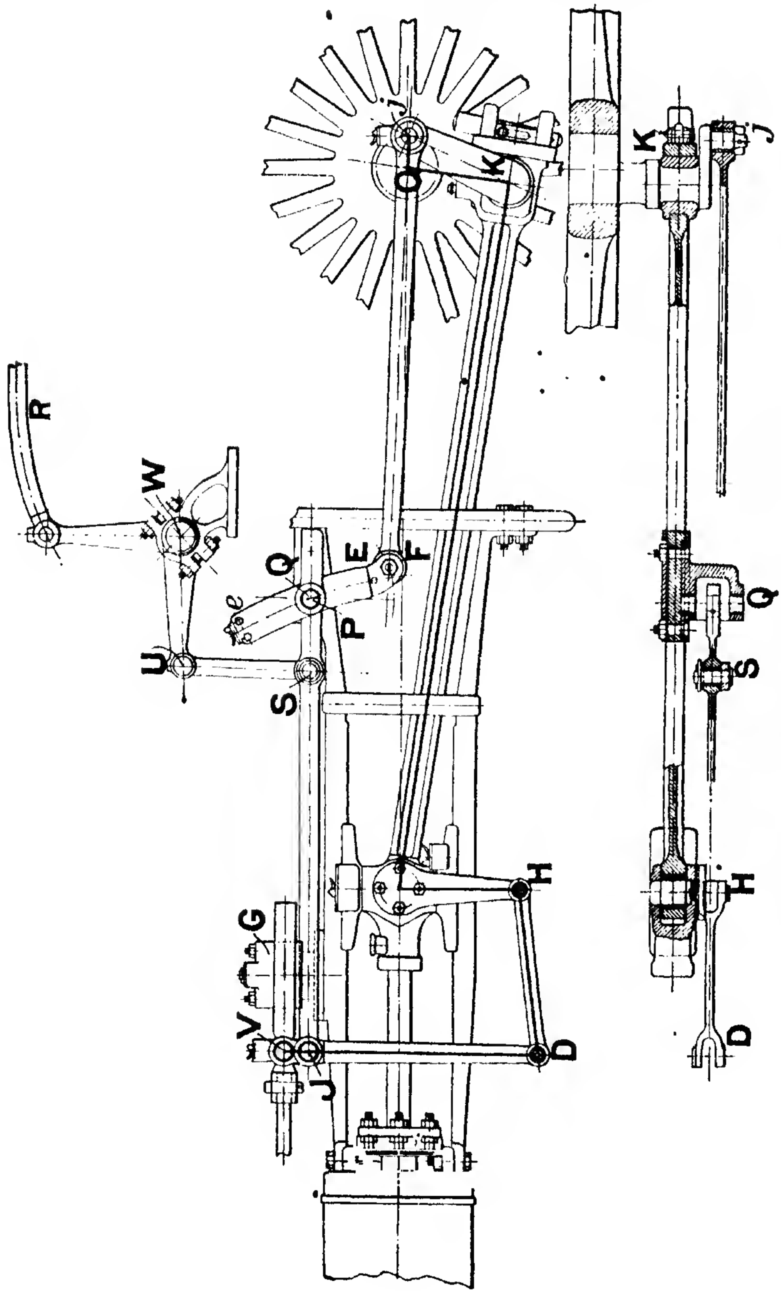


FIG. 206.—Walschaerts valve gear. Northern Railway of France.

90° and is the B component. The respective amplitudes of these components depend upon the position of the point V along the rod. The point must be selected so that the link DJ, turning about J, produces an angular oscillation of sufficient magnitude to make the semi-amplitude equal to the lap plus the lead. That is to say

$$A = \frac{VJ}{DJ} \times OD$$

and the component displacement curve produced by the connection of the point Y to V is

$$-\frac{VJ}{DJ} OD \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

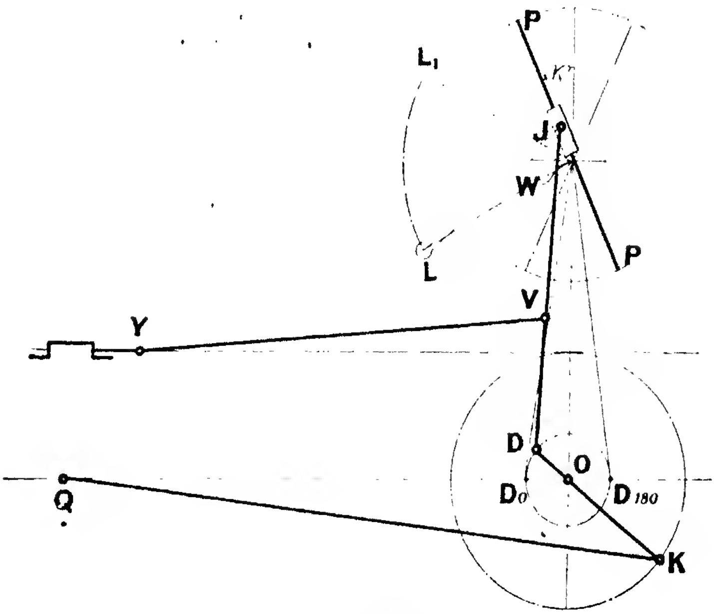


FIG. 207.—Hackworth gear.

Assuming that the fixed axis of the path PP is placed at the mid position of the displacement of J, the vertical displacement of J from its central position in the path is OD sin θ. When the path is inclined κ to the vertical, the horizontal projection of this is

$$\tan \kappa \times OD \sin \theta$$

and reducing this to the point V, turning about D, the 90° component is given by

$$\pm \frac{\tan \kappa \times VD \times OD}{JD} \sin \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The plus sign is prefixed when the upper part of the path is inclined to the left of the vertical, and the minus sign in the opposite case.

The actual valve displacement is then given by the sum of the displacements (1) and (2), and the value of the maximum ordinates of the component valve displacement curve are

$$A = -\frac{VJ \times OD}{JD}$$

and
$$B = \pm\frac{VD \times OD}{JD}\tan\kappa$$

The Marshall[1] gear is a gear of this type in which the straight path PP is replaced by a curved path, and the steam ports are modified, the object being to correct some of the inequalities of distribution. The Hackworth gear has been applied to marine engines in the forms described by Marshall and others. Reference may be made to an article by Mr. Dendy Marshall in the *Engineer* for June 20, 1913, for further information relating to the Hackworth gears.

177. The Joy Gear.—In this gear eccentric sheaves are dispensed with altogether and both the 180° and the 90° components are taken from the connecting rod.

A drawing of the gear is shown in Fig. 208. A link RS is jointed with the connecting rod at R, and with a link FS at S. The object of this link is merely to guide the end of the link RS in an approximately straight line. A second link DJV is jointed with RS at D, with a sliding block at J, and with the valve rod at V. The path PP in which the block at J slides is curved to the radius of the valve rod YJ, and it is centered at W, and it can be turned and clamped by the reversing screw connected to it by the rod and arm shown in the sketch.

The A or 180° component of the valve's motion is furnished by the oscillation of the link DJV about the centre J, and to a first approximation it is clear that the semi-amplitude of this, which must be equal to the lap plus the lead, is given by

$$\frac{VJ \times DS \times OK}{JD \times RS} = A = \text{lap plus lead} \quad . \quad . \quad . \quad (1)$$

and the corresponding component displacement curve is

$$x_1 = -A\cos\theta$$

This represents the actual motion of the valve when the path PP is vertical.

When the path is inclined, the slider constitutes the 90° component, just as in the case of the Hackworth gear, and the corresponding displacement is given by

$$x_2 = \mp\frac{VD \times QR \times OK}{JD \times QK}\tan\kappa\sin\theta$$
$$= \mp B\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The actual motion of the valve is found by adding the components (1) and (2) together.

[1] *Proc. Inst. Mech. Eng.*, 1880.

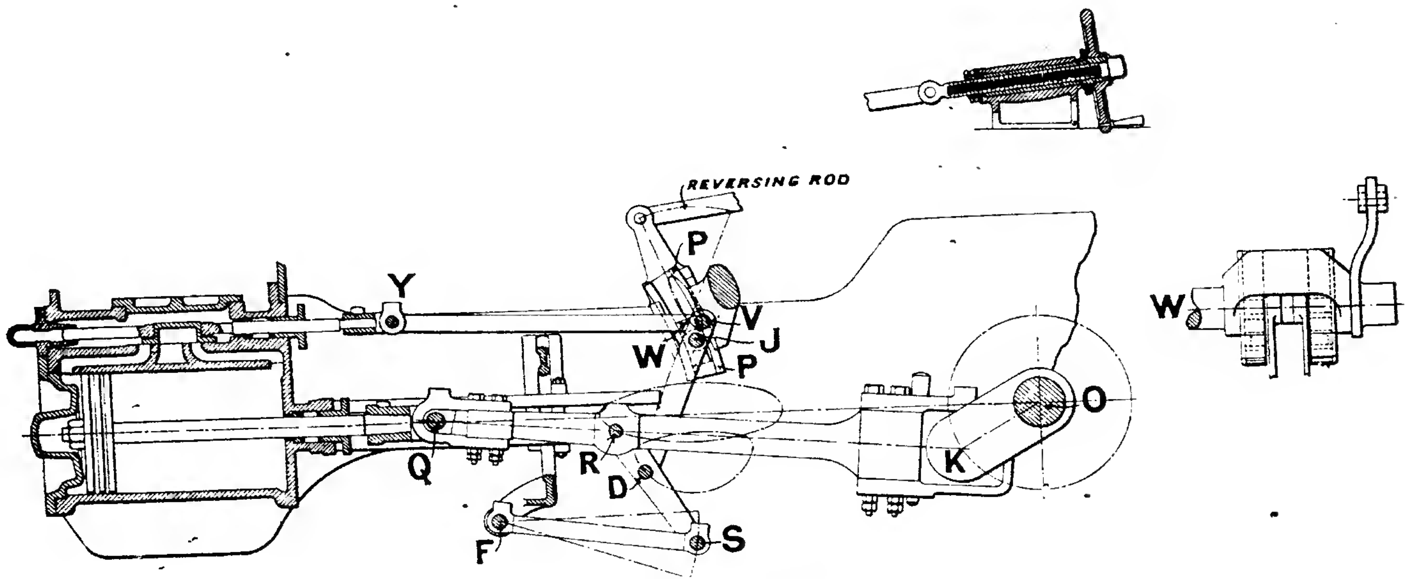


FIG. 208.—The Joy valve gear.

In these expressions A and B are the maximum ordinates of the component valve displacement curves, and these are to be found from the equivalent eccentric corresponding to a given distribution in full gear when a gear is to be designed.

At first sight it might appear that simplicity would be gained by attaching the rod DJV directly to the connecting rod, so saving the link RS. But it is precisely the introduction of this link which produces the regularity of distribution without which a gear would be useless.

It can easily be seen that if the end of DJV is connected directly to the connecting rod the slider will not be in the centre of its path when the main crank is at 0° or 180°, as it should be if a good distribution is to be secured. The link RS corrects this error by introducing an obliquity error in the opposite direction. By properly choosing the point D the error due to the obliquities of DJV and RS mutually cancel.

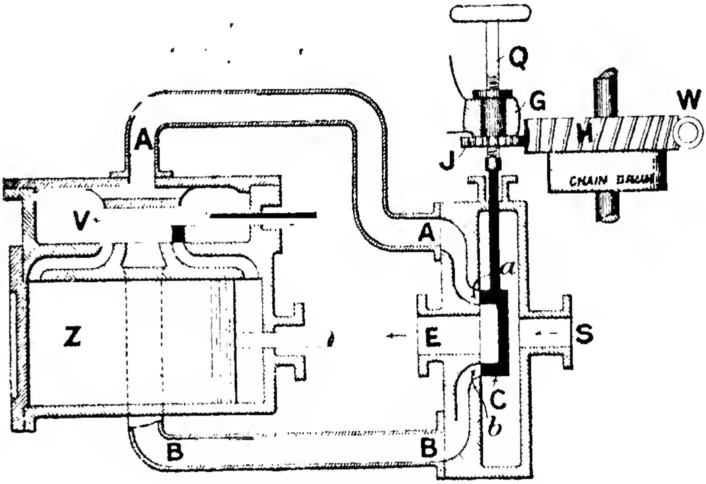
178. Reversing by the Interchange of the Steam and Exhaust.—When an engine is fitted with an eccentric set at 90° to the crank, the steam cannot be cut off until the end of the stroke and the economy incident to expansive working is lost. But another condition of great convenience is gained. For if the steam and the exhaust ports are interchanged, the engine will run in the opposite direction, still of course with admission of steam through the whole length of the stroke. Reversal in this way cannot be done unless the eccentric sheave is set at 90°. It is only necessary therefore to provide a controlling valve which enables this reversal of the direction of the steam supply to be accomplished in order to produce a reversible engine of extreme mechanical simplicity though wasteful considered as a steam engine. But the loss of economy is relatively insignificant in cases where engines, or steam motors, of this kind are used.

A diagram is shown in Fig. 209 of the general arrangement of an engine of this kind. Steam from the boiler is supplied to a supplementary valve chest at S. This chest contains a slide valve C, which can shut off steam altogether, or which can put either the passage A or B in communication with the steam supply, at the same time making a corresponding connection with the exhaust pipe E. The actual slide valve V must be fitted so that it is not pushed off its seat when the steam supply is admitted inside it through the pipe B. Engines of this kind are used in connection with power controlling gears for reversing the main engines of a ship and the combination of a "hunting gear" with this arrangement is used in the steam steering gear.[1]

The tiller chains are wound on the chain drum, and this drum is driven by the worm W and the worm wheel H. The worm W is driven by the engine of which Z is the steam cylinder. Gearing

[1] A description of steam steering gears of varying types will be found in the *Engineer*, June, 1901.

with the wheel H is a small wheel J which is fixed to a sleeve free to rotate in the fixed bearing G. The spindle Q, which operates the controlling valve, passes through this sleeve, engaging with it by means of a screw thread. When the sleeve J is at rest, a slight turn of the spindle Q by hand moves the valve C up or down and starts the engine in one direction or the other. Immediately the engine starts the wheel H begins to turn, and so turns the sleeve J. The rotation of J brings the valve C back to its central position, and unless the spindle Q is turned by hand at exactly the same rate as the sleeve J is turned by the engine, the engine will stop. In an actual steering gear the spindle Q is connected by shafts and gearing to the steering wheel on the bridge.



Reversal by interchange of steam and exhaust pipes.

FIG. 209.—Diagram of steam connections in combination with hunting gear.

The reader is referred to the author's book on "Valve and Valve-Gear Mechanisms"[1] for a more extended treatment of the subjects of this chapter. Valve diagrams are there given for the Walschaerts gear of the Northern Railway of France; for the Allan Link Motion of the London and North-Western Railway; for the Joy gear of the Lancashire and Yorkshire Railway together with rules of design. Statical and dynamical problems relating to mechanism in general, and to valve gears in particular, are also considered, and the problem of the inertia loading brought on to the links of a Joy gear fitted to a locomotive is worked out in detail for a speed of 60 miles per hour.

[1] Published by Edward Arnold, London, 1906.

CHAPTER XI

The Flow of Steam

179. Limiting Velocity of Flow.—When a fluid is allowed to flow freely from one state of temperature and pressure to another state, the available energy due to this difference in state is mainly transformed into kinetic energy.

The flow is not in general steady in the sense that the velocities of all the particles of the fluid are instantaneously equal. There will be eddies and vortices produced by abrupt changes of section, abrupt changes of direction, sharp corners, or obstructions or rough boundaries, so that the energy of motion is stored in the flowing fluid partly as rotational energy and partly as energy of translation. When the passage through which the flow takes place is properly proportioned, the production of eddies and vortices is small, and the kinetic energy stored in the rotational form is negligible in comparison with the kinetic energy corresponding to the mean linear velocity of flow.

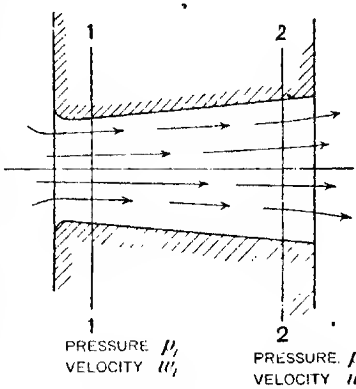


Fig. 210.—Flow of steam.

Part of the available energy corresponding to the state difference is lost by direct radiation of heat and by conduction, and part is lost in overcoming the frictional resistance to the relative motion of the particles of the fluid amongst themselves and against the walls of the passage through which the flow takes place.

Let the figures 1 and 2 in Fig. 210 mark two sections of a pipe through which fluid is flowing freely, and let the temperature at the first section be T_1 and at the second section T_2. Let U be the available energy due to this temperature range. Further, let w_1 be the mean velocity of flow through section 1, and w_2 the mean velocity through section 2. Then per pound of flow

$$U = \frac{1}{2gJ}(w_2^2 - w_1^2) + A + B \quad . \quad . \quad . \quad . \quad (1)$$

A is the heat energy used to overcome the frictional resistance between the sections 1 and 2, and B is the heat lost to the boundary by direct radiation and by conduction.

If it is assumed that there is no loss of heat to the boundary the flow is adiabatic, and B = 0, and if it is further assumed that there is no loss by friction the flow is said to be frictionless, and A = 0. With these assumptions, the whole of the available energy due to the temperature range between the sections is converted into kinetic energy of motion, and the increase of velocity can be calculated from equation (1) after putting both A and B equal to zero.

If the flow starts in section 1, w_1 is zero, and then

$$w_2 = \sqrt{2gJU} = 300\sqrt{U} \text{ ft. per sec.} \quad . \quad . \quad . \quad . \quad (2)$$

The velocity of flow caused by the temperature difference between the sections cannot be greater than this, and in any practical case it will be less, and it may therefore be called the limiting velocity of flow corresponding to the temperature range from T_1 to T_2.

If steam is the fluid, then the state of the steam, whether wet, dry and saturated, or superheated, must also be known as well as the temperature and the pressure, in order to define its initial and final state. The available energy U is the difference between the total energies of the steam in the initial state at section 1 and the final state in section 2. The limiting velocity is then

$$w_2 = \sqrt{2gJ(I_1 - I_2)} = 300\sqrt{I_1 - I_2} \text{ ft. per sec.} \quad . \quad (3)$$

The value of U can be obtained directly from the Total Energy-Temperature diagram or from the Total Energy-Entropy diagram when the initial state of the steam is assigned, and the final temperature or pressure is given, and then the corresponding limiting velocity can be scaled from the velocity scale on the diagram.

Or the total energy may be calculated after adiabatic expansion to the final condition 2 from

$$I_2 = T_2\phi_1 - G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$G = T_2\phi_{w_2} - I_{w_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

both these expressions being quoted from page 191.

Example.—Steam flows freely from a boiler in which the pressure is 150 lbs. per square inch into a vessel where the pressure is maintained at 1 lb. per square inch. Assuming that the flow is frictionless and adiabatic and that the steam is initially dry and saturated, calculate the limiting velocity of flow into the vessel.

The pressure at the last section is the pressure in the vessel, namely, 1 lb. per square inch. The pressure at the first section is the pressure in the boiler, namely, 150 lbs. per square inch. The total energy of the steam per pound as it passes through the first section is from the tables I_1 = 666·49 lb.-calories. The total

energy as it flows through the final section 2 after adiabatic drop of pressure from 150 to 1 lb. per square inch is from the Total Energy-Temperature diagram 489·08 lb.-calories.

The available energy U is therefore

$$666{\cdot}49 - 489{\cdot}08 = 177{\cdot}4 \text{ lb.-cals. per pound of steam}$$

and the limiting velocity is

$$w = 300\sqrt{177{\cdot}4} = 4000 \text{ ft. per sec.}$$

The calculation of I_2 by means of equations (4) and (5) is done as follows:—

From the tables ϕ_{w_2} and I_{w_2} are respectively for 1 lb. per square inch, 0·1323 and 38·63. T_2 is 311·84° C. absolute. With these values in (5), $G = 2{\cdot}62$.

The adiabatic constant ϕ_1 is the entropy of the steam in the dry saturated condition. This from the tables for 150 lbs. per square inch is $0{\cdot}5138 + 1{\cdot}0627 = 1{\cdot}5765$.

Therefore $I_2 = 1{\cdot}576T_2 - 2{\cdot}62 = 489{\cdot}08$ lb.-cals., when 311·84 is substituted for T_2.

180. Relation between Discharge and Area.—The area of a particular section of a pipe or nozzle which must be provided to pass a given flow depends upon the density of the steam at the section and the velocity of flow through the section. Let A_2 be the area in square feet of a particular section through which the velocity of flow is w_2 feet per second. Then, neglecting any water present with the steam, the flow across the section is A_2w_2 cubic feet per second. Let D_2 be the density of the fluid as it passes through the section, and let F be the weight flowing measured in pounds per second; then

$$F = D_2A_2w_2 \text{ lbs. per sec.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The subscript 2 is used to denote a particular section where the pressure has fallen from the initial value to a lower value p_2. No subscript is affixed to F because the total weight of the flow, that is, the weight of steam and condensed water held in suspension, is the same through every section, although, by equation (1), area is specifically provided only for the steam. The volume of water flowing with the steam is, however, negligible in comparison with the volume of the steam. The density of the steam is equal to the reciprocal of the volume per pound. If V_2 is the volume per pound of dry saturated steam at the pressure existing in section 2, and if q_2 is the dryness fraction there, then

$$D_2 = \frac{1}{q_2V_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the flow is assumed to be adiabatic and frictionless, then q_2 can be obtained directly from the Total Energy-Temperature diagram,

or from a Total Energy-Entropy diagram, or it can be calculated as follows. The total energy of a pound of wet steam is

$$I_2 = I_{w_2} + q_2 L_2$$

from which

$$q_2 = \frac{I_2 - I_{w_2}}{L_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Or q_2 may be calculated from equation (1), page 190. With q_2, and with the aid of the steam tables, the density D_2 corresponding to any assigned state can be found, and then equation (1) can be used to find the area for a given discharge F, or to find the discharge per square foot.

Example.—Steam flows freely from a boiler in which the pressure is 150 lbs. per square inch into a vessel where the pressure is maintained constant at 1 lb. per square inch. Assuming adiabatic and frictionless flow, calculate the limiting discharge per square foot through the last section of the pipe connecting the vessel with the boiler. $\omega_2 = 360$

The pressure at the last section of the connecting pipe is the pressure in the vessel, namely, 1 lb. per square inch, corresponding to 38·74° C. The temperature in the boiler is 181·31° C. The limiting velocity corresponding to this temperature difference, as calculated in the last section, is 4000 ft. per sec. This is the velocity of discharge into the vessel.

From the last section $I_2 = 489·08$. From the steam tables $I_{w_2} = 38·63$; $L_2 = 573·83$; $V_2 = 333·12$.

Substituting in (3) above,

$$q_2 = \frac{489·08 - 38·63}{573·83} = 0·7848$$

And $q_2 V_2 = 261·3$.

Therefore from (1)

$$F = \frac{4000}{261·3} = 15·3 \text{ lbs. per sq. ft. per sec,}$$

$\frac{\omega_2}{q_2 V_2} = \omega_2$ $= \omega_2$

A being 1 (ft)

181. Form of Nozzle for Frictionless Adiabatic Flow.—The areas of successive sections of a nozzle are determined by the condition

$$F = DAw,$$

which shows that the areas must be proportioned so that

$$A = \frac{F}{Dw} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The problem of finding the form of a nozzle therefore narrows down to the investigation of the way in which Dw, the product of the density and the velocity, varies as the flow along the nozzle proceeds.

Before the form of the nozzle can be found some assumption must be made regarding the relation between the pressure and the length of the nozzle. To begin with, it is convenient to assume that the pressure falls uniformly as the length increases. Then at any cross-section of the nozzle distant x from the entry the pressure can be calculated when the initial pressure and the back pressure, against which the nozzle discharges, are both given, after which the velocity of flow across the section, the density of the steam as it passes through the section, the product Dw, and the area required for a stated flow F, can be immediately calculated by the methods already illustrated in the two previous sections. The areas corresponding to a series of values of x then determine the form of the nozzle for the condition of frictionless adiabatic flow.

Stating a definite problem, find the longitudinal section of a nozzle connecting a boiler in which the pressure is 150 lbs. per square inch with a vessel in which the pressure is maintained at 1 lb. per square inch, assuming that the flow is 1 lb. of steam per second; that the flow is frictionless and adiabatic; and that the pressure falls uniformly along the axis of the nozzle.

Set out the nozzle axis, OP, Fig. 211. Since the pressure is assumed to fall uniformly along the axis, the line OP will also be a scale of back pressure on which O is 150 lbs. per square inch and P is 1 lb. per square inch.

Let it be required to find the area of the nozzle at the point B in the nozzle axis. The back pressure at the point B is read off from the pressure scale (it is 60 lbs. per square inch), and the state of the steam after adiabatic flow from the initial state to the back pressure is determined by calculation or from a chart.

It is then possible to calculate at the back pressure B:—

the available energy U;
the limiting velocity w;
the density $D = \dfrac{1}{qV}$;
the product Dw = the discharge per square foot per second;
the area per lb. of flow, $A = \dfrac{1}{Dw}$;
the diameter corresponding to A.

Referring to Fig. 211, E is a point on the curve of limiting velocity, and the ordinate BE represents the velocity w when the back pressure has fallen to B.

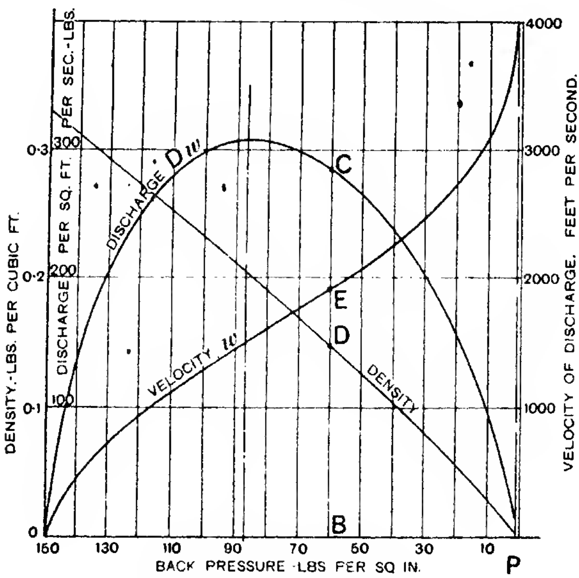
Similarly, D is a point on the density curve, and the ordinate BD represents the density when the back pressure has fallen to B.

Finally, C is a point on the curve showing the discharge per square foot of section in pounds per second, and the ordinate BC represents this discharge when the back pressure has fallen to B.

The calculated diameter, dd, is set out in Fig. 212, and d is thus a point on the curve showing the longitudinal section of the nozzle.

Table 30 gives the results of the calculations made with the series of back pressures shown in the first column.

FIG. 211.—Frictionless adiabatic flow of steam.



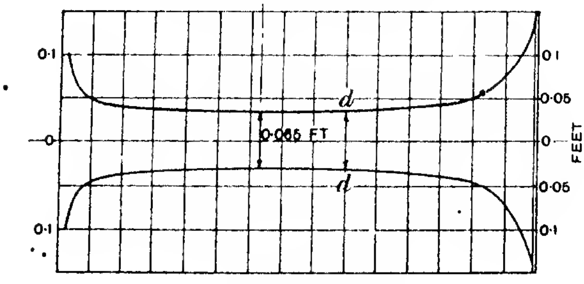


FIG. 212.—Section of nozzle.

The corresponding limiting velocity curve, the density curve, and the discharge curve are plotted in Fig. 211, and the form of the nozzle is shown in Fig. 212.

A remarkable result is disclosed by these curves, namely, that the nozzle first decreases in area to a minimum value, and then increases until the final area is large enough to reduce the adiabatic pressure

to the pressure maintained in the vessel into which the discharge is made.

The flow through each section is the same, namely, 1 lb. per second, so that the flow per square foot of area is a maximum at the smallest section. This is obvious from the curve (Fig. 211) showing the product Dw, which gives the flow per square foot per second.

TABLE 30.—NOZZLE AREAS FOR THE FRICTIONLESS ADIABATIC FLOW OF 1 LB. OF STEAM PER SECOND FROM AN INITIAL PRESSURE OF 150 LBS. PER SQUARE INCH.

Back pressure. Lbs. per sq. in.	U. Lb.-calories.	w. Velocity. Ft. per sec.	$D = \frac{1}{qV}$. Lbs. per c. ft.	Dw.	$A = \frac{1}{Dw}$.	Diameter for flow. 1 lb. per sec.	Proportional diameters.
					Sq. ft.	Ft.	
150	0	0	1/3·041				
130	6·0	735	1/3·448	213·1	0·004691	0·07729	1·2
110	13·8	1115	1/3·985	279·8	0·003575	0·06746	1·046
90	23·4	1452	1/4·742	306·2	0·003266	0·06448	1·000
80	28·6	1606	1/5·255	305·6	0·003273	0·06455	1·001
60	40·7	1914	1/6·759	283·2	0·003524	0·06705	1·04
40	57·2	2260	1/9·661	234·0	0·004275	0·07377	1·144
20	83·6	2746	1/17·82	154·1	0·006489	0·09088	1·41
10	107·8	3118	1/33·01	94·48	0·01059	0·1161	1·802
8	115·8	3231	1/40·35	80·07	0·01248	0·1261	1·957
6	124·8	3355	1/52·01	64·49	0·01551	0·1405	2·18
4	136·3	3506	1/75·08	46·69	0·02142	0·1651	2·561
2	158·0	3775	1/139·7	27·02	0·03702	0·2171	3·37
1	177·4	4000	1/261·3	15·31	0·06516	0·2880	4·475

U = available energy in lb.-calories corresponding to the pressure drop from dry saturated steam at 150 lbs. per square inch to the back pressure in column 1.

w = velocity of flow in feet per second.

D = density of flow in lbs. per cubic foot.

Dw = discharge in pounds per second per square foot of section.

A = area of section in square feet for a discharge of 1 lb. per second.

d = corresponding diameter in feet.

The proportional numbers are obtained by dividing the diameters given by the smallest diameter. The corresponding nozzle would have a throat whose diameter is 1 ft., and the discharge would be 240 lbs. per second.

The diagram (Fig. 212) gives the shape of the nozzle for frictionless adiabatic flow from 150 lbs. per square inch to any lower back pressure. The part of the whole nozzle shown in Fig. 212 included between a vertical through the origin O and a vertical through any assigned back pressure, as B, is the longitudinal section of the nozzle required. The velocity of discharge and the density and the value of the product Dw are also severally determined by the intercepts on the vertical through the assigned back pressure. If, for example, the discharge is to be made against a back pressure in the vessel of 60 lbs. per square inch, the nozzle would end at the section dd and the final velocity of discharge into the vessel would be BE.

The nozzle sketched in Fig. 212 expands the steam adiabatically down to a back pressure of 1 lb. per square inch, a pressure equal to the pressure maintained in the vessel into which the nozzle discharges.

What will happen if the back pressure to which the steam falls in the last section of the nozzle is different from the pressure in the vessel into which the steam is discharged? It has been found by experiment that when steam flows through a nozzle a state of flow approximating very nearly to frictionless adiabatic flow establishes itself automatically so that neglecting losses it may be assumed that the standard theoretical flow through any nozzle is frictionless and adiabatic. The pressure in the final section of the nozzle is therefore not the pressure maintained in the vessel into which the discharge is made, but the adiabatic pressure corresponding to the area of the last section of the nozzle. If this adiabatic pressure is greater than the pressure in the vessel there is a drop of pressure at discharge with resulting loss of energy. For example, suppose that the nozzle drawn in Fig. 212 stops at the section *dd* whilst the pressure in the vessel is still maintained at 1 lb. per square inch. The pressure in the last section of the nozzle is 60 lbs. per square inch and there is a drop of pressure at discharge from 60 to 1 lb. per square inch. The discharge through the nozzle will be unaltered, but the velocity of discharge into the vessel will now be only 1914 ft. per second instead of 4000 ft. per second, the limiting velocity corresponding to the whole pressure range. If also the pressure in the vessel is higher than the lowest adiabatic pressure in the nozzle, the flow will still be 1 lb. of steam per second, with a velocity of discharge of 1914 ft. per second. In the cases both of positive and negative drop at the end of the nozzle there is a loss of energy due to the formation of waves. When, however, there is no pressure drop at the last section the discharge from the nozzle is made steadily without the production of waves. These conditions of discharge have been verified experimentally by Professor Stodola, and particulars of his experiments on this point will be found in his book on Steam Turbines.

The pressure at the throat of the nozzle, Fig. 212, is 87 lbs. per square inch. This is 0·58 the initial pressure. If curves similar to those in Fig. 211 are plotted, using different values of the initial pressure, it will be found that in every case the ratio between the pressure in the section of minimum area and the initial pressure is about 0·58. This, together with other general expressions relating to frictionless adiabatic flow, will be demonstrated in a general way in the next section.

Meanwhile, consider the form of the longitudinal section shown in Fig. 212. The two sections of the nozzle which determine the conditions of discharge are the final section and the throat section. The form of the nozzle from entry to throat and from throat to exit was determined from the initial assumption that the fall of pressure along the nozzle was proportional to the length of the nozzle. If entry, throat, and exit sections are connected by a different form of longitudinal section the relation between the pressure and the length is changed, but by the principle that whatever the form of longitudinal section the conditions of adiabatic flow establish themselves automatically, the pressure at each section of the nozzle is

determined by the area of the section alone. If the flow were truly frictionless and adiabatic the longitudinal section of the nozzle between entry, throat, and final section would be immaterial. Nozzles of any form with equal throat and equal final areas would discharge equal quantities at equal velocities on the same pressure range.

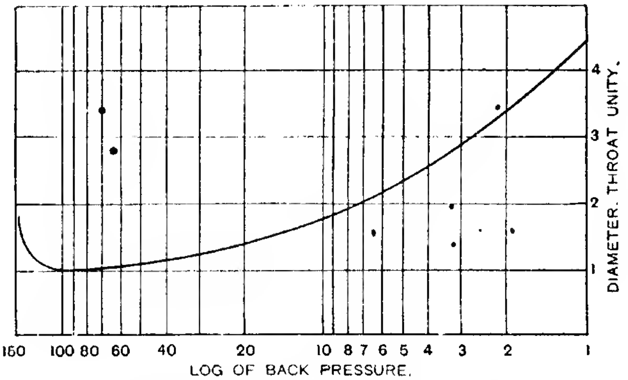


FIG. 213.—Proportional numbers for nozzle design.

The modifying effect of friction and heat loss produce differences in the efficiencies between nozzles of different lengths and shapes. The object to be aimed at is to reduce the frictional resistance by keeping the nozzle as short as possible, but at the same time to allow a gradual enlargement of area to the back pressure, so that the formation of eddies or waves is avoided.

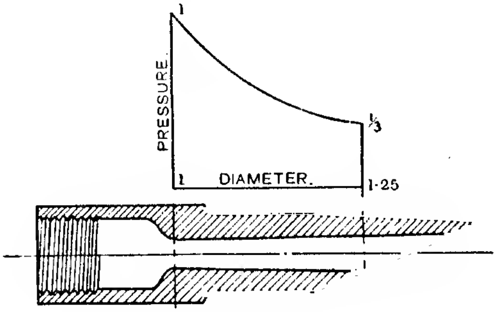


FIG. 214.—De Laval nozzle.

The length from entry to throat in the form shown in Fig. 212 is unduly long. A better shape is obtained by plotting the sections against the logarithm of the back pressure as suggested by Rateau,[1] in the way shown in Fig. 213. It will be seen that the length from the entry to the throat is reduced in this form and that the expansion from the throat to the back pressure is made more gradually than in the case of the nozzle, Fig. 212. It is a small step from this form to the more usual form in which the part of the

[1] "Experimental Researches on the Flow of Steam through Nozzles and Orifices." A. Rateau. A. Constable & Co., London, 1904.

nozzle after the throat, enlarges simply as a cone, the form of nozzle introduced by Laval in connection with the turbine which bears his name. A section of the Laval nozzle is shown in Fig. 214. The entry narrows to the throat by a short smooth curve and then the area increases uniformly with the length. The end of the nozzle is cut off obliquely because it is designed to discharge a jet of steam against a turbine wheel in a direction inclined to the plane of the wheel. The relation between the pressure and the length is shown by the diagram above the nozzle.

182. General Relations relating to Frictionless Adiabatic Flow.—The available energy U is represented by the area of the pressure-volume diagram drawn for one pound of the fluid expanding adiabatically from the higher to the lower temperature, as already explained on page 195.

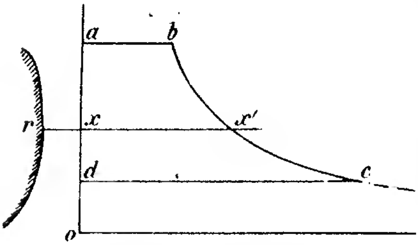


FIG. 215.—Pressure-volume diagram and flow.

For U is the difference between I, the total energy of the fluid in the initial state, and I_2, the total energy in the final state.

Let *ab*, Fig. 215, represent the volume of one pound of fluid at the pressure O*a*. Then its initial state is represented by the point *b*, and its total energy in the state *b* is represented by the area under the line *ab*, the indefinitely prolonged adiabatic curve *bc* and the axes.

Similarly the total energy of the fluid in its final state is represented by the area under the line *dc*; the indefinitely prolonged adiabatic curve through *c*, that is, the adiabatic curve *bc* prolonged; and the axes. The difference is the area of the pressure-volume diagram *abcd*.

Let Z stand for the area of the diagram measured in foot-pounds, then

$$Z = JU$$

and the limiting velocity of flow w can be calculated from

$$w = \sqrt{2gZ} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The velocity can be calculated for any intermediate value of the back pressure, as O*x*, because the area of the diagram above the horizontal line of constant pressure *xx'* represents the available energy between the initial state *b* and the final state *x'*.

When the diagram is actually drawn, the area down to any given back pressure can be measured directly by a planimeter or by any other convenient method.

Thus when the back pressure is O*x* the area *xabx'* represents the

kinetic energy of flow when the pressure has fallen from Oa to Ox, and this area is to be substituted for Z in the above equation. Also $\frac{1}{xx'}$, the reciprocal of the volume, is the density corresponding to the back pressure Ox. Then the flow per square foot of nozzle area is

$$F = \sqrt{2gZ} \times \frac{1}{xx'}$$

From this the area required for a given flow F is

$$A = \frac{F}{\sqrt{2gZ} \times \frac{1}{xx'}} \text{ sq. ft.}$$

The radius xr corresponding to this area is set out to the left of the vertical pressure axis. If this is done for a series of back pressures the nozzle shape determined is exactly the same as that given in Fig. 212.

In order to obtain general relations the area Z must be expressed analytically. The adiabatic curve bc is, for any fluid, represented with close approximation by an equation of the form

$$pv^n = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The index n has a nearly constant value for a particular fluid. With an equation of this form for the adiabatic curve the area between the initial pressure p_1 and the back pressure p_2 is given by

$$Z = \int_{p_2}^{p_1} v\,dp = p_1^{\frac{1}{n}} v_1 \int_{p_2}^{p_1} \frac{dp}{p^{\frac{1}{n}}}$$

$$= p_1^{\frac{1}{n}} v_1 \left\{ \frac{p_1^{1-\frac{1}{n}} - p_2^{1-\frac{1}{n}}}{1 - \frac{1}{n}} \right\}$$

$$= \frac{n}{n-1} \left\{ 1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \right\} p_1 v_1 \quad . \quad . \quad . \quad . \quad (3)$$

Substitute this expression for Z in equation (1). Then the limiting velocity can be calculated when the initial pressure, the pressure ratio $\frac{p_2}{p_1} = r$, and the initial volume per pound v_1, are given.

Again, the density D_2 after adiabatic expansion to p_2 is the reciprocal of the volume v_2. Thus, from (2)

$$D_2 = \frac{1}{v_2} = \frac{1}{v_1}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The flow per square foot of nozzle area in pounds per second is then

$$F = w_2 D_2$$

and substituting the value of w derived from (1) and (3) and the value of D from (4), and writing r for the ratio $\frac{p_2}{p_1}$, this product becomes

$$Dw = \sqrt{\left(2g \cdot \frac{n}{n-1} \cdot \frac{p_1}{v_1}\right)} \sqrt{\left(r^{\frac{2}{n}} - r^{\frac{n+1}{n}}\right)} \quad . \quad . \quad (5)$$

For given initial conditions the quantity under the first root sign is constant. Dw is therefore a maximum when the quantity under the second root sign is a maximum.

Differentiating the quantity under the second root sign with regard to r and equating to zero, Dw is a maximum when

$$\frac{2}{n} \cdot r^{\frac{2-n}{n}} - \frac{n+1}{n} \cdot r^{\frac{1}{n}} = 0$$

which gives

$$r = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

as the ratio between the back pressure and the initial pressure for which the discharge per square foot of area is a maximum.

When the fluid is steam the value of the index n in the adiabatic equation is 1·3 if the steam is initially superheated and remains superheated down to the back pressure.

If the steam is initially dry and saturated or wet, the value of n depends upon both the initial and the final pressure. Its value can be calculated from

$$n = \frac{\log \frac{p_1}{p_2}}{\log \frac{q_2 V_2}{q_1 V_1}}$$

an expression quoted from page 192, equation (5)

In this expression q_1 is the initial dryness, which is unity if the steam is initially dry and saturated, and q_2 is the dryness after adiabatic expansion to the pressure p_2. V_1 and V_2 are respectively the tabular volumes corresponding to the initial pressure p_1 and the back pressure p_2.

The index n calculated from this equation determines a curve which passes through two points on the real adiabatic curve, namely, the points corresponding to the pressures p_1 and p_2. It will be found that in the case of dry saturated steam expanding adiabatically to one-half its initial pressure, n varies from 1·153 to 1·11 when the initial pressure varies from 200 to 10 lbs. per square inch.

From 150 to 75 lbs. per square inch the calculated index is 1·15.

Therefore for calculations relating to the flow of steam through the throat of a nozzle an average value of 1·15 may be taken to be the constant value of the index.

Using this value of n in equation (6)

$$r = 0{\cdot}574 = 0{\cdot}58 \text{ approximately.}$$

This result generalizes for all initial pressures the result found by plotting the curves, Fig. 211, page 590, corresponding to an initial pressure of 150 pounds per square inch, namely, that the area is a minimum when the back pressure has fallen to 0·58 of the initial pressure or thereabouts. Generally, therefore, whatever be the initial pressure p, the pressure found in the throat of the nozzle will be very nearly $0{\cdot}58p$.

Introducing, $n = 1{\cdot}15$, and $\frac{p_2}{p_1} = r = 0{\cdot}574$ into equation (3), and then using the value of the area Z so found in equation (1), the expression for the velocity of flow through the throat of a nozzle, that is, the section of minimum area, is

$$w_0 = 5{\cdot}86\sqrt{p_1 v_1} \text{ ft. per second} \quad . \quad . \quad . \quad . \quad (7)$$

In this expression p_1 is expressed in pounds per square foot, and v_1 in cubic feet. If p_1 is expressed in pounds per square inch the constant is changed so that

$$w_0 = 70{\cdot}4\sqrt{p_1 v_1} \text{ ft. per second} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

With the same values of r and n in equation (4) the corresponding value of the density is

$$D_0 = 0{\cdot}6177\frac{1}{v_1} \text{ lbs. per cubic foot} \quad . \quad . \quad . \quad . \quad (9)$$

The discharge per second per square foot of the minimum area is then found, either by multiplying together the expressions for w_0 and D_0, or by direct substitution of n and r in equation (5). In either case the result is the same, and the discharge is

$$F_0 = D_0 w_0 = 3{\cdot}62\sqrt{\frac{p_1}{v_1}} \text{ lbs. per sq. ft. per second} \quad . \quad (10)$$

In this expression p_1 is in pounds per square foot. Expressed in pounds per square inch the constant is changed, so that

$$F_0 = 43{\cdot}4\sqrt{\frac{p_1}{v_1}} \text{ lbs. per sq. ft. per second} \quad . \quad . \quad (11)$$

A very simple approximate expression may be found for the discharge across the minimum area in the following way. Multiply the numerator and the denominator of the quantity under the root sign in equation (11) by p_1. Then

$$F_0 = 43{\cdot}4\, p_1\sqrt{1/p_1 v_1} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The term $\sqrt{1/p_1v_1}$ would be constant if the product p_1v_1 were constant. As may be tested by the aid of the steam tables the value of the term varies very little for wide variations in the value of p_1. For example, when p_1 is 200 lbs. per square inch the value of the term is 0·0464, and this only changes to 0·0476 when p_1 is changed to 100 lbs. per square inch. Selecting therefore 0·047 as a mean value and using it in (12)

$$F_0 = 43{\cdot}4 \times 0{\cdot}047 \times p_1 = 2{\cdot}04p_1 \quad . \quad . \quad . \quad . \quad (13)$$

Even if the initial pressure is as low as 10 lbs. per square inch the factor increases only to 0·0508, giving $F_0 = 2{\cdot}20p_1$.

These formulæ have been obtained on the assumption of frictionless adiabatic flow. In practice the discharge would be somewhat less than this. With this in mind, expression (13) shows that the discharge per second per square foot of the minimum area can be estimated with considerable accuracy by merely doubling the initial pressure when the pressure is given in pounds per square inch.

For example, if the initial pressure is 150 lbs. per square inch the discharge per second through the section of minimum area at the throat of a nozzle is approximately 300 lbs. per square foot.

This may be tested by a reference to Table 30, page 591, where the maximum discharge, that is the discharge per second per square foot of minimum area, shown in the column headed D*w*, is just over 300.

If the table had been calculated for an initial pressure of 75 lbs. per square inch, then it would have been found that the maximum discharge in the table would have been a little over 150 lbs. per second per square foot at the throat.

When the back pressure is greater than 0·58 of the initial pressure the expressions for the discharge per square foot of minimum area do not apply. In these circumstances the velocity, density, and discharge from the final or smallest section of the nozzle must be calculated respectively from (3) and (1); (4); and (5), and the value of r to be substituted in (5) is the actual ratio between the given back and the given initial pressures. Or the calculations may be made by the processes already exemplified in Sections 180 and 181, pages 587 *et seq.*

Experiments[1] show that the flow from a convergent nozzle is slightly greater than the flow calculated from the assumptions made in the preceding investigation. The difference is, however, only of the order 1 per cent. In the discussion of a paper by Prof. Henderson,[2] Prof. Stodola[3] pointed out that the discrepancy was probably due to the steam being supersaturated at the throat of the nozzle; and Prof. Callendar[4] has recently shown that the difference is accounted for by

[1] Rateau, "Experimental Researches on the Flow of Steam". A. Constable & Co., London, 1904.

[2] "Theory and Experiment in the Flow of Steam through Nozzles," *Proc. Inst. Mech. Eng.*, Feb., 1913.

[3] *Proc. Inst. Mech. Eng.*, Feb., 1913, p. 313.

[4] "Flow of Steam through a Nozzle," *Proc. Inst. Mech. Eng.*, 1915.

this hypothesis. We proceed to apply Callendar's correction for supersaturated flow to an expanding nozzle. It has been tacitly assumed above that the steam condenses without time lag, notwithstanding that the time taken to reach the throat of the nozzle is only of the order 0·0001 second. The probability is that the steam flows through the throat and onwards into the expanded part of the nozzle in a supersaturated condition. It continues in this state, falling in temperature and pressure, until condensation can no longer be delayed. Condensation then takes place rapidly and the heat derived from the water formed increases the temperature and the volume of the steam, and equilibrium is obtained when the temperature has risen to the saturation temperature corresponding to the pressure. The change from the supersaturated to the normal condition takes place gradually at no well-defined pressure. The effect of this supersaturated state on the size of the nozzle is conveniently studied by assuming that the flow remains supersaturated beyond the throat up to a section where the pressure has fallen to about one-third of the initial pressure, and then to assume that condensation takes place suddenly and continues until the temperature has risen to the temperature of saturation, after which the flow proceeds normally. The areas of the part of the nozzle which contains the throat on the side of this section are then proportioned for supersaturated flow. The areas of the enlarged part of the nozzle on the other side of the section are proportioned for frictionless adiabatic flow from the conditions as regards wetness which result after condensation has taken place. The section at which condensation is assumed to take place has thus two calculated areas, the one for supersaturated flow through it, the other corresponding to the conditions after the supersaturated condition has changed by condensation into the normal condition. A smooth curve joining the two parts of the nozzle produces a mean sectional area and modifies the areas of the two parts joining in the section so that the real conditions of the change are more nearly provided for.

The design of a nozzle taking into account this condition of supersaturation, reduces to the calculation of the area of the throat assuming supersaturated flow; the selection of a pressure at which the change from the supersaturated to the normal state takes place; the calculation of the state of the steam at this section after the equilibrium state is attained; and then the calculation of the area of the final section for frictionless adiabatic flow from the state found after condensation at the section of discontinuity.

It has been shown on page 193 that the relation between the pressure and the volume for superheated steam expanding adiabatically is

$$p(v - b)^{1\cdot 3} = \text{a constant}$$

The equation applies equally to the adiabatic expansion of supersaturated steam. The volume, $b = 0\cdot016$ cubic foot, is so small in relation to the volume v that it becomes negligible if v is at all large, so that the relation for the flow of the steam up to the section at

which supersaturation changes to the normal flow becomes with sufficient accuracy

$$pv^{1\cdot 3} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Introducing this index for n in equation (6), page 596, the ratio between the pressure in the throat and the initial pressure is

$$\frac{p_0}{p_1} = r = 0{\cdot}5457 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Substituting this value of r for $\frac{p_2}{p_1}$, together with the value 1·3 for n in equation (3), page 595, the velocity of the flow at the throat is

$$w_0 = 6{\cdot}02\sqrt{p_1 v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

And substituting these values of r and n in equation (4), page 595, the density at the throat in lbs. per cubic foot is

$$D_0 = 0{\cdot}628\frac{1}{v_1}$$

Remembering that p_1 is expressed in pounds per square foot, the discharge per second in lbs. per square foot of the minimum area at the throat is then

$$F_0 = D_0 w_0 = 3{\cdot}78\sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The throat area required for a flow of F lbs. per second is then calculated from

$$A_0 = \frac{F}{F_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Let the pressure at the section where the supersaturated state changes to the normal state be p_d. Then the volume per lb. of steam in the supersaturated condition can be calculated from

$$(v_d - b) = (v_1 - b)\left(\frac{p_1}{p_d}\right)^{\frac{1}{1\cdot 3}} \quad . \quad . \quad . \quad . \quad (19)$$

And from equation (6), page 193, the temperature to which the supersaturated flow has fallen can be found from

$$T_d = \left(\frac{p_d}{p_1}\right)^{\frac{3}{13}} T_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The total energy of the steam in this condition can then be calculated from equation (5), page 175, after substituting therein the values of p_d and T_d found above.

Neglecting losses, the condensation takes place at constant total energy. And condensation must take place to the extent required to produce equilibrium at the pressure p_d. The total energy I_d of 1 lb. of steam at the pressure p_d in this condition is expressed by $I_w + q_d L_d$. Therefore

$$I_d = I_w + q_d L_d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

from which q can be found, and then by the aid of the tables the volume can be calculated.

From this point onwards to the final section the flow proceeds normally, and the final section can be found from the conditions p_d, a suitable value of n, and the volume calculated with q_d, or by the thermodynamic method given in Section 181, page 588.

Consider the case of steam initially dry and saturated at 150 lbs. per square inch flowing into a reservoir where the pressure is maintained constant at 1 lb. per square inch, and assume that the flow is supersaturated until the pressure has fallen to 50 lbs. per square inch, at which pressure condensation takes place and the flow then proceeds normally along the expanded part of the nozzle. This case has been fully worked out for frictionless adiabatic or normal flow, and the results are tabulated on page 591.

First find the minimum area at the throat.

From equations (17) and (18) above, the minimum area at the throat is

$$A_0 = \frac{1}{3 \cdot 78}\sqrt{\frac{v_1}{p_1}} \text{ sq. ft. per pound of flow}$$

From the tables $v_1 = 3{\cdot}041$ cubic feet, and the pressure $p_1 = 144 \times 150$ lbs. per square foot. Therefore $A_0 = 0{\cdot}00313$ square foot. The area required per lb. of flow at the throat is thus about 4 per cent. less than if condensation were proceeding without lag.

Again, condensation is supposed to begin when the pressure has fallen to 50 lbs. per square inch. The volume at this pressure for supersaturated flow is, from equation (19), above

$$(v_d - 0{\cdot}016) = (v_1 - 0{\cdot}016)\left(\frac{p_1}{p_d}\right)^{\frac{1}{1\cdot 3}} = 3{\cdot}025 \times 3^{\frac{1}{1\cdot 3}} = 7{\cdot}042 \text{ cub. ft.}$$

$$\text{Therefore } v_d = 7{\cdot}058$$

From equation (20) the temperature in the supersaturated state is

$$T_d = 454{\cdot}5\left(\frac{50}{150}\right)^{\frac{3}{13}} = 352{\cdot}5^\circ \text{ absolute}$$

$$= 79{\cdot}5^\circ \text{ C.}$$

The total energy of the steam, calculated from equation (5), page 175, after substituting these values of p and T, and also the value of c corresponding to the temperature 79·56°, which from Steam Table 2, page 742, is 0·509, is

$$I_d = 0{\cdot}4772 \times 352{\cdot}5 - 0{\cdot}1028\left\{\frac{13}{3}c - 0{\cdot}016\right\}50 + 464 = 621$$

Then $$I_w + q_d L_d = 621$$

from which $q = 0{\cdot}936$.

The tabular volume at 50 lbs. per square inch is 8·52 cubic feet.

Therefore the volume per pound after condensation has produced equilibrium is 7·98 cubic feet.

From this point the flow proceeds normally and the nozzle areas can be worked out by either of the methods explained above.

The state of steam during frictionless adiabatic but supersaturated flow changing at a definite pressure into normal frictionless adiabatic flow can be followed on the total energy-temperature diagram. Calculations relating to the flow can be made most easily by the aid of the diagram.

In the total energy-temperature diagram the network above the saturation curve relates to superheated steam. The continuation of this network across the saturation curve into the region occupied by the network relating to wet steam provides a diagram on which the change of state during supersaturated flow can be followed. After the position of the state point has been identified in this diagram, condensation at constant total heat energy moves the point along a line of constant total heat to a position of equilibrium in the wet network corresponding to the pressure at which condensation is assumed to begin. An adiabatic curve sketched through this point to the pressure in the final section determines the final state of the steam. Only the lines which are necessary to fix the position of the state point need be continued across the boundary from the superheated region.

Let it be required to find the area of the final section of a nozzle with the data above, namely, flow starting with steam dry and saturated at 150 lbs. per square inch and ending when the pressure has fallen to 1 lb. per square inch, the flow remaining supersaturated until the pressure has fallen to 50 lbs. per square inch.

The initial position of the state point is at A, Fig. 215A, corresponding to steam dry and saturated at 150 lbs. per square inch. The position of the state point in the supersaturated network is found by producing from the superheated network the adiabatic line which runs through A until it meets in B the 50 lbs. pressure line produced from the superheated network. Through B draw a constant total heat line, namely, a line at 45 degrees, to cut the horizontal pressure line corresponding to 50 lbs. per square inch in C. Through C draw an adiabatic to cut the horizontal pressure line corresponding to 1 lb. per square inch in D. The points A, B, C, and D fix the conditions of the flow. From the diagram the total energy corresponding to each of these four points is read off. Thus

	Total energy.	q.
A	666·5	—
B	621·0	—
C	621·0	0·936
D	491·5	0·788

The values of the dryness at the points C and D are also added. w_2, the velocity across the last section, is $300\sqrt{175 \cdot 5} = 3980$ ft. per

second. D_2, the density at the last section, is $\frac{1}{0{\cdot}788 \times 333{\cdot}12} = \frac{1}{262{\cdot}5}$. And the discharge is 1 lb. per second. Therefore the final area is

$$A_2 = \frac{1}{Dw} = \frac{262{\cdot}5}{3980} = 0{\cdot}066 \text{ sq. ft.}$$

The throat area is calculated above. A smooth curved surface or even a straight conical surface joining the throat to this final area

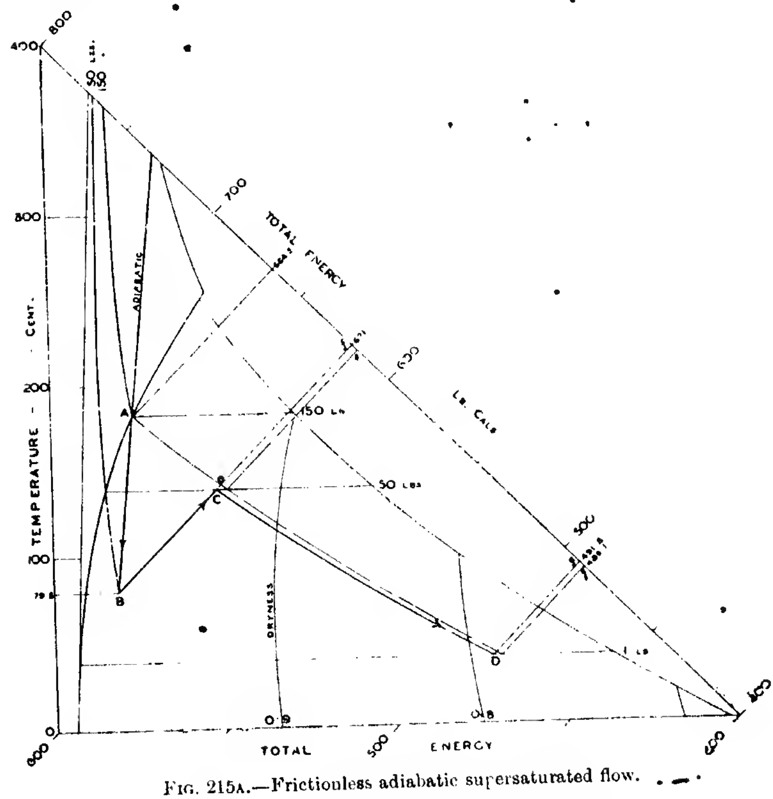


Fig. 215a.—Frictionless adiabatic supersaturated flow.

will form the enlarged part of the nozzle proportioned for the state of flow assumed. Comparing the areas found with the areas given in the table above it will be seen that the throat is about 3 per cent. smaller, and the final section about $1\frac{1}{2}$ per cent. larger.

An alternative way to fix the point B is to calculate first the temperature to which the steam falls when the pressure has fallen to the temperature at which condensation is assumed to begin from equation (20) above, and then to calculate the total energy at this

temperature from the equation of the adiabatic through the point A. In the superheated and supersaturated regions the adiabatics are, for all practical purposes, straight lines. From page 206 it will be seen that the general equation of an adiabatic curve for superheated steam is with sufficient accuracy

$$I = \left\{K_p^{\circ} - \frac{144(n+1)}{1400}\frac{c_a p_a}{T_a}\right\}T + 464$$

Along an adiabatic, $\frac{c_a p_a}{T_a}$ is a constant, so that the value of this constant can be calculated from the pressure, the temperature, and the co-aggregation volume, corresponding to any point along the adiabatic. Choosing the values corresponding to 150 lbs. per square inch, and substituting 0·4772 for K_p°, the equation for the adiabatic through A becomes

$$I = 0{\cdot}445T + 464 \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

When the pressure falls by supersaturated flow to 50 lbs. per square inch, the corresponding temperature, as shown above, is 352·5° C.

The insertion of this temperature in equation (22) gives

$$I = 621$$

The position of the point B is then determined on the chart.

The dotted adiabatic curve through A is added to the diagram, so that a comparison of the states through which the steam passes in the normal flow and in the supersaturated flow, can be made.

The process of condensation which brings the steam from the unstable state resulting from supersaturated flow to the stable state corresponding to the pressure at which condensation takes place, is irreversible. Referring to the diagram, this irreversible process is represented by the passage of the state point from B to C. In section 60, page 216, it was explained that a certain amount of the heat energy available is absorbed during an irreversible process and appears as an addition of heat to the working substance at the end of the process. This addition of heat increases the entropy. In the diagram, Fig. 215A, *ff* represents the energy absorbed in the irreversible process BC; and C*c* shows the corresponding increase of entropy. The energy absorbed during the irreversible process is equally shown by the distance *gg*. The distance *gg* represents 2·4 lb.-calories. The gain of entropy C*c* is given approximately by dividing this quantity by the mean temperature of condensation, which is 382° absolute. Therefore the increase of entropy is about $\frac{2{\cdot}4}{382} = 0{\cdot}006$.

183. Summary of Equations relating to Frictionless Adiabatic Flow.

Limiting velocity of flow, w, in feet per second.

$$w = \sqrt{2gJU} = 300\sqrt{U} \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$J = 1400$. U = the available energy $(I_1 - I_2)$ in lb.-calories.

Or approximately, from the pressure volume diagram

$$w = \sqrt{2gZ} = \sqrt{2g}\sqrt{\frac{n}{n-1}\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}144p_1v_1} \quad . \quad (b)$$

p_1 is the initial pressure in pounds per square inch;
p_2 is the back pressure in pounds per square inch;
v_1 is the initial volume in cubic feet;

and

$$n = \frac{\log \frac{p_1}{p_2}}{\log \frac{q_2V_2}{q_1V_1}}$$

in which V_1, V_2, are respectively the tabular volumes corresponding to the initial and to the final pressures.

A suitable value of n to use when the steam is initially dry and saturated is 1·15.

The limiting velocity of flow w_0 across the section of minimum area at the throat where the pressure is $0·58p_1$ is

$$w_0 = 70\sqrt{p_1v_1} \text{ ft. per second} \quad . \quad . \quad . \quad . \quad (c)$$

p_1 is in lbs. per square inch.

Density; pounds per cubic foot D.

$$D_2 = \frac{1}{q_2V_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

q_2 = dryness after adiabatic expansion from the initial to the back pressure. V_2 = tabular volume corresponding to the back pressure.

Density at the section of minimum area at the throat

$$= 0·62 \cdot \frac{1}{v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

Discharge, F; *per second in pounds per square foot of sectional area of a nozzle.*

$$F = D_2w_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

Discharge per second in pounds per square foot of the minimum area at the throat.

$$F_0 = 43·4\sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$

p_1 is the initial pressure in pounds per square inch
v_1 is the initial volume in cubic feet

Or more approximately $\quad F_0 = 2·04p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (h)$

Or for estimating purposes $\quad F_0 = 2p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$

When the back pressure is less than or equal to 0·58 times the initial pressure the discharge can be reckoned on the section of minimum area by (g), (h), or (i).

If the back pressure is greater than 0·58 the initial pressure the discharge must be reckoned from the final section using expression (f) after calculating D_2 from (d) and w_2 from either (a) or (b) for the assigned back pressure. In all cases the velocity of discharge across the final section of the nozzle is to be calculated from equation (a) or (b) except in the particular case where the back pressure is equal to 0·58 p_1. Then the velocity can be calculated directly by equation (c).

184. Frictionally Resisted Adiabatic Flow.—In the preceding articles, expressions for the limiting velocity of flow and for the corresponding limiting discharge have been found for the condition of frictionless adiabatic flow. In a particular example it was shown that steam flowing in this ideal manner from a pressure of 150 to 1 lb. per square inch approaches a limiting velocity of 4,000 ft. per second. The energy required to overcome the frictional resistance at velocities like this is a sensible fraction of the kinetic energy of flow, but the high velocity at the same time tends to reduce the direct loss of heat to the wall of the nozzle, because the wall surface is so small in relation to the weight of steam flowing, and the steam is in contact with the wall for such a short time, that there is little possibility of any serious transfer of heat taking place. The actual condition of flow at these high velocities is therefore nearly adiabatic in the sense that there is no exchange of heat between the steam and the walls. But heat is produced internally equivalent to the energy of flow spent in overcoming the frictional resistance against which the flow is made. Prof. Stodola,[1] experimenting with an expanding nozzle, found that about 10 per cent. of the available energy was lost when the pressure had fallen from 150 to 15 lbs. per square inch, and this loss increased to 20 per cent. at exit when the pressure had fallen to 2·8 lbs. per square inch. The condition of frictionally resisted adiabatic flow has been considered generally in Section 60, page 216, in connection with the subject of irreversible processes, and reference may be made to that section for the general theory involved. It is sufficient to state here that if during an adiabatic change the heat which is transformed into mechanical energy is not wholly used in doing external work, but some of it is retransformed into heat by friction, then the heat so retransformed appears in the working agent at the end of the process, and the final state of the agent is therefore to be found so that this quantity of heat is included.

The effect of the addition of heat by friction as the flow proceeds is twofold. In the first place part of the total energy U is sacrificed to overcome the frictional resistance, in consequence of which the limiting velocity of flow is reduced. In the second place the heat produced by friction dries the steam if it is wet and increases the

[1] "The Steam Turbine," 2nd edition.

temperature if it is dry, in consequence of which the volume per pound is increased and the density is therefore reduced.

The discharge across a square foot of section per second is therefore reduced because each of the factors in the expression on which it depends, namely, Dw, is reduced.

If the heat which is required to overcome the frictional resistance between two sections of a nozzle is expressed as a fraction of the available energy due to the state difference between the sections, the state of the steam and the velocity of flow can be traced step by step from section to section along the nozzle, and points can then be found on the velocity curve, the density curve, and the discharge curve corresponding to any given frictional loss.

Consider two sections of a nozzle in which the pressures are respectively p_1 and p_2; and assume that the steam flows across the first section with the velocity w_1 and with the dryness q_1. Let it be given that the fraction η of the available energy corresponding to the state difference is used against the frictional resistances. The problem is to find the velocity w_2 with which the steam flows through the second section and its dryness there.

By calculation, or from a steam diagram, find the total energy I_1 of a pound of steam at pressure p_1 and dryness q_1. Next find the total energy I_2 and the dryness q_2 after frictionless adiabatic expansion from the initial state defined by p_1 and q_1 to the pressure p_2. If U is the available energy

$$U = I_1 - I_2$$

Of this, by hypothesis, the fraction η is used to overcome the frictional resistances, leaving the part $1 - \eta$ for the production of velocity. Therefore

$$w_2 = \sqrt{2gJ(I_1 - I_2)(1 - \eta) + w_1^2} \quad . \quad . \quad . \quad (1)$$

Owing to friction the dryness q_2 is increased by the quantity $\dfrac{\eta(I_1 - I_2)}{L_2}$, and therefore

$$Q_2 = \frac{q_2 L_2 + \eta(I_1 - I_2)}{L_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This dryness fixes the initial condition for the next stage of the expansion to a section where the pressure is p_3. The process is repeated with the initial conditions p_2 and Q_2 to find the velocity w_3 and the dryness Q_3.

In each stage in succession the available energy due to frictionless adiabatic expansion, namely $I_n - I_{n+1}$, is first found from the initial conditions p_n, Q_n, and the final pressure p_{n+1}; and then equations (1) and (2) are used to find w_{n+1} and Q_{n+1} for the given value of η.

Strictly, the pressure intervals should be chosen so that the value of $I_1 - I_2$ is small, but actually the expressions may be applied to large heat drops without incurring errors of practical importance.

The general problem may be illustrated by a numerical example.

Calculate the discharge per square foot per second from the final section of a nozzle where the pressure is 1 lb. per square inch, when the flow is from 150 to 1 lb. per square inch, and when 10 per cent. of the available energy, reckoned to any value of the variable back pressure, is used to overcome the frictional resistance to adiabatic flow.

Divide the flow into two stages, first from the initial section of the nozzle to the section where the pressure has fallen to 20 lbs. per square inch; and, secondly, from this section to the final section in which the pressure is 1 lb. per square inch. Assume the steam to be initially dry and saturated in the first section. Call the sections Nos. 1, 2, and 3—

I_1 = total energy of dry saturated steam at 150 lbs. per square inch . . .	= 666·5 lb.-cals.
I_2 = total energy after frictionless adiabatic expansion to 20 lbs. per square inch .	582·9 ,,
And the dryness q_2 = 0·89. Available energy	= 83·6 ,,
Less 10 per cent.	8·36 ,,
$(I_1 - I_2)(1 - \eta)$ = energy available for increasing the velocity	= 75·24 ,,

Therefore the velocity of flow through section 2, since the initial velocity through section 1 is zero, is from (1)—

$$w_2 = 300\sqrt{75\cdot24} = 2604 \text{ ft. per sec.}$$

From (2) $$Q_2 = \frac{0\cdot89 \times 533\cdot8 + 8\cdot36}{533\cdot8} = 0\cdot903$$

The tabular volume for 20 lbs. per sq. inch is 20·075 cub. ft. Therefore the density of the steam is $\dfrac{1}{Q_2 V_{tab}} = \dfrac{1}{18\cdot1}$.

The discharge Dw is then $\dfrac{2604}{18\cdot1}$ = 144 lbs. per square ft. per second, as against 154 lbs. at 2746 ft. per second if the flow is frictionless and adiabatic, as will be seen from a reference to Table 30, page 591.

The initial conditions in the second section are $p_2 = 20$ and $Q_2 = 0\cdot903$. It follows that

$$I_2 = 591\cdot26 \text{ lb.-cals.}$$
$$I_3 = 495\cdot00 \text{ lb.-cals. and } q_3 = 0\cdot705$$

$$I_2 - I_3 = 97\cdot26 = \text{available energy}$$
Less 10 per cent. $9\cdot72$

$$(I_2 - I_3)(1 - \eta) = 87\cdot54$$

From (1) $w_3 = \sqrt{2gJ(87\cdot54) + 2604^2} = 3830$ ft. per sec.

From (2) $Q_3 = 0\cdot811$

And the tabular volume at 1 lb. per square inch is 331·1. Therefore the discharge product Dw is $= \frac{3830}{0·811 \times 333·1} = 14·1$ lbs. per square ft. of section 3 per second as against 15·31 at 4000 ft. per second when the expansion from the initial pressure is frictionless and adiabatic, as will be seen from Table 30, page 591.

If the calculation is carried out through a greater number of sections the velocity would be slightly greater than that found for two stages, and the value of the product would also be slightly greater. The velocity and the density approach a limiting value as the number of stages is indefinitely increased. If equations (1) and (2) are applied to the whole drop from 150 to 1 the results at the last section are—

$$U = 177·4\ ;\ w_2 = 3790\ ;\ D_2w_2 = 13·9\ ;\ q_2 = 0·785\ ;\ Q_2 = 0·816,$$

which shows that for practical purposes the calculation of the final velocity and the final density can be made, between the initial and final conditions, omitting intermediate stages, without introducing serious error; especially considering the approximate nature of many of the assumptions made.

The process of finding the final conditions in a single stage are illustrated on the entropy temperature diagram in Fig. 216, both for saturated and for superheated steam. The line BCH is a constant pressure line for the higher pressure and APK is the constant pressure line for the lower pressure. The area ABCD represents the available work $U = I_1 - I_2$ per lb. of steam expanding from 150 lbs. to 1 lb. per square inch, when the expansion is frictionless and adiabatic, neglecting the small correction between the total energy and the total heat of the steam at C, mentioned on page 196. Since 10 per cent. of the total work is lost in friction and since the equivalent appears in the steam at the end of the process, the point E is chosen so that the area DEFG is one-tenth of the area ABCD. The energy producing the velocity of flow is 0·9 of the area ABCD. The dryness at the final section is increased by the amount $DE = \Delta\phi$, with corresponding increase of volume and reduction of density.

This construction is made considering the heat drop as a whole and omitting intermediate stages. If the construction is made in two stages in which the intermediate back pressure is *ad*, *de* × G*d* is the area representing $\frac{1}{10}$ of the area *a*BC*d*. The available energy for the second stage is then represented by the area under A*ae*. Working in this way a curve of condition C*e*E can be obtained which only differs slightly from the straight line joining C to the point E found from the whole drop.

Analytically if η is the fraction of the available energy used against friction, it is clear from the figure that

$$\Delta\phi T_2 = \eta U \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The value of $\Delta\phi$ calculated from this relation approximates to its true value the smaller the value U, since the value of U to be used in this

equation is not ABCD, but ABCE. An approximation of sufficient accuracy is made, however, by using the value of U corresponding to the area ABCD even for large values of U. A curve of condition is the locus of a point, E, moving in such a way that the area under DE is always the fraction η of the area ABCE, or, with sufficient approximation, the area ABCD.

The case of steam initially superheated and remaining superheated after falling to the back pressure is also illustrated in Fig. 216.

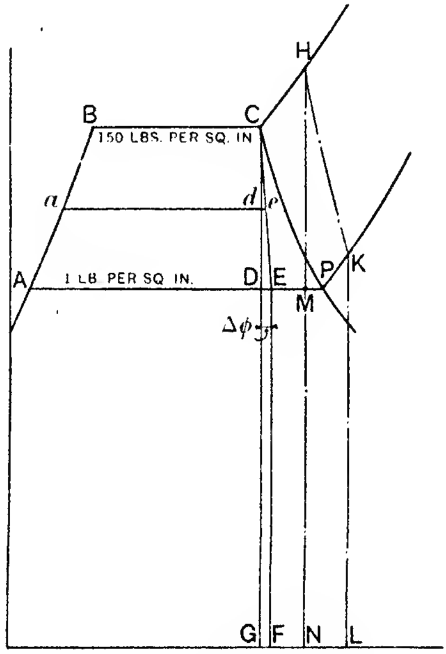


FIG. 216.—Frictionally resisted adiabatic flow.

The saturated part of the diagram is drawn to scale, but the superheated part is not to scale, in order to make the diagram clearer. The point H represents the state of a pound of steam formed and superheated along the constant pressure path BCH. The available energy, assuming frictionless adiabatic expansion, is represented by the area ABCHMA. The area NMPKL represents the fraction of this energy which is lost against friction and K is the corresponding final state of the steam. HK is the path of the state point plotted by finding such points as K, the corresponding area under MPK being with sufficient approximation the fraction η of the area representing the available energy.

Although the entropy temperature diagram usefully illustrates

the principles involved when friction is included in the flow, in practice it is far easier to obtain the final state of the steam for a given percentage of loss either by the Total Energy-Temperature diagram or by the Total Energy-Entropy diagram.

The process of finding the final state of the steam for a given percentage frictional loss per stage is shown on the Total Energy-Temperature diagram in Fig. 217, both for initially dry and saturated steam and for superheated steam.

The constant pressure lines PAH and QRS are drawn for 150 lbs. per square inch and for 1 lb. per square inch respectively. First let

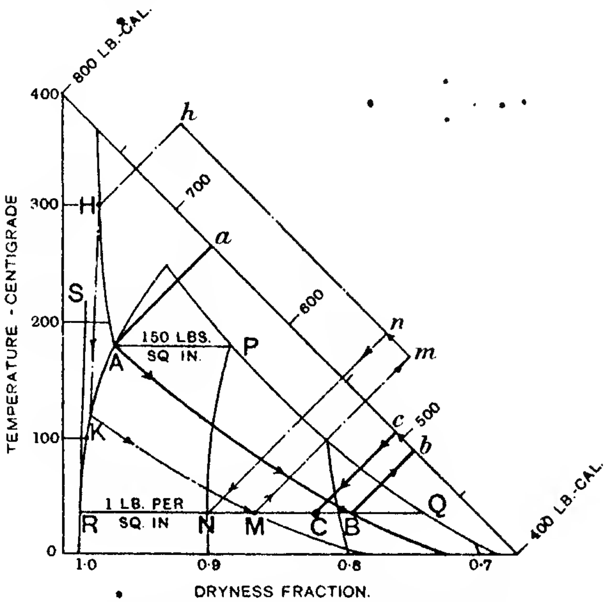


FIG. 217.—Final state in frictionally resisted adiabatic flow determined from the Total Energy Temperature diagram.

the steam be dry and saturated. Locate the state point A on the saturation curve for the given initial pressure, and then by a 45-degree projector transfer A to the energy scale. The intersection a gives the total energy of the steam in its initial condition. Starting again from A, trace the adiabatic path AB down to the given back pressure, and then refer B by a 45-degree projector to the energy scale, thus fixing the point b. The length ab on the scale is the available energy for frictionless adiabatic expansion. Take c so that bc is the energy lost against friction. In this case $bc = 0{\cdot}1\ ab$. Transfer c back to the back-pressure line, thus fixing the point C. The point C determines the final state of the steam, and at the same time the initial state of the steam for the next stage.

After fixing the point *a* from A, C is found by tracing the path A*Bbc*C shown in thick lines, as indicated by the arrows, although the movement of the tracing point must be interrupted to fix the point *c* from the given conditions.

Secondly, suppose the steam is initially superheated at the constant pressure corresponding to the line PAH to the temperature H. Then H marks the initial position of the state point. The process of finding the final state for a given back pressure and frictional loss is exactly similar to that just explained, and the successive steps are indicated by the chain-dotted lines in the diagram. The energy scale is displaced upwards to *mh* in order to avoid confusion. Having located H fix the point *h* on the energy scale, and then, starting from

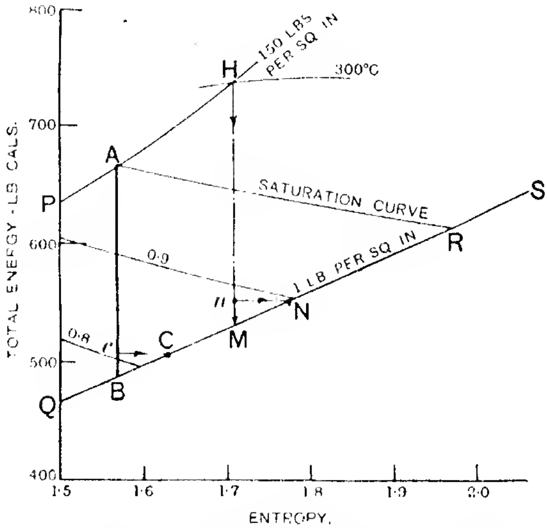


FIG. 218.—Final state in frictionally resisted adiabatic flow determined from the Total Energy-Entropy diagram.

H, trace out the adiabatic path HKM; project M to *m*; fix *n* so that $\frac{mn}{mh}$ is equal to the given fractional loss in friction: project *n* to the back-pressure line, thus fixing N. The point N gives the final state of the steam after frictionally resisted adiabatic expansion from the initial condition H to the back pressure QR, when the fraction $\frac{mn}{mh}$ of the total energy *mh* is lost in friction. The energy *nh* is the part of the available energy which is used in producing the velocity of flow.

In some problems it may happen that the adiabatic path from H cuts the back-pressure line to the left of the saturation curve. This

indicates that the steam is still superheated when it has fallen to the back pressure. The process of finding the final state is, however, just the same. Wherever the adiabatic path cuts the back-pressure line the point of intersection is referred to the energy scale and reprojected, after allowing for the loss, to the back-pressure line. If the point falls in the superheated region, it shows an increase of temperature instead of an increase of dryness. It may happen that the state point at the end of frictionless adiabatic expansion falls in the wet region, but that after transference to and re-transference from the energy scale it falls in the superheated region.

The process is even simpler to carry out on the Total Energy-Entropy diagram, because the adiabatic path which must be traced is a straight vertical line from the initial position of the state point to the back pressure. The process is illustrated for both saturated and superheated steam in Fig. 218. The constant-pressure lines PAH and QRS are drawn for 150 lbs. and 1 lb. per square inch respectively. AR is the saturation curve above which lies the region of superheat, and below which is the wet region.

A is the state point for a pound of dry saturated steam formed at the constant pressure PH. AB is the available energy for frictionless adiabatic expansion from the upper to the lower pressure, and B*c* is the loss due to friction. A*c* is the part of the available energy which is used to produce velocity, and projecting *c* horizontally to the back-pressure line, C is the state point which determines the final condition of the steam.

Similarly H is the state point for a pound of steam formed at constant pressure and superheated to H; HM is the available energy; M*n* is the frictional loss, and N is the final condition of the steam.

No difficulty will be found in applying these constructions through a series of small stages to find the final condition, though in practical work such refinement is seldom necessary.

185. Ejectors and Injectors.—Two familiar examples of the use made of the kinetic energy of a steam jet are afforded by the Vacuum Brake Ejector and the Boiler Injector. In both cases the energy of the steam is converted into kinetic energy of flow by means of nozzles. And in both cases the energy is shared with a second substance with the object of setting this substance in motion.

The sketch, Fig. 219, shows an ejector used by the Vacuum Brake Company, who have kindly supplied the drawing from which the sketch is made. The drawing shows two ejectors, a large and a small, combined in one casing. The small ejector is powerful enough to maintain the vacuum in the train pipe and reservoirs against leak when steam flows through it continuously, and this continuous flow is regulated by the valve V. The large ejector is only brought into use when a vacuum is to be formed quickly after the application of the brakes in order to take the brake off again. The supply of steam to the ejector is regulated by the driver's valve, which is not shown in

the sketch. Both the large and the small ejectors are similar in construction and in action. Considering the large ejector, steam is admitted through the annular cone formed by the inner nozzle N

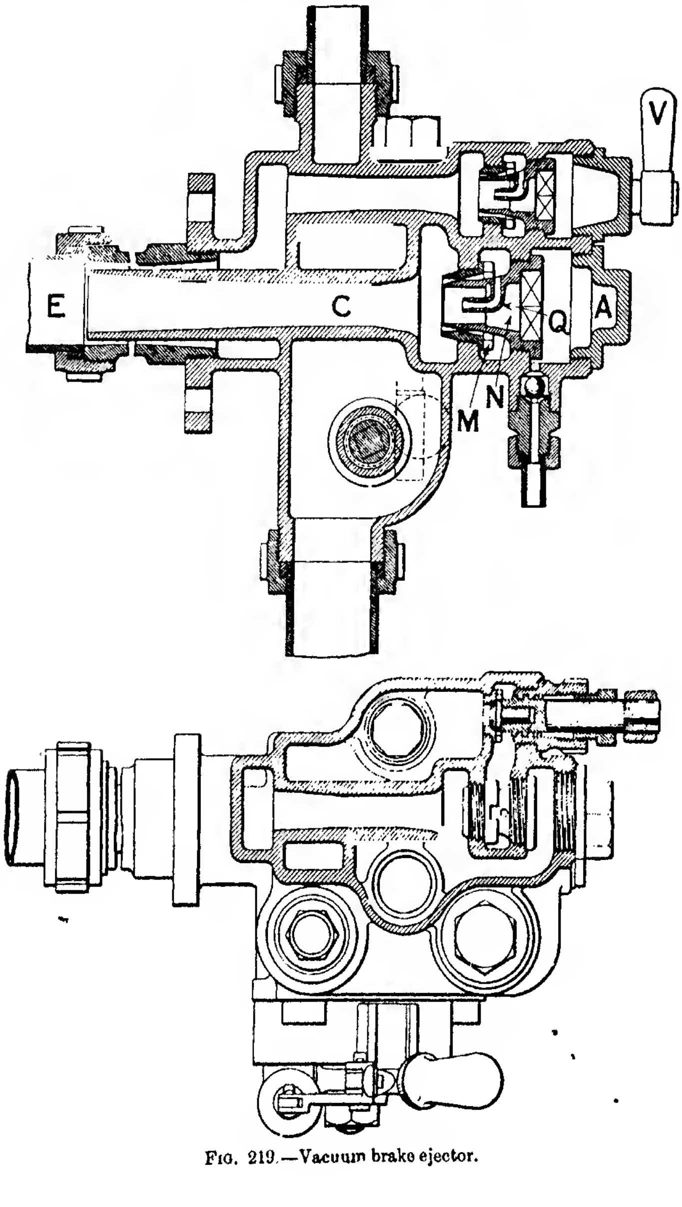


FIG. 219.—Vacuum brake ejector.

and the outer nozzle M. The steam flows as an annular cone into the combining nozzle C, where it mingles with the air drawn through the centre of the middle steam nozzle N from the space A, which is in communication with the train pipe. The combined jet of steam and air flows away through the exhaust pipe E. A small central steam nozzle Q assists the action of the large outer nozzles.

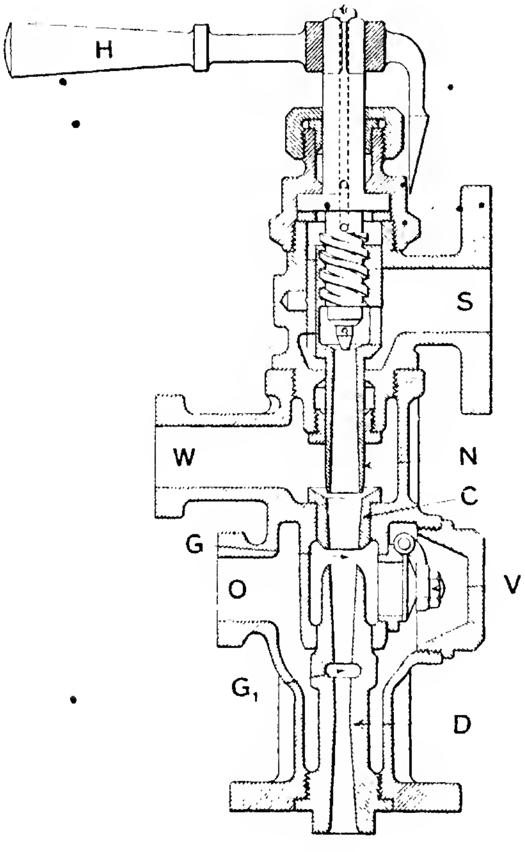


FIG. 220.—Water injector.

Fig. 220 shows a sketch of a water injector of the kind made by Messrs. Holden & Brooke, Ltd., to whom the author is indebted for the drawing from which the sketch was produced. Steam is admitted to the nozzle N through the supply branch S. The pressure of the steam and the distance of the nozzle N from the entrance to the combining cone C, which regulates the water supply, are both regulated by a single movement of the handle H. Water is drawn into the

combining nozzle through the branch W. Steam and water mingle in C and in its continuation D. The part D gradually enlarges to its exit into the feed pipe in order to transform part of the kinetic energy of flow of the combined jet into pressure energy, and the pressure produced is sufficiently above the boiler pressure to enable the stream to lift the clack valve and flow into the boiler.

When starting the injector the first rush of steam escapes through the overflow valve V and away through the overflow outlet O. When the flow is established and the jet is formed, a vacuum is produced in the gap G, and the valve V therefore closes; any further overflow taking place through the gap G_1, which is also in communication with O. The proper working is established when the handle H is turned into a position where the overflow of water at O just ceases. Should the jet be accidentally broken by jolting, or by the admission of air with the water supply, the vacuum is destroyed in the gap at G and the valve V is free to open, in consequence of which a rush of steam takes place through it and the injector immediately restarts.

The mutual action of the steam jet and the water in the combining nozzle is of an indeterminate character. All that can be said is that 1 lb. of steam combines with x lbs. of water and produces $x + 1$ lb. of water flowing with sufficient energy to enable the reversed nozzle at D to bring the pressure above the boiler pressure.

The injector illustrated can lift the water from a tank below it, and it thus acts as an ejector to the feed tank as well as an injector to the boiler. In general, it is better to fix the injector below the feed tank so that the water flows from the tank into the injector. The injector is then easier to start and troubles are avoided. In some cases two injectors are combined in series so that the reversed or delivery cone of the first leads the water to the combining chamber of the second, and it is the reversed cone of the second injector which finally leads the water into the boiler feed pipe. Exhaust steam injectors are those which are fed with exhaust steam from the engine and are not supplied with boiler steam directly. The range of pressure available for producing the velocity of flow is small and the steam nozzles are larger. An exhaust steam injector may be used in series with a live steam injector, and so increase the final pressure of delivery.

An injector returns nearly all the heat of the steam to the boiler with the feed water, providing the water supply is adjusted so that there is no overflow, so that although it is an inefficient heat engine, since it uses only about one per cent. of the heat energy of the steam supply to force the water into the boiler, it is an efficient feed water heater.

CHAPTER XII

STEAM TURBINES

186. Introduction.—The Hon. Sir Charles Algernon Parsons, within a period of thirty years, by the invention and the development of the steam turbine, has revolutionized the practice of Motive Power Engineering. The steam turbine has displaced the reciprocating steam engine from the premier position which it occupied as a prime mover in power stations, and from its long unchallenged position as the prime mover in fast merchant ships and in warships.

The first Parsons steam turbine was constructed in 1884. It is to be seen in the South Kensington Museum. It was non-condensing and developed 10 H.P., using about 150 lbs. of steam per horse-power hour. In 1907, only twenty-three years afterwards, Parsons turbines of 74,000 H.P. were installed in the *Mauretania* and in the *Lusitania*, and the consumption of steam is stated to be 14·4 lbs. per shaft horse-power hour. Since 1907, British warships have been fitted with turbine engines instead of with reciprocating engines. The success of the steam turbine for the propelling machinery of fast ships is complete. The development of the Parsons steam turbine since 1884 has been by increasingly large steps. In 1888, turbines running at high speeds to drive specially designed alternators were constructed of about 120 H.P. The first condensing steam turbine was constructed in 1892, and it developed about 150 H.P. at 4800 revolutions per minute. In 1900, two 1000 kilowatt turbo-alternators were supplied to the City of Elberfelde in Germany. Between 1907 and 1910 plants of 5000 kilowatts were constructed, using about 13·2 lbs. of steam per kilowatt hour, equivalent to 10·2 lbs. of steam per horse-power hour. And in 1913, a 25,000 kilowatt Parsons turbo-alternator,[1] with a guaranteed consumption equivalent to 8·1 lbs. of steam per shaft horse-power hour, was built by Messrs. C. A. Parsons & Co. for the Commonwealth Edison Company of Chicago.

Other types of steam turbine have been developed on the Continent and in America during the period of the growth of the Parsons turbine, notably the Laval, the Curtis, the Rateau, the Zoelly, and many firms have entered the manufacturing field, but the premier position is, however, to-day occupied by the Parsons steam turbine.

What are the broad differences between the steam turbine and

[1] *Engineering*, October 17, 1913.

the reciprocating engine which have enabled the steam turbine to so rapidly supersede its ancient rival?

There is first the mechanical advantage that the steam acts to produce a pure rotation of the shaft without the intervening mechanism of the crank and connecting rod, and the necessarily accompanying valve gear and distributing valves. There is next the thermodynamic advantage that a steam turbine can utilize ranges of pressure ending at a lower pressure than it is commercially practicable to go to with a reciprocating engine. When the steam has been expanded in a reciprocating engine to about 7 lbs. per square inch the low-pressure cylinder has reached a limiting size. The cost of increasing the size to reduce the pressure to a lower value more than counterbalances the gain due to the increased expansion. But in a turbine, steam can be effectively expanded to the lowest pressure which can be produced in the condenser, and is thus able to realize to a greater extent the thermodynamic advantage that pressure differences at low pressures furnish more available energy than equal differences at higher pressures.

For example, the available energy corresponding to the pressure range from 200 to 194 lbs. per square inch is approximately 4 lb.-cals. per pound. If the expansion is continued adiabatically to 1 lb. per square inch, it will be found that the range from 7 to 1 lb. furnishes about 47 lb.-cals. per pound; 12 times as much as that obtained from the same pressure difference at the higher pressures.

This point may be further illustrated by tabulating the theoretical gain due to a continued expansion through the ranges 2½ to 2 lbs. per square inch; 2 to 1½; 1½ to 1, and from 1 to ½ lb. per square inch, assuming that at the beginning of expansion the steam is dry and saturated at 200 lbs. per square inch.

Initial pressure 200 lbs. per square inch.

TABLE 31.—Theoretical Gain per Inch Increase of Vacuum.

Back pressure. Pounds per square inch.	Available energy from 200 lbs. per square inch to the back pressure in column 1. Lb.-calories.	Gain.
	U.	
2½ = 25 ins. vacuum	163	
2 = 26 ,,	170	7 = 4·3 per cent.
1½ = 27 ,,	178	8 = 4·7 ,,
1 = 28 ,,	189	11 = 6·2 ,,
½ = 29 ,,	206	17 = 9 ,,

This table shows that the pressure rate at which energy is produced goes on increasing as the pressure falls.

From 2½ to 2 lbs. the rate is 7 units per ½ lb. fall of pressure, *i.e.* 4·3 per cent. of the available energy, viz. 163 lb.-cals., at the pressure 2½ lbs. per square inch. From 1 to ½ it is 17 units per ½ lb.

fall of pressure, equivalent to 9 per cent. of the available energy at 1 lb. per square inch, viz. 189 lb.-cals.

A large proportion of this theoretical gain is realized in practice, and since the turbine can utilise the heat energy of steam down to the lowest pressure it follows that a feature in all modern turbine plants is an efficient condenser.

187. Elementary Turbine Pairs. Impulse and Reaction Pairs. Steam Turbines.—The elementary mechanical element of a steam turbine is a pair, consisting of a fixed nozzle and a ring of blades. The nozzle is fixed to the casing and directs a jet of steam on to the blades at an angle to the plane in which they move. The blades are secured to the periphery of a wheel or drum, carried by and keyed to the turbine shaft.

The moving blades change the motion of the jet of steam as it flows between them, and in so doing a pressure is produced between the steam jet and themselves, the resolved component of which in the direction of motion of the blades is effective in doing work. The rate at which work is done is the product of the resolved component of the pressure in the direction of motion of the blades multiplied by the mean velocity of the blades.

A pair, consisting of a nozzle and a ring of blades, is shown in Fig. 221 in diagrammatic form. The straight line *aa* shows the path in which the jet of steam flows from the nozzle when the blade ring is taken away. The curved line *bb* shows a path which the stream has been compelled to take by the action of the blades upon it. The shape of the path depends upon the shape of the blades and the speed at which they move. The effect of the action of the blade ring on the jet is to change the velocity of the jet from the magnitude w in the direction *aa* to the magnitude u in the direction *bc*.

Each blade in succession passes under the action of the jet, receives its share of the flow, whose velocity it changes from w to u, and passes on.

The blades are set close together in the ring in order, as far as possible, to compel every particle of the fraction of the flow caught between a pair of blades to keep to the path defined jointly by the shape of the blade and the speed. In practice the blades are set closer than indicated in the diagram. It will be assumed in the discussion of the general principles that every particle of steam flowing on to the blade ring passes into the blade channels with equal velocity, and leaves them with equal velocity, so that the motion of every particle is equally changed by the blades.

When only a single nozzle is used a blade is acting on the steam jet for a mere fraction of the time of a revolution. For the rest of the revolution it is idle. In order to keep a blade at work continuously, a ring of nozzles must be set facing the ring of blades, each nozzle touching its neighbours so that the nozzles become a series of guide blades. With this arrangement a cylindrical sheet of steam is projected on to the blade ring with its particles moving

in a helical whorl, and the action of the blades is to change the direction of the helical whorl, and in so doing to produce a uniform and continuous pressure between the steam and every blade in the ring.

Fig. 222 shows a pair consisting of a ring of nozzles or guide blades and a ring of blades. In this case the nozzles are indistinguishable from the blades. In fact, in the Parsons turbine the nozzles and blades of a pair are made of precisely the same section.

Figs. 221 and 222 illustrate more than the mechanical difference of arrangement between pairs. They are intended to illustrate two

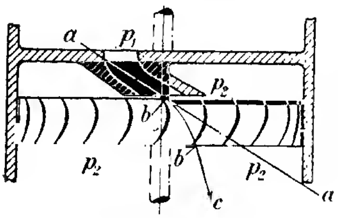


FIG. 221.—Diagram of impulse pair.

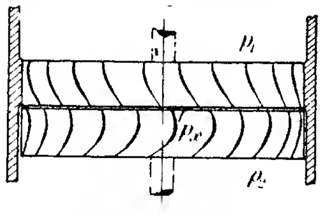


FIG. 222.—Diagram of reaction pair.

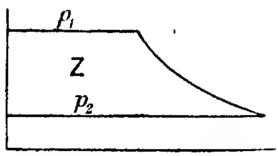


FIG. 223.—P.-V. diagram. Area Z.

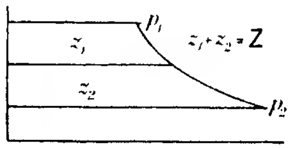


FIG. 224.—P.-V. diagram. Area $z_1 + z_2 = Z$.

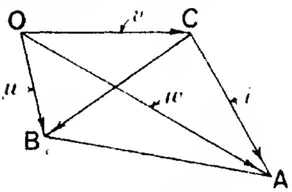


FIG. 225.—Velocity diagram. Impulse pair.

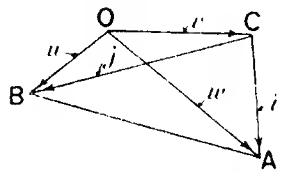


FIG. 226.—Velocity diagram. Reaction pair.

different modes of working; modes which class a pair either as an **impulse pair** or a **reaction pair.** Fig. 221 is representative of an impulse pair; Fig. 222 of a reaction pair.

Let it be assumed that each of the two pairs is to utilize the same proportion of the available energy U, corresponding to the pressure range from p_1 to p_2, and represented by the equal areas Z of the pressure volume diagrams shown in Figs. 223 and 224 below the turbine elements in Figs. 221 and 222. In these diagrams expansion from the higher to the lower pressure is along an adiabatic curve.

In the first case, neglecting losses, the whole of the available

energy U, represented by the area Z, is converted into kinetic energy by the nozzle, so that the steam flows into the blade ring with the limiting velocity due to the pressure range. No further reduction of pressure is made, and the pressure throughout the whole region in which the vane ring revolves is uniform and equal to p_2. The blades in this case merely change the direction of the velocity of flow as the steam passes through them, no change being made in the magnitude of the velocity relative to the blades. The steam flows through the blade channels as if it were flowing through a pipe with uniform velocity. The actual velocity of the steam at any point in the flow is the resultant of this uniform relative velocity of flow through the channel between the blades, and the velocity of the blade.

Thus, let w be the actual velocity with which the steam flows from the nozzle on to the blade ring; and let u be the actual velocity with which the steam emerges from the blade ring. Take any pole O (Fig. 225). From O set out OA parallel to aa, and of length proportional to the magnitude of w. Then OA represents the velocity w in magnitude and direction. Similarly set out AB to represent in magnitude and in direction the change in the velocity of the steam which is produced by blade action. Join OB. Then OB will represent the actual velocity of the steam as it leaves the blades. OB is therefore equal to u.

Usually w and u are given, and then the construction may be regarded as giving the vector difference between the two velocities w and u. Put in this way, the general rule to find the difference of two velocities is:

Choose an origin O; from the origin set out two vectors to represent the two velocities. Then the line joining their ends is the vector difference between them, and the arrow-head denoting the way of action of the change must be placed in circuit with the original velocity.

Thus, generally, the change in the velocity of the steam produced by the blades is found by subtracting the original velocity of the steam from the final velocity of the steam, the way of action of the change being in circuit with the original velocity.

Further, let v be the velocity of the mean circumference of the blade ring. Then the actual velocity w is the resultant of the velocity of flow relatively to the blade at entrance and the actual velocity of the blade. Set out from O the vector OC to represent by its direction and magnitude the velocity of the blade v. Join CA. Then in the velocity triangle OAC, OA is the actual velocity of every particle at entrance to the blade channels; OC is the actual velocity of the blade channel itself; and CA is the velocity of flow relative to the blade; that is to say, it is the velocity with which the steam would appear to flow towards a point, or station, of observation on the receiving edge of the blade.

Similarly join C with B. Then in the velocity triangle OBC, OB is the actual velocity of the steam as it emerges from the blades; OC is the velocity of the mean circumference at the discharging edges of

the blades (assumed to be equal to the velocity of the mean circumference of the receiving edges); and CB is the velocity of flow in the blade channels at exit, relative to the channels themselves, the velocity with which the steam would appear to leave a point, or station, of observation on the blade at the discharging edge.

The vector difference between the velocities of flow at entry and exit, relative to the blade ring, is AB; and this is also the vector difference between the actual velocities and is the change of velocity produced by the blades. When the steam flows from the nozzle with the limiting velocity, no change in the magnitude of the relative velocity can take place as the steam flows through the blade channel, since all the available heat energy has been converted into kinetic energy by the nozzle, and there is no energy left to produce any further change in the speed. Therefore, the length CA is equal to the length CB. In its passage through the blade channels the velocity of the steam is altered only in direction and not in magnitude, and in these circumstances the pair is called an **impulse pair.**

In the second mode of working the total available energy U is not wholly converted into kinetic energy in the nozzle by the pressure drop taking place there; part only of the pressure drop occurs in the nozzle, the remainder taking place whilst the steam is passing through the blade channels themselves. There are thus two consecutive pressure ranges, viz. from the initial pressure p_1 to some arbitrary intermediate pressure p_x, and from p_x to the final pressure p_2. The available energy U is therefore divided into two parts, U_1 and U_2, represented respectively by the areas z_1 and z_2, Fig. 224.

The nozzle is now designed to convert into kinetic energy the energy corresponding to the first pressure range, and represented by the area z_1. The blade ring receives the steam flowing with the velocity corresponding to z_1, and with the possibility of further increase of velocity as the steam expands through the second pressure range. This part of the expansion is carried out in the blade channels themselves at the expense of the heat energy corresponding to the area z_2. In fact, in this mode of working, a nozzle and a corresponding pair of blades may be regarded as originally one nozzle, designed to expand the steam from the pressure p_1 to the pressure p_2, and which, after its design, is cut into two parts which are suitably curved and set at an angle with one another, the one part as a guide or fixed nozzle, the second part as a pair of blades in the blade ring. The result of the division is that the blade ring must not only provide a pressure for changing the direction of the velocity produced by expansion in the first part of the nozzle, but must also furnish the reaction required for the increase in the velocity of the flow in that part of the nozzle formed by the blade ring.

The corresponding velocity diagram is shown in Fig. 226. It will be seen from this that the conditions of working may be arranged to produce the same change in velocity AB, but that the main difference in the diagram is that the relative velocity increases from CA at the entry to the blade channel to the larger magnitude

CB at exit from the channel. This increase of magnitude is in fact equivalent to the energy U_2 corresponding to the area z_2.

Let the initial relative velocity CA be represented by i and the final relative velocity CB be represented by j.

Then $$\frac{j^2}{2g} - \frac{i^2}{2g} = U_2,$$

assuming that the diagram corresponds to the flow of 1 lb. of steam per second.

For an impulse pair $U_2 = 0$; and therefore $i = j$.

A pair of nozzle and blade rings working in this second mode is called a **reaction pair,** because the blade ring provides a reaction against which the magnitude of the velocity of the steam in the blade channels is increased.

In a reaction pair the pressure in the space between the nozzle ring and the blade ring is p_x, and the pressure in the space below the blade ring is the lowest pressure p_2. There is, therefore, a tendency of the steam to pass round the edges of the blades through the clearance spaces which must be provided between the ends and the casing. This leak depends upon the magnitude of the pressure difference on the two sides of the blades and on the magnitude of the clearance space itself. The proportionate leak is small with long blades, because the ratio between the leakage area and the area through the blade channel is small. Mechanical reasons limit the diminution of the clearance, and therefore the proportion of leakage area increases as the blade length shortens. Consequently, a turbine pair of this type can only work economically on a small pressure difference.

It is clear also that all of the nozzles must be in action simultaneously, otherwise the pressure difference between the two sides of the blade ring cannot be maintained.

In the case of the impulse turbine pair, where the pressure all round the blade ring is constant, it is immaterial whether a single nozzle is in use or whether a whole ring is supplying steam to the blades. The power will, of course, vary as the number of nozzles in action, but the power corresponding to one nozzle can be developed by the blades with all other nozzles closed. That is to say, in the case of an impulse pair the admission may be partially round the circumference, and may be varied from the admission given by one nozzle to complete admission from a ring of nozzles. The regulation of the power of impulse turbines can therefore be conveniently made by varying the number of nozzles in action. In the case of the reaction pair, since the passages must always be full of steam, regulation of the power is effected by supplying the steam in a series of gusts in the way originated by Parsons, and which will be discussed more fully below.

Summarizing, and calling the force exerted on the moving blades by the steam flowing through the channels between them the **action,** and calling the equal and opposite force exerted by the

blades on the steam the **reaction**, the action and reaction together constitute the pressure between the steam and the moving blades. The **reaction** changes the velocity of the steam. A velocity is defined by its magnitude and its direction. So that in general the **reaction** changes both the magnitude and the direction of the velocity of the steam. In these circumstances the turbine pair is called technically a **Reaction Turbine.** When the **reaction** changes the direction of the velocity only and not the magnitude, the turbine is technically called an **Impulse Turbine.** The impulse turbine is therefore merely a particular case of a reaction turbine; that is to say, all turbines are reaction turbines. Usage has, however, established the distinctive names impulse and reaction turbines. These names may therefore be regarded as merely defining a particular mode of working, and as they are convenient the names will be adhered to in the following pages. In practice, however, the names do not always denote the sharp distinction indicated above. In some cases impulse turbines are arranged to work so that there is a slight change in the magnitude of the velocity as it passes through the moving blade channels. Practically, therefore, the reaction turbine shades into the impulse turbine without a sharply defined limit. In the Parsons turbine the change of the magnitude of the velocity produced by the fixed blades is approximately equal to the change of the magnitude of the velocity produced by the moving blades.

188. Rate at which Work is done by a Pair.—The dynamical principles involved in the production of a pressure between the steam flowing through the blade channels and the blades are contained in Newton's "Laws of Motion".

The natural mode of motion of a jet as it leaves a nozzle is in a straight line with uniform velocity (Law 1). Force must be applied to change the motion of the jet. This force is applied by the blades and its application involves the appearance of an equal and opposite force acting from the jet to the blades (Law 3). The two aspects of the force constitute the pressure between the steam and the blades. The magnitude of the pressure is found from the second law, which gives

$$\text{force} = \text{rate of change of momentum},$$

and the force acts in the direction of the change of momentum produced.

Consider a pair consisting of a nozzle and a ring of blades. Let the nozzle discharge steam to the blades at the rate W pounds of steam per second at a velocity of w feet per second. Then the momentum received by the ring per second is $\frac{W}{g}w$ units.

Let the blades change the velocity from w to u. Then the momentum discharged from the blades per second is $\frac{W}{g}u$ units.

The change of momentum per second is then $\frac{W}{g}(w - u)$ units. But the blade ring moves with uniform velocity, so that the quantity of fluid discharged per second is equal to the quantity received per second. Therefore $\frac{W}{g}$ is constant, and finally :

$$\text{Force} = \frac{\text{flow}}{g} \text{ (change of velocity produced by the blades)} \quad (1)$$

In this equation, the flow is measured in pounds of fluid per second and is represented by W; and the force is measured in pounds weight. The change of velocity is measured in feet per second.

Referring to Fig. 225, in the diagram of velocities there shown the change of velocity effected by the blade is AB. Its length to scale divided by 32·2 gives the force exerted by the blade on the jet per pound of steam flowing through the blade ring. And the force acts in the direction AB.

Fig. 227 is a velocity diagram like the diagram of Fig. 226, in which the relative velocity is changed both in magnitude and direction.

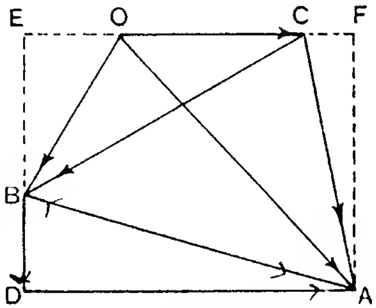


Fig. 227.—General velocity diagram.

AB is the change of velocity produced, and it acts in the direction AB. The force acting from the steam to the blade is the equal and opposite force BA. The resolved component of this force in the direction of motion of the blade is the component which is effective in doing work.

Resolve therefore BA into the two components by drawing BD and DA respectively perpendicular to, and parallel to OC, the direction of motion of the blade.

Then,

$$\left.\begin{array}{l}\text{Effective component of the force in}\\ \text{the direction of motion}\end{array}\right\} = \frac{W}{g} \times \text{DA lbs. wt.}$$

This force acts at the mean circumference of the blades.

If, for example, DA is 200 feet per second, and W, the flow, is 2 lbs. per second, the magnitude of the force is $\frac{2 \times 200}{32 \cdot 2} = 12 \cdot 4$ lbs. wt.

The work done per second by a force is the effective component of the force in the direction of motion multiplied by the velocity of the point of application. The velocity of the point of application is the velocity of the mean circumference of the blades, and this is

equal to OC in the velocity diagram. Therefore the work done per second by the effective component of the force is

$$\frac{\text{W . DA . OC}}{g} \text{ ft.-lbs. per second} \quad . \quad . \quad . \quad . \quad (2)$$

Or, stated in words,

$$\left.\begin{matrix}\text{Work done per}\\ \text{second in ft.-lbs.}\end{matrix}\right\} = \frac{\text{flow}}{g}\left\{\begin{matrix}\text{change of velocity pro-}\\ \text{duced by the blades in}\\ \text{the direction of motion}\end{matrix}\right\} \times \left\{\begin{matrix}\text{mean velocity}\\ \text{of blades}\end{matrix}\right.$$

when the flow, W, is measured in pounds per second.

The corresponding rate of working in horse-power units is

$$\frac{\text{W}}{g} \cdot \frac{\text{DA . OC}}{550} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

DA may be found graphically or by calculation, from data sufficient to determine the velocity diagram.

It is interesting to notice that the expression for the rate of working, namely $\frac{\text{W}}{g}$. DA . OC, can be obtained directly from the velocity diagram and the principle of the conservation of energy. Thus, reckoning in terms of a flow of W lbs. of steam per second: the rate at which kinetic energy is supplied to the blades by the nozzles is $\frac{\text{W}w^2}{2g}$ ft.-lbs. per second; the rate at which kinetic energy is produced in the blade channels by the further expansion of the steam is $\text{W}\left(\frac{j^2}{2g} - \frac{i^2}{2g}\right)$ ft.-lbs. per second; and the rate at which kinetic energy is discharged from the blades is $\frac{\text{W}u^2}{2g}$ ft.-lbs. per second.

Then the rate at which energy is converted into mechanical work by the blades is

$$\frac{\text{W}w^2}{2g} - \frac{\text{W}u^2}{2g} + \frac{\text{W}j^2}{2g} - \frac{\text{W}i^2}{2g} \text{ ft.-lbs. per second} \quad . \quad . \quad (4)$$

In the velocity diagram, Fig. 227,

OC = v, the velocity of the mean circumference of the blades;
CA = i, the relative velocity of steam at entry to blade ring;
CB = j, the relative velocity of steam as it glides off the blades;
OA = w, the actual velocity of discharge from the nozzles;
OB = u, the actual velocity of discharge from the blades;
DA = the resolved component of BA parallel to OC.

Then through D and A draw perpendiculars to meet OC produced, in E and F. Let CF = a; OE = c.

Then $$w^2 - i^2 = v^2 + 2av$$
$$j^2 - u^2 = v^2 + 2cv$$
so that $$w^2 - i^2 + j^2 - u^2 = 2v(a + c + v) = 2v\text{DA}$$
giving that the rate of working is

$$\text{W} . \frac{\text{DA} . \text{OC}}{g} \text{ ft.-lbs. per second,}$$

which is the equation (3) above.

The vertical component BD represents the force with which the steam acts on the blades in a direction parallel to the axis.

Its magnitude is

$$\frac{\text{W}}{g}\text{BD lbs.}$$

This force, when it exists in an actual turbine, is included with the total steam thrust in the direction of flow, and this thrust is balanced by suitable devices.

189. Shape of Blades and Blade Channels.—It has been tacitly assumed hitherto that the blades change the velocity of the steam without waste of energy. To realize this the steam must glide on to the vanes without shock or the formation of eddies.

A particle of steam on the surface of a blade can have no motion perpendicular to the surface. The only motion possible to it is along the surface. The direction of the velocity of a particle relative to the blade is therefore equally the direction of the element of the blade surface at the point corresponding to the velocity.

It follows that the direction of the receiving element of the blade surface should be the direction of the velocity of flow from the nozzle relative to the blade.

And also that the direction of the discharging element of the blade surface is the direction of the velocity of discharge relative to the blade.

Both these directions are given by the velocity diagram. For example, in Fig. 225, page 620, CA is the direction of the receiving element of the blade, and CB the direction of the discharging element of the blade surface for the conditions defined by the velocity diagram. Similarly, CA and CB, Fig. 226, fix the direction of these surface elements for the conditions shown in that velocity diagram.

If the receiving element of the blade surface is set at any other angle an on-coming particle in the stream will have a component velocity at right angles to the surface of the element. But a velocity at right angles to the surface cannot take effect, and its prevention produces a sudden pressure normal to the surface, and this pressure, which is in the nature of a shock, produces consequent local disturbances of the flow, and the formation of eddies.

The directions of the receiving and the discharging elements of the blade must change into one another by a smooth curve of such a

nature that the actual path of the flow which it determines is without abrupt changes of velocity.

Therefore, for the preliminary step in the design of a blade, assume a depth; determine from the velocity diagram the directions of the surface elements at the receiving and the discharging edges; join these directions by a smooth curve.

To ascertain whether the curve chosen is suitable, construct the actual path of a particle of the steam stream flowing along the surface, and so find if the path is smooth and without abrupt changes in direction.

In order to construct the path, draw the blade in a series of positions corresponding to equal intervals of time; plot on the blade

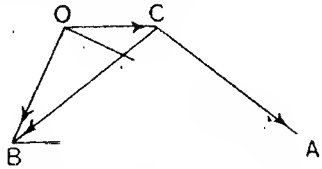


FIG. 228.—Velocity diagram. Impulse pair.

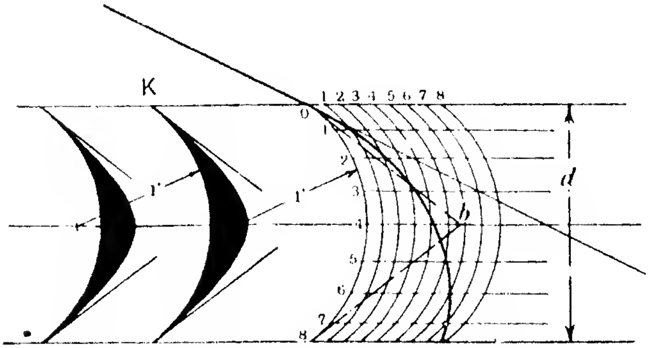


FIG. 229.—Blade form and path of flow. Impulse pair.

in each position the corresponding position of the particle. The curve joining these particle positions is the actual path which the particle follows in space during its passage through the blade ring.

Consider first the case of an impulse pair working in conditions determined by the velocity diagram, Fig. 228.

Take two parallel lines at a distance d apart, Fig. 229. At any point O draw a line Ob parallel to the direction of relative velocity CA. And since the relative velocity of discharge is equally inclined in the other direction, draw a line $8b$ parallel to BC in the velocity diagram through the point numbered 8 taken vertically below O. Draw the circular arc of radius r to which these directions are tangential. This circular arc gives a suitable form; but any curve joining the directions would do equally well providing that it

determines a smooth flow of the steam, a point we are about to examine for the blade form selected.

Let s be the length of the blade surface measured from 0 along the surface to the point 8. Then

$$s = it$$

where i is the velocity of the particle relative to the blade, and t is the time. Since it is an impulse pair this velocity is constant during the whole flow along the blade, and its magnitude is given by the length CA or CB in the velocity diagram.

Let x be the displacement of the blade along its path. Then

$$x = vt$$

where v is given by the vector OC in the velocity diagram. The particle occupies the time $\frac{s}{i}$ in moving from entry to exit along the blade surface, consequently the blades move a distance

$$x = v\frac{s}{i} = s \times \frac{OC}{CB}$$

in the same time.

Therefore, measure the length of the blade surface by stepping along it from 0 to 8 with a pair of dividers; compute the ratio $\frac{OC}{CB}$ from the velocity diagram, and then calculate the value of x and set out 08, the distance so found, along the blade path. Divide the whole displacement of the blade into n equal parts. $n = 8$ in the diagram.

The particle moves along equal distances of the blade surface in equal times, since the relative velocity is constant. Therefore divide the surface from 0 to 8 into $n = 8$ equal parts, and through each point draw a horizontal line. The intersection of a line with the correspondingly numbered blade surface is a point in the path of flow. The thick line in the diagram shows the path. It is smooth, and therefore a cylindrical surface of which the section is shown in the figure is a suitable form for the back or concave surface of the blade. The front or convex surface is now to be designed, so that the channel between successive blades is approximately constant in cross-section. The actual section of a blade in which this condition is fulfilled is shown at K in Fig. 229.

In practice, to ensure that the steam issuing from a nozzle does not strike the front of the blade and so produce a component of force which retards the motion of the blades, the inclination of the receiving elements of the back surface is increased a few degrees, thus slightly opening out the entry to the channel between the blades. In consequence of this there is a slight expansion of the steam as it passes through the moving channels.

Only those particles of steam which are close to the concave surface of the blade form a stream line similar in shape to that determined above as the path of the flow. The particles more

remote from the concave surface follow a path slightly different, a path determined by the compression or crowding of the steam particles on to the receiving elements with subsequent expansion on the way to the discharging elements. If the blades are not set close enough, the steam will tend to pass through the channel in a straighter path than that determined by the blade form. Usually the blades are pitched at about half their depth, so that a blade 1 in. deep would be set round the mean circumference at ½ in. pitch.

The geometry is not quite so simple in the case of a reaction pair, because the velocity of a particle relative to the blade increases as the flow proceeds, and also the blade channel must be designed as a nozzle to give the variation of cross-section necessary to increase the velocity of flow. It is necessary as a preliminary step, therefore, to define the rate at which the increase of relative velocity is to take place.

Let the velocity diagram OCAB, Fig. 230, show the conditions of working for a reaction pair. The relative velocity CA at entry changes to CB at discharge. Assume that the vector CA swings into the position CB with constant angular velocity, and that it increases in magnitude uniformly.

Divide the angle ACB into eight equal parts by lines through C. Starting from the position CA, increase the length of successive vectors by one-eighth of the difference CB − CA, thus defining the curve AB. Bisect each of the intermediate angles as shown by dotted lines in the figure. Then each dotted vector gives the average velocity of the particle during the time interval Δt occupied by the vector in turning through the angle $\frac{ACB}{8}$.

Let S be the length of the surface of the blade; and let Δs be the length of the element of this surface which is described in time Δt with average velocity i. Then $\Delta s = i\Delta t$ for each element. Summing these elements together we have,

$$\begin{aligned} \Delta s_1 &= i_1\Delta t \\ \Delta s_2 &= i_2\Delta t \\ \Delta s_8 &= i_8\Delta t \\ S &= \Sigma i\Delta t \end{aligned}$$

where $S = \{\Delta s_1 + \Delta s_2 \ldots \Delta s_8\}$

and $\Sigma i = \{i_1 + i_2 \ldots i_8\}$

These partial equations give

$$\frac{\Delta s_1}{i_1} = \frac{\Delta s_2}{i_2} \ldots = \frac{S}{\Sigma i} = \text{a constant } K = \Delta t,$$

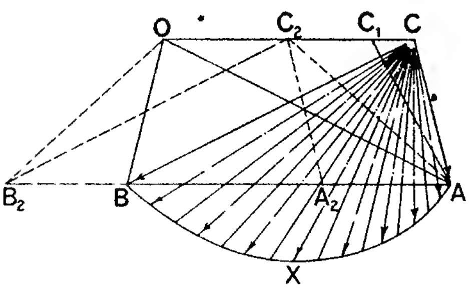
so that $\Delta s_1 = i_1 K$

$$\Delta s_2 = i_2 K, \text{ etc.}$$

In these equations, S is the length of an ideal surface midway between the actual surfaces of a blade, and may be called the mean surface. The lengths Δs_1, Δs_2, are elements of this mean surface.

The construction for the blade and the path it determines is now as follows:—

Select any depth d; draw the initial and the final directions, $0b$, $8b$ (Fig. 231), parallel respectively to CA and CB in the velocity diagram; join these by a smooth curve; step out the length of this curve to find S; add together the lengths of the eight dotted vectors in the velocity diagram to find Σi; calculate the value of the constant K; calculate separately the lengths of the surface elements,



	CROSS SEC.	i MEAN VEL.	ΔS DISTANCE	Δt
0				
1	1·0	47½	5·80	
2	0·87	54½	6·66	
3	0·77	61½	7·52	
4	0·69	68½	8·37	
5	0·63	75½	9·23	
6	0·57	82½	10·10	
7	0·53	89½	10·95	
8	0·49	96½	11·80	
		576	(70·43) 70·50	0·1225
		Σi	S	

FIG. 230.—Velocity diagram. Reaction pair.

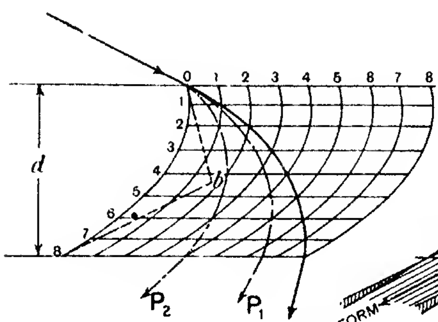


FIG. 231.—Blade form and path of flow. Reaction pair.

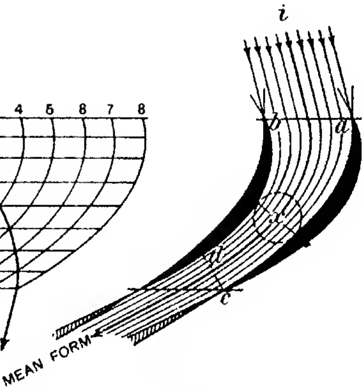


FIG. 232.—Channel form.

Δs_1, Δs_2, etc.; step out these lengths along the blade surface, thus fixing eight points through which to draw horizontal lines; finally, set out the blade in eight positions corresponding to the equal time intervals Δt, and draw the path of the particle through correspondingly numbered intersections of the blade surface with the horizontals.

The constant K = Δt; therefore the distance 08 on the paper is 8K × OC.

As an example of the method the figures relating to the large

drawing from which Fig. 231 was produced may be given. The lengths of the average velocity vectors were respectively 47½, 54½, 61½, 68½, 75½, 82½, 89½, and 96½ mm.

Therefore $\Sigma i = 576$ mm.

The length of the blade surface set out as a preliminary trial, the surface shown in the figure, measured 70·5 mm.

Therefore $K = \frac{70 \cdot 5}{576} = 0 \cdot 1225 = \Delta t$

So that $s_1 = 47\frac{1}{2} \times 0 \cdot 1225 = 5 \cdot 8$ mm.

$s_2 = 54\frac{1}{2} \times 0 \cdot 1225 = 6 \cdot 7$ mm.,

and so on.

These numbers fix the points 1 to 8 on the assumed surface. The distance OC in the velocity diagram, that is, the blade velocity, measured 78 mm. Therefore, $08 = 8 \times 78 \times 0 \cdot 1225 = 76 \cdot 4$ mm. This distance is divided into eight equal parts, and a blade form is drawn in through each division. The path of the flow can then be sketched in.

The forms of the back and the front of the blade have now to be chosen, so that the back of one blade and the front of the adjacent blade form a channel whose cross-section varies correspondingly with the variation of the magnitude of the velocity defined by the velocity diagram.

The drop of pressure through a reaction pair is small, so that it may be assumed that the density of the steam during its flow through the pair is constant. With this assumption the cross-section of the channel varies inversely as the velocity relative to the blade surface.

Two blades are shown in Fig. 232, blackened in. Since the dimension of the blade perpendicular to the paper is constant, the area of the initial cross-section is proportional to ab; and the area of the final cross-section is proportional to cd. Then the ratio $ab : cd$ must be CB : CA, the ratio of the final and the initial relative velocities. That is,

$$cd = ab \times \frac{CA}{CB}$$

An intermediate cross-section as x must have the value

$$x = ab \times \frac{CX}{CB}$$

where CX is the corresponding value given by the velocity diagram.

The actual surfaces were set out by calculating the width x, corresponding to each velocity like CX, and starting with an assumed width at entry, setting off half of these several values on either side of the mean surface. A channel of the desired variation of section can then be drawn, and the curves defining it are respectively the front and the back curves of a blade surface. If these curves do not fit about the mean surface, another blade form must be assumed, and the whole process repeated.

In Fig. 232 the path of the flow relative to the blade channel is drawn in. It will be seen that although the mid-channel stream line passes into the channel without abrupt change of direction, those stream lines in contact with the blade surfaces cannot enter without shock, because of the modification of the proper angle at entry produced by the blade thickness on the receiving elements of surface, both at the front and back of the blade.

If the blade velocity is reduced, then the direction of relative velocity gradually approaches the actual angle of entry at the front of the blade. In the figure this condition is reached when the velocity is reduced to OC_1, where C_1A is the direction of the front of the blade at *b*. The reduction of velocity also reduces the length of the path through the blade ring, as will be seen from Fig. 231, where the chain dotted curve OP_1 shows the mean path corresponding to the blade speed OC_1. The curve OP_2 shows the mean path when the velocity is reduced to one half the velocity OC. This is about the shortest path which the flow can follow through the blade ring. The corresponding velocity diagram is $OC_2A_2B_2$.

By reducing the speed of the blade ring without changing the form of the blades, there is shock at entry and consequent loss. The blade length for the proper angle at entry is greater with reduced speeds than that shown in the figure, and the loss by friction along the blades is therefore increased. Experience proves that the loss by shock at entry is relatively less important than blade friction. In practice, therefore, turbine pairs are run at speeds considerably lower than those corresponding to the angles at entry to the blades.

The notion of shock as relating to liquids, requires considerable modification when applied to the case of compressible fluids. A gas impinging on a surface is compressed, and it subsequently expands and gives back part of the energy of compression, so that with steam the shocks are all softened by its elasticity. In fact, the action between the wrongly inclined surface element and the steam is analogous to the action between a piston and the clearance steam in a cylinder. The steam cushions itself, and the loss of energy caused by this elastic impact is probably small, within the limits of correct and incorrect inclination of the receiving elements of turbine blades.

The design of the nozzle, or guide blade ring, as it is sometimes called, is done in a similar way. In the velocity diagram, Fig. 230, the directions OA and OB, which determine the final and initial directions of the fixed blades, assuming that steam is received in the direction in which it is discharged, are symmetrically inclined with the direction of relative velocity CA and CB. Therefore the nozzle may be the same in shape as the moving blades, so that the pair, Fig. 232, reversed in direction, does equally well for the guide blades.

190. Axial Velocity of Flow.—The component of the velocity of the steam parallel to the axis of a pair is the velocity with which

the steam flows through the turbine in a direction parallel to the axis at right angles to the direction of revolution of the blades.

This is the velocity with which the steam is brought to, flows through, and flows away from the pair.

Let A be the area of the channels in square feet in a direction at right angles to the axis; and let the flow be W pounds per second of steam whose density is D. Then if f is the axial velocity of flow

$$f = \frac{WD}{A} \text{ ft. per second} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This gives the value of f at any point in the flow when the corresponding area A is substituted.

In a reaction pair the area A at the entry to the guide blades or nozzles is equal to the area of the annulus providing that the blade tips are sharp. Also at exit from the guides; at entry to the moving blades; and at exit from the moving blades the area is equal to the annulus in which the blades revolve—that is, the annulus between the blade drum and the casing. As the flow proceeds through the wheel, the area A first decreases and then increases owing to the thickness of the blades. The actual area in any position is the area of the annulus between the drum and the casing less the thickness of the blades measured in a direction at right angles to the axis. The velocity of flow f in the space just above entry to the fixed blades gradually increases as the flow proceeds through the guide blades, and then decreases again towards the discharge from the guide blades into the space between the guides and the moving blades, where it is for the moment a minimum, since it begins to increase directly it flows into the moving blades, increasing to a maximum at the plane of minimum area and finally falling to a minimum at the point of discharge. The change in the magnitude of the velocity of flow should take place gradually.

The change through the moving blades and equally through the guide blades is given by the vertical component of the velocities CA to CB in Fig. 230 for shape of the blade there shown.

In the case of the impulse pair the axial component is constant along the nozzle, and its value depends upon the inclination of the nozzle, but it increases to a maximum and again falls as it passes through the wheel.

The axial component of the velocity at discharge from a pair is determined by the angle of the blade. If the angle is small, then the axial component of the flow is small, and the area required to carry a given weight of steam along is correspondingly great.

If the discharge angle is increased, the velocity of flow is increased and the necessary area A is therefore reduced. In reaction pairs working at the low-pressure end of a turbine the discharge angle is usually increased in order to reduce the area necessary for the quantity of steam flowing.

191. Conditions of Maximum Rate of Working.—Energy corresponding to the velocity of flow is of necessity rejected from a turbine pair. A pair is working at its maximum rate when the actual velocity of the steam as it leaves the blades is in a direction parallel to the axis. In any other direction its velocity is greater than the axial velocity, and it therefore rejects more energy than it need. Another way of stating the matter is that when the actual velocity of discharge is axial, that is, at right angles to the direction of motion of the blades, it has no component in that direction of motion and is therefore incapable of doing any more work on the moving blade.

Consider an impulse pair set to work on a pressure range corresponding to U lb.-cals. per lb. and supplied with steam at an axial velocity of flow u corresponding to U_s lb.-cals.* Assume also that the blade form is such that the change of motion produced is in the direction of motion of the blades. Then the axial velocity of discharge will be equal to the axial velocity along the nozzle.

Referring to the diagram, Fig. 233, these conditions determine first, that the vector OB is at right angles to the direction of motion OC; that the vector representing the change of velocity passes through B in a direction parallel to OC. And further, since the diagram is for an impulse pair, the relative velocity is constant in magnitude.

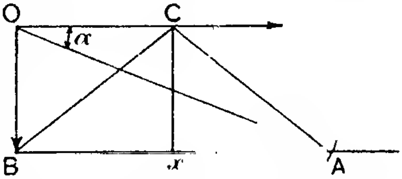


FIG. 233.—Velocity diagram for maximum rate of working. Impulse pair.

Before the diagram can be constructed the magnitude of the axial velocity of flow must be given.

Also since the energy supplied to the pair is U from the pressure range, and U_s from the velocity of supply, the velocity of discharge from the nozzle is given by

$$w = \sqrt{2gJ(U + U_s)} = OA \quad . \quad . \quad . \quad . \quad (1)$$

The construction of the velocity diagram giving the conditions for maximum rate of working, having given f, U (U_s is calculated from f), is then as follows:—

Set out OC, Fig. 233, in the direction of motion of the blades; OB at right angles to OC and equal in magnitude to f; calculate the magnitude of w from (1), and from O as centre and with radius w draw an arc to cut a line through B parallel to OC in A. Then AB is the change of velocity which is produced by the vane, and the condition is satisfied that there shall be no component of the velocity of discharge in the direction AB.

A point C is now to be found such that CA = CB. Therefore bisect AB in x, and through x erect a perpendicular to cut OC in C. This is the required point, and it fixes the corresponding blade velocity OC, equal to half the change of velocity AB.

It also fixes the angle COA, the angle of the nozzle, and the directions CA and CB of the blade elements at entry and exit.

It is clear from the diagram that

$$BA^2 = OA^2 - OB^2 = 2gJ(U + U_x) - 2gJU_x = 2gJU$$

Therefore

$$BA = \sqrt{2gJ}\sqrt{U}\text{; so that the force} = \frac{1}{g}\sqrt{2gJ}\sqrt{U}\text{ lbs. per lb. of flow.}$$

$$OC = \frac{BA}{2} = \sqrt{\frac{gJ}{2}}\sqrt{U}\text{ ft. per second} \quad . \quad . \quad . \quad (2)$$

$$\left.\begin{array}{r}\text{And the maximum}\\ \text{rate of working}\end{array}\right\} = \frac{1}{g}\sqrt{2gJU} \times \sqrt{\frac{2gJ}{4}}\,.\,U = JU\text{ ft.-lbs. per sec.}$$

In order to find the conditions giving the maximum rate of working of a reaction pair, the proportion in which the energy U, corresponding to the pressure difference, is divided between the nozzle and the moving blades must be given in addition to the two conditions that the direction of the actual velocity at discharge is at right angles to the direction of motion of the blades, and that the change of velocity produced by the blades is in the direction of motion.

Consider a reaction pair set to work on a pressure range corresponding to U lb.-cals. per pound of flow and supplied with steam at an axial velocity corresponding to U_x lb.-cals. per pound of flow. Assume that the energy U is divided between the fixed nozzles and the moving blades so that the part U_1 is converted into kinetic energy in the nozzles and the part U_2 in the blades.

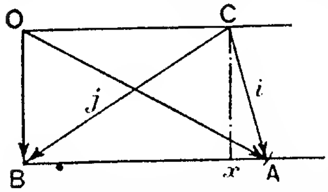


FIG. 234.—Velocity diagram for maximum rate of working. Reaction pair.

Referring to the diagram, Fig. 234, these conditions determine that the vector OB is at right angles to the direction of motion OC; that the vector representing the change of velocity passes through B in a direction parallel to OC.

The pair is supplied with the total energy U_x from the velocity of supply and U from the pressure range, but of this only the part U_1 is converted in the fixed nozzles, so that the velocity of discharge from the nozzles to the moving blades is

$$w = \sqrt{2gJ(U_1 + U_x)} = OA \quad . \quad . \quad . \quad . \quad (3)$$

From O as centre, with radius w, draw an arc to cut a line through B parallel to OC in A. Then AB is the change of velocity produced by the blade when there is no component of the velocity of discharge in the direction AB.

It remains to find the point C.

Assume that C is determined. Let i and j be the initial and final relative velocities. Then, since $OB^2 = 2gJU_s$,

$$BA^2 = OA^2 - OB^2 = 2gJ(U_1 + U_s - U_s) = 2gJU_1$$

And by the geometry of the figure

$$j^2 - i^2 = 2 \,.\, BA \,.\, Bx - BA^2 = 2gJU_2$$

Therefore

$$Bx = OC = \frac{2gJ(U_2 + U_1)}{2\sqrt{2gJU_1}} = \frac{\sqrt{2gJ}}{2} \,.\, \frac{U}{\sqrt{U_1}} = 150\frac{U}{\sqrt{U_1}} \quad (4)$$

From which the blade velocity corresponding to the maximum conditions of working with the division of the total energy assigned is known.

If $U_1 = U_2$

$$OC = \sqrt{gJ}\sqrt{U} \text{ or } \sqrt{2gJ}\sqrt{U_1} = AB$$

Therefore, since OC in these circumstances = AB, the direction of relative velocity at entry is radial, and the velocity diagram becomes a rectangle OCAB whose diagonals are OA and CB, giving that the velocity of discharge from the fixed nozzle OA has the same magnitude as the relative velocity of discharge from the moving blades, namely CB.

192. General Expression for the Efficiency of an Impulse Pair.—A pair is working at maximum efficiency when the velocity of discharge is at right angles to the direction of motion of the moving blades. An inspection of the velocity diagram will show that the magnitude of the velocity of discharge OB, on which the energy of discharge from the turbine alone depends, is not materially increased if it is moderately inclined to the direction corresponding to maximum efficiency in the way shown in Fig. 235, that is, to the left of the vertical. When the final velocity is so inclined the blade velocity v is reduced. Let λ be the ratio between v, the velocity of the moving blades, and w the velocity of discharge of the steam from the nozzles. That is

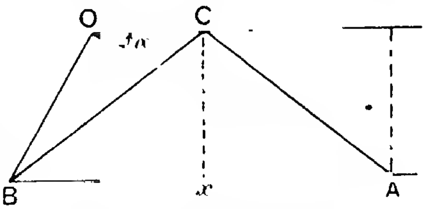


FIG. 235.—General velocity diagram. Impulse pair.

$$\lambda = \frac{v}{w}$$

From the geometry of the velocity diagram for maximum efficiency (Fig. 233, page 635), it will be seen that the value of λ for the maximum rate of working is $\frac{1}{2}\cos\alpha$. An expression will now be found giving the efficiency of the pair in terms of λ, assuming

that the velocity of supply is axial and that the flow is frictionless adiabatic.

Let Fig. 235 be a velocity diagram for an impulse pair, in which the velocity of discharge is inclined to the axial direction. Let OC = v; OA = w; the angle AOC = a.

Then the force exerted by the jet on the blades per pound of steam flowing per second is

$$\frac{BA}{g} = \frac{2}{g}(w \cos a - v) \text{ pounds}$$

The work done per second per pound of flow per second is then

$$\frac{BA \times OC}{g} = \frac{2v}{g}(w \cos a - v) \text{ ft.-lbs.}$$

The energy supplied per pound of flow per second is $\frac{w^2}{2g}$ ft.-lbs.

The efficiency is therefore

$$\eta = \frac{4v(w \cos a - v)}{w^2} = 4\lambda \cos a - 4\lambda^2 \quad . \quad . \quad . \quad (1)$$

This is clearly a maximum when

$$\lambda = \tfrac{1}{2} \cos a$$

giving that $v = \frac{1}{2}w \cos a = \frac{1}{2}$BA, and this carries with it the condition that the direction of discharge is at right angles to the direction of motion as stated above.

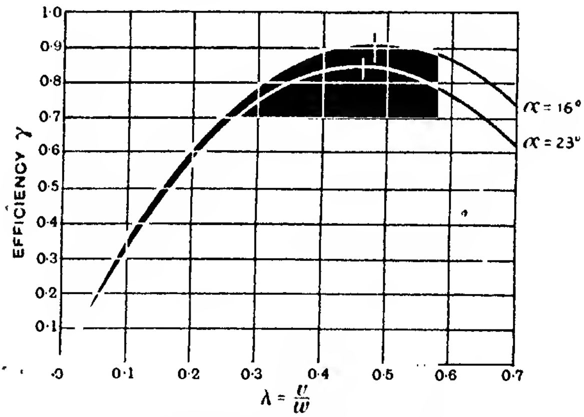


Fig. 236.—Efficiency curves. Impulse pair.

Let the angle a be fixed at 20°, giving cos 20° = 0·94. Then for maximum efficiency $\lambda = \frac{1}{2}$ cos 20° = 0·47.

Then from (1) the value of the maximum efficiency for this inclination of the nozzle is

$$\eta = 0{\cdot}88$$

4×·47×·94 − 4×·47×·47

4+·47×·47

If λ is reduced to 0·3, about $\frac{2}{3}$ of the value for maximum efficiency, the actual efficiency is only reduced to 0·77; or a reduction of 33 per cent. in the blade speed only reduces the theoretical efficiency by about 11 per cent.

The influence of the ratio λ on the efficiency is shown by the curves in Fig. 236 for the two cases where the nozzle inclination is respectively 16° and 23°.

The maximum efficiency for the value $a = 16°$ is 0·92, and the corresponding value of λ is 0·48.

When $a = 23°$, the maximum efficiency is reduced to 0·84, and the value of λ is 0·46.

It will be seen from the curve that λ can be considerably reduced without a too serious reduction of the efficiency.

The efficiency given by equation (1) is the efficiency for frictionless adiabatic flow, and furnishes a standard with which to compare the actual performance of a single impulse pair working in the conditions assumed.

193. Efficiency of a Reaction Pair.—The case of most practical interest is that in which the blading in the wheel is identical in form with the fixed blading, but reversed in direction, so that if the fixed blading discharges to the right, the moving blades discharge to the left, as in the Parsons turbine.

A velocity diagram corresponding to these conditions is shown in Fig. 237. For identical blading the angle COA = the angle OCB. OA is the velocity of discharge from the fixed blading; OC is the velocity of the moving blading; CB is the velocity of discharge from the moving blades relative to the moving blades; OB is the actual velocity of discharge from the pair, and CA is the relative velocity at which the steam flows from the fixed to the moving blade. Assume, further, that the change of velocity produced by the moving blades is parallel to the direction of motion OC, then

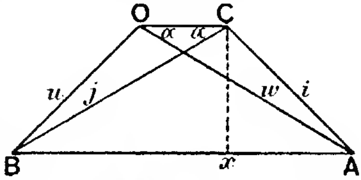


FIG. 237.—Velocity diagram. Reaction pair.

BA is parallel to OC,
OA is equal to CB,
CA is equal to OB.

Also when the pair forms one in a series, in the way explained below, the velocity of supply OB is equal to the velocity of discharge from the pair, and this is also equal to OB.

Let OB = u; OA = w; OC = v; CA = i; and CB = j.

Let U_s be the energy corresponding to the kinetic energy of supply, then

$$U_s = \frac{u^2}{2gJ}$$

Let U_1 be the energy required to change the magnitude of the velocity of supply u to the magnitude w of the velocity of discharge from the fixed blading. The direction of the velocity is changed by the fixed blading. Then

$$U_1 = \frac{w^2 - u^2}{2gJ}$$

Let U_2 be the energy required to change the magnitude of the relative velocity i to the magnitude j of the relative velocity of discharge from the moving blades. The direction of the velocity is changed by the moving blades. Then

$$U_2 = \frac{j^2 - i^2}{2gJ}$$

From the symmetry of the figure it is clear that

$$2gJU_1 = w^2 - u^2 = j^2 - i^2 = 2gJU_2 \quad . \quad . \quad . \quad (1)$$

from which $U_1 = U_2$. And U_s is the energy corresponding to the velocity of supply. Therefore $U_1 + U_2$ is the heat energy received by the pair from the heat fall on which it is working. If there were no frictional losses $U_1 + U_2$ would be equal to U, the available energy of the fall corresponding to the pressure difference. Allowing for losses the fraction ηU is the energy actually used in producing the change in velocities. Assuming equal losses in the fixed and the moving blades

$$U_1 = \frac{\eta U}{2} = U_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) since $u^2 = 2gJU_s$

$$w^2 = 2gJ\left(U_s + \frac{\eta U}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also the energy required to change i into j is given by

$$j^2 - i^2 = 2gJ\frac{\eta U}{2} = AB^2 - 2AB \cdot Ax \quad . \quad . \quad . \quad (4)$$

But from the figure it is clear that

$$AB = 2w \cos a - v \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$Ax = w \cos a - v \quad . \quad . \quad . \quad . \quad . \quad (6)$$

so that $$j^2 - i^2 = 2wv \cos a - v^2$$

Therefore the energy utilized by the pair in terms of the velocities reduces to

$$J(U_s + U_1 + U_2) = \frac{1}{2g}(w^2 + 2wv \cos a - v^2) \quad . \quad . \quad (7)$$

The work done by the pair per second per pound of flow per second is given by the product $\frac{AB \cdot v}{g}$, where AB is the change of

velocity shown in the velocity diagram. Substituting the values of AB from (5), this reduces to

$$\left.\begin{array}{l}\text{Work per second per pound}\\ \text{of flow per second}\end{array}\right\} = \frac{1}{g}(2wv\cos a - v^2) \quad . \quad . \quad (8)$$

Dividing (8) by (7), and dividing through the numerator and the denominator by w^2, and writing λ for the ratio $\frac{v}{w}$, the efficiency of the pair is

$$E = \frac{2(2\lambda\cos a - \lambda^2)}{1 + 2\lambda\cos a - \lambda^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This is a maximum when the velocity OB is at right angles to OC, giving that $\lambda = \frac{v}{w} = \cos a$. Substituting $\cos a$ for λ in (9),

$$E_{max} = \frac{2\cos^2 a}{1 + \cos^2 a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The efficiency given in expression (9) is the efficiency for frictionless adiabatic flow for the values of λ chosen, and furnishes a standard with which to compare the actual performance of a single pair working in the conditions assumed.

Similar expressions can be obtained for different conditions of working, as, for example, when the velocity of supply is axial (see Example 183, page 732), but they are not of practical interest in connection with steam turbines, because reaction pairs are always used in series, and the efficiency of a pair forming one of a series is defined in a different way, as will appear below.

If the available energy ηU, corresponding to a pressure difference is given, the value of $(U_s + U_1)$ the energy producing the velocity w can be calculated as follows :—

From the diagram, Fig. 237, it will be seen that the velocity of supply OB $= u$ is equal to the relative velocity CA $= i$.

And

$$CA^2 = (w\cos a - v)^2 + w^2\sin^2 a = w^2 - 2wv\cos a + v^2 = u^2.$$

Therefore

$$\frac{u^2}{w^2} = 1 - 2\lambda\cos a + \lambda^2 = \frac{U_s}{U_s + U_1} \quad . \quad . \quad . \quad (11)$$

From which

$$U_s + U_1 = U_1\left(\frac{1}{2\lambda\cos a - \lambda^2}\right) = \frac{w^2}{2gJ} \quad . \quad . \quad . \quad (12)$$

194. On the Blade Velocity v and the Value of λ. Compound Turbines.—The blade velocity is given by the relation

$$v = \lambda w$$

where w is the velocity of discharge from the nozzle to the blade ring, and v is the speed of the mean circumference of the blades.

The available energy corresponding to a pressure range from 150 lbs. to 1 lb. per square inch is 177 lb.-cals. per pound of flow. Steam issues from a nozzle designed to expand it between these pressures at $300\sqrt{U} = 4000$ ft. per second neglecting losses.

Suppose an impulse pair is set to work on this pressure range with a nozzle inclination of 23°. A reference to the curve, Fig. 236, page 638, shows that the maximum efficiency which is possible to the pair is 0·84, with a corresponding value of $\lambda = 0{\cdot}46$; giving for the blade velocity, $v = 0{\cdot}46 \times 4000 = 1840$ ft. per second.

This is an impracticable speed, because the centrifugal tension in a ring running at this speed is about 158 tons per square inch. The magnitude of the centrifugal tension for an independent ring is given by the expression

$$t = \frac{Dv^2}{g}$$

where t is the tension in pounds per square foot, and D is the density in pounds per cubic foot. Taking D for steel equal to 485 lbs. per cubic foot, and reducing the result to tons per square inch, the expression becomes,

$$\text{tons per square inch} = \frac{v^2}{21{,}400}$$

To realize such high velocities as those found above it is clear that a new metal is required which possesses the strength of steel with the lightness of aluminium.

In practice the difficulty is avoided by dividing the pressure range between the boiler and the condenser into a series of small consecutive pressure ranges, and then setting a turbine pair to work on each range. The velocity w and therefore the blade speed v are in this way brought within practical limits.

The wheels of successive pairs are keyed to the same shaft, so that the wheels have a common angular velocity. The nozzles or guide blades of the pairs are fixed to the same casing. The first pair receives its supply of steam from the boiler, and its exhaust is the supply for the next pair, which in its turn supplies steam to the next pair; the exhaust from the last pair flows into the condenser. The complete turbine is in fact a Compound Steam Turbine formed of a chain of pairs working in series.

The number of divisions made in the total fall is greater in the case of reaction pairs in series than in the case of impulse pairs in series, because the smaller the pressure difference the smaller the leak through the clearance spaces at the blade tips. On the other hand, the greater the number of pairs the longer the steam path between boiler and condenser, and the greater the loss by friction. The point at which the gain by subdivision exceeds the loss is a matter which can only be settled by experience.

The boiler pressure in the *Mauretania*[1] is 150 lbs. per square inch by gauge, giving a pressure range from 165 lbs. to 1 lb. per

[1] *Engineering*, 1907.

square inch approximately. There are 183 reaction pairs inserted in this range.

In the *Chester*[1] 98 reaction pairs are inserted in the range from the boiler pressure of 200 lbs. per square inch by gauge to the condenser pressure. The subdivision is not carried so far in this case as in the case of the *Mauretania* because the speed is higher, being 558 revolutions per minute as against 190 in the *Mauretania*.

In the *Salem*, a sister ship to the *Chester*, the pressure range from the boiler pressure of 250 lbs. per square inch by gauge to the condenser is divided into seven stages only. Four impulse pairs are inserted in the first stage, and three in each of the remaining six, making 19 impulse pairs in all on the total range. The reduction of the velocity v which can be made by the use of a series of impulse pairs within a pressure range is explained in detail in Sec. 197, page 654.

195. Division of the Pressure Range. Efficiency of a Chain of Pairs.—This is made so that each pair of the series receives available energy in some fixed proportion. Usually in a series of reaction pairs the division is made so that each pair receives approximately the same fraction of the total quantity available. The way in which the pressure range may be divided so that each pair receives a predetermined quantity of energy is illustrated in Fig. 238, page 644.

OP is a pressure axis. The pressure at O is the initial pressure. The ordinate set up at any pressure lower than the initial pressure represents the available energy U corresponding to adiabatic expansion from the initial pressure to the lower pressure through which the ordinate is drawn. The data for plotting the available energy curve OE is taken from Table 30, page 591.

Let the range which is to be divided be from 150 to 1 lb. per square inch. Through the point on the pressure axis giving the lower pressure of the range draw a vertical FE. Set off FG to represent U_0, the energy used to produce the axial flow, assuming it to remain constant throughout the series, and then divide the distance GE in the proportion in which each turbine pair is to receive energy. Through the points so found draw horizontals to cut the curve OE; and through the intersections of OE with the horizontals, drop perpendiculars to the pressure axis, in this way fixing the consecutive pressure ranges corresponding to the energy intervals.

For example, suppose that 16 pairs are to be inserted in the range, and that each pair is to receive one-sixteenth of the total energy after allowing for the axial velocity of flow.

In practice the number of reaction pairs inserted on a range of this magnitude would be more numerous, but the number 16 will serve for the purpose of illustration.

Set out FG to represent U_0, the energy corresponding to the axial

[1] Report of the U.S. Navy Department, *International Marine Engineering*, Aug., 1910, pp. 338, 344.

velocity of flow; and again for the purpose of illustration, take it equal to 2 lb.-cals. In practice U_0 is of the order of 5 per cent. of the total energy utilized by a pair.

Divide GE into 16 equal parts. The horizontals through the points so found intersect the energy curve OE in the points through which perpendiculars are drawn to the pressure axis as shown in the diagram. The pressure ranges are so small after 15 lbs. per square inch that to make the pressure intervals clearer the peak of the diagram is plotted to a larger pressure scale above the main diagram.

From the diagram it will be seen that the pressure range on which the first pair works is from about 141 to 110 lbs. per square inch after allowing for the range of 9 lbs. required to produce the axial velocity of flow. But the pressure range on which the last

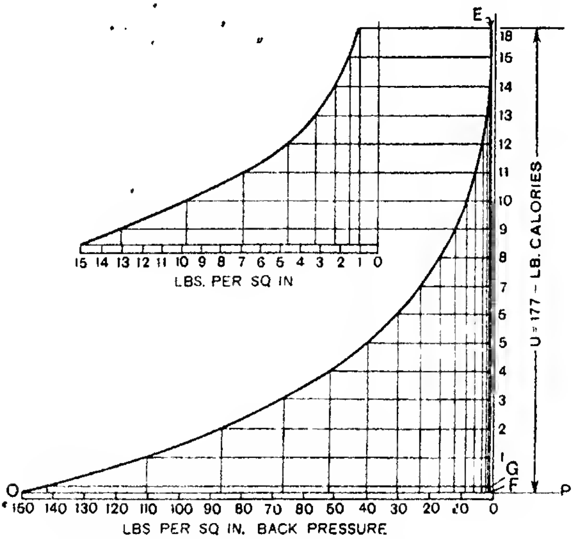


FIG. 238.—Pressure ranges for equal division of work between 16 pairs.

pair works is only about ½ a lb. per square inch, and yet each pair theoretically contributes equal quantities of work to the total work done on the shaft by the whole series.

The diagram brings out several important points.

In the first place, it may be noticed that the velocity of flow having been provided for at the first pair, no further provision of energy is needed for this purpose; each pair receives the energy U_0 as kinetic energy of axial flow and passes an equal quantity on to its neighbour. In practice the values of U_0 increase somewhat towards the low-pressure end.

In the second place, it is clear that with the large pressure range at the high-pressure end, a reaction pair there is not working under

such advantageous conditions as at the low-pressure end, where the pressure difference per pair, on which the leak per pair depends, is so much smaller, and also this smaller pressure difference has a relatively much smaller clearance area through which to produce a leak. The economy of reaction pairs working on high pressure ranges is in fact not so good as when they are set to work on the low pressure ranges of the turbine. For this reason in recent designs impulse pairs are introduced at the high-pressure end of a range, and reaction pairs at the low-pressure end, reaction blading beginning after the pressure has fallen to approximately the atmospheric pressure.

Another method is to combine a compound reciprocating engine with a low-pressure turbine, as in the case of the *Olympic*[1] and other large vessels built by Harland & Wolff. The expansion of steam cannot conveniently be carried below 7 lbs. per square inch in a reciprocating engine, but there is no difficulty in continuing below 1 lb. per square inch with reaction pairs. And the compound reciprocating engine is economical at the higher pressure. Therefore, the division of the total pressure range between a reciprocating engine and a low-pressure turbine is likely to be an efficient combination.

A third point brought out by the diagram, and already mentioned in page 618, is the importance of reducing the condenser pressure to the lowest possible value. A pressure range from $1\frac{1}{2}$ to 1 lb. is as valuable as a pressure range of 31 lbs. per square inch at the upper end of the scale. The addition of a few reaction pairs to the turbine does not add proportionately to the cost, though it does add proportionally to the work done. The increase in size of a low-pressure cylinder of a reciprocating engine to enable the expansion of the steam to be carried to the low pressure of 1 lb. per square inch increases the cost, however, out of all proportion to the work gained. With turbine installations, therefore, it is desirable to install very efficient condensing plant.

It will also be noticed from the diagram that nearly one half of the total available energy of 177 lb.-cals. per pound of flow is transformed into work after the pressure has been reduced to 15 lbs. per square inch. The exact proportion is 47 per cent. A reaction turbine provided with a good condenser is therefore an efficient addition to any plant where there is much exhaust steam available at atmospheric pressure from non-condensing engines or other sources.

A reference to Table 30, page 591, will show that the volume per pound of flow increases from 3 cubic ft., the initial volume at 150 lbs. per square inch, to 261 cubic ft., after the expansion has proceeded adiabatically to 1 lb. per square inch. The annulus between the drum and the casing must therefore be increased in the same proportion to provide the area necessary to accommodate the flow at a given axial velocity. As, however, the velocity of axial flow is increased towards the low-pressure end, the increase of the area of the annulus is not quite in proportion to the increase of volume. Theoretically each pair in succession must increase in size in order

[1] For description of the *Olympic*, see *Engineering*, 1910, vol. 90.

to provide sufficient area for the flow. In practice, for constructive reasons, this increase in size is made in a series of steps. The blade heights increase correspondingly. In the case of the *Mauretania* turbine the height of the blades in the first pair is 2·375 ins., and this is increased by steps to 22 ins. in the last pair.

Special problems arise in connection with pairs used in series, and one of the most important is the determination of the change of quality of the steam as it flows through the turbine. It has been assumed in the general consideration of the subject up to now that the quality of the steam at any point in the flow corresponds to that in frictionless adiabatic flow. In practice the flow is frictionally resisted and it is not adiabatic. Moreover, the efficiency of each pair in the series is not the same. A further discussion of these problems is, however, reserved until the general design of a Parsons turbine is considered.

In Sections 192 and 193, pages 637 and 639, expressions have been obtained for the efficiencies of an impulse pair and of a reaction pair in terms of the ratio $\frac{v}{w} = \lambda$. The energy required to produce the velocity of flow in the case of a single pair is a material fraction of the whole available energy corresponding to the pressure range on which the pair works. The ideal performance of a single pair falls considerably below the efficiency of the Rankine engine of comparison, corresponding to the pressure range, as λ is reduced, because in the Rankine engine there is no loss corresponding to the kinetic energy of discharge from the steam turbine, a loss which is inevitable and which increases as λ is reduced.

The case is altogether different when a pair is considered as one of a chain of pairs. In this case, the velocity of supply originally created to supply the first pair, and created by heat drawn from the available heat of the range, continues through the chain and no more energy is taken from the range for the purpose of producing supply velocity, providing that its magnitude is not increased. Now when the pairs in a chain are numerous, as they always are in practice, the energy required for the purpose of producing the velocity of supply to the first pair is relatively small, so that the efficiency of the chain approaches very nearly to the efficiency of the Rankine engine of comparison corresponding to the pressure range on which the chain works.

Each pair of the chain after the first, then receives from the pair next above it energy of supply U_s, corresponding to the kinetic energy of flow, and heat energy U from the steam flowing through it, and if the flow is frictionless and adiabatic, the whole of U is converted into work, providing that the velocity of discharge is equal to the velocity of supply, so that it rejects to the pair next below it kinetic energy equal in amount to that which it received from the pair above it. In these circumstances the ideal efficiency of the pair, reckoned with respect to the quantity of heat which it receives from the available energy of the fall, is unity.

The actual efficiency of the pair is of course less than this. Call it η. Then if the pair converts U lb.-cals. into work it withdraws from the stock of available energy the quantity $\frac{U}{\eta}$, where η is an actual efficiency obtained from experimental data.

For example, if a pair receives kinetic energy equivalent to 2 lb.-cals. from the pair above it, and converts 3 lb.-cals. of the energy flowing through it with the steam into work, it withdraws from the steam the quantity $\frac{3}{0{\cdot}6}$ to do so, assuming that the efficiency of the chain is 0·6 and that each pair in it is equally efficient. The total energy which the pair receives is thus 7 lb.-cals., but of this only 5 are actually withdrawn from the available energy of the range. Considered, therefore, as a single pair its efficiency is $\frac{3}{7}$; but considered as a pair in a chain, its efficiency is $\frac{3}{5}$, and this is the efficiency of the whole chain, assuming each pair to be equally efficient and neglecting the energy producing the velocity of supply.

The negligibly small effect on the efficiency of this velocity when there are a considerable number of pairs is seen by making the calculation when there are, say, 100 pairs in the chain. The total work done is then equivalent to 300 lb.-cals. The total heat received is 500 lb.-cals. + 2 for the production of the velocity of supply. The efficiency is thus $\frac{300}{502}$, which is practically equal to $\frac{3}{5}$.

Therefore the ideal efficiency of a chain of pairs is practically the same as the efficiency of the corresponding Rankine engine of comparison.

The case is different when a number of impulse pairs are placed in a chain in the usual manner. The efficiency of each then approximates to the efficiency given in equation (1), page 638, because the velocity of flow is practically destroyed in each wheel chamber, and requires to be created anew at the entry to the nozzles of each pair in the chain.

196. General Design of a Chain of Reaction Pairs to utilize a Given Heat Fall.—The first step is to calculate the available energy corresponding to the fall, from the given pressure range and the initial state of the steam. The next step is to find how many pairs are required to utilize the available energy of the fall for stated conditions of working.

Let U be the available energy corresponding to the given pressure range. And let U_s be the energy corresponding to the velocity of flow. Then the energy supplied to each pair in succession is

$$\frac{U - U_s}{x} + U_s \text{ lb.-cals. per pound of flow} \quad . \quad . \quad (1)$$

Or with sufficient accuracy, since $\frac{U_s}{x}$ is negligibly small,

$$\frac{U}{x} + U_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The total supply per pair from the heat fall is equally divided between the fixed and the moving blades, and assuming equal efficiency, U_1, the energy actually used in changing the velocity from u to w is equal to $\frac{\eta U}{2x}$. Substituting this for U_1 in equation (12), page 641,

$$\frac{w^2}{2gJ} = \frac{\eta U}{2x} + U_s = \frac{\eta U}{2x}\left(\frac{1}{2\lambda \cos a - \lambda^2}\right) \quad . \quad . \quad . \quad (3)$$

By definition

$$\lambda = \frac{v}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Therefore, squaring each side of (4) and substituting for w^2 its value from (3) and solving for x,

$$x = \frac{45000\lambda\eta U}{v^2(2 \cos a - \lambda)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For example, if $U = 100$; $\eta = 0{\cdot}75$; $\lambda = 0{\cdot}4$; $a = 20°$, and $v = 150$ ft. per second, then—

$$x = \frac{45000 \times 0{\cdot}4 \times 0{\cdot}75 \times 100}{150^2 \times 1{\cdot}48} = 40$$

In practical cases the value of v is not constant during the whole range, neither is the value of λ. Average values of these quantities used in (5) give an approximate value of x sufficiently exact for a preliminary consideration of a design. Average values of v and λ may be applied to various sections of the chain when the value of U proper to that section is given. For example, in marine turbines the chain may be divided into three separate groups of pairs, each group being enclosed in a casing to form a turbine. In this way a single chain of pairs may be used to drive three separate propeller shafts. In such a case values of v and λ must be chosen appropriate to the group and to the quantity of energy U assigned to the group.

For given values of λ, a, η and U the equation may be written

$$xv^2 = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

a form often used in practice, values of the constant to meet various conditions being tabulated.

The mean diameter of the blading in the first pair of the chain is given by the relation

$$d = \frac{60v}{\pi N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where N is the speed in revolutions per minute.

The next step is to fix the height of the blades in the first pair. To do this the rate at which steam must be fed to the turbine in order to produce the power which the turbine is required to develop must be estimated. This estimate is based on previous experience and will depend upon the value of v chosen and upon the size of the turbine. In general it may be taken that steam must be supplied

at the rate of from $8\frac{1}{2}$ to 16 lbs. per shaft horse-power hour, if the turbine is a large one, the lower figure applying to land turbines driving electric generators where v is relatively high, and the higher rates to marine turbines of the class which are coupled directly to the propeller shafts where v is relatively small. A reference to expression (6) shows that with v small x is large, and consequently the frictional path is long and the efficiency is relatively smaller than in the case where v can be taken large.

Let F be the feed in pounds per second; and let A be the area of the annulus in which the blade of height h rotates; and let f be the axial velocity of flow.

Then, quoting from page 587, the fundamental relation between these quantities is given by

$$F = DAf \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where D is the density of the steam as it enters the turbine. The area A is equal to πdh. The axial velocity of flow f is equal to $w \sin a = \frac{v}{\lambda} \sin a = \frac{2\pi N d}{60 \times 2\lambda} \sin a$. Substituting these values for A and f in (8) and solving for h, and then multiplying the right side by 12 so that h is expressed in inches,

$$h = \frac{73F\lambda}{DNd^2 \sin a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In this expression h is in inches; F is the feed in pounds per second; D is the density of the steam at entry; d is the mean diameter of the blading in feet; N is the speed in revolutions per minute.

The value of the density D is found from a line of condition which must be plotted on a steam diagram; and the value of d is found from equation (7).

The next step is to find the height of the blading, and the diameter of the last pair in the chain or the part of the chain enclosed in one casing. As the flow proceeds the density decreases and the blade height increases, but it must not be allowed to increase beyond from $\frac{1}{5}$ to $\frac{1}{10}$ of the diameter of the rotor, and the latter value is to be preferred. The fixing of the value of this ratio furnishes the additional condition which enables both the blade height and the final diameter to be calculated.

Let $$d = ah \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The value of a will range from 5 to 10.

Then substituting for d^2 in (9) its equal value $\frac{a^2h^2}{144}$ and solving for h, the equation becomes

$$h^3 = \frac{10500F\lambda}{a^2DN \sin a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

from which h can be found and afterwards d from (10).

If $a = 20°$, equation (9) becomes

$$h = \frac{213F\lambda}{DNd^2}, \text{ say } \frac{220F\lambda}{DNd^2} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

allowing for thickness of blades in reducing the annular area ; and equation (11) becomes

$$h^3 = \frac{30700F\lambda}{DNd^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If $a = 30°$, (11) becomes

$$h^3 = \frac{21000F\lambda}{DNd^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If $a = 40°$, the relation is

$$h^3 = \frac{16340F\lambda}{DNd^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The cube roots of the constants in the last three equations are in the proportion 1 : 0·88 : 0·81, showing that the adoption of the larger angles reduces the blade height in those proportions, and at the same time the diameters d in the same proportions.

Expression (9) may be called the blading equation. It shows how the blade height depends upon the steam consumption, and the speed when a, N, and d are fixed. It gives the height of the blades for the first pair in the chain. Equation (11) gives the height of the blades in the last pair of the chain, so that the mean diameter shall be a given multiple of the blade height. Intermediate blade heights must satisfy the relation

$$hd^2 = c\frac{\lambda}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where c is a constant for the chain and equal to $\frac{73F}{N \sin a}$. As the flow proceeds D decreases, and therefore, if λ is kept constant, the product hd^2 must increase for each pair. In practice the pairs are, for constructional reasons, divided into a series of groups of identical dimensions, each group being called an expansion. Thus during the flow through a group or expansion hd^2 is constant. And therefore λ decreases as D decreases. When λ has fallen to an assigned minimum value a step up must be made either by increasing h or d, or by increasing both h and d to bring λ back to an assigned maximum value at the beginning of a new expansion. In practice a definite number of steps is fixed arbitrarily from experience, together with a value of λ for the beginning of each group or expansion, and the corresponding value of the density D is found from the line of condition plotted on a chart. For the last few pairs, where D increases rapidly, the product hd^2 is reduced by reducing the value of the constant c, and this is done by increasing the angle a from its

normal value of about 20°, to 30°, or 40°, forming what is called technically, normal, semi-wing, and wing blading.

The available energy U in the equations of this section may for preliminary calculations be taken equal to the available energy of the Rankine engine. The effect of friction must be brought into the detail calculations, and this, as has been shown in Section 184, page 606, reduces the velocity below that corresponding to frictionless adiabatic flow, and increases the dryness as the flow proceeds. The heat-fall on which the turbine chain is to work is divided into a series of sections. Then the energy spent in overcoming the frictional resistance to flow in one section appears in the next stage and dries the steam. The way to draw a line which shows the condition of the steam as the flow proceeds for a given percentage loss in friction and irreversible operations has been explained and illustrated in Section 184.

An example will illustrate the method of applying the equations and results to find the general dimensions of a chain of reaction pairs of the Parsons type.

Find the general dimensions of a high-pressure turbine to work on a pressure range from 180 to 15 lbs. per square inch with steam initially superheated to 200° C. to develop 10,000 H.P. at 750 revolutions per minute, assuming a frictional loss of 25 per cent. as the flow proceeds. Take the efficiency constant at 0·75; the blading symmetrical, with a common angle of 20°; and $\lambda = 0{\cdot}5$. And let v, the blade velocity, be 150 ft. per second in the first pair of the chain.

Assume the condenser pressure to be $\frac{1}{2}$ lb. per square inch, so that the range of pressure on which the low-pressure turbine is to work will be from about 15 lbs. per square inch to $\frac{1}{2}$ a lb. per square inch. The available energy corresponding to the whole range from 180 lbs. per square inch at 200° C. to the condenser pressure is practically 210 lb.-cals. This is the heat which would be transformed into work by a Rankine engine working on the range. Therefore, the Rankine engine would on this range require $\frac{1414}{210} = 6{\cdot}74$ lbs. of steam per horse-power hour. An efficiency of 0·75 per cent. is assumed, so that the actual engine would require at least 9 lbs. of steam per horse-power hour. Allowing for leakage at glands and joints, the flow may be estimated at 10 lbs. of steam per horse-power hour, and this gives a flow of 27·8 lbs. per second for a turbine of 10,000 horse-power, and is the F in the general equations.

The line of condition for 25 per cent. frictional loss, starting from a temperature of 200° C., and a pressure of 180 lbs. per square inch, is shown, plotted on a Total Energy-Entropy diagram, in Fig. 239. H is the state point for the initial conditions. The range is divided into eight stages corresponding to a heat drop of 12·5 lb.-cals. per stage. It will be found that, following the general construction explained on page 612, the final state point falls at N on the 17 lbs. per square inch pressure line. The dryness is 0·921.

The pressure at the beginning of each stage is shown in Table 32, together with the dryness, the tabular volume, and the actual volume. The actual volume is the reciprocal of the density D.

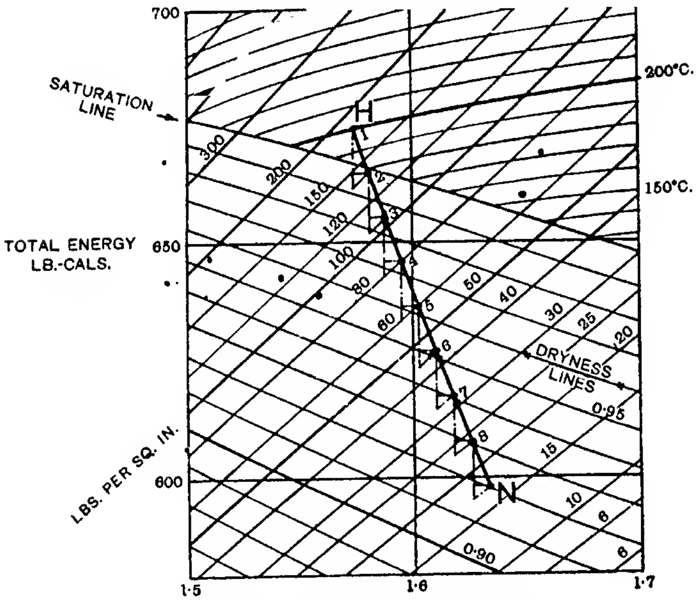


Fig. 239.—Line of condition. 25 per cent. loss.

TABLE 32.—Example of a Chain of Reaction Pairs. Irreversible Losses assumed to be 25 per cent.

Stage.	Pressure. Lbs. per sq. inch, p.	Dryness, q.	Volume from tables, v.	Cubic feet per pound of flow, $qV = \frac{1}{D}$.	$hd^2 = C$ eq. (18), p. 653.	Height of blade, h.	Mean diameter of blading.
						ins.	ft.
1	180	Superheated	2·62	2·62	10·35	0·71	3·82
2	130	0·998	3·48	3·47	13·70	0·93	3·84
3	106	0·986	4·21	4·15	16·40	1·10	3·85
4	80	0·974	5·48	5·33	21·00	1·4	3·88
5	60	0·963	7·18	6·92	27·30	1·8	3·91
6	44	0·952	9·60	9·14	36·1	2·3	3·95
7	32	0·941	12·94	12·15	48·0	3·0	4·01
8	23½	0·931	17·29	16·10	63·6	3·8	4·09
Last pair	17	0·921	23·37	21·50	84·8	4·89	4·17

Angular velocity, 78·5 rad. per second.
F = 27·8 lbs. per second.
Diameter of drum, constant, 3·76 ft. = 45·1 ins.
λ at beginning of each stage, 0·5.
Mean value of λ, 0·44 app.
Mean velocity of the blades in the last stage, 163 ft. per second.
Mean velocity of the blades in the first stage, 150 ft. per second.
Geometric mean = 156 ft. per second.

Let it be assumed that the turbine pairs constituting a stage are to be made of identical dimensions. From equation (7), page 648, the mean diameter of the blading of the first pair is

$$d = \frac{60 \times 150}{750 \times \pi} = 3{\cdot}82 \text{ ft.}$$

From equation (9), page 649, with the given data,

$$h = \frac{73 \times 27{\cdot}8 \times 0{\cdot}5 \times 2{\cdot}62}{750 \times 3{\cdot}82^2 \times 0{\cdot}34} = 0{\cdot}71 \text{ in.} = 0{\cdot}0595 \text{ ft.}$$

The diameter of the drum on which the blades are fixed is thus

$$3{\cdot}82 - 0{\cdot}059 = 3{\cdot}76 \text{ ft.} = 45{\cdot}1 \text{ ins.}$$

Let it be assumed that the diameter of the drum is to be kept constant in each of the eight stages. Then the mean diameter of the blading in any one stage will be

$$d = \frac{(45{\cdot}1 + h)}{12} \text{ ft.}$$

Then equation (9) becomes

$$\left(\frac{45{\cdot}1 + h}{12}\right)^2 \times h = \frac{73\text{F}\lambda}{\text{DN} \sin \alpha} = \frac{73 \times 27{\cdot}8 \times \lambda}{750 \times 0{\cdot}34} = \frac{7{\cdot}9\lambda}{\text{D}} \qquad (17)$$

And this with $\lambda = 0{\cdot}5$ reduces to

$$\left(\frac{45{\cdot}1 + h}{12}\right)^2 h = \frac{3{\cdot}95}{\text{D}} = \text{C} \quad . \quad . \quad . \quad . \qquad (18)$$

The value of the constant C at the beginning of each of the eight groups is shown in column 6 of the table.

The values of h are found from this equation by trial. Values of h calculated on the assumption that the mean diameter of the blading is constant and equal to 3·82 ft. guide quickly to the trial value to be used in the equation.

Values of h which satisfy this equation are shown in column 7 of Table 32. The mean diameters are given in the next column.

Since the pairs constituting a stage are to be of identical dimensions, hd^2 is constant for a stage. And with the data of the example

$$hd^2 = \frac{7{\cdot}9\lambda}{\text{D}}$$

giving that

$$\lambda = \frac{hd^2\text{D}}{7{\cdot}9}$$

The values of hd^2 are given in column 6. At the end of the first stage, therefore, since hd^2 is constant and the density has decreased to $\frac{1}{3{\cdot}47}$, λ has fallen in value to 0·377, giving an average value for the stage of 0·44. It will be found that the average of λ for each of the eight stages is approximately 0·44. The mean diameter of the blading is shown in column 8 of the table, so that the mean blade velocity in any stage, with the given data, is

$$v = 39{\cdot}25d \text{ feet per second} \quad . \quad . \quad . \quad . \qquad (19)$$

The number of pairs in each stage can now be calculated by the aid of equation (5), page 648, substituting for λ the mean value 0·44, and for U, 12·5.

In the first stage

$$x = \frac{45000 \times 0 \cdot 44 \times 0 \cdot 75 \times 12 \cdot 5}{150^2 \times 1 \cdot 438} = 5 \cdot 7$$

So that six pairs would be used.

For the last stage, in which the mean value of λ is still 0·44, but in which the blade velocity, calculated from (19) after substituting therein the value of d in Table 32, is 163 ft. per second, $x = 5$. Calculating in a similar way it will be found that the number of pairs will be, starting from the first stage,

$$6 + 6 + 6 + 6 + 6 + 6 + 5 + 5 = 46$$

The continuation of the chain into the low-pressure part of the turbine can be designed in a similar way. The diameter of the drum would be increased, and the last few pairs of the chain would possibly be constructed with blades set at an angle greater than 20° in order to avoid too great a height.

197. Chains of Impulse Pairs. Compounding the Velocity within a Pressure Range.—The development of the Compound Impulse Turbine is largely due to Professor Rateau.[1] In the 5000-kilowatt Rateau turbines at the Greenwich Power Station there are twenty-four impulse pairs placed in series. The turbines work on a range from 180 lbs. per square inch by gauge to 27 ins. vacuum, the guaranteed consumption being 14½ lbs. of steam per kilowatt hour. The Zoelly turbine is a compound impulse turbine in which each pair in the chain works on a pressure difference of which the lower pressure is made equal to 0·58, the higher pressure, and therefore the nozzles used to produce the velocity of flow from stage to stage end at a throat and have no enlargement.

The division of the available energy of a pressure range between the pairs forming a chain may be made in the way explained above for a reaction chain, namely, by drawing a line of condition either on the Total Energy-Temperature chart or on the Total Energy-Entropy chart, and from this line finding the condition of the steam at entry to each stage. The nozzles and the wheel channels can then be designed, stage by stage.

A simpler method may be used, based on the assumption that the steam in its expansion through the turbine follows the law, PV = a constant.

Consider a pressure-volume diagram. The total area Z represents the available energy of the pressure range p_1 to the back pressure. The area of a horizontal strip of this diagram, corresponding to a fall of pressure from, say, p_7 to p_8, represents the energy available to a

[1] "Different Applications of Steam Turbines," Rateau, *Proc. Inst. Mech. Eng.*, part 3, 1904, p. 737.

turbine pair working between these pressure limits. The area of the strip is

$$\int_{p_8}^{p_7} v\,dp = p_1 v_1 \log_e \frac{p_7}{p_8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$p_1 v_1$ is the constant for the expansion curve and is usually obtained from the initial conditions, for which reason the subscript 1 is used.

The whole area of the diagram is then equal to

$$p_1 v_1 \left(\log_e \frac{p_1}{p_2} + \log_e \frac{p_2}{p_3} \ . \ . \ . \ \log_e \frac{p_n}{p_{n+1}} \right) \quad . \quad . \quad . \quad (2)$$

In this series p_{n+1} is the back pressure in the condenser.

The conditions that the areas shall be equal are

$$\frac{p_1}{p_2} = \frac{p_2}{p_3} = \ . \ . \ . \ \frac{p_n}{p_{n+1}} = \text{a constant} = \frac{1}{r}$$

Therefore

$$p_2 = r p_1$$
$$p_3 = r^2 p_1$$
$$p_{n+1} = r^n p_1$$

Solving the last relation for n

$$n = \frac{\log \dfrac{p_{n+1}}{p_1}}{\log r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which the number of pairs, n, can be calculated to give equal distribution of power between them.

In the Zoelly turbine the ratio r is selected equal to 0·58. It follows that each of the nozzles in each diaphragm stops short at a throat without subsequent enlargement of area, because 0·58 is the pressure ratio which gives the maximum discharge per square foot of cross section, as shown in the previous chapter.

If the range of pressure is from 150 to 1 lb. per sq. inch, the ratio $\frac{p_{n+1}}{p_1} = \frac{1}{150}$, the logarithm of which is $-$ 2·1761. And if r is 0·58, $\log r = -$ 0·2366. Substituting these values in (3) the next whole number is 10. Thus 10 single velocity stage impulse turbines in series would be sufficient to exhaust the available energy corresponding to the range from 150 to 1 lb. per sq. in.

The maximum efficiency corresponding to a given pressure range and velocity of axial flow may be realized at a blade speed considerably lower than that which is necessary in the case of a single impulse pair working on the same range, by compounding the velocity in two or three impulse pairs in series in the way developed by Curtis. The velocity due to the pressure difference is produced in the nozzles of the first pair of the series, after which the nozzles of the succeeding pairs are used merely as guiding channels to change the direction of the actual velocity of discharge from any one pair into a direction suitable for supplying the next pair.

Neglecting frictional losses and heat losses, the magnitude of the velocity is unchanged by the guiding channels. The way in which the reduction of blade velocity is brought about will be understood by considering the velocity diagrams of two impulse pairs in series working on a given pressure difference.

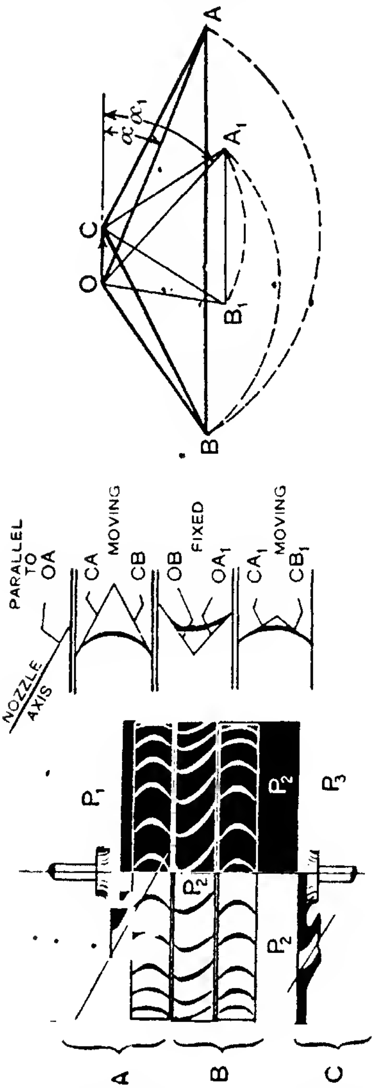


FIG. 240.—Velocity diagram. Two impulse pairs in series.

A diagrammatic sketch is shown in Fig. 240 of two impulse pairs, A and B, arranged in series between a pressure range from p_1 to p_2. The pressure throughout the chamber containing the two wheels and the set of guiding channels belonging to the second pair, is constant and equal to p_2. The chamber communicates either with the exhaust, or its floor contains a set of nozzles belonging to a third pair C, the wheel of which is in the chamber below the nozzles. Only one nozzle is shown in each case.

Assume that the wheels of the two turbine pairs are equal in diameter so that the circumferential velocities of the blades are equal. Assume also that the angle a is given. The velocity w is found from the pressure range when the quality of the steam supply is given. With these assumptions the velocity diagram is drawn as follows:—

Set out OC to represent the blade velocity, and OA at the angle a with OC to represent the velocity w. OC is taken considerably less than the value it should have to secure the maximum efficiency for a single pair. Join CA. Through A draw AB parallel to OC. With C as centre and CA as radius cut the line AB in B. Join OB

and CB. Then OCAB is the velocity diagram for the first pair. With centre O and radius OB draw the arc OA_1. Select a point A_1 on this arc and join it to O. Then OB is the direction of the actual velocity of discharge from the first pair, and it is therefore the direction of the elements of the receiving surfaces of the blades forming the guide channels: OA_1 is the direction of the discharging elements of surface of the guide, and is the direction of the actual velocity of the steam flowing on to the second wheel. OC is also by hypothesis the velocity of this second wheel circumference. Therefore join CA_1: through A_1 draw A_1B_1 parallel to OC: from centre C and with radius CA_1 cut the line A_1B_1 in B_1. Then OB_1 is the final direction of discharge from the combination.

The corresponding angles are shown to the right of the diagram of the turbine.

The change of velocity effected by the first pair is AB; by the second pair A_1B_1. The work done per second by the combination is then $\frac{1}{g}$ $(BA + B_1A_1)$ OC ft.-lbs. per pound of flow. The final actual velocity of the steam issuing from the combination OB_1 is nearly at right angles to the direction of motion, and yet the blade speed is considerably less than that which is necessary to determine the same final velocity with a single pair for the same inclination of the nozzle and the same velocity w.

The nozzle angles and the blade angles are different in each pair. It is more convenient in practice to keep the blade angles constant and make the necessary changes in the nozzle angles, because there are fewer nozzles and fewer guides than there are moving blades.

The following numerical example illustrates the way to draw the diagram of velocities for a given pressure difference in order to secure the maximum efficiency of the combination, having given the angle of the last guiding channel in the series of pairs and also the condition that the blade angle is constant in each wheel of the series.

The given data are :—

Number of pairs to be inserted within the pressure range, 3.

Quality of the steam supplied to the combination, dryness = 0·99.

Pressure of steam supplied to the combination, 200 lbs. per square inch.

Pressure in wheel chamber, 160 lbs. per square inch.

This gives a pressure difference of 40 lbs. per square inch.

Angle of inclination of the third guiding nozzle at discharge, 20°.

Efficiency to be a maximum, so that the actual velocity of discharge from the combination is at right angles to the direction of motion.

Blade angles to be constant in the moving wheels.

Finally, assume that the flow is frictionless and adiabatic.

From either a Total Energy-Temperature chart or a Total-Energy-Entropy chart, or by calculation, it will be found that 1 lb. of steam expanding adiabatically from 200 to 166 lbs. per square inch furnishes 10 lb.-cals. of available energy when the initial dryness is 0·99.

The corresponding velocity w is $300\sqrt{10} = 950$ ft. per second.

This velocity cannot, however, be immediately used, because the diagram is drawn from the given final conditions. It is, however, ultimately used to fix the scale of the velocity diagram.

To draw the diagram, set out (Fig. 241) OC and OB_3 at right angles, and then OA_3 at 20° with OC: take B_3 in any position on OB_3, and through it draw B_3A_3 parallel to OC: bisect A_3B_3 in x, and through x draw a perpendicular to cut OC at C.

This fixes the point C and therefore the blade velocity OC, and also β, the common blade angle.

Join CA_3 and CB_3 and produce them indefinitely: with centre O and radius OA_3 draw an arc cutting CB_3 produced in B_2: join OB_2: through B_2 draw B_2A_2 parallel to OC, cutting CA_3 produced in A_2: join OA_2.

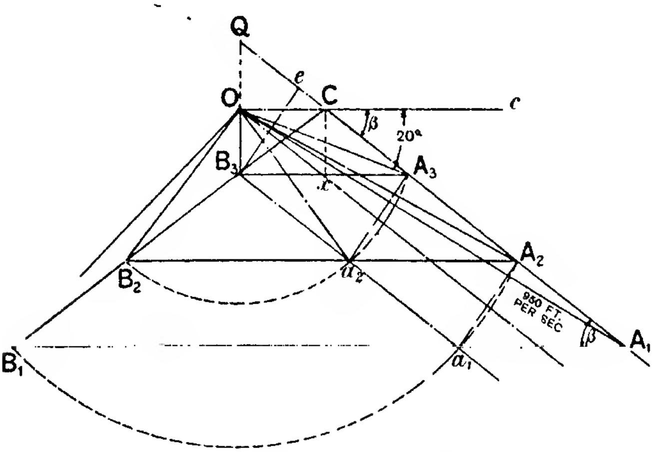


FIG. 241.—Velocity diagram. Three impulse pairs in series.

Then OCA_3B_3 is the velocity diagram for the last pair in the series, and OCA_2B_2 is the velocity diagram for the second pair in the series. Continue the construction and obtain the velocity diagram of the first pair OCA_1B_1.

The line OA_1 is the direction of discharge from the first nozzle. But the velocity which this line is to represent is 950 ft. per second, and therefore the scale of the diagram is determined because the length OA_1 represents 950 ft. per second.

OC, the blade velocity, now scales 178 ft. per second.

OB_3, the final velocity of discharge, scales 130 ft. per second.

The angle COA_1 of the first nozzle measures 30°, and COA_2, the angle of discharge of the guiding nozzle of the second pair, is 27°; the corresponding receiving angles of the guiding nozzles, namely, COB_1 and COB_2, are respectively 45° and 51°. The common blade angle cCA_1 measures 36°.

The total change of velocity effected by the moving blades of the combination is $A_1B_1 + A_2B_2 + A_3B_3 = 1300 + 835 + 355 = 2490$ ft. per second. The force on the moving blades is then $\frac{1}{g}$ 2490 = 77·3 lbs. Therefore the work done per second by the combination is $77·3 \times 178 = 13,750$ ft.-lbs. per pound of flow. The corresponding horse-power is 25.

The efficiency of the combination is $\frac{13,750}{10 \times 1400} = 0·98$. The volume of 1 lb. of steam after adiabatic expansion from the initial conditions to 160 lbs. per square inch is 2·85 cub. ft. per pound. The wheel channels and the guiding channels and the first nozzle must therefore be designed to have a sectional area which at the proper velocities measured from the diagram will determine a flow of 2·85 cub. ft. per second per 25 horse-power.

It will be understood from Fig. 241 that since the scale of the diagram is fixed by the single velocity $OA_1 = w$, a succession of similar combinations could be placed on a common shaft without change in the angles, providing that the pressure ranges in the succession are regulated so that each pressure range produces 10 lb.-cals. of available energy, and that the sectional areas are increased as the volume per pound increases.

The data from which to construct the diagram may be given in a variety of ways.

A simple relation may be derived directly from the geometry of the figure. Produce B_3O and A_1C to meet in Q. Through B_3 draw B_3a_1 parallel to QA_1. Then since $B_3O = OQ$ and $B_3x = xA_3$, Ox produced lies midway between the parallels QA_1 and B_3a_1. It follows that a perpendicular from A_3 to these parallel lines passes through the intersection of B_3a_1 and B_2A_2; and a perpendicular from A_2 passes through the intersection of B_3a_1 and B_1A_1.

Also $B_3A_3 = a_2A_2 = a_1A = 2v$. So that $A_1A_2 = A_2A_3 = A_3e = 2v \cos\beta$, where B_3e is perpendicular to A_1C produced.

Thus with 3 pairs $A_1e = 3 \times 2v \cos\beta$, and generally if n is the number of pairs in series

$$Ae = 2vn \cos\beta$$

This simple relation may be applied to find the number of pairs which must be placed within a given pressure fall in order to realize the maximum efficiency with given values of w, the blade speed v, and the common blade angle β. From the diagram it will be seen that $OA_1 = w =$ approximately eA_1, from which

$$n = \frac{w}{2v \cos\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Bearing the geometrical property proved above in mind, it will be seen that when the constant angle β is given as one of the data instead of the final angle α, the diagram is constructed as follows: Draw a horizontal line and select a point C anywhere in it. Set out the angle $cCA_1 = \beta$ and $OCB_1 = \beta$. Draw B_3A_3 parallel to OC at any

convenient distance from it. Through B_3 draw B_3O at right angles to OC, thus fixing the origin O and the direction OA_3 of the discharging elements of the blades of the last guide channel.

Through B_3 drop a perpendicular to A_3C, cutting it in e. Set out as many equal divisions from e, each equal to eA_3, as there are pairs in the series, thus obtaining a range of points A_3, A_2, etc. Through these points draw lines parallel to CO to meet CB_3 produced, thus fixing the range of points B_3, B_2, etc., and then the whole diagram is determined.

It will be seen that the diagram depends essentially upon the relative position of the two origins O and C and the perpendicular B_3e.

It is possible to arrange the angles in a series so that the change of velocity produced by each pair is constant and is not of diminishing magnitude through the pairs. The number of pairs required to realize this condition is, however, excessive.

The velocity diagrams and expression (1) have been obtained, assuming that the flow of steam is frictionless and adiabatic.

If the available energy of the pressure difference on which the combination of pairs works is utilized to produce the velocity of flow in the first ring of nozzles, the energy which must be used to overcome the frictional resistance against which the flow proceeds through the combination is taken from the kinetic energy of flow. The effect of friction is therefore to reduce gradually the velocity of flow and to produce a slight pressure difference across each ring of blades, a difference corresponding to the head lost in friction as the steam flows through the ring. The loss is greatest through the moving blade channels, because the loss at entry is included. Assuming the loss is the same in both the nozzles and the moving channels through a combination of two pairs and equal to 10 per cent. of the kinetic energy of flow through each of the four nozzle and blade channels through which the steam makes its way, the modification of the velocity diagram produced by the loss when the velocity of discharge is at right angles to the direction of motion is illustrated in Fig. 242.

The diagram is drawn as follows :—

Take any pole O and set out OB_2 at right angles to OC, the direction of motion of the blades. Then take C so that the angle OCB_2 is any convenient value. The value 33° is used in the figure. Join B_2C, taking OB_2 equal to unity. Through B_2 draw a line parallel to OC.

The velocity CA_2 is now greater than CB_2 because, by the assumption of the problem, the velocity CB_2 corresponds to 10 per cent. less energy than CA_2, the velocity at entry to the moving blades of the second pair. Therefore in general

$$\frac{CB_2^2}{2g} = \eta \frac{CA^2}{2g}$$

from which

$$CA_2 = \frac{CB_2}{\sqrt{\eta}}$$

In the diagram CB_2 is 1·87, and taking $\eta = 0·9$, $CA_2 = 1·97$. Therefore, with this value as radius, and from C as centre, cut the line through B_2 in the point A_2. Join A_2 to C, and A_2 to O. It will be found that the angle β measures 31°, and that the angle α measures 17°.

OA_2 is the velocity of discharge from the guide blades of the second pair. This enlarges to $\frac{OA_2}{0·95} = OB_1$ at entry. Similarly CB_1, the relative velocity of discharge from the first pair, enlarges to $\frac{CB_1}{0·95} = CA_1$ in the first pair. The lines CA_2 and CA_1 are practically coincident. The corresponding velocity of discharge from the nozzles of the first pair is OA_1, and this is enlarged to $\frac{OA_1}{0·95} = OA$ at entry to the nozzles.

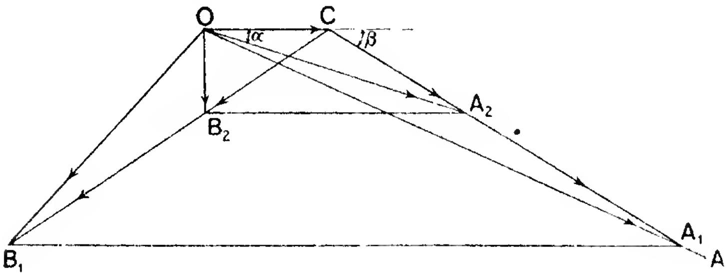


FIG. 242.—Two impulse pairs in series. 10 per cent. loss.

The scale of the diagram is fixed when any one velocity is given. The velocities from the diagram in terms of OB_2 = unity are :

1st pair	Entry to nozzles	OA	= 6·89
	Discharge from nozzles	OA_1	= 6·55
	Entry to wheel	CA_1	= 5·13
	Discharge from wheel	CB_1	= 4·87
2nd pair	Entry to guide nozzles	OB_1	= 3·62
	Discharge from nozzles	OA_2	= 3·44
	Entry to wheel	CA_2	= 1·97
	Discharge from wheel	CB_2	= 1·87
	Actual velocity of discharge from the second wheel and from the combination in a direction at right angles to the direction of motion of blades	OB_2	= 1

Change of velocity produced by first pair $= A_1B_1$ = 8·5

Change of velocity produced by second pair $= A_2B_2$ = 3·27

Velocity of blades . . . $= OC$ $= 1.59$

Work done per second by the combination, per pound of flow . . . $= \dfrac{OC(B_1A_1 + B_2A_2)}{g}$ ft.-lbs. $= 0.581$

Energy received per second $= \dfrac{OA^2}{2g}$ ft.-lbs. per pound of flow $= 0.738$

Efficiency of combination $= 0.79$

This is obtained with the value of λ . . . . . . $= \dfrac{1.59}{6.89}$ $= 0.23$

This shows the value of the combination in cases where the speed of rotation is required to be slow.

The diagram OA_2B_2C is a velocity diagram for maximum efficiency, allowing 10 per cent. loss in the fixed nozzles and 10 per cent. loss in the moving blades.

Work done per second by a pair of which this is the velocity diagram is

$\dfrac{OC \times B_2A_2}{g}$ ft.-lbs. per pound of flow . . . . $= 0.1605$

Energy received per second . . $= \dfrac{OA_2^2}{2g} = 0.203$ ft.-lbs. per pound of flow

Efficiency of pair . . . . . . $= 0.79$

This is obtained with a value of $\lambda = \dfrac{1.59}{3.62} = 0.44$.

Suppose that the blade speed is fixed at 600 ft. per second. Then for the combination

$$w = \frac{600}{0.231} = 2600 \text{ ft. per second}$$

corresponding to 75 lb.-cals. per second per pound of flow.

For the single pair

$$w = \frac{600}{0.44} = 1365 \text{ ft. per second}$$

corresponding to 21 lb.-cals. per second per pound of flow.

The differences between a chain of reaction pairs and a chain of impulse pairs designed for equal ranges of pressure are broadly that fewer impulse pairs are used in the chain; that admission to the pairs at the beginning of the chain is usually through nozzles occupying only a segment of the complete ring, the segment increasing gradually to the limiting case of a ring in the last pairs of the chain as the flow proceeds and the density of the steam decreases; that the velocity of flow through the chain is created only at the first pair of a reaction chain, but in general it is re-created at the entrance to each nozzle, segment, or ring in the case of an impulse chain.

These results illustrate the general principles on which impulse pairs can be used in combination to exhaust the available energy of a

given pressure range. The velocity diagram, Fig. 242, page 661, shows generally that with the conditions assumed two impulse pairs combined for velocity gradation absorb 75 lb.-cals. per second per pound of flow, and give an efficiency of 0·79 with a corresponding value of $\lambda = 0{\cdot}23$. Or a single pair with the angles and with the losses assumed, absorb 21 lb.-cals. per second from the available energy and give an efficiency of 0·79 with a value of $\lambda = 0{\cdot}44$.

For example, if the range is from 150 to 1 lb. per square inch the available energy is 177 lb.-cals. per second per pound of flow. Two impulse pairs combined, as in the figure, would absorb 75, leaving say 100 to be absorbed by another such pair and a single pair. Or the 100 lb.-cals. may be divided between say 5 single pairs, giving a chain in which there were 7 impulse pairs in all, the first two being combined for velocity gradation within a pressure stage, the remainder being single stage impulse pairs, each on its own pressure range. The energy converted into work neglecting all but the 10 per cent. loss in the channels would be $0{\cdot}79 \times 177 = 140$ lb.-cals. per second per pound of flow. In practice there would be other losses to be included, and the efficiencies of the pairs would not be the same, but the example above sufficiently illustrates the basis on which the calculations for impulse chains can be made.

198. General Construction of a Compound Impulse Turbine. Impulse Blading.—Each wheel of a chain of impulse pairs is enclosed in a separate casing. The separate casings are formed by diaphragms carried on the shaft common to the wheels carrying the moving blades, and the diaphragms are fixed at their peripheries to the main casing of the turbine. Nozzles are formed through the diaphragms near their peripheries. A pair is thus formed of a diaphragm and a wheel. The pressure in each wheel chamber is practically uniform, though there may be a small pressure drop through the wheel channels. The wheel in any one chamber may carry a single row of blades, or it may carry two or more rows in order to reduce the velocity produced by the nozzles supplying the chamber in two or more stages in the way explained in the previous section.

Figs. 243, 244, and 245, reproduced from a paper by Zoelly,[1] by the kind permission of the Council of the Institution of Mechanical Engineers, illustrates these points of construction. Fig. 243 shows a general view of a Zoelly turbine formed of a chain of eight impulse pairs. It is designed to give from 1500 to 1800 H.P. at 3000 revolutions per minute. The wheels are sectioned black, and the diaphragms separating them are cross-hatched. The nozzles are seen at the diaphragm peripheries. The nozzle size increases through the turbine to allow for the increasing volume of the steam as the flow proceeds. The detailed construction of one pair is shown in Fig. 244. A is the main casing of the turbine; S is the common shaft; C is a diaphragm fixed at its periphery to the main casing and

[1] "Steam Turbines," H. Zoelly, *Proc. Inst. Mech. Eng.*, July, 1911.

packed on the boss of the turbine wheel W. N is a nozzle in the diaphragm; the nozzles are formed by means of the thin plates *n*, *n*, *n*, shown in the plan of the blading. These plates are cast in the diaphragm. B is a moving blade channel formed by the blades *b*, *b*, *b*, shown in plan. N_1 is a nozzle in the next diaphragm through which the steam expands to the pressure in the chamber of the next wheel. Fig. 245 shows the rim of a wheel carrying blading for two velocity stages. In the plan *n*, *n*, *n*, are the thin plates cast in the diaphragm to form the nozzles; B, B are the two rows of moving blades shown in perspective on the wheel rim which carries them; and G is the row of fixed guide blades. The combination forms

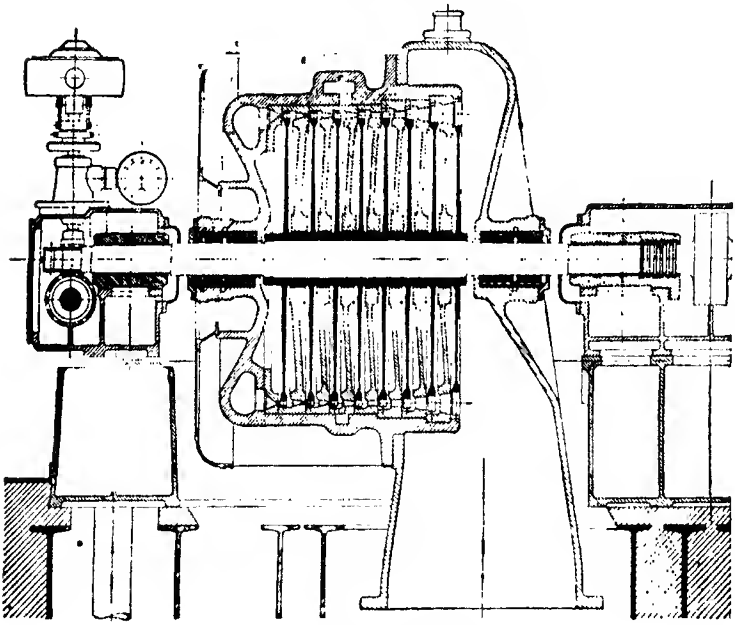


FIG. 243.—Zoelly turbine.

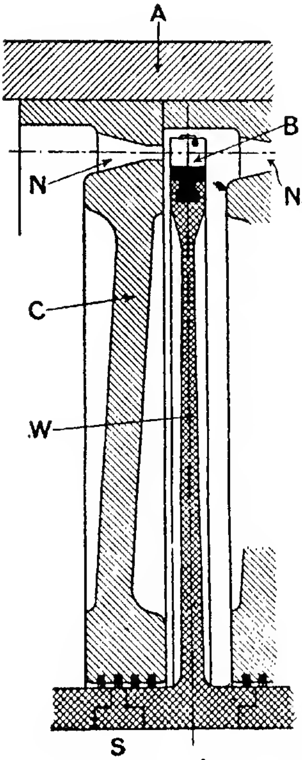
two pairs within one pressure stage. Velocity stages are used in cases where the blade speed is low.

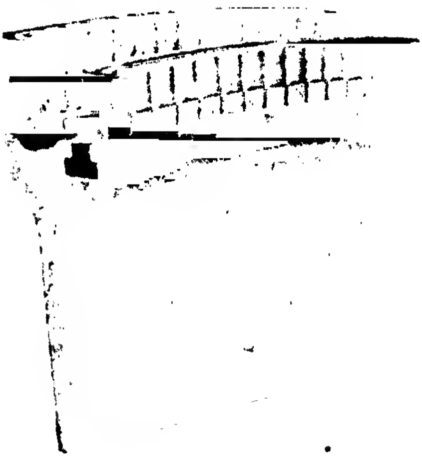
The nozzles in the first diaphragm of the high speed Zoelly turbine extend round a part of the circumference only, whilst in the remaining diaphragms the nozzles usually extend completely round the circumference. Governing is done by throttling the steam supply. In the Zoelly marine turbine two-stage velocity wheels are fitted like the wheel illustrated in Fig. 245, since with the necessarily low speed, too many single-stage impulse turbines would be required to exhaust the available energy of the range. The low-pressure turbine is constructed rather differently. The

nozzle and diaphragm system is replaced by impulse blading on a drum.

Impulse pairs placed in series, each on its own pressure fall and with full peripheral admission give rise to impulse blading.

The difference between impulse blading and reaction blading is that in each of a series of impulse pairs the fixed blading is so proportioned in section that the whole of the heat energy corresponding to the pressure fall is converted into kinetic energy between the channels of the fixed blading, whilst in reaction blading only a fixed proportion of the energy is so transformed, the remainder





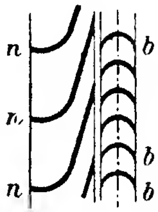


FIG. 244.—Impulse pair. Zoelly turbine.

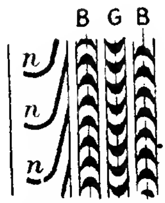


FIG. 245.—Rim of wheel with blading for two velocity stages. Zoelly marine turbine.

being transformed in the channels between the blades of the moving wheel as already explained above. For example, Fig. 222, page 620, may represent either an impulse pair with full peripheral admission or a reaction pair, which of necessity must have full peripheral admission.

Each channel between the fixed blades is in impulse blading a nozzle, and it must be proportioned accordingly.

The economy of the Zoelly turbine is indicated by quoting from a table given in the paper mentioned above. A 1700 kilowatt turbine working on a range from 217 to 1 lb. per square inch, the steam being initially superheated to 370°, required 8·91 lbs. of steam per horse-power hour, excluding the steam required for the condensing plant.

199. Impulse Pair in which the Velocity is compounded along the Circumference of One Wheel.—An impulse pair equivalent to a series of pairs within a pressure fall is formed by guiding the steam after its discharge from the first wheel, not on to a second wheel, but back again by suitable guiding channels on to the same wheel. The flow is no longer axial, but it follows a spiral path

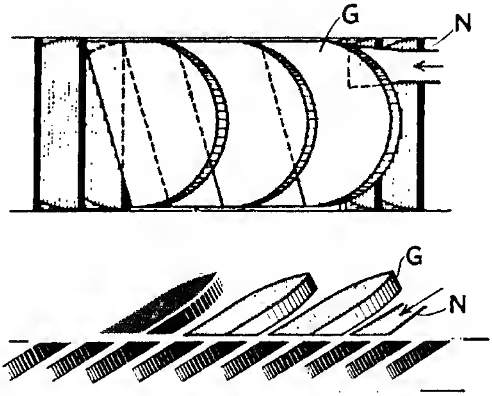


Fig. 246.—Velocity compounding along wheel circumference.

round the circumference of the wheel. There is no blading. Buckets, like the bucket of a Pelton Water Wheel, are milled into the periphery of the wheel in the way illustrated in Fig. 246, which shows a part of the periphery developed into a straight line. A nozzle N discharges steam on to the buckets, which are nearly semicircular in form, and these buckets change the direction of the velocity through 180° and discharge the steam from their edges into semicircular guiding channels of which G is one. The steam caught by the channel is guided back to be discharged at the proper angle on to the wheel again. The process may be repeated to any extent by the addition of more guiding channels. In this way the steam is guided again and again to the wheel buckets, until its actual velocity is reduced to a small magnitude, following along the periphery of the wheel a spiral path, the coils of which engage partly with the wheel buckets and partly with the guiding channels. An ordinary

spiral spring laid lengthwise along the circumference of the wheel roughly illustrates the actual path followed by the steam. Fig. 246 is merely a diagram drawn to illustrate the general principle of the action of impulse pairs of this kind.

A non-condensing turbine developing 170 H.P. at 2000 revolutions per minute, constructed on this principle, is described, in *Engineering*, July 11, 1913. It is built by the Sturtevant Engineering Co. Six nozzles are equally spaced round the circumference of the turbine wheel. Combined with each nozzle are four guiding channels, so that each jet of steam is returned to the circumference four times, each time with a smaller velocity. The buckets are semicircular and are milled out of the wheel circumference. From the results given it appears that the steam consumption is 40 lbs. per brake horse-power when steam is supplied at 180 lbs. per square inch by gauge. This is, of course, reduced by superheating or by combining the turbine with a condenser.

200. The De Laval Turbine.—The Laval turbine consists of one impulse pair set to work on the whole pressure range between the boiler and the condenser without velocity compounding. Steam is expanded right down to the condenser pressure in diverging nozzles of the kind discussed in the previous chapter, and it was in connection with the development of this class of turbine that De Laval patented the expanding nozzle in 1889. The turbine is made in standard sizes from 3 to 600 B.H.P.

The nozzles are carried in the casing which encloses the wheel, and the particular arrangement depends upon the size of the turbine. Fig. 247 shows a section through the axis and nozzle of a large turbine. The nozzles are drilled in a ring R, fitted into the turbine case, and are set out in groups of two or three round the ring, each group being provided with its own steam chest, S, and shutting off valve V. Steam is admitted to the turbine through a stop valve, and after flowing through a strainer, and then through a double-beat throttle valve, it finds its way into the supply belt P, shown in section, from which it is admitted to as many nozzle steam chests as may be necessary by opening the shutting-off valves of which V is one. The steam flows through the nozzles and the blade channels into the exhaust belt indicated at H, from which it flows either to the condenser or direct to the atmosphere as the case may be.

The steam issues from the nozzles at a high velocity, and therefore the blade velocity must be high in order to realize a reasonable economy. From data given in a paper by Mr. Konrad Andersson,[1] it appears that the peripheral speed, reckoned at a circumference passing through the middle of the blades, varies from 515 ft. per second in the 5 H.P. turbine to 1378 ft. per second in the 300 H.P. turbine. The diameter of the 5 H.P. wheel is 4 ins., so that it runs at nearly 30,000 revolutions per minute. The diameter of the

[1] "Steam Turbine with special reference to the De Laval Type of Turbine," *Trans. of the Inst. Engineers and Shipbuilders of Scotland*, Oct., 1902.

300 H.P. wheel is 30 ins., giving therefore, at a blade speed of 1378 ft. per second, nearly 10,600 revolutions per minute. The blades on this wheel are 3 ins. long.

If the 300 H.P. turbine works on a range from 150 to 1 lb. per square inch, the velocity with which the steam issues from the nozzles is about 4000 ft. per second. The value of λ is then $\frac{1378}{4000}$ $= 0{\cdot}345$. A reference to Fig. 236, page 638, will show that the theoretical efficiency corresponding to this value of λ is 80 per cent.

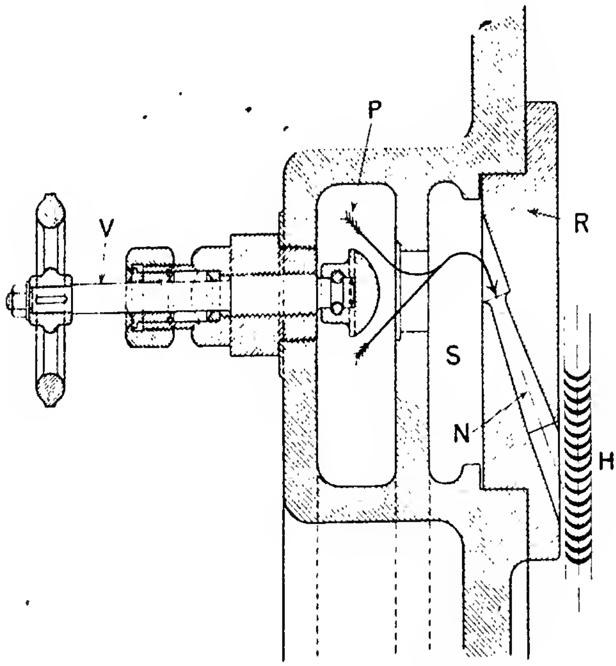


FIG. 237.—Nozzle in Laval turbine.

when the nozzle angle is 23°. The actual efficiency approaches reasonably near to this, as the results quoted below will show.

The turbine wheel shaft is supported at one end by a spherical bearing, and at the other end by a bearing fixed in direction, and the shaft is prolonged through this bearing into a gear box, in which a double helical pinion on the end of the shaft gears with wheels on a second shaft parallel to the turbine shaft. The gearing is proportioned to give velocity ratios of from 10 to 1 in the smaller turbines, to 13 to 1 in the larger sizes. The gear wheel shaft is prolonged outside the gear case, and is in fact the shaft which is coupled to the motor which the turbine is intended to drive. This

shaft runs at from 3000 revolutions per minute in the smaller turbines to 600 in the larger sizes.

The construction of the wheel used in the larger turbines is shown in Fig. 248. B is one of the blades to an enlarged scale. Each blade is dovetailed into the rim sideways so that it can be removed without disturbing any of its neighbours. The exceptionally high peripheral speed produces a condition of stress which is met by making the wheel solid at the centre and thickening it laterally as indicated in the figure. A groove is turned in the wheel on each side, just under the rim at the roots of the blades, so that if the wheel should race, the buckets and the rim beyond the grooves burst off in small fragments and thus prevent the fracture of the wheel itself into pieces large enough to damage the casting.

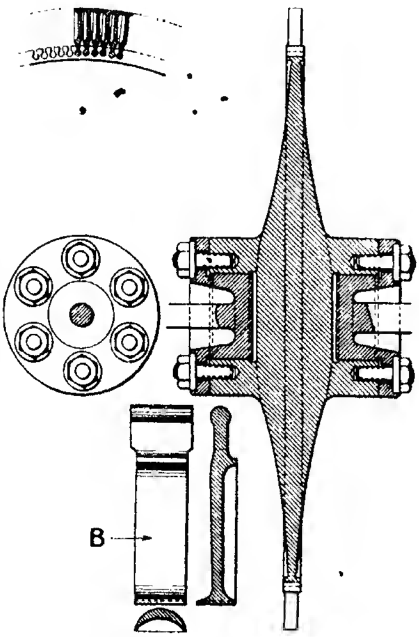


FIG. 248.—Laval turbine wheel.

The high speed of rotation of the Laval Turbine wheels is assisted by the flexible shaft designed by de Laval. Let a mass M like a turbine wheel be carried on a shaft so that when the shaft is deflected the plane of the wheel remains at right angles to the axis through the centres of the bearings. Assume also that the mass centre of the wheel is on the geometric axis of the shaft, which is itself coincident with the centre line through the bearings. Whilst the wheel is turning at ω radians per second, let a force be applied to deflect it laterally so that when the force ceases to act its mass centre is at a distance y from the centre line of the bearings. Two forces now act simultaneously on the wheel, the one against the other. There is first the centrifugal force at the mass centre which tends to increase the deflection. Neglecting the mass of the shaft itself the magnitude of this force is $M\omega^2 y$ acting radially at the mass centre. Secondly, there is the force acting on the wheel by the deflected shaft, a force which is proportional to the deflection and may be written equal to ay, where a is a constant depending upon the size of the shaft and the way it is constrained at the ends. Assuming that the force applied

by the shaft is the greater, the force acting to restore the shaft to its central position may be written

$$\text{Restoring force} = ay - M\omega^2 y = y(a - M\omega^2) \quad . \quad . \quad (1)$$

Under the action of this restoring force the shaft will execute oscillations about its central position which will finally die out, leaving the shaft rotating stably in the central position. As the speed of rotation increases, the restoring force decreases, and the period of the oscillation which follows a lateral deflection is increased. A limiting condition is reached when the restoring force vanishes. The slightest deviation will then produce what is called centrifugal whirling, a state of unstable motion in which the deflection of the shaft goes on increasing as ω increases, and a broken shaft may be the consequence.

The limiting speed is reached in the conditions assumed above when the quantity in the brackets of eq. (1) vanishes, giving for the whirling speed

$$\omega = \sqrt{\frac{a}{M}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The quantity a is the force required to produce unit deflection in the shaft. For a circular shaft freely supported at the ends the value of a is $\frac{48EI}{Wl^3}$, by the aid of which and equation (2) the approximate speed of whirling of a single wheel or pulley can be calculated for the simple conditions assumed.

Entirely new conditions are established if the shaft is driven at a speed considerably higher than the whirling velocity. The high speed may be regarded as produced by the sudden application of a couple to the wheel by the shaft, in which case, assuming flexibility so great in the shaft that the force applied to the wheel in consequence of a small deflection is small and has small influence during the time in which the impulsive couple is acting, rotation about an axis through the mass centre of the wheel will be established. If when rotating in this condition a force is applied to produce a lateral displacement, the axis of rotation is merely moved parallel to itself and may still be regarded as passing through the displaced mass centre of the wheel, in consequence of which there is only one force acting on the wheel when the applied force has ceased to act, namely, the force ay applied to the wheel by the deflected shaft. The restoring force is thus by equation (1) merely ay, since there is no displacement of the mass centre from the axis of rotation to produce a centrifugal force on the deflected wheel. The shaft must be sufficiently flexible to allow this state of rotation to be established, and the de Laval shaft is made so flexible that the critical speed at which centrifugal whirling occurs is only about $\frac{1}{7}$ of the actual speed at which the shaft is driven. For example, the critical velocity of a shaft intended to run at 30,000 revolutions per minute is about 4000 revolutions per minute. In passing from the first kind of rotation to the second some restraint must be applied to the shaft as

it passes through the limiting or critical velocity to prevent an excessive lateral deflection, otherwise the shaft may break unless it is driven through the critical speed very quickly. The general consideration of the motion of a flexible shaft involves the question of the change in the direction of the plane of the motion of the wheel which accompanies a lateral deflection, and any mathematical consideration becomes complicated. A mathematical investigation has been published by Föppl.[1]

Problems relating to the critical velocity at which centrifugal whirling occurs have been investigated by Rankine,[2] Greenhill,[3] Dunkerley,[4] and one of the most recent papers on the subject is by Dr. Chree.[5] Dunkerley's paper contains the results of many experiments and also useful empirical formulæ for the calculation of the critical speed in the case of a shaft loaded with several loads, and on account of the importance of this question in the design of turbine motors the expression may be mentioned here. Let N_2 be the frequency of the critical velocity calculated for a shaft from expression (2), neglecting the mass of the shaft; and let N_1 be the frequency of the unloaded shaft calculated from the ordinary elastic equations and following Greenhill's method. Then the frequency N of the critical speed of the combination is

$$\frac{1}{N^2} = \frac{1}{N_1^2} + \frac{1}{N_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Dunkerley extended this method to several loads, there being a term in the series on the right for each load on the shaft.

The steam supply to the Laval turbine is roughly adjusted to the load by the opening of a sufficient number of nozzle steam chests, in this way increasing the number of nozzles blowing steam on to the blades. The finer regulation is done by means of a spring-loaded governor attached directly to the gear wheel shaft and working through a linkage to the double-beat throttle valve through which the steam flows to the steam belt.

When the turbine exhausts into a condenser, the velocity of the steam issuing from the nozzles is so high, and the resistance of the turbine is so low, that special precautions are taken to prevent a runaway in case the governor fails to act when the load is thrown off. An air inlet valve is fitted to the condenser, so that when the speed increases to about 5 per cent. above the normal it opens automatically and reduces the vacuum; this instantly reduces the velocity of flow from the nozzles, and the speed falls to the normal again and the air valve then closes.

When the turbine is connected to a large condenser used in

[1] "Das Problem der Laval'schen Turbinwelle," Von A. Föppl, *Der Civilingenieur*. Leipzig, 1895.

[2] Rankine, *Engineer*, April 9, 1869.

[3] Greenhill, *Proc. Inst. Mech. Eng.*, 1883.

[4] Dunkerley, S., "On the Whirling and Vibration of Shafts," *Phil. Trans.*, A., 1894.

[5] Chree, Dr. C., "The Whirling and Transverse Vibrations of Loaded Shafts," *Proc. Phys. Soc.*, London, vol. xix.; also *Phil. Mag.*, May, 1904.

common by other engines, a small air valve would not reduce the vacuum sufficiently to ensure safety. In such cases a throttle valve is fitted in the exhaust pipe in addition.

The throttle valve is connected by a link to a small piston, the upper side of which is exposed to the full pressure in the exhaust pipe. The lower side is also exposed to the exhaust pressure, but through a pipe connected with the air valve. Normally, therefore, the pressure is the same above or below the piston, and a spring keeps the piston down at the bottom of the cylinder, in which position it holds the throttle valve open. An increase of speed of 5 per cent. results in the opening of the air valve, in consequence of which the pressure below the piston increases; the piston is pushed up and the throttle valve in the exhaust pipe is closed. The rise in back pressure immediately reduces the velocity of the steam from the nozzles.

The results of trials of Laval turbines working in different conditions are recorded in Mr. Andersson's paper mentioned above. In one set of trials on a 300-H.P. turbine a consumption of 14·35 lbs. of steam per B.H.P.-hour is recorded when the steam was superheated through about 35° C., and this increased to 15·17 when the supply was dry and saturated In both cases the pressure range was practically the same, namely, from 222 lbs. per sq. inch to approximately $1\frac{1}{2}$ lbs. per sq. inch, and the power developed was in the superheated trial 352 B.H.P., and in the trial with dry saturated steam 333 B.H.P.

201. The Parsons Steam Turbine.—The Parsons turbine[1] consists essentially of a chain of reaction pairs communicating at one end with the boiler and at the other end with the condenser. The moving blades are secured to a common shaft or drum, commonly called a rotor. The fixed blades are attached inside the casing in which the rotor rotates.

As mentioned above, the necessary increase of size of each pair in the chain in succession, which is necessary if the flow is to take place at constant axial velocity, is made in a series of steps.

Each step is technically called an **expansion.**

An **expansion** is constituted by a group of pairs of identical dimensions.

A general arrangement of a large turbine constructed by Messrs. C. A. Parsons & Co., for the Commonwealth Edison Company of Chicago, is shown in Fig. 249.[2]

The turbine is designed for a continuous load of 25,000 kilowatts at 750 revolutions per minute. The guaranteed consumption at 20,000 kilowatts is 11·25 lbs.[3] of steam per kilowatt hour output from

[1] An historical account of the development of the Parsons turbine is given in "The Evolution of the Parsons Steam Turbine," by Alex. Richardson. London, 1911.

[2] A detailed account of the turbine together with the generator will be found in *Engineering*, October 17, 1913. Fig. 249 is reduced from the two-page plate illustrating the description.

[3] Sir Charles Parsons informs me that the consumption came out under the guarantee; and that contracts for 10,000 kilowatt plants are accepted with a guarantee of 10·75 lbs. of steam per kilowatt hour.

the alternator to which it is coupled, when the steam supply, superheated through 111° C., is at 200 lbs. per square inch by gauge, and the pressure in the condenser corresponds to 1 in. of mercury, which is about $\frac{1}{2}$ lb. per square inch. This is equivalent to 8·1 lbs. of steam per shaft horse-power hour.

In this turbine the chain of pairs divides into two similar chains after the pressure has fallen to about 25 lbs. per square inch, so that the flow from the boiler to the condenser is first through a chain of pairs in series and then through two chains in parallel. The single chain is enclosed in one casing, and the combination is called the high-pressure turbine. The two chains in parallel are enclosed in a separate casing, and the combination is called the low-pressure turbine.

Steam enters the high-pressure turbine at S. If the load exceeds 20,000 kilowatts a further supply is admitted automatically by the governor gear through S_1 direct to the second expansion. The steam leaves the high-pressure turbine at E, and is led through two pipes, each 30 ins. diameter, to the central belt F formed in the low-pressure casing, from which space it flows right and left through the similar chains of pairs in parallel into the condenser at C and C. The condenser is placed immediately beneath the low-pressure turbine, and is provided with 39,300 sq. ft. of cooling surface.

The high-pressure rotor is supported on two bearings B, B, and it passes out of the casing through the glands G and G. The end of the rotor shaft passes into a small thrust block T, which takes up unbalanced steam thrust in the direction of flow, and is designed so that it enables slight adjustment to be made in the relative axial position of the rotor and the casing in order to regulate the clearances between the fixed and moving blades.

Similarly, the low-pressure turbine rotor is carried in two bearings *b* and *b*, and it leaves the casing through the glands *g* and *g*. It is coupled to the high-pressure rotor on the left and to the alternator which it is designed to drive on the right.

The first pair of the chain is at A, and the last pairs of the parallel chains are at Z and Z. There are six expansions in the chain in the high-pressure casing, and six expansions in each of the chains in the low-pressure casing. The blades in the first expansion are $2\frac{3}{4}$ ins. high, and in the sixth expansion $6\frac{1}{2}$ ins. high. In the seventh expansion, that is, the first expansion in the low-pressure casing, the blades are $2\frac{3}{4}$ ins. high, and in the last expansion 19 ins. high. The changes in height in the high-pressure turbine are accompanied by changes in the rotor diameter, but the diameter of the low-pressure rotor is constant throughout. The blades which are exposed to the action of superheated steam are made of copper; the remainder are of brass. The number of pairs in the several expansions beginning at the high-pressure end of the chain are :—

$$19 + 16 + 10 + 9 + 5 + 5 = 64 \text{ in the high-pressure casing}$$

$$\left.\begin{array}{l} 4 + 4 + 4 + 4 + 3 + 5 = 24 \\ 4 + 4 + 4 + 4 + 3 + 5 = 24 \end{array}\right\} = 48 \text{ in the low-pressure casing}$$

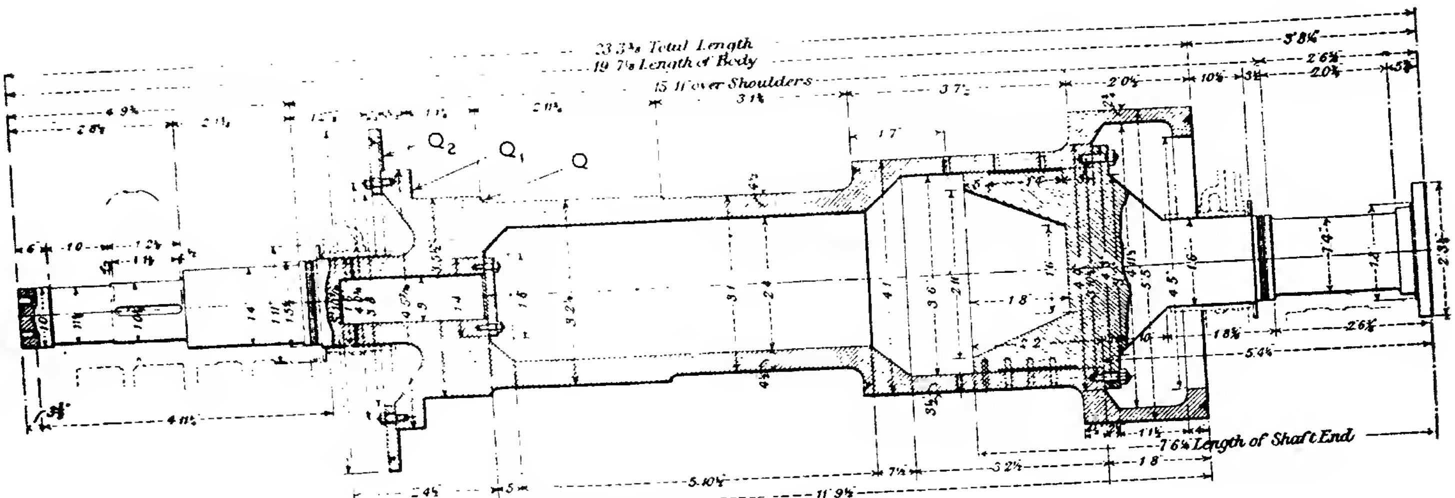


FIG. 250.—High-pressure rotor. Parsons turbine.

There are therefore altogether 112 turbine pairs in the two casings. It will be seen that the increase of annular area in the high-pressure casing is made partly by increasing the blade height and partly by increasing the rotor diameter. In the low-pressure casing the necessary increase of area is obtained by progressively increasing the height of the blades on a rotor of constant diameter. The last two pairs carry respectively semi-wing and wing blading.

The rotor for the high-pressure turbine is shown separately in Fig. 250. It is constructed of forged steel. The rotor is shrunk over and bolted to the separate shaft at the exhaust end. The rotor for the low-pressure turbine is shown in Fig. 251. The cylindrical drum is secured to a steel casting at the centre, and this casting is keyed to the shaft. Each end is further supported by a flexible steel diaphragm in the manner illustrated. The drum is thus free to expand each way from the centre as the temperature increases, without involving the shaft itself. The low-pressure shaft is coupled rigidly to the high-pressure shaft. Provision for expansion of the casing is made by supporting the inlet end of the high-pressure turbine on a sliding joint formed on the top of the pedestal.

The governor gear is of the relay type explained and illustrated on page 403, and includes steam admission valve, emergency valve, and bypass valve, to supply steam direct to the second expansion when the load exceeds 20,000 kilowatts. For full details of the turbine reference should be made to *Engineering*, October 17, 1913. A separate reprint of the article is also published.

202. The Balance of the Steam Thrust in the Direction of Flow.—The steam flowing through a chain of turbine pairs exerts a thrust on the rotor due to the fall of pressure as it passes through the moving blades. The thrust contributed from each turbine pair is measured by the product of the drop of pressure through the moving blades multiplied by the area of the annulus in which the blades rotate. The thrust due to the pressure drop through the fixed blades is exerted on the casing.

In a Parsons turbine it may be assumed that the drop of pressure through the moving blades of a pair is half of the total drop through the pair. This assumption may be extended to each pair in an expansion, so that if P_1 is the pressure at which steam enters an expansion, and P_2 is the pressure at which it leaves it, and if A is the area of the annulus common to all the pairs forming the expansion, the thrust exerted on the rotor by the steam as it flows through the expansion is approximately

$$\tfrac{1}{2}(P_1 - P_2)A. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Each expansion contributes to the total thrust an amount which may be calculated from this expression after substituting appropriate values for the pressures and the area. To the thrust found in this way must be added the thrust of the steam on each solid annulus caused by the enlargement of the rotor diameter. Let p be the

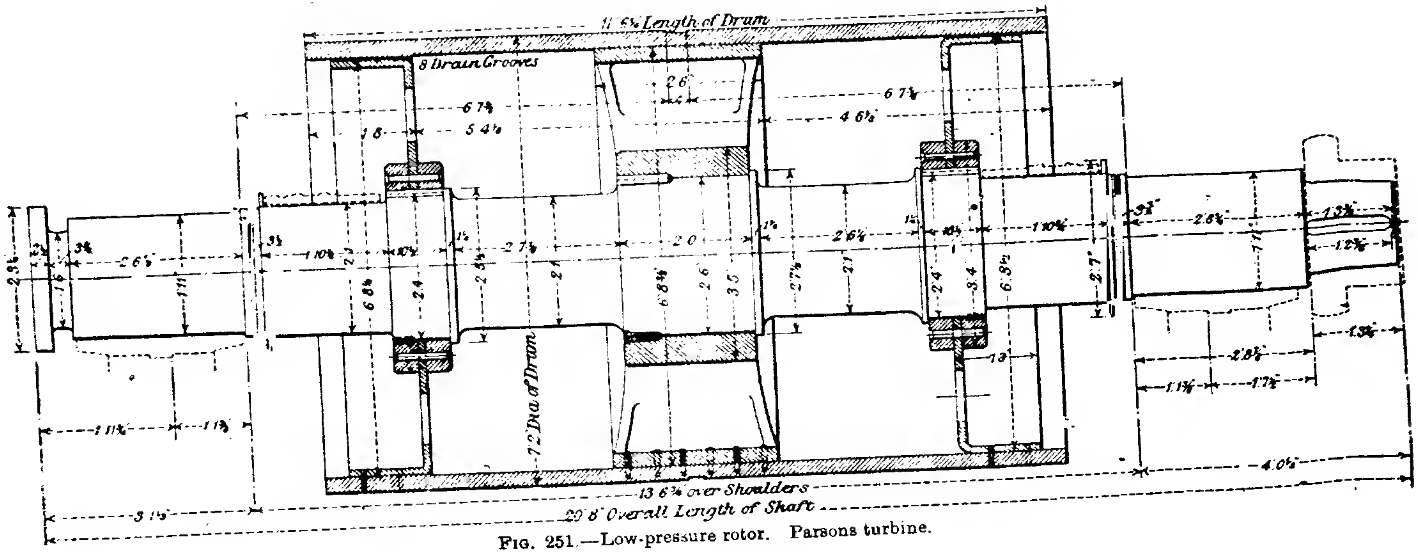


FIG. 251.—Low-pressure rotor. Parsons turbine.

pressure acting on a solid annulus of area a. The total thrust on the rotor is then

$$\Sigma\tfrac{1}{2}(P_1 - P_2)A + \Sigma pa \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This thrust can be balanced either by arranging two equal chains of turbine pairs on one rotor, end for end, and introducing the steam at the middle of them, so that the steam thrust from the one is balanced by the steam thrust from the other, the method originally used by Parsons; or by contriving balancing pistons at the high-pressure end of the turbine so proportioned that the steam pressure acting on them produces a longitudinal force equal and opposite to the steam thrust. The disadvantage of the first method is that it produces a turbine consisting of two single turbines in parallel, each of half the total power, and thus the disadvantages of small turbines are incurred to a greater extent than is absolutely necessary. These disadvantages disappear as the size of the turbine increases. Both methods of balancing are illustrated in Fig. 249, the first in the low-pressure part of the turbine, the second in the high-pressure part.

In the low-pressure turbine the steam flows right and left from the central supply belt through similar chains of pairs, and thus the steam thrust from the one balances the steam thrust from the other.

Referring to the high-pressure turbine, Q, Q_1, Q_2 are balancing pistons. The rotor, Fig. 250, is correspondingly lettered. Q is acted upon by the initial steam pressure in the supply belt S. Q_1 is acted upon by the pressure in the annulus at q_1, being connected therewith by the pipe s_1. Q_2 is acted upon by the pressure in the annulus q_2, being connected therewith by the pipe s_2. The ends of the rotor are connected through the pipe e, so that the rotor as a whole is balanced as regards end pressures. It will be seen that the balancing pistons, or balancing dummies as they are called, are stepped to correspond with each enlargement of the rotor, so that each section of the rotor is balanced separately. This method secures that variations in initial steam pressures and in load do not materially affect the balancing system.

The area of the annulus of any one dummy can be calculated as follows: Let Q be the area of the annulus of a dummy which is to balance a rotor section on which there are two expansions beginning with a rotor enlargement which presents a solid annulus of area a. Let the annulus at the enlargement be connected by a pipe to the dummy chamber, so that the pressure acting on the dummy is equal to the pressure acting on the enlargement. This will also be the pressure acting on the first turbine pair on the rotor after the enlargement: call it P_1. Then if P_2 is the pressure at the end of the first of the two expansions and P_3 the pressure at the end of the second expansion

$$P_1Q = \tfrac{1}{2}(P_1 - P_2)A_1 + \tfrac{1}{2}(P_2 - P_3)A_2 + P_1a \qquad (3)$$

from which Q can be found when the pressures and areas are given.

203. Mechanical Details.—The leakage of steam past the cylindrical peripheries of the dummy rings is reduced to negligible amount by means of "labyrinth packing," a sketch of which is shown in Fig. 252. Grooves are turned in the periphery of the dummy into which project rings *r*, *r*, *r*, turned to the section shown in the drawing, and so arranged that each ring almost touches one side of the groove into which it projects. Escaping steam is throttled at each ring, so that the quantity escaping through a series of rings is small.

Labyrinth packing is applied to the glands through which the

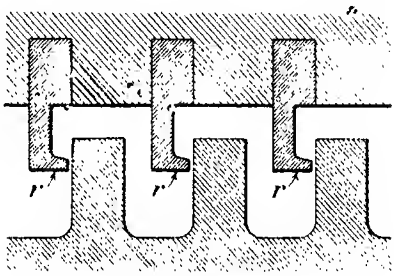


Fig. 252.—Labyrinth packing. Facial clearance.

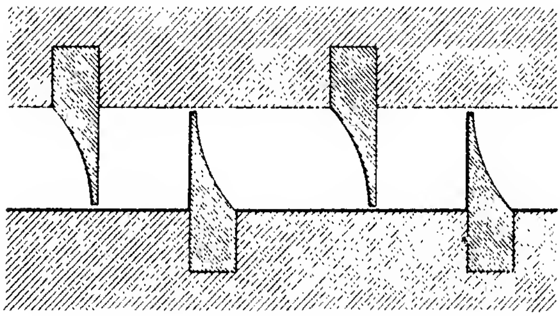


Fig. 253.—Labyrinth packing. Radial clearance.

rotor-spindle emerges from the casing. At the low-pressure end steam is supplied to the gland at a suitably reduced pressure, so that steam leaks into the vacuum instead of air.

The labyrinth is formed in various ways, and the form shown in Fig. 253 with radial clearance is adopted to give more freedom of differential expansion, owing to changes of temperature, than the type shown in Fig. 252. For this reason a labyrinth with radial clearance is used where the bearing is remote from the thrust block, whilst the form shown in 252 can be used for bearings and dummies close to the thrust block where relative axial displacement is small.

The thrust rings in the thrust block are cut and are arranged in the block so that the upper segments are in contact with the left of the grooves, and the lower segments are in contact with the right of the grooves which are turned in the rotor shaft. The block can therefore take up unbalanced steam thrust from the rotor in either direction without end play. Devices are fitted to the thrust block to enable slight axial adjustment to be made in order to regulate the amount of clearance between the fixed and the moving blades.

Bearings are formed in the usual way and are generally arranged for forced lubrication. In the earlier turbines the speed of rotation was exceptionally high, and steadiness of running was obtained by forming the bearing of three concentric bushes placed somewhat loosely over one another, the whole being immersed in a bath through which oil was forced under pressure. The bearing formed in this way possessed sufficient elasticity to allow the rotor to turn about its principal axis, and is equivalent to the flexible shaft of Laval.

The blades are in general either of brass or copper, drawn to a section similar to that shown in Fig. 232, page 631. The blades are inserted into grooves in the rotor with distant pieces between them shaped to hold the blades at the proper distance apart, and are then caulked up tightly. The tips of the blades are thinned off to a knife-edge so that accidental contact between the moving blades and the casing, or between the fixed blades and the rotor, merely blunts the edges and so prevents stripping. A ring of wire is added to stiffen the longer blades. The ring is notched into the blades and then each blade is bound by wire to the ring, and all the joints formed in this way are subsequently brazed. As the blades increase in length more stiffening rings are added. For example, three rings would probably be used to stiffen 24-in. blading, placed one ring near the tips of the blades and the remaining two so that the blades were held at about one-third and at two-thirds of their length. Modified methods of securing the blades are in use, the object being to save time in the actual mechanical process of inserting and securing the blades in position.

204. Parsons Vacuum Augmenter.—Sir Charles Parsons has contrived an ingenious and effective apparatus for the removal of air from the condenser beyond the point at which the ordinary air pump can pump it out with the condensed steam. The condensing plant includes, besides the usual elements, a steam ejector and a small surface condenser called an augmenter condenser. A diagram of the plant is shown in Fig. 254. M is the main condenser, N the augmenter condenser, and J the ejector. The air pump A draws condensed steam and as much air as it can from the condenser at E through the water seal W. Steam is supplied to the ejector, and the jet flowing through suitably proportioned cones extracts the air remaining in the condenser, and the combined jet of steam and air is delivered into the augmenter condenser N, where the steam is

condensed, and the resulting water and air pass into the suction pipe of the air pump at S. The action of the ejector is thus by the removal of air to reduce the pressure in the main condenser and to deliver the air against the higher pressure in the augmenter condenser. The steam required by the ejector is small compared with the effect produced. Comparing this plant with that described on page 318, it will be seen that the ejector is equivalent to a dry air pump.

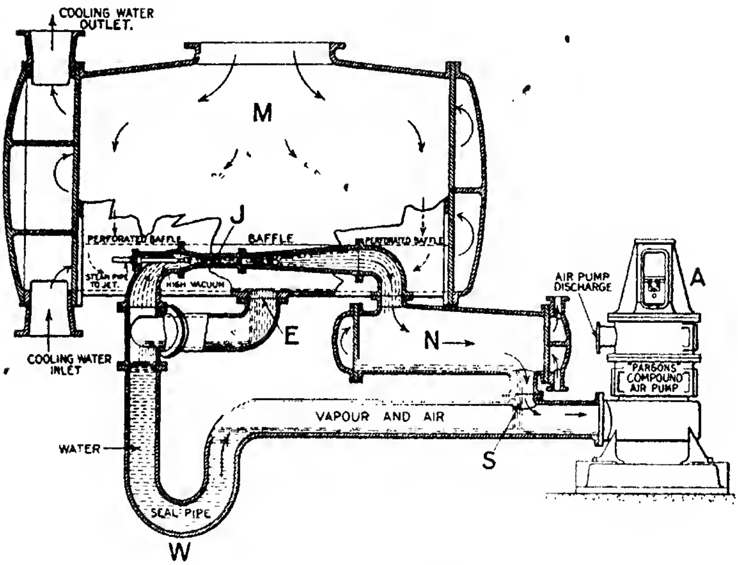


FIG. 254.—Vacuum augmenter.

205. The Marine Steam Turbine.—The design problem of the marine turbine is to secure an efficient compromise between the contradictory conditions that the peripheral speed of the blades must be high to realize steam economy, and that the speed of the propeller shaft must be low to obtain propeller efficiency. A solution of the problem was first put to the test by Sir Charles Parsons in the *Turbinia*. It was soon found that at high speeds of rotation the water round the screw is thrown away from contact with the blades, and forms a cave round it so that the screw has not the full area corresponding to its diameter to thrust against. This phenomenon is called "cavitation," and it was noticed by Sir John Thorneycroft and Mr. S. W. Barnaby in 1893 during the trials of the *Daring*, a 27-knot torpedo-boat destroyer. The limitations of speed and screw diameter imposed by "cavitation" determined finally the use of three separate shafts in the *Turbinia*. Each shaft carried one or more small propellers, and was driven by an independent turbine, but the pairs in

each of the turbines formed part of a single chain of pairs in series. The high-pressure turbine was coupled to the starboard shaft, the intermediate turbine to the port shaft, and the low-pressure turbine to the centre shaft. The port and starboard shafts are often called wing shafts. For going astern a separate turbine was fitted to the centre shaft.

In a marine turbine the steam thrust is partially balanced by the propeller thrust, so that balancing dummies are only provided to take the difference of thrust, the thrust block taking minor differences due to departures from the conditions assumed in working out the balancing. The rotors for marine turbines are generally shorter and stiffer than in land turbines owing to the division of the chain of pairs to form separate turbines for the separate propeller shafts. For this reason smaller clearances can be used without risk of stripping.

The economy of the *Turbinia* was established by tests made by Sir Alfred Ewing. The *Turbinia*[1] was 100 ft. long, 44½ tons displacement. She developed 2300 H.P., using for all purposes 15 lbs. of steam per shaft horse-power hour, and achieved a speed of 32·75 knots.

The success of this little pioneer boat wrought a change in marine engine practice which the most sanguine would scarcely have ventured to predict at the time.

The torpedo-boat destroyers *Viper* and *Cobra* were built next, each just over 200 ft. long and about 370 tons displacement. Both vessels were fitted with Yarrow boilers, and remarkable results were obtained. Both vessels were lost through circumstances quite unconnected with the turbine equipment. The *Cobra* went down in a gale in the North Sea on September 18, 1901. The *Viper* ran ashore on the Channel Islands during a fog on August 3, 1901.

The *Velox*, fitted with two sets of triple-expansion engines for cruising at 12 knots, and the *Eden*, a similar boat, but fitted with separate turbines for cruising purposes, marked the next steps in development; but the trials of the *Amethyst*, a third-class cruiser of 3000 tons displacement, in comparison with a sister ship fitted with reciprocating engines, may be said to have brought the experimental stage to a conclusion, for the results obtained determined the Admiralty to fit Parsons steam turbines in the *Dreadnought*. For main propelling machinery the reciprocating engine has since then been generally abandoned in the Royal Navy.

The *Dreadnought* is fitted with four separate propeller shafts. Each of the central shafts is connected directly to a low-pressure turbine, and to a cruising turbine in a separate casing. An astern turbine is placed in the casing of each low-pressure turbine. Each wing shaft is directly connected to a high-pressure turbine, and to a high-pressure astern turbine in a separate casing. There are therefore in all eight separate turbines on four shafts. The astern turbines on each shaft give full power of manœuvring, whilst the

[1] "The Steam Turbine." The Rede Lecture, 1911. Sir Charles A. Parsons. Cambridge, 1911.

cruising turbine on the central shafts give sufficient power for cruising.

Sir Henry Oram stated[1] that in the *Dreadnought* the consumption was 13·48 lbs. of steam per shaft horse-power hour, and in the later cruisers of the *Invincible* class this was reduced to 12·03 lbs. per shaft horse-power hour as against 16 with reciprocating engines. It was found that exhaust steam from all the auxiliaries could with advantage be turned into the low-pressure turbines instead of into the condensers, because it could there be further and economically expanded to a lower pressure. In fact, it was found that in some battleships the exhaust steam from the auxiliaries could, when used in the turbines, drive the ship at 5 knots.

One of the problems to be solved in connection with the application of steam turbines to the propulsion of warships was that the turbines should be equally efficient at the maximum speed and the much lower cruising speed. A turbine designed for efficiency at the maximum speed is considerably less efficient at the cruising speed. The problem was met by adding a supplementary chain of pairs at the high-pressure end of the main chain. Enclosed in a separate casing these pairs formed the "cruising turbine," and these turbines, together with the main turbines, formed an efficient chain of pairs for the lower speeds.

For example, suppose a turbine designed for an average blade speed of 100 ft. per second at maximum speed with average values of $\lambda = 0·35$; $a = 20°$; $U = 170$; $\eta = 0·75$. Then from equation (5), page 648, x, the number of turbine pairs required is 131. If the value of v at the cruising speed is reduced to 70, then x should be 270. The chain therefore requires lengthening, and it is lengthened in practice by the addition of the pairs forming the cruising turbine. When not in use the cruising turbine is disconnected from the steam supply, and is connected to the condenser so that its rotor revolves in vapour of extreme tenuity, and thus adds only a negligibly small resistance to the whole resistance of the plant.

The development of turbine-propelled machinery in the mercantile marine has been as rapid as in warships. After the efficiency of the *Turbinia* had been established, the *King Edward* was built in 1901 to demonstrate the advantages of the turbine for fast passenger steamers. The *King Edward* displaced 650 tons, and reached a speed of 20 knots at an estimated horse-power of 3500. The *Queen Alexandra*, 750 tons displacement, built in 1902; the *Queen*, built in 1903; the *Virginian*, 13,000 tons displacement, built in 1905; the *Carminia*, displacement 30,000 tons, built in 1905, all mark steps in the development of the turbine for the propulsion of vessels. All these were about 20-knot boats. The building of the *Mauretania* and the *Lusitania* in 1907, each 40,000 tons displacement, and equipped with turbines of 74,000 shaft horse-power to drive the boats at 26 knots, indicate a truly extraordinary rapid development. To pass from the 2300 H.P. *Turbinia* in 1897 to the 74,000 H.P.

[1] *Engineering*, vol. 88, p. 703.

Mauretania in 1907 is an achievement unparalleled in the history of the development of any form of prime mover.

The following table, quoted from Sir Charles A. Parsons' Rede lecture at Cambridge, shows the results obtained with notable ships. Below, for convenience of comparison, is added a second table showing results obtained with Parsons turbo-alternator.

TABLE 33.—PERFORMANCE OF NOTABLE SHIPS OF DIFFERENT EPOCHS WITH PARSONS TURBINES.

Date.	Name of ship.	Length.	Displacement.	Horse-power.	Steam consumption per S.H.P. hour for all purposes.	Speed in knots.
		feet.	tons.		lbs.	
1897	*Turbinia*	100	44½	2,300	15	32·75
1901	*King Edward*	250	650	3,500	16	20·48
1905	H.M.S. *Amethyst*	360	3,000	14,000	13·6	23·63
1906	H.M.S. *Dreadnought*	490	17,900	24,712	15·3	21·25
1907	*Mauretania* and *Lusitania*	785	40,000	74,000	14·4	26·00

TABLE 34.—PERFORMANCE OF PARSONS TURBO-GENERATORS AT DIFFERENT DATES.

Date.	Power, kw.	Steam per kilowatt hour. lbs.	Estimated [1] equivalent steam per S.H.P. hour. lbs.	Vacuum (Bar. 30″). ins.	Superheat. Deg. C.	Steam pressure by gauge Lbs. per sq. in.
1885	4	200	144	0	0	60
1888	75	55	39·5	0	0	100
1892	100	27·00	19·4	27	28	100
1900	1,250	18·22	13·1	28·4	70	130
1902	3,000	14·74	10·6	27	130	138
1910	5,000	13·20	9·5	28·8	67	200
To which may be added the turbine illustrated in Fig. 249.						
1913	20,000	11·25 [2]	8·1	29	111	200

206. Geared Turbines.—The introduction of gearing between the turbine shaft and the propeller shaft removes conditions hampering the design when the two are directly coupled. The turbine speed can be increased, and the speed of the propeller shaft reduced, and a speed for each can be chosen to secure the greatest efficiency. Gearing has always been used in the Laval turbine to reduce the turbine speed from about 30,000 to 3000 revolutions per minute on the working shaft, and has been found efficient, and therefore there is no reason why helical gearing of the same kind should not prove equally efficient in large turbines. Propelling machinery consisting of a combination of high-speed turbines coupled by gearing to a low-speed propeller shaft enables the turbine to be economically applied to the propulsion of steamers at low speeds.

[1] Multiplier used for converting kw. hours to S.H.P. hours = 0·72, allowing therefore 0·965 for generator efficiency.

[2] Guaranteed.

The combination was first tried on a large scale in the *Vespasian*,[1] a vessel of 4350 tons displacement, fitted originally with triple-expansion engines of 900 I.H.P. Trials of coal and water consumption were made with the existing machinery, after which it was taken out and replaced by geared turbines, the propeller shafting and boilers remaining the same. With the geared turbine there was a saving of 15 per cent. in the total water consumption. This was increased to 22 per cent. by changing the propeller. The new machinery consisted of a high-pressure and a low-pressure turbine, each driving a pinion at 1400 revolutions per minute gearing into a spur wheel on the propeller shaft turning at 70 revolutions per minute. The gearing is enclosed in a casing, and is continually sprayed with oil by a pump.

The use of geared turbines has extended to warships, where by this means efficient designs can be made for cruising speeds. The cruising turbine and the high-pressure turbine are in some cases geared to the shaft, whilst the low-pressure turbine is coupled directly to the shaft. For low speeds the cruising turbine, the high-pressure turbine, and the low-pressure turbine are in series, thus forming a chain with a sufficient number of pairs in it to utilize the available energy efficiently at the cruising speed. At full speed the cruising turbine is cut out, and steam is admitted directly to the high-pressure turbine. There are thus fewer pairs in series in the chain, but the number is sufficient to give the full-load efficiency at the higher speed of rotation.

Although the geared turbine was primarily designed to solve the problem of producing turbine-propelling machinery for low-speed vessels, its success for this purpose has led to its extension to high-speed boats. For example, geared turbines are fitted to the twin-screw 25-knot channel steamer on the Newhaven and Dieppe service of the London, Brighton and South Coast Railway Company.[2] The turbine chain is divided into four units, namely, the high and the intermediate pressure turbines, and two low-pressure turbines in parallel. The high-pressure and one low-pressure turbine are geared with the port propeller shaft through helical gearing enclosed in a case. The speeds of rotation are: high-pressure rotor, 2610; low-pressure rotor, 1849; propeller shaft, 435 revolutions per minute. The intermediate and the second low-pressure turbine are geared with the starboard propeller shaft. The speed of the rotor of the intermediate turbine is 2610, the low-pressure rotor 1849, and the propeller shaft 435 revolutions per minute. The shaft horse-power developed is 14,000. Yarrow boilers are fitted, and a moderate forced draught is used.

Geared turbines[3] have been fitted in the S.S. *Normannia* and in

[1] Sir C. A. Parsons, "On the Application of the Marine Steam and Mechanical Gearing to Merchant Ships," *Proc. Inst. Naval Arch.*, 1910.

[2] *Engineering*, December 5, 1913.

[3] Particulars of the machinery of these ships will be found in a paper entitled "The Geared Turbine Channel Steamers *Normannia* and *Hantonia*," by Prof. J. H. Biles, *Trans. Inst. Naval Arch.*, March 28, 1912.

the S.S. *Hantonia*, each ship being of 1900 tons displacement. The steamers run in the service of the London and South-Western Railway Company between Southampton and Havre. In each case the turbines develop 5000 shaft horse-power at a speed of 18 knots. Sir Charles Parsons, in a paper read at the 1913 Spring Meeting of the Institution of Naval Architects, stated that, compared with other turbine steamers on the same service, these two ships show an economy of about 40 per cent.[1] This is due partly to the increased efficiency of the turbines owing to the higher blade speed at which they can be run, partly to the increased efficiency of the propellers owing to the lower revolutions at which they can be run, and partly to the improved form of the vessel made possible by the reduction of the number of boilers and the adoption of twin screws. In this paper it is also stated that there were under construction in March, 1913, turbines mechanically geared representing the transmission of over 120,000 H.P., including two installations of over 20,000 H.P. each, and examples are given of typical designs of geared turbine propelling machinery for a torpedo-boat destroyer of 20,000 H.P., for battleships of 40,000 H.P. and 60,000 H.P. respectively, and for an Atlantic liner of 60,000 H.P.

Sir C. A. Parsons, in this paper, describes a method of cutting gearing devised by himself and his colleagues, which secures great accuracy of pitch, and tends, therefore, to noiseless running; and it is stated that when examined after a voyage of some 26,000 miles, the gearing of the *Normannia* was found to be in perfect condition, no sign of wear being detected.

The system of geared turbines is extending rapidly to both warships and to merchant vessels, and to land installations also.

There are other methods of transmitting the power from a high-speed turbine to a low-speed working shaft. In the hydraulic system of Dr. Föttinger[2] the turbine shaft drives what is essentially a centrifugal pump, the delivery from the pump driving a turbine connected to the propeller shaft. Both pump and turbine are enclosed in one casing, and are designed in such a way that the water passages form a closed circuit for the circulation of the water between the pump and the turbine.

In ships two transmitters of this kind must be fitted, one for ahead and the other for astern running.

Electrical methods of transmission are also in use. The high-speed turbine drives a dynamo, the current from which drives a motor on the slow-speed shaft.

207. The Curtis Turbine.—The first 600-kw. Curtis turbine built by the General Electric Company of Schenectady, the firm who took up and developed the Curtis turbine, consisted generally of

[1] "Mechanical Gearing for the Propulsion of Ships," by the Hon. Sir C. A. Parsons, *Proc. Inst. Naval Arch.*, March 13, 1913.

[2] Dr. H. Föttinger, "Recent Development of the Hydraulic Transformer," *Trans. Inst. Naval Arch.*, 1914.

a chain of six impulse pairs arranged in two pressure stages, there being three pairs in each stage combined for velocity gradation in the way explained on page 656. The results of trials of this turbine, carried out by Mr. Emmet,[1] and published in 1902, led to the design of a 5000-kilowatt vertical type in which the electric generator was placed on the top of the turbine. In this large type there were eight impulse pairs in the chain arranged in two pressure stages, there being four pairs within each pressure stage combined for velocity gradation. Quoting from Mr. Emmet's paper in the *Proceedings of the Institution of Mechanical Engineers*, a 2000-kilowatt turbine of this type, running at 750 revolutions per minute on a pressure range from 155 lbs. per square inch gauge to a vacuum of 28·7 ins. and the steam initially superheated through 117° C., developed 2000 kilowatts when using steam at the rate of 15·3 lbs. per kilowatt hour.

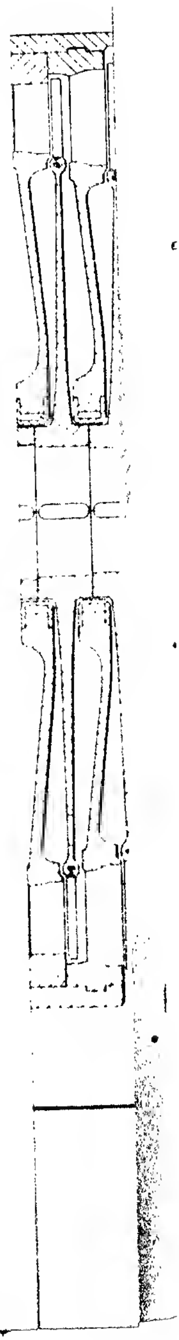
A feature of the design of a Curtis turbine is the way in which the steam supply is regulated to the load. The nozzles in the first pair extend only partially round the circumference and are divided into groups, each group being arranged in an independent steam chest which is supplied from the main steam chest through a valve operated by the governor. The first group of nozzles supply sufficient steam to run the turbine at no load. As the load comes on the turbine the stop valves are opened in succession by the governing mechanism.

A feature which the Curtis turbine shares with all turbines constructed with chains of impulse pairs is that there is only slight longitudinal thrust on the turbine rotor. This is due to the slight fall of pressure through the blades due to the frictional resistance of the flow. A thrust block is fitted to the shaft to take up the longitudinal thrust and at the same time to prevent longitudinal play of the shaft, so that the moving blades can be maintained in their proper relative position to the fixed blades.

Curtis turbines have been developed in America and in this country both for stationary and marine engines. In the marine type impulse blading on a rotor is sometimes used for the low-pressure end, and then the size of the drum can be arranged so that the longitudinal thrust of the steam on the drum nearly balances the propeller thrust. Particulars of a comparative trial of the efficiency of the reciprocating engine, a Curtis marine turbine, and a Parsons marine turbine will be found in a report issued by the United States Navy Department.

Three sister ships, the *Birmingham*, the *Salem*, and the *Chester*, were fitted respectively with reciprocating engine, Curtis turbines, and Parsons turbines. Trials were made in 1909. At full power the *Birmingham* reached a speed of 24·3 knots, the *Salem* at 25·9 knots, and the *Chester* at 26·5 knots. When all three vessels were driven at 22½ knots and devoloping about 10,000 H.P., the *Birmingham*

[1] American Philosophical Society, April, 1902. See also "The Curtis Steam Turbine," *Proc. Inst. Mech. E.*, June, 1904.



ran 2·47 miles per ton of coal, the *Salem* 2·73, and the *Chester* 2·83. The Parsons turbines showed, therefore, a slight advantage over the Curtis, and both turbines were distinctly better than the reciprocating engines.

The Curtis turbine in the *Salem* developed about 20,000 H.P., and consisted of 22 pairs arranged in seven pressure stages, four pairs being combined for velocity gradation in the first stage and three in each of the remaining stages. Steam is supplied to the first stage through twenty nozzles, each opened by a separate valve. There was no impulse blading on a rotor, so that a thrust block of the ordinary kind was fitted.

The general arrangement of a 6000-kilowatt Curtis turbine constructed by the British Thomson Houston Co. of Rugby, to whom the author is indebted for the drawing, is shown in Fig. 255. The turbine runs at 1500 revolutions per minute, and the range on which it is designed to work is from 200 lbs. per square inch by gauge to a vacuum of 28½ ins., with steam superheated initially to 343° C.

The chain is composed of twelve pairs. The first two pairs are compounded for velocity, the remaining ten being each a single-stage pair working on a definite pressure difference.

Each wheel rotates in a chamber formed by diaphragms which divide the wheels from one another, and in the peripheries of which the nozzles are placed. The first diaphragm is packed round the shaft, the remainder being packed round the bosses of the wheels which they separate. The packing is formed of brass segments held together by an encircling garter spring, but with freedom to move radially in order to allow for the deflection of the shaft.

The nozzles of the first pair, of which N is one, extend only partially round the circumference. They are divided into groups, each group being independently supplied from the steam pipe S through a valve, one of these valves being shown, V.

These valves are actuated by the governor, and the number of them open at any time depends upon the load on the turbine.

The second pair is formed of the segment of fixed guiding channels G and the ring of moving blades B_2. The moving blades B_1 of the first pair and those of the second pair B_2 are carried on one wheel, which rotates within the first pressure chamber.

The third and subsequent pairs are similar in construction. Each is formed of a diaphragm carrying a complete ring of nozzles and a wheel carrying at its periphery a single row of blades. The area of the nozzles and of the blade channels increase as the flow proceeds and the density falls.

The wheels are carried on a common shaft supported by two main bearings. The bearings are white metalled, and oil under pressure is forced into them and spreads either way to the ends, from which it is caught and led back to the pump.

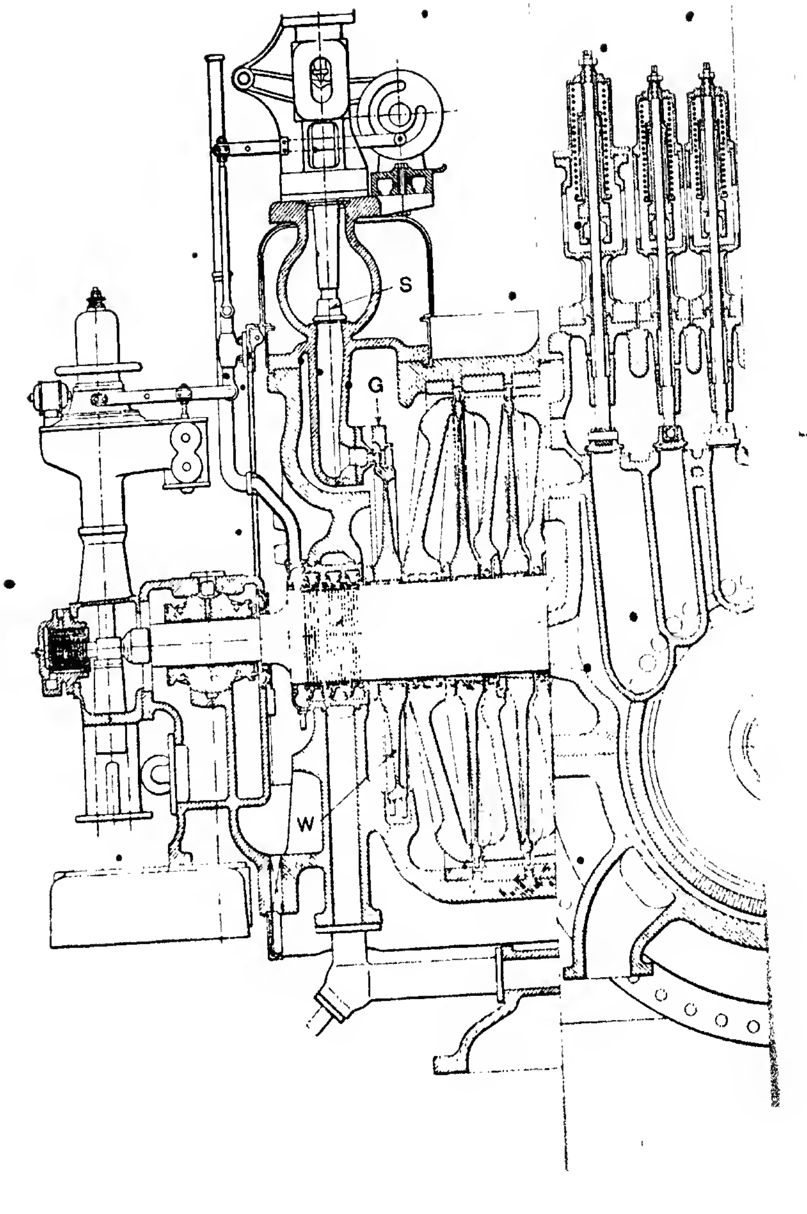
The shaft is packed at each end by means of a series of carbon rings C, C, C, C, – C, C, arranged each in its own chamber. Each

ring is formed of segments butted together and held in contact with the shaft by an encircling garter spring. Each ring chamber is connected by a small pipe with a pressure chamber of the turbine, and the pressure chambers are selected so that the pressures in the ring chambers fall towards the atmospheric pressure. Steam leaking along the shaft is thus prevented from escaping to the outside, since it is caught in the ring chambers and then flows back into the turbine itself. At the low-pressure end the steam connections are arranged so that the steam leaks into the turbine and so prevents air leak and a reduction of the vacuum.

The blading of a turbine of this kind must be strong enough to resist bending due to the torque which they transmit to the wheel, and to resist the centrifugal force due to the high speed of rotation. The concave surface is in section a circular arc, and the convex surface is formed by tangents to this arc joined by a circular curve. The blade is rooted in the rim by dovetail connections. A dovetail groove is turned in the wheel rim, and a slot is cut in the rim at two or more places, so that the dovetailed end of the blade may be inserted in the groove and after it a dove-tailed packing piece, and both are pressed into close contact with the blades already in place by means of a special machine. The ends of the blades are afterwards riveted into a ring which encircles their tips.

The guiding nozzles G for the second pair, which is velocity compounded with the first pair, are similar in form to the blades and are similarly secured in dovetail grooves in cast-iron segments which are afterwards bolted to the inside of the turbine casing. The nozzles in the first pair and in the diaphragms are rectangular in cross section, and are formed by casting a series of bent nickel steel plates in the diaphragms. The plates are first bent to the required angle and are then laid round the mould at the pitch required, after which the metal is poured. The rectangular nozzles so formed present cast-iron surfaces in the radial direction, but nickel steel surfaces in the direction in which the nozzle acts to change the direction of motion of the flow.

As already mentioned, the nozzles leading the steam to the first wheel are divided into groups and each group is supplied by an independent admission valve. The admission valves are arranged in a line parallel to a shaft supported on roller bearings. Keyed to this shaft are cams, one cam to each admission valve. The section of the cam shaft A, one cam K, and the plunger P, and lever L connecting the cam with the admission valve V, are shown in Fig. 255. The cams are arranged on the shaft with angular lag, so that the turning of the shaft opens the admission valves in succession. It requires the greater part of a revolution to open up all the valves. Fitted at the end of the cam shaft is a small rotatory motor operated by oil under pressure which is admitted to and exhausted from the motor cylinder by means of a slide valve connected to the governor. As the governor sleeve rises or falls the slide valve is moved to one or other side of its central position and so regulates the oil supply


S
G
W

and through it the position of the rotatary piston, and so finally the angular position of the cam shaft. This shaft in its turn determines by its angular position the number of admission valves opened to steam. In this way the governor is able to regulate the steam supply from partial to full admission. The number of nozzles in the first group admits sufficient steam to drive the turbine at no load.

208. The A.E.G. Impulse Turbine.—The two views, Figs. 256 and 257, show the general arrangement of a compound impulse turbine constructed by the Allgemeine Electricitats Gesellschaft of Berlin, to whom I am indebted for the drawings from which the sketches were prepared and for the following particulars:—

The turbine is designed to develop 5000 kilowatts at 2400 revolutions per minute, and is coupled to three-phase alternators. Steam, superheated to 327° C., is supplied at a pressure of 205 lbs. per square inch (190 by gauge), and the final pressure is about 1·3 lbs. per square inch. The available energy corresponding to this range is 213·5 lb.-cals. per pound of flow. The efficiency of the Rankine engine of comparison on this range is 0·3. Allowing 1900 lb.-cals. per kilowatt hour, the Rankine engine would require $\frac{1900}{213 \cdot 5} = 8 \cdot 9$ lbs. of steam per kilowatt hour.

The steam consumptions guaranteed by the company depend upon the temperature of the cooling water and upon the load, and are as follows:—

TABLE 35.—CONSUMPTION OF A.E.G. TURBINES.

Load in kilowatts.	Pounds of steam per kilowatt hour with cooling water at— 21° C.	26·7° C.
2500	13·45	13·1
3750	12·85	12·5
5000	12·55	12·2
6250	12·85	12·5

All these consumptions are ± 3 per cent.

Comparing the consumption at 5000 kilowatts and cooling water at 21° with that of the Rankine engine, it will be seen that the efficiency is $\frac{8 \cdot 9}{12 \cdot 55} = 0 \cdot 7$. About 70 per cent. of the available energy is therefore utilized.

From Fig. 256 it will be seen that the turbine consists of a chain of seven pairs. The first two pairs of the chain are velocity compounded, and the moving blades are carried on the wheel W. The admission is partial, and the guide blades corresponding to the second pair are indicated at G. The remainder of the chain consists of five single-stage impulse pairs, each pair being constituted by a wheel

and diaphragm as indicated. The nozzles of the first pair are divided into groups, each group being supplied with steam through an independent stop valve, one of which is shown at S, Fig. 256. There are seven groups of nozzles, and the seven valves supplying them are shown in the sectional end view of the turbine, Fig. 257. M is the main stop valve admitting steam to the steam chest H, from which

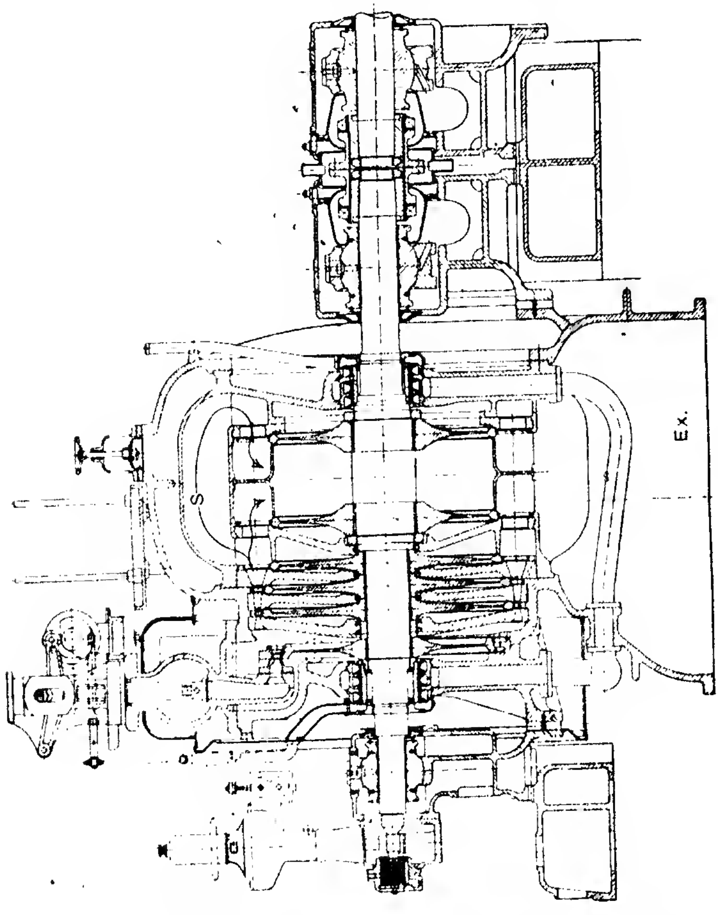


FIG. 258.

steam is admitted to the groups of nozzles by the independent supply valves. These valves are operated by cams carried on a common shaft, the end of which is seen at K. The cam shaft is connected to the governor through a special hydraulic motor, so that the valves are opened up in succession as the load comes on the engine. The general construction of the turbine is clearly shown in the drawings.

It is instructive to analyse the distribution of energy through

a turbine of this kind in a general way, neglecting the direct consideration of the details of the losses and the effect of the frictional reproduction of heat as the flow proceeds through the turbine. For this purpose assume that the blading conditions are worked out in connection with the velocity diagram, Fig. 242, page 661.

Assume that the mean diameter of the blade rings in the two velocity compounded pairs is about 4 ft. At 2400 revolutions per minute the corresponding blade speed is about 500 ft. per second, giving with the value of λ for the combination in Fig. 242, namely 0·23, a velocity of discharge through the nozzles of the first pair of 2175 ft. per second. This corresponds to 52·6 lb.-cals. per second per pound of flow. Allowing 10 per cent. for losses not included in the velocity diagram, the combination absorbs from the range about 60 lb.-cals. per pound of flow. There remains 213 − 60 = 153 to be distributed between five pairs. Assuming that the distribution is made equally, then each pair absorbs, say, 30 lb.-cals. per second per pound of flow, and deducting 10 per cent. this leaves 27 lb.-cals. for the production of the velocities.

Assume that the mean diameter of the blade rings of the last five pairs of the chain is about 5·2 feet. This corresponds to a blade velocity of about 654 ft. per second. An impulse pair of the kind considered in connection with Fig. 242 has a value of $\lambda = 0·44$, from which it appears that the corresponding velocity of discharge from the nozzles in each pair is 1486 ft. per second. With these assumptions each pair absorbs 25 lb.-cals. per second from the range for the production of the velocities, a number sufficiently near the 27 found above to show that the distribution found in this general way approximates to the actual distribution. The height of the blading can then be determined when the state of the steam at exit from each nozzle ring is known, and this can be found from one of the steam charts.

The volume of the steam increases so rapidly as the pressure falls that the blade height of the last pair is excessive if a very low vacuum is to be utilized. In practice this excessive blade height is sometimes avoided by increasing the nozzle angle, the effect of which is to increase the axial velocity of flow into the condenser. This, of course, reduces the efficiency of the last pair. An A.E.G. turbine, designed so that a high vacuum can be utilized with moderate blade height in the final pair, is illustrated in Fig. 258. Two pairs at the end of the chain are arranged in parallel, the effect of which is to provide the area necessary to pass the large volume of steam corresponding to a high vacuum. The chain consists of seven pairs. The first two are velocity compounded. The next three are in series. The last two are in parallel. The arrows in the upper part of the diagram show the course of the steam through the last two pairs in the chain.

209. Exhaust-Steam Turbines. Mixed-Pressure Turbines.—Exhaust-steam turbines are those which derive their supply of

steam from the exhaust steam of reciprocating engines, and are better described as turbines working on a low-pressure range. Wherever low-pressure steam is available either from a number of small reciprocating engines, or from any auxiliary plant, the steam can be used to develop power in a turbine working with an efficient condenser. There is no difference between an exhaust-steam turbine and any other kind of turbine. It is designed and proportioned to the range on which it is to work.

A mixed-pressure turbine is one designed to utilize low-pressure steam, but with the addition of a few turbine pairs, through which when necessary higher pressure steam can be turned from the boiler. If there is sufficient low-pressure steam available to produce the power required, it is supplied to the turbine at a point in the chain of pairs of suitable dimensions to receive the flow, the pairs above this in the chain revolving idly. When the supply of low-pressure steam is insufficient the supplementary supply of high-pressure steam is turned into the turbine at the beginning of the chain, and so flows first through the pairs specially added to expand it down to the pressure of the supply of low-pressure steam. The two streams then unite and flow as one through the low-pressure part of the chain to the condenser.

EXAMPLES.

1. The gauge on the boiler of a steam plant indicates a pressure of 200 lbs. per square inch. The gauge on the condenser of the plant shows 26·9 ins. vacuum. The barometer reads 29·3 ins. Find the absolute pressures which condition the working of the plant, and write against each steam pressure the corresponding temperature.

29·3
26·9
2·4

Answers.—Absolute boiler pressure, 214·35 lbs. per square inch. Temp. 197·6° C.
Absolute condenser pressure, 1·19 lbs. per square inch. Temp. 42° C.

2. Calculate the cost of electrically driving a train at an average of 360 H.P. for a distance of 20 miles at an average speed of 30 miles per hour when a Board of Trade unit of electrical energy costs one penny.

Answer.—14*s*. 11*d*.

3. What is the cost of stopping a steamer of 20,000 tons from a speed of 20 miles per hour when energy costs 2*d*. per horse-power hour?

Answer.—£2 10*s*. 5*d*.

4. An engine developing 1500 I.H.P. requires steam at the rate of 15 lbs. per indicated horse-power hour. Each pound of water requires 620 lb.-cals. to evaporate it. Each pound of coal burned produces 8000 lb.-cals of heat energy, two-thirds of which is transferred across the heating surface of the boiler to evaporate the steam. The cost of the coal is 12*s*. 6*d*. per ton. What is the cost of fuel for one day's work of twelve hours? What is the cost of fuel per indicated horse-power hour? How many pounds of water can be evaporated for one penny? How many foot-pounds of work does the engine do for one pennyworth of fuel?

Answers.—£8 15*s*.
0·117*d*. per I.H.P. hour.
128 lbs. of water for a penny.
16,950,000 ft.-lbs. ,,

5. The indicator diagram below was taken from an engine cylinder 50 ins. diameter and 4 ft. stroke, and having a clearance volume equal to 4 per cent. of the effective volume. Barometer 29·9 inches.

(*a*) Find the maximum pressure and the minimum pressure in the pressure-volume cycle which the diagram represents.

(*b*) Find the mean pressure.

(*c*) Find the work done during the cycle.

(*d*) Assuming that the engine is single acting and that the diagram is repeated without sensible change at each revolution of the crank shaft, find the rate at which work is done on the piston when the crank shaft is driven at 80 revolutions per minute—

(1) In ft.-lbs. per second.
(2) In ft.-tons per minute.
(3) In horse-power.

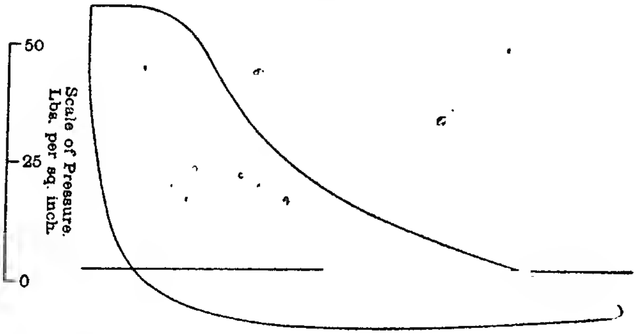


FIG. 1.

(*e*) Assuming that the engine is double acting; that the indicator diagram shown in Fig. 2 was taken from the end of the cylinder opposite to that from which Fig. 1 was taken; that the diagram is repeated without sensible change at each revolution of the crank shaft, and that a piston rod 9 inches diameter passes through the end of the cylinder from which the diagram was taken; find the rate at which work is done on the piston when the crank shaft is driven at 80 revolutions per minute—

(1) In ft.-lbs. per second.
(2) In horse-power.

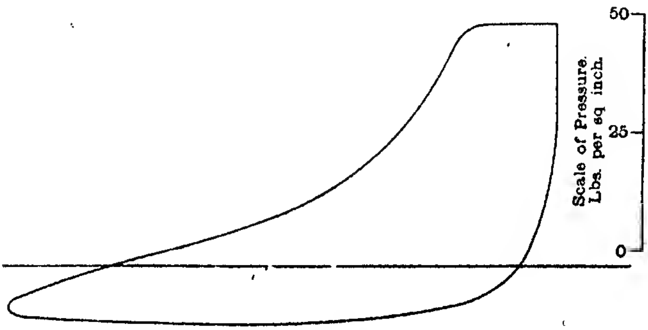


FIG. 2.

(*f*) Find the net force, in tons, acting on the piston at $\frac{1}{10}$ stroke; at $\frac{1}{2}$ stroke; and at $\frac{9}{10}$ stroke, from the diagram Fig. 1.

(*g*) Calibrate the diagram Fig. 1 for pressure intervals of 10 lbs. per square inch, and for volume intervals of 5 cub. ft.

Answers.—(*a*) 71 lbs. per square inch. 3 lbs. per square inch.
(*b*) 28·6 lbs. per square inch.
(*c*) 224,700 ft.-lbs. per cycle.
(*d*) (1) 299,600 ft.-lbs. per second.
(2) 8020 ft.-tons per minute.
(3) 545 I.H.P.
(*e*) (1) 277,700 + 299,600 = 577,300 ft.-lbs. per second
(2) 505 + 545 = 1050 I.H.P.
(*f*) 58 tons. 22 tons. 0 tons.

6. The torque exerted by the crank shaft of an engine is 100,000 lbs.-ft. Calculate the rate at which the crank shaft does work when it is driven at 70 revolutions per minute—

(1) In ft.-lbs. per second.
(2) In ft.-tons per minute.
(3) In horse-power.

Answers.—(1) 733,000 ft.-lbs. per sec.
(2) 19,630 ft.-tons per min.
(3) 1333 horse-power.

7. A rope brake is placed on the flywheel of an engine which is running at 300 revolutions per minute. The rope is held in equilibrium by a weight of 200 lbs. hanging freely, so that its line of action is 3·1 ft. from the centre of the shaft; and a pull on the spring balance of 18 lbs. in a line 3 ft. from the centre of the shaft.

Calculate (1) The torque exerted by the shaft.
(2) The brake horse-power.

Answers.—Torque = 566 lbs.-ft.
B.H.P. = 32·3.

8. A brake made of rope $\frac{3}{4}$ in. diameter is placed on an engine fly-wheel. The wheel is 7 ft. diameter. Equilibrium is established by 300 lbs. hanging from the free end of the brake, and a pull on the spring balance of 20 lbs. Calculate the brake horse-power when the crank shaft is driven at 160 revolutions per minute.

Answer.—B.H.P. = 30·1.

9. Calculate the torque corresponding to 10,000 I.H.P. when the crank shaft is driven at 80 revolutions per minute.

Answer.—657 × 10^3 lbs.-ft.

10. The propeller shaft of a marine engine is 10 ins. diameter, and it is observed to twist 1° in 20 ft. when driven at 120 revolutions per minute. The modulus of rigidity of the material of the shaft is 5000 tons per square inch. Calculate the horse-power transmitted by the shaft.

Answer.—1519 horse-power.

11. Using the data given in the steam tables, plot on a temperature base, curves showing—

(1) The pressure.
(2) The volume per pound.
(3) The water energy per pound.
(4) The total energy per pound.

12. Using the data given in the steam tables, plot the pressure against the volume per pound of dry saturated steam.

13. The relation between the pressure and the volume of dry saturated steam can be approximately represented by the equation

$$PV^n = \text{a constant} = k.$$

Find the constants n and k between the pressure limits of 50 and 200 lbs. per square inch by means of the steam tables and logarithmic plotting.

Answer.—$n = 1{\cdot}06$; $k = 490$.

14. Using the steam tables, find the heat which must be supplied to produce from water at 15° C.

1 lb. of dry saturated steam at 180 lbs. per square inch.
1 lb. of wet steam with a dryness fraction of 0·9.
1 lb. of steam evaporated at 180 lbs. per square inch, and afterwards superheated at constant pressure to 250° C.

Answers.—653·5 lb.-cals.
605·9 lb.-cals.
702·9 lb.-cals.

15. An engine is supplied with 355 lbs. of dry saturated steam per hour at a pressure of 180 lbs. per square inch. The temperature of the water in the hot well is 20° C.

Calculate the mean rate at which heat is absorbed from the heating circuit, assuming that the feed temperature is 20° C.

Calculate also the mean rate at which heat is absorbed by the cooling circuit, having given that the circulating pump causes a flow through the condenser of 10,000 lbs. of water per hour, and that the temperature of the water at entry is 16° C. and at leaving the condenser 30° C.

Assuming that the rate at which heat is converted into work in the cylinder is equal to $\frac{1}{3}$ of the difference between the rate at which heat is received by the motive power circuit from the heating circuit, and rejected by the motive power circuit to the cooling circuit, find the indicated horse-power and calculate the thermal efficiency, reckoning from the hot-well temperature.

Answers.—230,260 lb.-cals. per hour.
140,000 lb.-cals. per hour.
I.H.P. = 21·27.
Efficiency = 0·13.

16. An engine developing 128 I.H.P. is supplied with steam at the mean rate of 2330 lbs. per hour. The steam is produced at 150 lbs. per

square inch, and is superheated at constant pressure to 250° C. The observed temperatures are as follows:—

In the exhaust pipe, 47·0° C.
In the hot well, 41·5° C.
In the feed tank, 28·3° C.

Calculate the thermal efficiency of the engine reckoned from (1) The exhaust temperature; (2) The hot-well temperature; (3) The feed temperature.

Answers.—11·8 per cent.
11·7 per cent.
11·5 per cent.

17. An engine uses 20 lbs. of steam per indicated horse-power hour. The boiler pressure is 150 lbs. per square inch, and the feed temperature is 20° C. Assuming that the steam supplied to the engine is dry and saturated, calculate the thermal efficiency.

Answer.—10·9 per cent.

18. An engine developing 217 I.H.P. uses 3772 lbs. of water per hour. When the indicated horse-power is reduced to 102 by throttling the pressure of the steam supply, the consumption falls to 1920 lbs. of water per hour. Assuming that the Willans law applies, find the constants of the Willans line.

If the relation between the brake horse-power and the indicated horse-power is also assumed to be linear, find the constants in the equation

$$\text{I.H.P.} = a + b\ \text{B.H.P.},$$

having given that the brake horse-power corresponding to the indicated horse-powers given above are respectively 209 and 97.

Calculate the probable brake horse-power corresponding to a steam supply at the mean rate of 2000 lbs. per hour.

Answers.—Willans line is $s = 277 + 16{\cdot}1 \times \text{I.H.P.}$
$a = 2{\cdot}4.$
$b = 1{\cdot}025.$
$\text{B.H.P.} = 102.$

19. Assuming a linear law between the brake horse-power and the indicated horse-power, and also between the indicated horse-power and the mean rate of steam supplied per hour, calculate the constants w, k, a and b in the equations

$$\text{Steam supplied per hour} = w + k\ \text{I.H.P.}$$
$$\text{I.H.P.} = a + b\ \text{B.H.P.}$$

from the following data:—

Brake horse-power at full load, 300;
Indicated horse-power at full load, 335;
Steam supplied per hour at full load, 5700 lbs.;
Indicated horse-power at no load, 30;
Steam supplied at no load, 660 lbs. per hour.

Calculate the probable brake horse-power when the steam supply is 4000 lbs. per hour.

Answers.—$w = 164\cdot 2$.
$k = 16\cdot 5$.
$a = 30$.
$b = 1\cdot 016$.
B.H.P. = 199.

20. The temperature in the steam pipe of an engine is 200° C., and the steam is dry and saturated. The temperature of the feed is 40° C. The engine uses 15 lbs. of steam per indicated horse-power hour.

(1) Calculate the thermal efficiency of the engine.

(2) Apply the second law of thermodynamics to find the efficiency of a perfect engine which receives all the heat into the motive power circuit at the constant temperature 200° C., and rejects heat to the cooling circuit at 40° C.

(3) Calculate how much heat would be taken from the heating circuit by the perfect engine per indicated horse-power hour.

(4) Calculate the steam supply to the perfect engine in pounds per indicated horse-power hour.

Answers.—(1) 14·9 per cent.
(2) 33·8 per cent.
(3) 4180 lb.-cals.
(4) 6·63 lbs.

21. Assuming the frictional resistance of an engine to be constant at all loads and equal to 7·5 I.H.P. at no load, calculate the mechanical efficiency of the engine when the indicated horse-power is 65·5.

Answer.—Efficiency is 88·6 per cent.

22. Corresponding values of the indicated and the brake horse-powers of an engine running at 380 revolutions per minute are 99 and 78 respectively, and values found at 370 revolutions per minute are 241 and 213 respectively. At no load the engine runs at 390 revolutions per minute. Assuming a linear law of engine resistance, find the no-load indicated horse-power; the mechanical efficiency when the indicated horse-power is 160 at 370 revolutions per minute, and the mechanical efficiency when the indicated horse-power is 75 at 380 revolutions per minute. Find also the expression for the engine resistance T_f in terms of the crank shaft torque T, corresponding to the B.H.P.

Answers.—

No load horse-power, 17·2.

Mechanical efficiency when indicated horse-power is 160, 85 per cent.

Efficiency at 75 I.H.P., 73 per cent.

No load torque = 232 lbs.-ft.

$T_f = 232 + 0\cdot 055T$.

23. An engine develops 17,000 I.H.P. when running at 119 revolutions per minute, and the power wasted in friction is 18 per cent. of the indicated horse-power, the no-load friction accounting for 8 per cent. Estimate the mechanical efficiency of the engine when it develops 2000 I.H.P. at 60 revolutions per minute, and write down the expression for

engine resistance T_f in terms of the crank shaft torque T, corresponding to the B.H.P.

Answers.—Efficiency, 58·6 per cent.
$T_f = 60{,}000 + 0{\cdot}122T$.

24. A safety valve is 4 ins. diameter. It is required to blow off at 150 lbs. per square inch by gauge. It is held down by a lever carrying a weight at the end. A perpendicular through the fulcrum of the lever is 6 ins. from the centre of the valve; and a perpendicular through the centre of gravity of the weight is 24 ins. from the centre of the valve in the opposite direction. The lever weighs 18 lbs., and a perpendicular through its centre of gravity is 10 ins. from the centre of the valve. Calculate the weight of the mass at the end of the lever.

Answer.—367 lbs.

25. A safety valve 4 ins. diameter is loaded with a spring. Calculate the strength of the spring so that when blowing off with a lift of 0·1 in. the steam pressure shall not increase more than 5 per cent. of the boiler pressure, which is 150 lbs. per sq. in. by gauge. Calculate also the initial compression of the spring.

Answers.—Strength of spring, 941 lbs. per inch of compression.
Initial compression, 2 ins.

26. The lever of a set of Ramsbottom safety valves weighs 40 lbs., and its centre of gravity is 10 ins. from the centre line between the pillars, which are 10 ins. apart. Each valve is 4 ins. diameter. Find the distance from the centre line between the pillars at which the spring must be applied to the lever so that the load exerted on each valve is equal; and find the pull which the spring must exert so that the valves just begin to lift when the pressure reaches 150 lbs. per square inch by gauge.

Answers.—Distance from centre line of pillar towards the pillar more remote from the centre of gravity of the lever, 0·1 in.
Pull exerted by spring, 3725 lbs.

27. A locomotive burns coal at the rate of 80 lbs. per square foot of grate per hour; and air is drawn in the furnace at the rate of 16 lbs. of air per pound of coal burnt. The grate area is 25 sq. ft. Calculate the velocity with which the air must flow into the ash pan through damper openings 4 sq. ft. in area, assuming that the air temperature is 17° C. and that the atmospheric pressure is 14·7 lbs. per square inch.

Answer.—Velocity of flow is 29¼ ft. per second.

28. A U tube contains a liquid whose density is 60 lbs. per cubic ft. It shows a difference of level of ¾ in. when connected to a flue. What is the corresponding pressure difference in pounds per square inch?

Answer.—0 026 lb. per square inch.

29. Air is supplied to a furnace at the rate of 20 lbs. per pound of fuel burned. The barometric pressure is 14·7 lbs. per square inch. The temperature at a point in the flue is 500° C. as measured on a platinum

thermometer. The consumption of fuel is 800 lbs. per hour. Calculate the volume of the furnace gas produced per hour, and find the flue area so that the velocity of flow is 20 ft. per second.

Answers.—Volume in cubic feet produced per hour, 561,000. Area, 7·8 sq. ft.

30. Find the density of air at 17° C. and at 300° C., when the barometric pressure is 14·7 lbs. per square inch.

Answers.—0·076; 0·0385.

31. Find the draught in inches of water produced by a chimney 130 ft. high when the mean temperature of the hot gas is 300° C. as measured on a platinum thermometer and the temperature of the outside air is 17° C., and the barometric pressure is 14·7 lbs. per square inch. What is the draught if the chimney temperature is reduced to 200° C.?

Answers.—0·93 in.; 0·73 in.

32. Find the head expressed in hot gas of a pressure difference due to a temperature of 300° C. at the base of a chimney and 17° outside air, for a chimney 120 ft. high.

Answer.—117 ft.

33. Find the velocity of discharge for the data of the previous question, assuming that the velocity is 20 per cent. of the velocity due to the head of hot gas.

Answer.—17 ft. per second.

34. Calculate the discharge in pounds of gas per second per square foot of area for a chimney 100 ft. high working at a temperature corresponding to the maximum discharge, and calculate also the discharge when the temperature is reduced to 200° C. Temperature and pressure of outside air 17° C., and 14·7 lbs. per square inch respectively. Take $c = 2$.

Answers.—0·76; 0·74.

35. A chimney is 60 ft. high and 7 ft. diameter. The temperature at the base is 420° C. and the mean temperature in the chimney is 177° C., both temperatures being measured by a platinum thermometer. The temperature of the outside air is 17° C., and the barometric pressure is 14·7 lbs. per square inch. The air supply per pound of fuel burnt is 13 lbs.

Calculate from the mean chimney temperature the draught in inches of water; the density of the gas at the base of the chimney; the head equivalent to the draught expressed in terms of the mean temperature in the chimney, the velocity of flow into the chimney, using a constant $c = 2$; the total weight of gas discharged per hour; the weight of fuel which can be burnt per hour; the horse-power of the plant, allowing 2 lbs. of fuel per indicated horse power hour.

Answers.—0·31 in.; 0·0318 lb. per cub. ft.; 33 ft.; 11·5 ft. per second; 51,000 lbs.; 3640 lbs. per hour; 1820 I.H.P.

36. The draught observed at the base of a chimney 57 ft. high is 0·22 in. of water when the outside air is 17° C. and the barometric pressure 14·7 lbs. per square inch. Calculate the mean temperature of the

furnace gas in the chimney which will produce this draught, and calculate the corresponding head of hot furnace gas. The chimney is 7 ft. diameter, and the discharge into it is 60,600 lbs. of furnace gas per hour. Find what fraction of the head due to the mean temperature is utilized to produce the velocity of flow, assuming that the temperature at the base of the chimney is equal to the mean temperature.

Answers.—Mean temperature, abs. 395° C.
Head, 20·6 ft.
$\frac{1}{21}$ of the head.

37. Find the diameter of a round chimney for a steam plant which is required to develop 2000 H.P., allowing a total coal consumption of 5 lbs. per horse-power hour, with the production of 20 lbs. of furnace gas per pound of coal burnt. The chimney to be 175 ft. high.

Answer.—Area, F = 55 sq. ft.; diameter, 8·36 ft.; or allowing for the sooty surface, 8·7 ft.

38. Calculate the size of the orifice of a blast pipe so that an engine may work continuously at 600 I.H.P., having given that the velocity of flow through the orifice is to be about 1000 ft. per second.

Answer.—4·45 ins. diameter.

39. The water equivalent of a coal calorimeter is 227 grams, and it is immersed in 1400 c.c. of water. By burning 1·274 grams of dry coal in a steady stream of oxygen and allowing the products of combustion to bubble through the water, the temperature of the water is raised from 12·6° to 19° C. Calculate the calorific value of the dry coal.

Answer.—8170 lb.-cals. per pound.

40. The following observations were taken from a gas calorimeter. Weight of water flowing through the calorimeter in 5 minutes, 8·74 lbs.; inlet temperature of the water, 8·6° C.; outlet temperature, 27·9° C. Gas burnt in 5 minutes, 0·6 cub. ft. Pressure of the gas above the atmosphere, 1·75 ins. of water; and the temperature of the gas flowing into the calorimeter, 11·6° C. Barometer, 30·7 ins. of mercury. Water condensed from the products of combustion, 13·5 c.c. at a temperature of 16·2° C.

Calculate the higher and the lower calorific values of the gas per cubic foot, as measured, and find also these values per cubic foot at normal temperature and pressure (0° C. and 30 ins. of mercury).

Answers.—
Higher value as measured, 281 lb.-cals. per cub. ft.
Lower ,, ,, 250 ,, ,,
At normal temperature and pressure the values are: Higher, 285; and Lower, 254 lb.-cals. per cub. ft.

41. Calculate from the following coal analysis:—
(a) The theoretical air supply per pound of coal;
(b) The weight of the products formed by combustion of a pound of coal;

(c) The higher and the lower calorific values of the coal.

Carbon	86·80	per cent. by weight
Hydrogen	4·25	,, ,,
Sulphur	0·83	,, ,,
Oxygen	3·06	,, ,,
Ash	5·06	,, ,,
	100·00	

Answers.—(a) 11·48 lbs.
(b) 12·43 ,,
(c) 8360 and 8150 respectively.

42. What is the available calorific value of the coal the analysis of which is given in the previous question, when it is burned in the furnace of a steam boiler in which the pressure is 200 lbs. per square inch absolute? If air is supplied to the furnace at the rate of 20 lbs. per pound of coal burnt, what per cent. of the available calorific value is used in heating the excess air? Air temperature, 18° C.

Answers.—Available calorific value, 7600 lb.-cals. per pound.
4·8 per cent.

43. Calculate from the following oil analysis:—

(a) the theoretical air supply per pound of oil;

(b) the weight of the products formed by the combustion of a pound of the coal;

(c) The higher and the lower calorific values of the oil.

Carbon	88	per cent. by weight
Hydrogen	10·75	,, ,,
Oxygen	1·25	,, ,,
	100·00	

Answers.—(a) 13·9 lbs.
(b) 14·9 ,,
(c) 10,770 and 10,190 respectively.

44. What is the available calorific value of the oil the analysis of which is given in the previous question, when it is burned in the furnace of a steam boiler in which the pressure is 200 lbs. per square inch absolute?

If air is supplied to the furnace at the rate of 20 lbs. per pound of oil burnt, what per cent. of the available calorific value is used for the heating of the excess air? Air temperature, 18° C.

Answers.—Available calorific value, 9565 lb.-cals. per pound.
2·7 per cent.

45. The following is a percentage analysis by volume of dry flue gas:—

Carbon dioxide	11·9	per cent.
Carbon monoxide	0·1	,, ,,
Oxygen	6·2	,, ,,
Nitrogen	81·8	,, ,,
	100·0	

Calculate the air drawn through the furnace per pound of coal fired: (*a*) assuming that no analysis of the fuel is available; (*b*) assuming that the analysis of the fuel is that which is given in question 41 above.

Answers.—(*a*) 19·9 lbs.
(*b*) 17·5 ,,

46. At what temperature will 10 lbs. of air at 25 lbs. per square inch pressure fill a volume of 100 cub. ft.? Air constant $c = 96$.

Answer.—102° C.

47. What weight of hydrogen at a pressure of 500 lbs. per square inch will fill a glass bottle 3·5 cub. ft. capacity, and at a temperature of 15° C.? The constant c for hydrogen is 1382 with pound and foot units.

Answer.—0·634 lb.

48. Three cubic feet of air at 200 lbs. per square inch pressure and 175° C. expand at constant pressure to a volume of 9 cub. ft. Find the temperature at the end of expansion; the heat absorbed during expansion; the work done during expansion; and the change of internal energy. The air constant is 96. The specific heat at constant volume is 0·17.

Answers.—1071° C.; 429 lb.-cals.; 172,800 ft.-lbs.; 306 lb.-cals.

49. One pound of air at a pressure of 15 lbs. per square inch and at 15° C. temperature is compressed adiabatically until its pressure is 400 lbs. per square inch. Find what is then its volume; its temperature; and calculate the work done during compression, and the ratio of compression.

Answers.—1·226 cub. ft.; 462° C., 107,250 ft.-lbs.; ratio of compression, 10·4.

50. The characteristic equation of a gas is $PV = 100T$, where T is the absolute temperature, and P and V are in pounds and feet units. One pound of the gas at a pressure of 60 lbs. per square inch expands from a volume of 5 cub. ft. to a volume of 20 cub. ft. Find the flow of heat between the gas and the jacket when the expansion takes place—

(1) Isothermally;
(2) Along the curve $PV^{1\cdot2}$ = a constant;
(3) Along the curve $PV^{1\cdot5}$ = a constant.

The specific heat of the gas at constant volume is 0·17.

Find also the index for adiabatic expansion, and calculate the final adiabatic pressure and temperature, and also the work done during adiabatic expansion from 5 to 20 cub. ft.

Answers.—(1) 42·77 lb.-cals. from the jacket.
(2) 19·51 lb.-cals. ,, ,,
(3) 5·86 lb.-cals. to the jacket.
Index is 1·42.
Final pressure, 8·39 lbs. per square inch.
Final temperature, 241·6° C. absolute.
Work done, 32·4 lb.-cals.

51. A double-acting air compressor compresses from the atmospheric pressure of 15 lbs. per square inch to 60 lbs. per square inch, at which pressure the delivery valve lifts and the air is discharged into the reservoir. The volume swept out by the piston is 10 cub. ft., and the clearance volume is at each end 3 per cent. of this. The characteristic equation for air is $PV = 96T$; and $K_v = 0{\cdot}17$.

Assuming that the expansion and compression curves in the cycle of operations follow the law $PV^{1{\cdot}3}$ = a constant, calculate—

	Answers.
(1) The volume of air drawn in per stroke.	9·43 cub. ft.
(2) The work done on the piston per cycle.	33,340 ft.-lbs.
(3) The horse-power at 80 cycles per minute.	161·6 H.P.
(4) The temperature of the air at the end of compression assuming a suction temperature of 15° C.	124° C.
(5) The heat given to the jacket water per stroke from the beginning of the compression to the opening of the delivery valve.	5·0 lb.-cals.
(6) The weight of air drawn per suction stroke.	0·736 lb.

52. Half a pound of air passes through a Carnot cycle between the temperatures of 205·1 and 100·58, and in which the ratio of isothermal expansion is three, and the initial pressure of the air is 300 lbs. per square inch. Draw the indicator diagram for the cycle and calculate the thermal efficiency.

Answer.—Efficiency = 0·22.

53. One pound of steam at an initial pressure of 250 lbs. per square inch passes through a Carnot cycle in which the lower temperature is 15 lbs. per square inch. Draw the indicator diagram for the cycle, using the equation $PV^{1{\cdot}135}$ = a constant with which to draw the adiabatic curves. Calculate the thermal efficiency.

Answer.—Efficiency = 0·22.

54. Let a perfect gas expand from an initial state A along an arbitrary expansion curve to the final state B. Show that the flow of heat between the gas and the jacket may be found in the following way:—

> Through the initial point A draw an adiabatic expansion curve to cut the line of constant volume through B in the point C. Then the heat-flow is the algebraical sum of the heat corresponding to the work area ABC and the product found by multiplying the specific heat at constant volume into the difference of temperature between B and C.

Verify the results (1), (2), (3) of question 50 by this method.

The work area is positive when the adiabatic AC falls below the path AB, and the product is positive when the temperature at B is greater than the temperature at C.

55. Let a perfect gas expand from an initial state A along an arbitrary expansion curve to the state B on the PV diagram. Draw an adiabatic through B to cut an isothermal through A in the point C. Draw ordinates Aa, Bb, Cc. Show that the change of internal energy is represented by the area $bBCc$.

(*Proof.*—AC is a line of constant internal energy for a perfect gas. Therefore the internal energies at the state points A and C are the same. Therefore the internal energy in the state A is represented by the indefinitely prolonged area enclosed by the adiabatic through C, the ordinate C*c*, and the volume axis. Similarly the internal energy in the state B is the area enclosed by the adiabatic through B, the ordinate B*b*, and the volume axis. But the adiabatic through B passes through C and therefore the difference between the internal energies is represented by the area *b*BC*c*.)

56. Draw an isothermal curve for 1 lb. of air at the temperature 300° C. Through a point on it corresponding to 1 cub. ft. draw an adiabatic curve. Draw a second adiabatic curve through the point on the isothermal corresponding to 2 cub. ft. Calculate the ratio $\frac{Q}{T}$ between the two curves, and then draw a third adiabatic at such a distance from the second that $\frac{Q}{T}$ is the same, and find the volume at which it cuts the original isothermal.

Sketch in a second isothermal corresponding to 100° and verify that $\frac{Q}{T}$ along this isothermal between the three adiabatics is constant and equal to the value along the 300° isothermal.

Answer.—$\frac{Q}{T} = 0{\cdot}0476$.

57. Using the steam table, plot a part of the pressure temperature curve about the pressure 200 lbs. per square inch, and deduce from it the value of $\frac{dp}{dT}$ where T is the absolute temperature. Use this and the tables to calculate the volume of 1 lb. of dry saturated steam at 200 lbs. per square inch pressure by means of Clapeyron's equation (Eq. (2) page 156).

Calculate the volume also by means of the Callendar characteristic equation (Eq. (3), page 172) for steam, using the saturation temperature corresponding to the pressure given in Steam Table 1.

58. Calculate, by the Callendar characteristic equation the volume of 1 lb. of steam at a pressure of 200 lbs. per square inch superheated to 400° C. (Eq. (3), page 172).

Answer.—3·56 cub. ft.

59. Calculate the specific heat at constant pressure, and at constant volume, of steam at 200 lbs. per square inch and 400° temperature (Eqs. (2), (3), page 174).

Answers.—$K_p = 0{\cdot}503$; $K_v = 0{\cdot}038$.

60. Calculate the total energy of 1 lb. of dry saturated steam at 200 lbs. per square inch, and also the total energy when superheated at constant pressure to 400° C. (Eq. (5), page 175).

Answers.—$I_s = 669{\cdot}7$; $I = 780{\cdot}2$.

61. Show that at 400° C. the total energy decreases at the rate of 0·0246 lb. cal. per pound per square inch increase of pressure.

62. Calculate the saturation pressure corresponding to 250° C. (Eq. (21), page 180).

63. Find by the aid of the tables, the total energy; the volume; the external work; the internal energy; and the entropy of 1 lb. of wet steam with a dryness fraction of 0·9 when formed at a pressure of 200 lbs. per square inch.

Answers.—622·47 lb.-cals.; 2·088 cub. ft.; 42·65 lb.-cals.; 579·82 lb.-cals.; $\phi = 1{\cdot}453$.

64. Calculate the heat which must be added per pound to steam formed at 240 lbs. pressure per square inch in order to superheat it to 400° C. Calculate also the internal energy of the steam in the superheated condition, and the increase of the internal energy produced by superheating. (Vol. after superheat, by Eq. (8), page 176 = 2·96 cub. ft. Total external work, 72·60 lb-cals.)

Answers.—107·61 lb.-cals; 706·64 lb.-cals.; 82·76 lb.-cals.

65. Calculate the entropy of 1 lb. of steam formed at 200 lbs. pressure per square inch and superheated to 400° C., having given that the entropy at the saturation temperature is 1·554.

(Form an equation from (22), page 181, by taking the difference between the entropy required and the entropy given, thus eliminating the constant. Verify your result by recalculation from equation (23), page 181.)

Answer.—$\phi = 1{\cdot}752$.

66. Plot on a pressure-volume diagram a state point A corresponding to 1 lb. of dry saturated steam at a pressure of 150 lbs. per square inch. Choose for the scales of the diagram 1 in. to represent 1 cub. ft. and 3 ins. to represent a pressure of 100 lbs. per square inch.

Through the state point A draw an adiabatic expansion curve to cut an isothermal corresponding to 40 lbs. per square inch pressure in B, thus forming a pressure-volume diagram for a Rankine cycle between the limits of temperature corresponding to pressures of 150 lbs. and 40 lbs. per square inch.

(1) Plot the adiabatic curve from the express PV^m = a constant, calculating an appropriate value of m from equation (5), page 192.

(2) Plot the curve also by calculating first the dryness and then the volume at different points.

In both (1) and (2) calculate the volumes corresponding to the pressures 130, 110, 90, 80, 70, 60, 50, 40 lbs. per sq. in.

Having obtained the pressure-volume diagram, find the work, U, corresponding to it.

(1) By direct measurement of diagram.

(2) By integrating with P as the independent variable from the equation PV^m = a constant.

(3) Compare the values of U so found with the value found by computing the difference of the total energy at A and at B.

Compute I_B first by using the dryness fraction q (page 190), secondly, equation (3), page 191.

U = approximately 57 lb.-cals.

67. Plot on the pressure-volume diagram a state point A corresponding to 1 lb. of steam formed at 150 lbs. pressure per square inch and superheated to 400° C. Through this state point draw an adiabatic curve to cut a line of constant pressure corresponding to 40 lbs. per square inch. Find the value of U for the diagram.

Answer.—From the characteristic equation $v = 4{\cdot}75$ cub. ft.
Use the expression $p(v - 0{\cdot}016)^{1{\cdot}3}$ = a constant, to obtain points on the adiabatic curve.
U = 84·3 lb.-cals.

68. If in the previous question the expansion of the steam is carried into the wet region, find the volume at which it becomes dry and saturated. That is, find the position of the state point in the pressure-volume diagram where the expansion curve crosses the boundary curve.

(The entropy is constant during adiabatic expansion. Therefore, find the entropy in the initial condition and with it identify a line in Steam Table 1.

Calculate the entropy either from the exact equation or by the approximate method, obtaining a mean value of the specific heat by the aid of Steam Table 3.)

Answers.—Mean specific heat $\dfrac{(I' - I_s)}{(t' - t_s)} = 0{\cdot}526.$

Entropy at 400° C. and 150 lbs. pressure per square inch; 1·7836 corresponding to the line in Steam Table 1, where the pressure is 10·5 lbs. per square inch. Hence $v = 36{\cdot}67$ cub. ft.

69. By calculation or by use of a steam diagram find the efficiency of the Rankine cycle in the following conditions:—

(1) Dry saturated steam at 200 lbs. per square inch expanding to 15 lbs. per square inch.

(2) Steam formed at 200 lbs. per square inch and superheated to 350° C. expanding to 15 lbs. per square inch.

(3) Dry saturated steam at 200 lbs. per square inch expanding to 1 lb. per square inch.

(4) Steam formed at 200 lbs. per square inch, and superheated to 350° C. expanding to 1 lb. per square inch.

Answers.—(1) 18·7 per cent.
(2) 20·3 ,,
(3) 29·9 ,,
(4) 31·4 ,,

70. Steam at 200 lbs. per square inch is throttled from the respective initial states: (1) dry and saturated; (2) wet, the dryness fraction being 0·92; (3) superheated to 300° C.; to a pressure of 20 lbs. per square inch in each case.

Assuming that throttling takes place at constant total energy, determine the final states.

Answers.—(1) Superheated to 162° C.
(2) Wet. Dryness fraction 0·98.
(3) Superheated to 284° C.
(Note.—Use a chart.)

71. An engine supplied with dry saturated steam at 200 lbs. per square inch takes 18 lbs. of steam per indicated horse-power hour. The pressure in the exhaust pipe is 5 lbs. per square inch.

Find the work done per pound of steam in pound-calories and also the work U of the Rankine ideal engine of comparison between the same limits of pressure.

Answer.—78·6 lb.-cals. U = 140 lb.-cals.

72. The following measurements were made from the calibrated mean indicator diagram of a double-acting engine running at 100 revolutions per minute.

Pressure at cut-off, 180 lbs. per square inch. Volume at cut-off, 3·5 cub. ft.

Pressure and volume at a point on the compression curve, 60 lbs. per square inch and 1·2 cub. ft. respectively.

Pressure and volume at release, 46 lbs. per square inch and 14 cub. ft. respectively.

The engine was supplied with 24,396 lbs. of steam per hour.

Find the missing quantity per hour at cut-off and at release.

Answers.—Missing quantity at cut-off, 927 lbs. per hour.
Missing quantity at release, 756 lbs. per hour.

73. Calculate the weight of steam condensed at 100° C. from dry saturated steam at 160° C. in three seconds on a cylinder cover 3 ft. diameter when the difference between the temperature of the steam and the temperature of the surface of the metal is steady and is 10° C., the steam being hotter than the metal.

Answer.—0·28 lbs.

74. Calculate the heat absorbed per minute by a cylinder cover 3 ft. diameter when the condensation area measures 123 in sixtieths of a cycle-centigrade unit, assuming that the area is measured from the mean wall temperature (Eq. (2), page 236).

Answer.—530 lb.-cals. per minute.

75. Take an indicator diagram from an engine and use it to find the probable maximum weight of steam which can be condensed per minute per square foot of surface up to cut-off.

76. Take an indicator card from an engine and at the same time measure the mean temperature of the cylinder covers and from these data calculate the probable weight of steam condensed per minute up to cut-off.

77. Given that the temperature variation on the inner surface of a cylinder cover 3 ft. diameter is 10° C., calculate the quantity of heat flowing into the walls per revolution, (1) at a speed of 25 revolutions per minute; (2) at a speed of 500 revolutions per minute, and in each case calculate the depth at which the temperature variation subsides to the mean value.

The conductivity $k = 5{\cdot}4$. The specific heat of the metal $= 0{\cdot}123$, and its density $D = 36{\cdot}3$, with area in feet and depth in inches.

Answers.—Heat absorbed per revolution at 25 revolutions per minute = 27·7 lb.-cals.
Heat absorbed per revolution at 500 revolutions per minute = 6·2 lb.-cals.
Wave length at 25 revolutions per minute = 0·78 in.
Wave length at 500 revolutions per minute = 0·174 in.

78. Using the data of the previous example, calculate the heat flowing into the cover per minute.

Answers.—Heat absorbed per minute = 692 lb.-cals. at 25 revolutions per minute.
Heat absorbed per minute = 3100 lb.-cals. at 500 revolutions per minute.

79. It is observed that at a depth of 0·04 in. from the inner surface of a cylinder the range of the variation of temperature is 2·62° C. when the speed is 100 revolutions per minute. Assuming that the temperature variation on the surface is simple harmonic, calculate the range on the surface and then calculate the flow of heat per square foot per minute.

Answers.—Range on surface, 5°.
Flow of heat, 98 lb.-cals. per minute.

80. A small non-condensing engine, supplied with dry saturated steam at 100 lbs. per square inch absolute, required 39·5 lbs. per indicated horse-power hour. The steam supply was reduced to 21 lbs. per indicated horse-power hour by superheating the steam to 310° C.; and to 19 lbs. per indicated horse-power hour by superheating to 352° C. Calculate in each case the efficiency of the engine; the efficiency of the Rankine engine of comparison; and the efficiency ratio, assuming for both the actual and the Rankine engine the lower temperature of 100° C.

Answers.—Efficiencies of actual engine, 0·064; 0·105; 0·113.
Efficiencies of Rankine engine, 0·137; 0·152; 0·160.
Efficiency ratio, 0·47; 0·69; 0·71.

81. One pound of dry saturated steam expands from an initial pressure of 150 lbs. per square inch along the expansion curve $PV^{1\cdot1}$ = a constant to a pressure of 2 lbs. per square inch. Find the heat-flow between the steam and the walls of the cylinder during expansion, assuming that there is no leak.

Answer.—57 lb.-cals. flowing from the walls to the steam.

82. Find the volume of a cylinder to give 200 indicated horse-power at a speed of 150 revolutions per minute. The engine is double acting and is to work with full compression. The initial pressure of the steam in the cylinder is 175 lbs. per square inch and the back pressure is 18 lbs. per square inch and the ratio of expansion is 4. Diagram factor, 1·1; clearance factor, 0·07. Find the effective volume and the diameter, assuming a stroke of 2 ft., and find also the fraction of the stroke at which cut-off takes place.

Answers.—Volume = 2·93 cub. ft.
Effective volume, 2·72 cub. ft.
Diameter, 15·85 ins.
Cut-off, 19·4 per cent. of the stroke.

83. Calculate the volume and the effective volume of a cylinder of a double acting engine from the following data: Indicated horse-power, 1370. Speed, 60 revolutions per minute. Initial pressure, 75 lbs. per square inch. Ratio of expansion, 5. Back pressure 5 lbs. per square inch, and pressure at the end of compression 50 lbs. per square inch. Diagram factor, 1·11. Clearance factor, 0·03. Find also the diameter of the cylinder, assuming a piston rod 10 ins. diameter and a stroke 7 ft., and also calculate the fraction of the stroke at which cut-off takes place.

Answers.—Volume = 96·7 cub. ft.
Effective volume = 93·8 cub. ft.
Diameter of the cylinder, 50·5 ins.
Cut-off, 17·5 per cent. of stroke.

84. Calculate the volume, the effective volume, and the cylinder diameter with the data of the last question, except that the clearance factor is 0·15.

Answers.—Volume = 221 cub. ft.
Effective volume, 188 cub. ft.
Diameter of cylinder, 71 ins.

85. For a two-stage two-cylinder double-acting compound engine find the ratio of the cylinder volumes uncorrected for clearance and compression and then find the actual volume of the low-pressure cylinder and the volume of the high-pressure cylinder corrected for clearance and compression, and finally find the effective volume of each cylinder from these data: Indicated horse-power (total), 540. Power to be equally distributed between the two cylinders; and the drop work to be equal in each cylinder. Speed, 114 revolutions per minute. Initial pressure 225 lbs. per square inch. Back pressure, 20 lbs. per square inch + 10 lbs. for receiver drop, that is 30 lbs. per square inch. Ratio of expansion, 4·34. Diagram factor, 1·1. Clearance factor, 0·1 for each cylinder.

Full compression in high-pressure cylinder. Pressure at end of compression in low-pressure cylinder to be equal to the mean between the back pressure and the initial pressure in that cylinder.

Answers.—Volume ratio, 1 : 2·74 = H.P. : L.P.
Volume of low-pressure cylinder, 7·04 cub. ft.; high-pressure, 2·71 cub. ft.
Effective volume, low-pressure cylinder, 6·34 cub. ft.; high-pressure, 2·44 cub. ft.

86. Calculate the quantity of heat transferred from the steam to the cooling water per minute in a condenser of an engine working at 2000 I.H.P. and using 14 lbs. of steam per indicated horse-power hour, assuming that the steam has a dryness factor of 0·85 at entry to the condenser, and that condensation takes place at the constant temperature of 50° C.

Answer.—225,000 lb.-cals. per minute.

87. If the condenser in Question 86 is of the surface type, and cooling water is supplied at a temperature of 9° C. and leaves at 30° C., find the weight in pounds which must be pumped by the circulating pump through

the condenser per hour, and state also the weight of water supplied per pound of steam condensed.

Answers.—643,000 lbs. per hour.
22·8 lbs.

88. If the condenser in Question 86 is a jet condenser and water is supplied at 9° C., calculate the weight of cooling water required per hour when the hot-well temperature is 35° C.

Answer.—536,000 lbs.

89. A surface condenser is fitted with 220 tubes, each ¾ in. diameter externally and 0·05 in. thick, and the end doors are arranged to cause a double flow of the circulating water through the tubes. The distance between the tube plates is 4 ft. Find the cooling surface reckoned on the external surface of the tubes, and the effective area of the water-way through the tubes.

Answer.—172 sq. ft.; 0·254 sq. ft.

90. Steam enters the condenser of Question 89 at a temperature of 51° C., and dryness 0·9. The hot-well temperature is 48° C., and the observed vacuum is 26·9 ins. at a 30 in. barometer, when the total weight of condensed steam leaving the condenser is 1800 lb. per hour. Circulating water 10° C. at entry, and 27° C. at exit from the condenser.

Calculate—

(*a*) The weight of circulating water which must be pumped through the condenser per hour.

Answer.—54,400 lb.

(*b*) The vacuum efficiency from the hot-well temperature, and also from the temperature at entry.

Answers.—Unity; 1·02.

(*c*) The velocity of flow through the tubes.

Answer.—0·95 ft. per second.

(*d*) The weight of steam condensed per square foot of cooling surface per minute, and the cooling water supplied per pound of steam condensed.

Answers.—0·174 lb.; 30·2 lbs.

91. Calculate the vacuum efficiency of a plant in which the recorded pressure in the condenser is 1·34 lbs. per square inch at a 30 in. barometer and the hot-well temperature is 43° C.

Answer.—0·99.

92. The temperature of condensation in a condenser is 40° C. The vacuum recorded is 26·5 ins. at a 30 in. barometer. Calculate the weight of air present per cubic foot of condenser volume.

Answer.—0·003 lb. per cubic foot.

93. If heat is abstracted from condensing steam at the rate of 0·12 lb.-cals. per second per degree difference of temperature between the steam and the metal surface, calculate the extent of cooling surface

required per indicated horse-power, assuming an engine uses 14 lbs. of steam per indicated horse-power hour, if the cooling water establishes a difference of temperature of 8° between the steam and the tube surface. Temperature of steam at entrance to the condenser, 45° C. Dryness factor, 0·9. Hot-well temperature, 40° C.

Answer.—2·1.

94. Calculate the magnitude of the acceleration of the piston when the connecting rod, which is $3\frac{1}{2}$ cranks long, subtends the crank angles 0°, 60°, 90°, 150°, and 180° respectively, using an approximate expression, and taking $\omega^2 r$ equal to unity (Eq. (8), page 338).

Answers.—1·286; 0·357; – 0·298; – 0·717 and – 0·714 ft. per second per second respectively.

Verify the results by using a graphical method and plot the acceleration curve on a piston displacement base. Compare results obtained by Eq. (5), page 338.

95. Draw a right-angled triangle ABC such that AB, the base, = 5 ins. and BC, the perpendicular measured downward, measures $\frac{1}{2}$ in. AC represents the line of stroke of a crosshead coupled by a connecting rod of length 3 to a crank centred at B whose radius is 1. Taking the crank 1 in. long, find by a graphical construction the velocity ratio between the crosshead and the crank pin, and the ratio of the accelerations of the crosshead and crank pin when the connecting rod subtends an angle at B of 45° between the crank and the line AB, and the crank is above the line AB.

Answers.—Velocity ratio = 0·95.
Acceleration ratio = 0·83.

96. The reciprocating masses of a single-cylinder engine weigh 650 lbs. The difference between the forward and the back pressure of the steam in the cylinder is 90 lbs. per square inch at the instant when the connecting rod subtends an angle 120° at the crank shaft. Find the turning couple on the crank when this angle is diminishing and the speed is 240 revolutions per minute. The length of the connecting rod is four times the length of the crank; and the crank is 1·25 ft. long. The cylinder is 18 ins. diameter.

Answer.—Turning couple = 12,220 ft.-lbs.

97. Prove that, neglecting the effect of the obliquity of the connecting rod, the ratio between the fluctuation of energy ΔE, and the total work done per revolution is approximately 0·01 in the case of a double-acting steam engine in which the cranks are at right angles, and the pressure acting on the piston to turn the crank against a constant resistance is constant throughout the stroke after being corrected for back pressure and inertia of the moving parts.

98. Find the moment of inertia of a flywheel to keep the speed of an engine within 1 per cent. above or below the mean speed of 120 revolutions per minute, when the fluctuation of energy as determined from a crank-effort diagram is 20 ft.-tons.

Answer. Moment of inertia = $6·34 \frac{W}{g}$ ft.² units.

99. Find the weight of the rim of the flywheel required by the data of the previous question (that is so that its moment of inertia is 6·34), and also the mean radius, having given that the speed of the rim at the mean radius is to be 70 ft. per second. Neglect the effect of the spokes.

Answers.—Weight of rim = 6·58 tons.
Mean radius = 5·57 ft.

100. What would be the weight of the flywheel required by the data of the previous question, if the wheel were made in the form of a solid disc the peripheral speed being 70 ft. per second?

Answer.—13·16 tons.

101. The turning couple exerted on the crank shaft of an engine which is working against a constant resistance is given by

$$\text{Couple} = 9 + 3 \sin 2\theta \text{ ft.-tons}$$

Find the moment of inertia of the flywheel necessary to keep the speed within $\frac{1}{4}$ per cent. above or below the mean speed, which is 120 revolutions per minute.

Answer.—Moment of inertia $= 3{\cdot}8 \dfrac{W}{g}$ ft.^{2} units.

102. An equal-armed Porter governor of the type shown in Fig. 111 has the following dimensions:—

Each arm is 10 ins. long and is jointed to the axis of revolution at a horizontal distance of 2 ins. Each ball weighs 5 lbs. Find—

(*a*) The loading on the sleeve required to produce equilibrium at a speed of 200 revolutions per minute when the distance between the horizontal line through the sleeve joints and the horizontal line through the top joints is 16 ins.

Answer.—27·7 lbs. per ball.

(*b*) The speed of equilibrium for the configuration corresponding to a sleeve position 1 in. higher and also 1 in. lower than in (*a*).

Answer.—209 and 191 revolutions per minute.

(*c*) The restoring force in the two positions of (*b*), when the displacement is made from the position (*a*) at the constant speed of equilibrium of (*a*), namely, 200 revolutions per minute.

Answers.—Upper position, 2·5 lbs.
Lower position, 3·0 „

(*d*) The time of an oscillation about the mean position, assuming that the restoring force is 3 lbs. per inch of the sleeve displacement from the mean position.

Answer.—One second nearly.

(*e*) The number of oscillations per revolution about the mean position (*a*).

Answer.—0·3

103. A governor of the Hartnell type, Fig. 113, is required to run at a speed of 300 revolutions per minute in the mean configuration where the ball arm is vertical. The movement of the sleeve from its corresponding mean position is to be $\frac{1}{10}$ ft. for an increase of speed of 3 per cent. Each ball weighs 4 lbs. The axis of the ball crank is 4 ins. from the axis of revolution. The ball arm is 6 ins. long and the sleeve arm is $2\frac{1}{2}$ ins. long. Find—

(*a*) The strength of the spring.

Answer.—163 lbs. per inch of compression.

(*b*) The compression at the bottom stop which is reached when the sleeve moves down $\frac{1}{20}$ ft. from its mean position.

Answer.—98 lbs.

(*c*) The diameter of the rod from which the spring is coiled, assuming the mean diameter of the coils to be 3 ins. and the maximum compression 300 lbs. and allowing a maximum stress of 25 tons per square inch.

Answer.—0·35 in.

(*d*) The length of the rod required to form the spring.

Answer.—45·75 ins.

104. The governor in the previous example is to be arranged as an emergency governor so that the sleeve remains in contact with the bottom stop until the speed reaches 320 revolutions per minute, when it moves rapidly through $\frac{1}{10}$ ft. into contact with the top stop and presses against it with a force of 10 lbs. Find the strength of the spring and the compression on the bottom stop.

Answer.—Strength, 125 lbs. per inch.
Compression on bottom stop 1·145 ins. corresponding to 143 lbs.

105. A governor of the Hartnell type, in which each ball weighs 4 lbs., has a spring acting on the sleeve such that it requires 180 lbs. to compress it 1 in. The compression at the bottom stop is 1 in. The ball arm is vertical when the sleeve has moved half an inch from the bottom stop, and in this position the radius in which the mass centres of the balls revolve is 4 ins. The ball arm is 6 ins. long and the short arm $2\frac{1}{2}$ ins. long. At what speed must the governor be driven to bring the ball arm into the vertical position?

Answer.—352 revolutions per minute.

106. The single spring in the previous question is replaced by two springs applied directly to the governor balls in the way illustrated in Fig. 114. What must be the strength of the new springs and what must be their extensions at the bottom stop?

Answers.—Strength of each spring, 3·9 lbs. per inch.
Tension at lower stop, 4·8 ins.

107. A supplementary controlling lever is applied to the sleeve of the governor of Question 105 above. The dimensions are such that the fulcrum is 6 ins. from the axis of the governor and the load is applied to the

lever by a dead weight so that the static push is increased. What dead weight must be applied to the lever at 15 ins. from the fulcrum in order that the speed may be increased 10 per cent. in the configuration where the ball arm is vertical?

Answer.—22·4 lbs.

108. A train weighs 400 tons. Calculate the horse-power required to produce an acceleration of 5 miles per hour in 40 seconds, first when the train is travelling at 5 miles per hour, and secondly when it is travelling at 70 miles per hour. Allow 7 per cent. for rotating parts.

Answer.—73 H.P. at 5 miles per hour.
1018 H.P. at 70 miles per hour.

109. Using the data of the previous question calculate the horse-power required to produce the accelerations when the train is travelling up a gradient of 1 in 350.

Answers.—107 H.P. at 5 miles per hour.
1496 H.P. at 70 miles per hour.

110. Find the horse-power to maintain a speed of 50 miles per hour up a gradient of 1 in 330 with a train composed of an engine weighing 100 tons and vehicles weighing 300 tons. Calculate the vehicle resistance per ton r_v and the engine resistance per ton r_e from the following expressions, in which V is the speed in miles per hour:—

$$r_v = 3 + 0{\cdot}08V + 0{\cdot}003V^2$$
$$r_e = 9 + 0{\cdot}1V + 0{\cdot}004V^2.$$

Answer.—1262 H.P.

111. Using the resistance formulæ of the previous question, calculate the speed which an engine developing 1200 I.H.P. will maintain up a gradient of 1 in 75, with a load of 300 tons.

(N.B.—Solve the resulting cubic equation by Horner's method; by plotting; or by trial and error.)

Answer.—28·5 miles per hour.

112. It was found that after correcting the results from a dynamometer car record for acceleration and gradient the tractive resistance in pounds per ton of load behind the car at speeds of 10, 30, and 70 miles per hour were respectively 4·66, 8·34, and 21·94. With these data calculate the coefficients in the expression

$$r_v = A + BV + CV^2$$

in which V is the speed in miles per hour.

Answers.—A = 3·6.
B = 0·08.
C = 0·0026.

113. Neglecting wear and tear and the energy in the rotating parts, calculate how much it costs to stop a train weighing 400 tons from a speed of 70 miles per hour when energy is estimated to cost one penny per horse-power hour.

Answer.—3*s.* ~~3*d.*~~

114. Assume that the engine resistance in pounds per ton can be represented by the expression

$$r_e = A + BV + CV^2$$

in which V is the speed in miles per hour, and determine the constants from the following data :—

Speed in miles per hour.	I.H.P. measured at the cylinders.	Draw-bar pull.	Gradient.
10	239	5680	1 in 100 up
25	350	3094	1 in 300 up
60	970	3000	level

Weight of engine and tender, 100 tons.
Weight of train, 200 tons.

Answers.—A = 9.
B = 0·09.
C = 0·0045.

115. The mean pressure at a cut-off of 20 per cent. of the stroke in a locomotive carrying a boiler pressure of 200 lbs. per square inch is, between the ordinary limits of speed, approximately expressed by

$$p = 75 - 0·038\,s \text{ lbs. per square inch,}$$

where s is the piston speed in feet per minute. Calculate the piston speed at which, with this cut-off, the engine will develop the maximum horse-power, and calculate the maximum horse-power, assuming that there are two cylinders each 18 ins. diameter and 24 ins. stroke.

Calculate the corresponding best speeds for an engine with 7 ft. driving wheels and also for an engine with 6 ft. driving wheels.

Answers.—990 ft. per minute.
570 I.H.P.
61·8 miles per hour.
53 miles per hour.

116. The following observations were made of speed and power on a locomotive of the 4–4–0 type with a boiler pressure of 180 lbs. per square inch, and having two cylinders each 16 ins. diameter and 24 ins. stroke and driving wheels 69¼ ins. diameter. With the regulator set at a constant opening and the valve gear set to cut-off at 26 per cent. of the stroke, the total indicated horse-power at 40 miles per hour was 524, and at 20 miles per hour 334. Assuming that for these conditions of working the mean pressure can be expressed as a function of the piston speed of the form

$$p = c + bs$$

use these data to calculate the values of the constant c and b when s is expressed in feet per minute.

Answers.—$b = -0·038$.
$c = 85$.

117. The crank arms and crank pin of a crank shaft are equivalent to a mass of 700 lbs. at 1 ft. radius. The shaft is supported in two bearings, 5 ft. centre to centre, and the centre of the crank is 1·5 ft. from the left-hand bearing. The diameter of the shaft at the journals is 8 ins. Find the dynamical load on the shaft and on the bearings for a speed of 240 revolutions per minute, and calculate the rate, in horse-power, at which

work is dissipated in heat at each bearing, assuming a coefficient of friction of 0·05.

Answers.—
Dynamical load on shaft, 13,708 lbs. weight. Load on bearings: left hand, 9595·6 lbs. weight; right hand, 4112·4 lbs. weight
H.P. loss: left-hand bearing, 7·30; right hand, 3·12; total, 10·42.

118. Draw the bending moment diagram and the shearing force diagram for the shaft of the previous question due to the revolution of the unbalanced mass, assuming the shaft to be a straight one. State the numerical value of the maximum bending moment.

Answer.—Maximum bending moment, 172,720 in.-lbs.

119. Find the single mass at 3 ft. radius which will balance the mass of Question 117. Find also the magnitudes of two masses which will effect balance when they are placed in the same plane of revolution as the disturbing mass, at radii of 4 ft. and 5 ft. respectively, these radii being inclined to the radius of the disturbing mass at 160° and 220° respectively.

Answers.—(1) $233\frac{1}{3}$ lbs.

(2) $\begin{cases} \text{130 lbs. at 4 ft. radius.} \\ \text{55 lbs. at 5 ft. radius.} \end{cases}$

120. Find the two masses which will balance the mass of Question 117, when they are placed at 4 ft and 5 ft. radii respectively—

(1) In planes of revolution, the first 1 ft. to the left of the plane of the given mass, the second 2 ft. to the right.

(2) In planes distant 1 ft. and 3 ft. respectively to the right of the given mass.

Answers.—
(1) Mass in plane to the left, 116·6 lbs. at 4 ft. radius, the radius being at 180° with the radius of the given mass.
Mass in plane to the right, 46·6 lbs. at 5 ft. radius, the radius being at 180° with the radius of the given mass.
(2) Mass in plane nearer the given mass, 262·5 lbs. at 4 ft. radius, the radius being at 180° with the radius of the given mass.
Mass in further plane, 70 lbs. at 5 ft. radius, the radius being at 0° with the radius of the given mass.

121. Draw the bending moment and shearing force diagrams for the shafts of the two balanced systems of Question 120, due to the dynamical loading alone, when the speed is 250 revolutions per minute.

122. Five pulleys, equally spaced at 2 ft. apart, are keyed to a shaft which is supported on bearings 12 ft. apart. The pulleys are out of balance to the following extent:—

No. 1, 5 lbs. at 1 ft. radius.
No. 2, 6 „ 2 „
No. 3, 7 „ 1 „
No. 4, 2 „ 2 „
No. 5, 6 „ 1 „

The angles between the several mass radii and the mass radius of No. 1 pulley are respectively 45, 90, 120, and 240°. Find the two masses which will balance the system—

(1) When placed in Nos. 1 and 5 pullies at 1 ft. radius.
(2) When placed in Nos. 2 and 4 pullies at 1 ft. radius.

Answers.—

$$(1)\ \begin{Bmatrix} 3{\cdot}84_{306^\circ} \text{ in No. 5.} \\ 15{\cdot}25_{225^\circ} \text{ in No. 1.} \end{Bmatrix} \quad (2)\ \begin{Bmatrix} 9{\cdot}1_{5^\circ} \text{ in No 4.} \\ 22{\cdot}7_{220^\circ} \text{ in No. 2.} \end{Bmatrix}$$

The angles are measured from the direction of No. 1 radius.

123. Find at what speed the maximum value of the unbalanced force of a two-cylinder locomotive is 2 tons, assuming that the revolving masses are balanced and that the reciprocating masses, which weigh 600 lbs. per cylinder, are unbalanced. Diameter of wheels, 4 ft. 6 ins. Stroke 2 ft. (Use equation (1), page 512.)

Answer.—20·2 miles per hour.

124. What is the speed in Question 123, if the revolving parts, which weigh 700 lbs. per cylinder, and are at 1 ft. radius, are unbalanced as well?

Answer.—13·7 miles per hour.

125. Calculate the maximum values of the swaying couples in Questions 123 and 124, assuming the cylinders to be inside and 2 ft. centre to centre. (Eq. (2), page 512.)

Answer.—4480 ft.-lbs in each case.

126. What is the speed in Question 123, if two-thirds of the reciprocating masses are balanced and all the revolving masses?

Answer.—35 miles per hour.

127. Calculate the speed in the previous question when the diameter of the driving wheel is 7 ft.

Answer.—54·4 miles per hour.

128. Calculate the maximum values of the respective swaying couples for a speed of 60 miles per hour and their respective periodic times, for the following engines, in each of which the revolving masses are balanced and the mass of the reciprocating parts per cylinder is 600 lbs., two-thirds of which is balanced. Stroke, 2 ft. (Eq. (2), page 512.)

(1) A 7-ft. inside single, cylinder 2 ft. pitch.
(2) A 7-ft. outside single, cylinders 6 ft. pitch.
(3) An 8-ft. outside single, cylinders 6 ft. pitch.
(4) A 5-ft. inside cylinder tank, cylinders 2 ft. pitch.

Answers.—(1)	5460 ft.-lbs.	0·25 second.
(2)	16,380 „	0·25 „
(3)	12,500 „	0·286 „
(4)	10,670 „	0·179 „

129. Two engines are built with similar sets of reciprocating parts, one as a 7 ft. outside single in which the cylinders are 6 ft. pitch, the other as an inside cylinder tank engine in which the cylinders are 2 ft. pitch. The revolving masses and two-thirds of the reciprocating masses are

balanced in each case. For what diameter of the driving-wheels would the swaying couple acting on the tank engine be equal to that acting on the single engine, when both engines are running at the same speed?

Answer.—4 ft. diameter approx.

130. Assuming that the tractive force exerted by an engine varies inversely as the speed, and that the tractive force is 2 tons when the speed is 30 miles per hour, and also that there are 200 lbs. reciprocating mass unbalanced per cylinder, find the speed at which the maximum value of the unbalanced force becomes equal to the average tractive force. Wheels, 7 ft. diameter. Stroke, 2 ft. (Unbalanced force, see Eq. (1), page 512.)

Answer.—44·6 miles per hour.

131. Find the balance weights for the inside single engine specified by the following data:—

Stroke, 26 ins. Cranks at right angles, left-hand crank leading.

All the revolving and two-thirds of the reciprocating masses are to be balanced.

Distance centre to centre of the cylinders	2 ft.	4 ins.
Distance between the planes containing the mass centres of the balance weights	4 ,,	$11\frac{3}{4}$,,
Mass of the reciprocating parts per cylinder	612	lbs.
Mass of the revolving parts per cylinder	720	,,

Answer.—

Left-hand wheel.—880 lbs. at 13 ins. radius at an angle of 160° measured from the left-hand crank, counter-clockwise, when facing the left-hand wheel.

132. Find the balance weights for the inside-cylinder four-coupled engine specified by the following data:—

Stroke, 24 ins. Inside cranks at right angles, right-hand crank leading. Outside cranks, 11 ins. radius, placed oppositely to the corresponding inside cranks. All the revolving and two-thirds of the reciprocating masses to be balanced. The mass of each coupling-rod to be divided equally between the driving and trailing wheels. The balancing mass for the reciprocating parts to be divided equally between the driving and trailing wheels.

Distance centre to centre of cylinders	2 ft.		
Distance between the planes of motion of the wheel-cranks	5·166 ft.		
Distance between the planes of motion of the coupling-rods	6·27 ,,		
Distance between the planes containing the mass centres of the balance weights	4·94 ,,		
Mass of reciprocating parts per cylinder	642	lbs. at	12 ins.
Inside revolving mass per cylinder	723	,,	12 ,,
Mass of each coupling-rod	224·5	,,	11 ,,
Mass of each wheel crank in driving and trailing wheels	117·6	,,	11 ,,
Mass of part of crank pin outside crank, together with the pin and washer for each outside crank	38·2	,,	

Answer.—

Left-hand driving-wheel.—493 lbs. at 12 ins. radius at an angle of 37°, measured from the outside crank, counter-clockwise, when facing left-hand wheel.

Trailing-wheel.—145 lbs. at 12 ins. radius at an angle of 144°, measured from the outside crank, counter-clockwise when facing the left-hand wheel.

133. Find the balance weights for the engine of the previous question when the two-thirds of the reciprocating masses are balanced entirely in the driving-wheels.

Answer.—

Left-hand driving-wheel.—651 lbs. at 12 ins. radius at an angle of 34° measured from the outside crank, counter-clockwise, when facing the left-hand wheel.

Trailing-wheel.—268 lbs. at 12 ins. radius at an angle of 175½°, measured from the outside crank, counter-clockwise, when facing the left-hand wheel.

134. Calculate the maximum value of the hammer-blow for the driving-wheel in the two preceding examples when the crank-shaft is making four turns per second (corresponding to 60 miles per hour with a 7-ft. wheel), and hence find the maximum and minimum load on the rail, supposing the static load to be 7½ tons. (Eqs. (3) and (4), page 512.)

Answers.—

For example 132, where one-third is balanced in the driving-wheel, hammer blow = 3185 lbs. weight.

Maximum load on the rail, 8·92 tons weight; minimum, 6·08 tons weight.

For example 133, where two-thirds is balanced in the driving-wheel, hammer-blow = 6370 lbs. weight.

Maximum load on the rail, 10·34 tons weight; minimum, 4·66 tons weight.

135. Find the maximum values of the unbalanced force and the unbalanced couple in terms of the revolutions per second, due to the reciprocating masses of a four-crank engine in which the cylinder pitches, reckoning from the left, are 10 ft., 12 ft., and 10 ft. respectively, the corresponding masses, reckoning from the left, being 2, 3, 4, and 2 tons, and in which the crank angles are, reckoning from the left, between cranks 1 and 2, 90°; between cranks 2 and 3, 90°; between cranks 3 and 4, 90°. Stroke, 4 ft.

Answer.—

Unbalanced force, $176{\cdot}5\,\dfrac{n^2}{g}$ lbs. weight.

Unbalanced couple in plane of reciprocation about an axis at the centre of the engine, $5920\,\dfrac{n^2}{g}$ ft.-lbs.

136. Find the unbalanced force and couple using the data of the previous question when the sequence of cranks is changed to the following,

reckoning from the left: angles between cranks Nos. 1 and 2, 180°; between cranks 2 and 3, 90°; between cranks 3 and 4, 180°.

Answer.—Unbalanced force, $176{\cdot}5\,\dfrac{n^2}{g}$ lbs. weight.

Unbalanced couple at the centre, $1275\,\dfrac{n^2}{g}$ ft.-lbs.

137. Reckoning from the left in order, let the letters A, B, C, D denote the cylinders of a four-crank engine. The distances between them are, 5 ft. between A and B, 8 ft. between B and C, 6 ft. between C and D. The revolving masses corresponding to A, B, C, D, are respectively 1, $1\frac{1}{2}$, $1\frac{1}{4}$, and 1 ton at crank radius. Given that the angle between the cranks of cylinders B and C is 105°, and that the reciprocating masses of cylinders B and C are respectively $2\frac{1}{4}$ and 2 tons, find the remaining crank angles and masses so that the reciprocating parts may be in balance amongst themselves, neglecting the obliquity of the connecting-rod. Find also the masses which must be added to the crank shaft at cranks A and B to balance it.

Answers.—

Angle between cranks C and A, $97\frac{1}{2}$° }
" " " A " D, 58° } measured in order.
" " " D " B, $99\frac{1}{2}$ }

Reciprocating mass at A, 1·615 tons.
" " D, 1·343 "

Revolving mass attached to crank shaft at A, $0{\cdot}07_{184°}$ tons.
" " " at D, $0{\cdot}158_{92°}$ "

The subscript directions being measured from crank B towards crank C.

138. Taking the data of the previous example, find the remaining crank angles and reciprocating masses, including the revolving masses with the reciprocating, so that the reciprocating and revolving masses are together in balance in the plane of reciprocation, but the revolving masses are unbalanced in the plane at right angles to it.

Answer.—

Angle between cranks C and A, $96\frac{1}{3}$ }
" " " A " D, $57\frac{1}{2}$° } measuring in order.
" " " D " B, $101\frac{1}{6}$° }

Reciprocating mass at A, 1·69 tons.
" " at D, 1·19 "

139. Taking the data of question 137, balance the reciprocating masses amongst themselves, and find what the corresponding revolving masses should be so that they may be in balance without the addition of balance weights, having given that the revolving mass at A is 1 ton.

Answer.—Revolving masses must be in the same ratio as the reciprocating masses.

Revolving mass at B, 1·394 tons.
" " at C, 1·238 "
" " at D, 0·833 "

140. Prove that in a three-cylinder locomotive, with cranks at 120° and with reciprocating masses of equal magnitude, the magnitude of the balance weight required in each driving wheel is given by the expression

$$M_1 = 0{\cdot}35\frac{r}{r_1}MD \text{ lbs.}$$

where M is the mass of the reciprocating parts per cylinder, D is the horizontal pitch of the three cylinders, r is the crank radius and r_1 is the radius of the mass centre of the balance weight. Distance between the planes in which the mass centres of the balance weights revolve, 59 ins.

141. Prove that if in a three-cylinder locomotive the outer cranks are at 90° with each other and the middle crank bisects the obtuse angle between them: and that if each outside reciprocating mass is $\frac{1}{\sqrt{2}}$ times the middle reciprocating mass then the balance weight required in each driving wheel is given by the expression

$$M_1 = 0{\cdot}2\frac{r}{r_1}MD \text{ lbs.}$$

where M is the mass of the middle set of reciprocating parts; r and r_1 and D have the same meaning as in the previous question, and the planes of the balance weights are 59 ins. apart.

142. Prove that in a four-cylinder locomotive with the cranks arranged in two 180° pairs at right angles, and with reciprocating masses of equal magnitude, the magnitude of the balance weight required in each driving wheel is

$$M_1 = 0{\cdot}29\frac{r}{r_1}MD \text{ lbs.}$$

where M is the mass in pounds of each set of reciprocating parts, r and r_1 are respectively the crank radius and the radius of the mass centre of the balance weights, and D is the horizontal distance between the cranks forming a 180° pair, and this distance is equal in each of the pairs. Distance between the planes in which the mass centre of the balance weights revolve, 59 ins.

143. The cylinders of a four-crank engine are arranged symmetrically. The pitch of the outer pair is 35 ft., and of the inner pair 15 ft. The mass of each set of reciprocating parts belonging to the outer cylinders is 6 tons. Find the crank angles and the inner masses, so that the reciprocating masses may be in balance for primary and secondary forces, and primary couples.

Answer.—

Let A, B, C, D indicate the cylinders taken in order.
Angle between cranks A and D, 61° 42′
" " " B " C, 108° 48′
Reciprocating mass corresponding to cylinders B and C, 8·844 tons per cylinder.

144. Details of the valve gear belonging to the engine of the previous question are given below:—

	Distance from crank A.	Mass at crank radius.
	Feet.	Tons.
Ahead sheave	− 5·0	0·6
Astern sheave	− 4·3	0·1
Crank A	0·0	
Crank B	10·0	8·844
Ahead sheave	14·3	0·6
Astern sheave	15·0	0·1
Astern sheave	20·0	0·1
Ahead sheave	20·7	0·6
Crank C	25·0	8·844
Crank D	35·0	
Astern sheave	39·3	0·1
Ahead sheave	40·0	0·6

Assuming that the angle between cranks B and C is 108° 48′, and that the corresponding masses are 8·844 tons per cylinder, as found in the previous question, find the remaining angles and the masses corresponding to cylinders A and D, including the effect of the valve gear.

(Take a reference plane at crank A and include the couples belonging to the valve-gear of crank A, by assuming that the direction of crank A is that found in the previous question.)

Answer.—
Angle between cranks A and D, 63° 53′
„ „ „ A „ C, 95° 56′
„ „ „ B „ D, 91° 23′
Mass at A, 6·3 tons.
„ at D, 6·0 „

145. The dimensions of the planed surface in which the steam ports are formed, and across which a slide valve moves, are $11\frac{3}{4}$ ins. in the direction of motion of the valve and 18 ins. in the direction at right angles, that is in the direction of the length of the steam ports, which are all 16 ins. long.

The steam ports, each $1\frac{1}{2}$ ins. wide and $7\frac{1}{2}$ ins. apart centre to centre, lie symmetrically on either side of the centrally placed exhaust port, which is $3\frac{1}{4}$ ins. wide.

The total width of the slide valve is in the direction of sliding $11\frac{1}{2}$ ins. and at right angles 18 ins., and the centrally placed exhaust cavity is 6 ins. wide.

Calculate the steam lap and the exhaust lap and sketch the valve in the series of positions corresponding to admission, cut-off, release, and compression for the two cycles, and mark on each sketch the displacement of the valve from its central position, its direction of motion, and the particular event its position determines together with the cycle to which the event belongs.

Answer.—Steam lap, 1·125 ins.
Exhaust lap, nil.

146. Using the data of Example 145 but increasing the distance apart of the steam ports to 10 ins. centre to centre, find the overall width of the slide valve and the width of the central cavity so that the steam lap and the exhaust lap are the same as in Question 145 with inside steam admission.

Answer.—Width of valve, $11\frac{1}{2}$ ins.
Width of exhaust cavity, $6\frac{1}{4}$ ins.

Sketch the valve in the series of eight positions corresponding to those specified in Exercise 145, page 723.

147. Using the dimension of the valve given in Exercise 145 together with the following dimensions of a steam passage as in the Trick valve, namely, width of each steam port $\frac{1}{2}$ in. and distance apart of the ports in the face of the valve, $9\frac{3}{4}$ ins. centre to centre, calculate the width of the planed surface along which the valve slides.

Answer.—Width, $12\frac{1}{2}$ ins.

148. The slide valve of Example 145, page 723, is coupled to an eccentric sheave of $2\frac{1}{4}$ ins. radius. What will be the maximum opening of the port for steam, and the maximum opening to the exhaust ?

Answer.—$1\frac{1}{8}$ in. for steam.
$1\frac{1}{2}$ in. to exhaust.

149. Using the dimensions given in Exercise 145 calculate the total pressure between the rubbing surfaces of the valve and cylinder just at cut-off when the pressure in the steam chest is 200 lbs. per square inch and the pressures on the steam and exhaust sides of the piston are respectively 180 lbs. and 20 lbs. per square inch.

Assuming a coefficient of friction of 0·1, calculate the horizontal resistance to sliding. Calculate the corresponding horizontal resistance when a relief ring puts 60 per cent. of the area of the back of the valve into communication with the exhaust cavity.

Answer.—Total pressure, 30,990 lbs.
Horizontal resistance, 3099 lbs.
With the relief ring, 912 lbs.

150. Given a slide valve with 1·125 ins. steam lap and no exhaust lap, find the radius of the eccentric sheave and the angular advance at which it must be set so that for the instroke cycle the valve cuts off the steam supply to the cylinder at 75 per cent. of the stroke. The lead is 0·125 in. and the connecting rod is five times the length of the crank. Neglect the obliquity of the eccentric rod.

Answer.—Radius of sheave, 2·55 ins.
Angular advance, $\phi = 119°$.

Find the percentage fractions of the stroke at which the eight events

of the double cycle take place, assuming that the valve is set with equa leads, and find the maximum port openings for steam.

Answer.—

	Instroke cycle.	Outstroke cycle.
Admission	99·9	99·9
Cut-off	75	81·9
Release	92·2	95
Compression	95	92·2

Maximum opening for steam, 1·43 ins.

151. How much would the lap have to be increased at the left edge of the valve in order that cut-off might take place at 75 per cent. of the stroke in the outstroke cycle, and could this increase be added to the valve? What is the greatest percentage correction which could be made to allow $\frac{1}{32}$ lead?

Answer.—0·38 in. No, because there would then be no lead. About $1\frac{3}{4}$ per cent.

152. Given that the angular advance is 120° and that the steam lap in the outstroke cycle is $\frac{15}{16}$ in. and the lead $\frac{1}{16}$ in., find by the Reuleaux diagram the steam lap for the instroke cycle in order that cut-off may take place at equal fractions of the stroke in each cycle. State what the fraction of the stroke is, and the lead necessary in the instroke cycle. The connecting rod is five cranks long.

Answer.—$\frac{5}{8}$ in.; $80\frac{1}{2}$ per cent.; $\frac{3}{8}$ in.

153. Find the steam lap, the radius of the eccentric sheave, and the angular advance so that cut-off takes place at 70 per cent. of the stroke for the instroke cycle, when the lead and maximum port opening for steam are respectively $\frac{1}{8}$ in. and $1\frac{1}{4}$ ins. Neglect the obliquity of the eccentric rod. The connecting rod is five cranks long. (Use the Bilgram diagram.)

Answers.—Lap, 1·17 ins.
Radius of eccentric sheave, 2·42 ins.
Angular advance, $\phi = 122\frac{1}{2}°$.

154. Draw the rectangular valve diagram with the data corresponding to the previous question. Plot the valve displacement curve separately on a sheet of tracing paper, and place it over the drawing of the piston displacement curve so that the axes of both curves coincide. Then study the effect on the distribution by changing the angular advance which is done by moving the tracing over the drawing in the direction of their common axes.

155. A piston valve distributes steam to a cylinder by inside admission. Cut-off is to take place at 70 per cent. of the stroke on the instroke cycle. The maximum opening of the steam port is to be 2 ins. and the lead is to be $\frac{1}{4}$ in. Release is to take place at 97 per cent. of the stroke. Find the steam and exhaust laps, the radius of the eccentric sheave, and the

angular advance, assuming that the length of the connecting rod is five times the length of the crank.

Answers.—Steam lap, 1·83 ins.
Exhaust lap, 0·96 in.
Eccentricity, 3·83 ins.
Angular advance, 303° or 57° angular lag.

Sketch the valve in its central position over the ports, using the following dimensions. Steam ports 3 ins. wide. Distance between the inner edges of the steam ports, 15 ins.

156. Set out a rectangular valve diagram for a valve gear of the Meyer type with the following data:—

Main valve:
Eccentricity of sheave, 1·8 ins.
Steam lap, 0·96 in. for each cycle.
Angular advance, 130°.

Expansion valve:
Eccentricity of sheave, 1·8 ins.
Angular advance, 180°.

Find from the diagram the laps required on the expansion valve to equalize the cut-off at 25 per cent. of the stroke in each cycle, and the exhaust laps on the main valve to equalize the compression at 80 per cent. of the stroke in each cycle.

The connecting rod is 5·5 cranks long.

Answers.—

	Instroke cycle.	Outstroke cycle.
Lap on expansion valve . . .	− 0·97 in.	− 0·78 in.
Exhaust lap on main valve . .	0·55 in.	0·29 in.

157. Draw on a piece of tracing paper a series of valve displacement curves for different eccentric radii on a crank base equal in length to that used in the previous exercise, and place it over the drawing of the displacement curves of the piston and the main valve so that the axes coincide. The effect of changing the angular advance of the expansion valve may be studied by moving the tracing paper along the axis, and the effect of changing the travel of the valve may be studied by means of the series of displacement curves on the tracing.

158. Using the data of Exercise 156 find the eccentricity and the angular advance of the imaginary eccentric sheave which gives the motion of the cut-off valve relative to the main valve. Use the imaginary eccentric with a Reuleaux diagram in order to find the laps to which the expansion valve must be set to equalize the cut-off in the two cycles at 10, 20, 30, 40, and 50 per cent. of the stroke.

Answers.—

	Instroke cycle.	Outstroke cycle.
10 per cent.	0·40 in. lap.	0·24 in. lap.
20 ,, ...	0·82 in. ,,	0·62 in. ,,
30 ,, ...	1·10 ins. ,,	0·91 in. ,,
40 ,, ...	1·30 ins. ,,	1·14 ins. ,,
50 ,, ...	1·42 ins. ,,	1·32 ins. ,,

Eccentricity, 1·52.
Angular advance, 245°.

159. Show that if n is the ratio between the length of the connecting rod and the length of the crank; R, the crank radius, and X, the displacement of the piston from its central position when the crank angle is θ, then,

$$\cos\theta = n - \sqrt{(n^2+1) - \frac{2n\mathrm{X}}{\mathrm{R}}}$$

is approximately true. Apply the expression to find the value of θ when $n = 4$, $\mathrm{R} = 1$, and $\mathrm{X} = -0{\cdot}73$.

Answer.—141°.

160. The equation to a valve displacement curve is

$$x = 3\cos(\theta + 120)$$

The equation to the corresponding piston curve is

$$\cos\theta = 4 - \sqrt{17 + 8\frac{\mathrm{X}}{\mathrm{R}}}$$

The steam lap is 1·375 ins. and the exhaust lap 0·1 in.

Calculate the fractions of the stroke at which cut-off, release, compression, and admission occur in the instroke cycle, and find the lead.

Answers.—Admission, 99·9 per cent.
Cut-off, 72·8 per cent.
Release, 92·7 per cent.
Compression, 94·1 per cent.
Lead, 0·125 in.

(Calculate the angles by the first expression, putting x = steam lap, for cut-off and admission, and x = exhaust lap for release and compression, and then use these angles in the second expression to find X. By rough plotting or by examining the sign of the differential coefficient of the first expression it may be found if x is increasing or decreasing.)

161. Using the data and results of the previous exercise, calculate the velocity of the valve at cut-off, assuming that the crank shaft is making three turns per second.

Answer.—4·2 ft. per second.

162. If r and r_1 are the respective radii of the eccentric sheaves working the main and the cut-off valves in a Meyer gear, and if ϕ and ϕ_1 are the corresponding angular advances, show that the displacement of the expansion valve from its central position relative to the main valve is given by the expression—

$$\text{Displacement} = \rho\cos(\theta + \psi)$$

where $\rho = \sqrt{\mathrm{C}^2 + \mathrm{D}^2}$

and $\tan\psi = \dfrac{\mathrm{D}}{\mathrm{C}}$

where $\mathrm{C} = r_1\cos\phi_1 - r\cos\phi$

and $\mathrm{D} = r_1\sin\phi_1 - r\sin\phi$

163. In a link motion of the Stephenson type the length of each eccentric rod is 60 ins.; the length of the link centre to centre of the pins jointing it to the eccentric rods is 20 ins.; the angular advance of

each eccentric sheave is $104\frac{1}{2}^\circ$, and the eccentric radius of each sheave is 3·48 ins. Calculate the equation giving the valve displacement when u, the distance of the motion block from the centre of the link, is 5·45 ins. for each of the following cases. (Eqs. (5), (6), (7), page 715.)

(1) Outside admission. Open rods.

Answer.—$x = -1{\cdot}26 \cos\theta - 1{\cdot}84 \sin\theta$.

(2) Inside admission. Crossed rods.

Answer.—$x = 1{\cdot}26 \cos\theta + 1{\cdot}84 \sin\theta$.

(3) Outside admission. Crossed rods.

Answer.—$x = -0{\cdot}48 \cos\theta + 1{\cdot}84 \sin\theta$.

164. A link motion with open rods and outside admission is to be designed so that cut off takes place at 70 per cent. of the stroke in full gear. The lead is to be 0·12 in. and the maximum opening of the port for steam is to be 1·2 ins. Assuming the half length of the link is 9·6 ins. and the distance of the motion block from the centre of the link is 6 ins., and that the length of the eccentric rod is 57·6 ins., find the eccentricity and the angular advances of the actual eccentric sheaves. Neglect the obliquity of the connecting rod.

Answers.—Eccentricity = 3·5 ins.
Angular advance, forward eccentric, $108\frac{1}{2}^\circ$.
Angular advance, backward eccentric, $251\frac{1}{2}^\circ$.
Lap = 1·32 ins.

165. With the data and results of the previous exercise plot the actual valve displacement curve for full forward gear, assuming the following data:—

Centre of weigh bar shaft, 42 ins. horizontally from crank shaft and 20 ins. vertically above the line of stroke. Arm of weigh bar shaft from which the link is suspended, 11 ins. long, and is parallel to the line of stroke at mid gear.

The link is suspended from the centre by a link 19·8 ins. long. Find the distribution of steam for both the instroke and the outstroke cycle when the connecting rod is 6 cranks long. In addition plot the displacement curve from the equivalent eccentric corresponding to the data in Question 161, and compare the distribution of steam deduced from it. (Equivalent eccentric, $\rho = 2{\cdot}52$; $\psi = 124^\circ\ 50'$.)

166. Using the data of Exercise 164 find the eccentricity and the angular advances when the link motion is designed with crossed rods to operate a piston valve with inside admission.

Answers.—Eccentricity = 3·5 ins.
Angular advance of forward eccentric, $288\frac{1}{2}^\circ$.
Angular advance of backward eccentric, $71\frac{1}{2}^\circ$.

167. Find the lap of the slide valve for outside admission and the eccentricity of the sheave of a Walschaerts valve gear so that cut off in full gear takes place at 80 per cent. of the stroke, and that the lead and maximum port openings for steam are respectively 0·25 in. and 1·35 ins., assuming the following dimensions: stroke, 20 ins.; distance centre to centre of pins connecting the eccentric rod and the valve spindle to the

lever driven from the crosshead, 3 ins., valve spindle uppermost; half length of link, 8 ins.; distance from centre of link to centre of motion block in full gear, 5 ins. below the centre. Neglect the obliquity of the connecting rod.

Answers.—Equivalent eccentric, $\rho = 2{\cdot}21$ ins.; $\psi = 120\frac{1}{4}°$.
Eccentric radius, 2·76 ins. Lap 0·86 in.

168. Referring to Fig. 208, page 582, show that in a Joy valve gear the equivalent eccentric may be calculated from the following expressions:—

$$A = -\frac{VJ \times DS \times OK}{JD \times RS}$$

$$B = \pm\frac{VD \times QR \times OK}{JD \times QK}\tan\kappa$$

where $A = \rho\cos\psi$

and $B = \rho\sin\psi$

169. Steam initially dry and saturated flows from a reservoir in which the pressure is maintained at 220 lbs. per square inch into the atmosphere (14·7 lbs. per square inch) through a nozzle. Assuming frictionless adiabatic flow, calculate the velocity of discharge into the air; the density of the steam in the final section of the nozzle, and the discharge per second per square foot of the final section of the nozzle (pages 586, 588).

Answer.—

Velocity of discharge = 3157 ft. per second.

Corresponding density = $\frac{1}{22{\cdot}88}$ lbs. per cubic foot.

Discharge per second per square foot = 136 lbs.

170. Given the data of Question 169, find the diameter of the throat and the diameter of the last section of an expanding nozzle to discharge $\frac{1}{2}$ lb. of steam per second, assuming that the pressure at the throat is equal to 0·58 the initial pressure.

Answer.—

Diameter of throat = 0·46 in.
Diameter of last section = 0·82 ,,

171. Calculate the velocity of flow, the density, and the discharge per second per square foot of sectional area of the final section of a nozzle from the pressure-volume diagram, assuming that the adiabatic curve is given by the expression pv^n = a constant. Calculate a value for n which will cause the approximate adiabatic curve to pass through the two points on the real adiabatic curve corresponding to p = 150 lbs. per square inch, and p = 1 lb. per square inch. Assume that the pressure at the final section is 1 lb. per square inch (Section 182, page 594).

Answers.—

$n = 1{\cdot}126$, $Z = 251{,}960$ ft.-lbs.; velocity of discharge = 4027 ft. per second.

Corresponding density = $\frac{1}{260{\cdot}4}$ lbs. per cubic feet.

Discharge per second per square foot = 15·4 lbs.

172. A nozzle discharges steam initially dry and saturated at 150 lbs. per square inch into a reservoir where the pressure is maintained constant at 1 lb. per square inch. The diameter of the nozzle at the throat is 0·4 in. Assuming that the flow is frictionless and adiabatic, calculate the weight of steam discharged per second into the reservoir. (Eq. (11), page 597.)

Answer.—0·27 lb. per second.

173. Assuming that the flow is frictionless and adiabatic, find the diameter of the throat and the diameter of the final section of a nozzle to deliver steam at the rate of 0·6 lb. per second into a reservoir in which the pressure is maintained constant at 2 lbs. per square inch when the steam is initially dry and saturated and at 100 lbs. per square inch. (Calculate U from $I_1 - I_2$, or use a chart.)

Answer.—Diameter at the throat, 0·73 in.
Diameter at the final section, 2·09 ins.

174. Find the size of the nozzle with the data of the previous question if the steam is initially superheated to 250° C.

Answer.—Diameter at the throat, 0·75 in.
Diameter at the final section, 2·11 ins.

175. Assuming frictionally resisted adiabatic flow and that the loss is 10 per cent. of the flow, find the size of a nozzle to deliver 0·6 lb. of steam per second into a reservoir where the pressure is maintained constant at 2 lbs. per square inch when the steam is initially dry and saturated and at 100 lbs. per square inch.

Answer.—Diameter at the throat, 0·75 in.
Diameter at the final section, 2·18 ins.

176. Find the size of the nozzle with the data of the previous question if the steam is initially superheated to 250° C.

Answer.—Diameter at the throat, 0·78 in.
Diameter at the final section, 2·21 ins.

177. In an impulse pair the nozzles discharge steam on to the wheel at an angle of 20° to the direction of motion of the blades; the velocity of discharge from the wheel is 600 ft. per second in a direction inclined 134° to the direction of motion of the blades. The change of velocity produced by the blades is parallel to the direction of motion. Draw the velocity diagram and from it find, neglecting losses:

	Answers.—
(*a*) The velocity of the discharge from the nozzles	1262 ft. per second.
(*b*) The mean velocity of the blades	385 " "
(*c*) The relative velocity of discharge from the blades	910 " "
(*d*) The blade angles at entry and discharge . .	28¼° and 151¾°.
(*e*) The change of velocity produced by the blades.	1602 ft. per second.
(*f*) The rate at which work is done per pound of flow	34·8 H.P.
(*g*) The heat supplied to the pair per second per pound of flow.	17·65 lb.-cals.
(*h*) The efficiency	0·77.

178. Using the same nozzle angle, namely 20°, and the same velocity of discharge from the nozzles, namely, 1262 ft. per second, as in the previous question, draw the velocity diagram corresponding to the maximum rate of working, and find the quantities (*b*) to (*h*), and in addition the velocity of discharge from the wheel.

> **Answers.**—(*b*) 593 ft. per second; (*c*) 732 ft. per second; (*d*) 36° and 144°; (*e*) 1186 ft. per second; (*f*) 39·7 H.P.; (*g*) 17·65 lb.-cals.; (*h*) 0·88.
> Velocity of discharge from wheel, 432 ft. per second.

179. In a reaction pair the fixed blades or nozzles discharge steam on to the wheel at an angle of 20° to the direction of motion of the blades; the velocity of discharge from the wheel is 300 ft. per second in a direction inclined at 134° to the direction of motion of the blades, the blade velocity is 300 ft. per second, and the change of velocity produced by the blades is parallel to the direction of motion. Draw the velocity diagram and from it find, neglecting losses—

	Answers.—
(*a*) The velocity of discharge from the fixed blades	631 ft. per second.
(*b*) The relative velocity at which the flow is received on to the moving blades	364 ft. per second at 36°.
(*c*) The relative velocity of discharge from the moving blades	552 ft. per second and at 157°.
(*d*) Change of velocity produced by the blades	801 ft. per second.
(*e*) Rate at which work is done per pound of flow	13·55 H.P.
(*f*) Heat supplied to the pair per pound of flow	6·32 lb.-cals.
(*g*) Heat used to increase the magnitude of the velocity through blades per pound of flow	1·91 lb.-cals.
(*h*) Efficiency	0·84.

180. Using the same angle for the fixed blades, namely 20°, and the same velocity of discharge from the fixed blades, namely, 631 ft. per second, as in the previous question, draw the velocity diagram corresponding to the maximum rate of working and find quantities (*b*) to (*h*), and in addition the velocity of discharge from the wheel and the blade velocity.

> **Answers.**—(*b*) 216 ft. per second at 90°; (*c*) 631 ft. per second at 160°; (*d*) 593 ft. per second; (*e*) 19·86 H.P.; (*f*) 8·30 lb.-cals.; (*g*) 3·89 lb.-cals.; (*h*) 0·94.
> Velocity of discharge, 216 ft. per second; blade velocity, 593 ft. per second.

181. In an impulse pair the ratio between the blade speed and the velocity of discharge from the nozzle to the moving blades is $0·3 = \lambda$. Calculate the efficiency of the pair when the nozzle is inclined 17° to the direction of motion of the blades. What is the value of λ corresponding to the maximum efficiency, and what is the maximum efficiency? (Page 638.)

> **Answers.**—Efficiency = 0·79.
> $\lambda = 0·48$; maximum efficiency = 0·92.

182. In a reaction pair with symmetrical blading the angle of discharge from the fixed to the moving blades is 17°. Calculate the efficiency of the pair when the value of $\lambda = 0{\cdot}5$. What is the value of λ for maximum efficiency, and what is the maximum efficiency?

Answers.—0·83; $\lambda = 0{\cdot}95$; maximum efficiency $= 0{\cdot}96$.

183. Steam is supplied to a reaction pair in a direction parallel to the axis, and is discharged from the pair at an angle to the direction of motion of the blades. Let U_s be the heat energy corresponding to the axial velocity of supply; U_1 the heat energy used to change the axial velocity of the supply to the velocity of discharge w from the fixed blades at an angle α to the direction of motion; U_2 the heat energy used to change the relative velocity of flow in the moving blade channels; and

$$\frac{1}{n} = \frac{U_s + U_1}{U_s + U_1 + U_2}$$

Assuming that the change of velocity effected by the blade is parallel to the direction of motion and neglecting losses, prove that the efficiency of the pair is given by the expression—

$$\eta = \frac{2}{n}\{(\lambda \cos \alpha - \lambda^2) + \sqrt{(\lambda \cos \alpha - \lambda^2)^2 + \lambda^2(n - 1)}\}$$

in which λ = the ratio between the blade velocity v and the velocity of discharge w from the fixed blades.

(This is a general expression for the efficiency of a turbine pair receiving energy U_s from the velocity of supply and energy $U_1 + U_2$ from the range. The efficiency of a pair working in a definite way can be derived from it by substituting therein the proper value of n. In particular for an impulse pair, $U_2 = 0$, therefore $n = 1$, and equation (1), page 638, follows. For a symmetrical pair, $n = 1 + 2\lambda \cos \alpha - \lambda^2$, and equation (9), page 641, follows.)

184. Steam initially superheated to 250° C. at 200 lbs. per square inch, expands adiabatically to 1 lb. per square inch. The available energy of the range is to be divided into 10 equal parts. Find the pressure intervals corresponding to the equal energy intervals, and find also the state of the steam corresponding to each pressure. What is the available energy of the range?

Answers.—

Available energy of range, 200·7 lb.-cals. per pound.

Pressure intervals (lbs. per sq. in.)	200	136	90	58	36	22	13	7	4	2	1
State	250° C.	206° C.	162° C.	Dryness = 0·972	0·945	0·920	0·895	0·872	0·850	0·827	0·806

185. Find the number of pairs required in an expansion of a chain of reaction pairs with symmetrical blading so that the expansion shall absorb 16 lb.-cals. from the range, having given that the mean value of the ratio $\lambda = 0{\cdot}4$; that the efficiency of each pair of the expansion is 0·75; that the blade speed is 160 ft. per second; and that the blading angle is 20°.

Find also the mean diameter of the blading and the height of the blades, assuming that the speed is 750 revolutions per minute; that the mean density of the steam flowing through the expansion is $\frac{1}{10}$ lb. per cubic foot; and that the flow is 45 lbs. of steam per second.

Answers.—Number of pairs = 6.
Mean diameter = 4·07 ft.
Height of blades, 3·09 ins.

186. Steam superheated initially to 250° C. at 200 lbs. per square inch is supplied to an impulse turbine in which the two first pairs of the chain are velocity compounded. Find the pressure drop and the temperature drop through the two velocity compounded pairs for a blade speed of 500 ft. per second, assuming that the loss by friction through each of the four rings of blades in the combination is 10 per cent. of the energy of flow and that the discharge is at right angles to the direction of motion and that the blading conditions are those shown in Fig. 242, page 661. State also the number of heat units absorbed from the range by the combination.

Answers.—

Pressure drops from 200 to 70 lbs. per square inch.

Temperature drops from 250° to 151° C. and the steam passes through the dry saturated condition, and is discharged from the combination with a dryness fraction, 0·985.

Energy absorbed from the range, 52 lb.-cals. per pound of flow.

APPENDIX

STEAM TABLES

[*Specially calculated by Prof. H. L. Callendar, F.R.S., from his own equations, to suit the methods of notation adopted in this book.*]

STEAM TABLES

[Specially calculated by Prof. H. L. Callendar, F.R.S., from his own equations, to suit the methods of notation adopted in this book.]

TABLE 1 shows the properties of steam against even values of the pressure, beginning with $\frac{1}{10}$ lb. per square inch and increasing by increments of $\frac{1}{10}$ lb per square inch to 2 lbs. per square inch, after which the increments become progressively larger up to 550 lbs. per square inch.

TABLE 2 shows the properties tabulated against even values of the temperature. The last column of the table gives values of a constant $G = (T\phi_w - I_w)$, which is useful in connection with the adiabatic expansion of steam.

TABLE 3 shows the total energy of superheated steam. This table is, in practice, more useful than a table of specific heats.

These tables are thermodynamically consistent, and are derived from the Callendar characteristic equation by applying to it the thermodynamic relations developed in Section 46, as illustrated in Section 48. The Callendar characteristic equation (Eq. (2), page 171) is,

$$(v - b) = \frac{JRT}{P} - c.$$

The constants used in the calculation of the tables are as follows:—

The unit of heat is the mean calorie, *i.e.* $\frac{1}{100}$ part of the heat required to raise 1 lb. of water from 0 to 100° Cent. The value of J, Joules' equivalent corresponding to this unit, is 1400 ft.-lbs. at Greenwich.

$R = 0{\cdot}11012$ mean calories; $RJ = 154{\cdot}168$ ft.-lbs.

c, called by Callendar the co-aggregation volume, is calculated from

$$c = C\left(\frac{T_o}{T}\right)^n$$

and for steam $C = 1{\cdot}192$ cub. ft. per lb. at the absolute temperature of $T_o = 273{\cdot}1°$ Cent., or $0{\cdot}4212$ at the absolute temperature $T_o = 373{\cdot}1$, that is, 100° Cent.

the index n is for steam $\frac{10}{3}$.

$b = 0{\cdot}016$ cubic feet per lb.

v = the volume in cubic feet of 1 lb. of steam either saturated or superheated.

P = pressure, lbs. per sq. foot. p = pressure, lbs. per sq. inch.

T = absolute temperature reckoned equal to $273{\cdot}1 + t$.

APPENDIX I

The characteristic equation for steam with these constants reduces to

$$v - 0{\cdot}016 = \frac{0{\cdot}11012JT}{P} - 1{\cdot}192\left(\frac{273{\cdot}1}{T}\right)^{\frac{10}{3}}.$$

Or when the pressure is in lbs. per square inch, and the constant C corresponds to 100° Cent.

$$v - 0{\cdot}016 = 1{\cdot}0706\frac{T}{p} - 0{\cdot}4212\left(\frac{373{\cdot}1}{T}\right)^{\frac{10}{3}}.$$

In order to determine the constants which are required in the calculations based upon this equation, certain experimental values are required, and these are—

L, the latent heat for a particular temperature. In this case L is equal to 539·3 lb.-cals at 100° Cent.

K_p°, the limiting value of the specific heat at constant pressure. This is stated by Callendar to be equal to

$$(n + 1)R = 0{\cdot}4772$$

and this is the value which has been used in the calculation of the steam tables.

The value of w given in Table 2 in terms of the temperature is based on the best available data. It is concerned in the calculation of I_w, the total energy of water given in that table.

Table 2 was calculated first, and then Table 1 was deduced from it by methods of interpolation.

Table 3 was calculated direct from the expression giving the total energy of superheated steam, equation 5, page 175, namely—

$$I = 0{\cdot}4772T - 0{\cdot}10286p\{(n+1)c - 0{\cdot}016\} + 464 \text{ lb.-cals. per pound.}$$

p = pressure in lbs. per square inch.

T = absolute temperature Cent.

Some values of c are given in Table 2.

$$n = \frac{10}{3}.$$

The mean specific heat of steam for any range of temperature can be obtained from this table when required by merely taking the difference of the total energy corresponding to the range and dividing by the temperature difference. The quantity required in most practical problems is the total energy of superheated steam, and this is given by the table direct.

TABLE 1. PROPERTIES OF DRY SATURATED STEAM.

Pressure. Lbs. per sq. inch.	Temperature. Centigrade.	Absolute temperature. Centigrade.	Energy in lb.-calories per lb.: Total energy of water.	Latent heat of steam.	Total energy of dry saturated steam.	External work.	Volume in cubic feet per lb.	Entropy per lb.: Water.	Increase due to addition of latent heat.	Pressure. Lbs. per sq. inch.
p	t	T	I_w	L	I_s	$\frac{144p}{J}(v-w)$	v	ϕ_w	$\frac{L}{T}$	p
·08922	0·0	273·10	0·0	594·27	594·27	30·063	3276·00	0·0	2·1760	·08922
0·1	1·59	274·69	1·5[illegible]	593·44	595·03	30·237	2940·00	0·0057	2·1605	0·1
0·2	11·69	284·79	11·67	588·14	599·81	31·357	1524·40	0·0417	2·0651	0·2
0·3	17·99	291·09	17·94	584·83	602·77	32·025	1037·70	0·0635	2·0092	0·3
0·4	22·66	295·76	22·60	582·[illegible]7	604·97	32·531	790·70	0·0794	1·9688	0·4
0·5	26·41	299·51	26·34	580·39	606·73	32·935	640·50	0·0922	1·9377	0·5
0·6	29·54	302·64	29·46	578·73	608·19	33·272	539·13	0·1025	1·9123	0·6
0·7	32·25	305·35	32·16	577·28	609·44	33·565	466·20	0·1113	1·8905	0·7
0·8	34·65	307·75	34·55	576·00	610·55	33·822	411·06	0·1191	1·8715	0·8
0·9	36·83	309·93	26·72	574·86	611·58	34·056	367·93	0·1262	1·8548	0·9
1·0	38·74	311·84	38·63	573·83	612·46	34·259	333·12	0·1323	1·8401	1·0
1·1	40·52	313·62	40·41	572·87	613·28	31·450	304·53	0·1379	1·8267	1·1
1·2	42·17	315·27	42·06	571·98	614·04	34·626	280·55	0·1432	1·8143	1·2
1·3	43·71	316·81	43·60	571·15	614·75	34·789	260·18	0·1481	1·8028	1·3
1·4	45·14	318·24	45·03	570·38	615·41	34·940	242·68	0·1526	1·7923	1·4
1·5	46·49	319·59	46·37	569·65	616·02	35·083	227·39	0·1568	1·7826	1·5
1·6	47·77	320·87	47·65	568·96	616·61	35·219	214·03	0·1604	1·7732	1·6
1·7	48·98	322·08	48·86	568·30	617·16	35·358	202·24	0·1646	1·7646	1·7
1·8	50·13	323·23	50·01	567·68	617·69	35·469	191·60	0·1683	1·7562	1·8
1·9	51·22	324·32	51·11	567·08	618·19	35·583	182·11	0·1715	1·7487	1·9
2·0	52·27	325·37	52·16	566·51	618·67	35·693	173·54	0·1747	1·7412	2·0
2·2	54·24	327·34	54·12	565·43	619·55	35·900	158·68	0·1807	1·7273	2·2
2·4	56·06	329·16	55·93	564·43	620·36	36·131	146·40	0·1863	1·7148	2·4
2·6	57·75	330·85	57·62	563·52	621·14	36·266	135·61	0·1914	1·7033	2·6
2·8	59·34	332·44	59·21	562·65	621·86	36·431	126·51	0·1962	1·6926	2·8
3·0	60·83	333·93	60·70	561·83	622·53	36·584	118·56	0·2007	1·6826	3·0
3·2	62·24	335·34	62·11	561·05	623·16	36·729	111·60	0·2049	1·6732	3·2
3·4	63·58	336·68	63·45	560·31	623·76	36·868	105·44	0·2089	1·6642	3·4
3·6	64·85	337·95	64·72	559·60	624·32	36·998	99·93	0·2127	1·6558	3·6
3·8	66·07	339·17	65·94	558·93	624·87	37·124	95·00	0·2162	1·6479	3·8
4·0	67·23	340·33	67·10	558·28	625·38	37·243	90·54	0·2197	1·6403	4·0
4·2	68·34	341·44	68·21	557·66	625·87	37·359	86·60	0·2230	1·6331	4·2
4·4	69·40	342·50	69·28	557·06	626·34	37·465	82·80	0·2260	1·6263	4·4
4·6	70·43	343·53	70·31	556·48	626·79	37·567	79·42	0·2290	1·6198	4·6
4·8	71·42	344·52	71·30	555·92	627·22	37·667	76·31	0·2319	1·6136	4·8
5·0	72·38	345·48	72·26	555·38	627·64	37·764	73·44	0·2346	1·6075	5·0
5·2	73·30	346·40	73·18	554·85	628·03	37·857	70·80	0·2372	1·6018	5·2
5·4	74·19	347·29	74·08	554·34	628·42	37·947	68·34	0·2398	1·5963	5·4
5·6	75·06	348·16	74·95	553·85	628·80	38·034	66·05	0·2423	1·5908	5·6
5·8	75·90	349·00	75·79	553·38	629·17	38·118	63·91	0·2448	1·5[illegible]	5·8

PROPERTIES OF DRY SATURATED STEAM. TABLE 1.

Pressure. Lbs. per sq. inch.	Temperature. Centigrade.	Absolute temperature. Centigrade.	Energy in lb.-calories per lb. Total energy of water.	Latent heat of steam.	Total energy of dry saturated steam.	External work.	Volume in cubic feet per lb.	Entropy per lb. Water.	Increase due to addition of latent heat.	Pressure. Lbs. per sq. inch.
p	t	T	I_w	L	I_s	$\frac{144p}{J}(v-w)$	v	ϕ_w	$\frac{L}{T}$	p
5·0	72·38	345·48	72·26	555·38	627·64	37·764	73·440	0·2346	1·6075	5·0
5·5	74·63	347·73	74·52	554·10	628·62	37·991	67·170	0·2411	1·5935	5·5
6·0	76·72	349·82	76·61	552·92	629·53	38·200	61·9[illegible]	0·2472	1·5805	6·0
6·5	78·67	351·77	78·56	551·81	630·37	38·395	57·442	0·2528	1·5685	6·5
7·0	80·49	353·59	80·39	550·76	631·15	38·575	53·593	0·2579	1·5574	7·0
7·5	82·21	355·31	82·11	549·77	631·88	38·745	50·242	0·2628	1·5472	7·5
8·0	83·84	356·94	83·75	518·82	632·57	38·904	47·298	0·2673	1·5376	8·0
8·5	85·38	358·48	85·30	547·93	633·23	39·054	44·687	0·2716	1·5285	8·5
9·0	86·84	359·94	86·76	547·08	633·84	39·197	42·358	0·2757	1·5199	9·0
9·5	88·24	361·34	88·17	546·27	634·44	39·334	40·273	0·2797	1·5118	9·5
10·0	89·58	362·68	89·51	545·50	635·01	39·461	38·388	0·2833	1·5041	10·0
10·5	90·87	363·97	90·81	514·73	635·54	39·585	36·676	0·2869	1·4967	10·5
11·0	92·10	365·20	92·05	544·00	636·05	39·702	35·110	0·2903	1·4896	11·0
11·5	93·29	366·39	93·24	543·29	636·53	39·816	33·680	0·2936	1·4829	11·5
12·0	94·44	367·54	94·40	542·61	637·01	39·926	32·365	0·2967	1·4764	12·0
12·5	95·55	368·65	95·52	541·95	637·47	40·029	31·150	0·2997	1·4702	12·5
13·0	96·62	369·72	96·60	541·31	637·91	40·131	30·030	0·3027	1·4642	13·0
13·5	97·66	370·76	97·65	540·70	638·35	40·228	28·987	0·3055	1·4585	13·5
14·0	98·66	371·76	98·66	540·12	638·78	40·322	28·020	0·3081	1·4530	14·0
14·5	99·64	372·74	99·64	539·52	639·16	40·412	27·114	0·3108	1·4476	14·5
14·689	100·00	373·10	100·00	539·30	639·30	40·448	26·788	0·31186	1·44546	14·689
15·0	100·58	373·68	100·58	538·95	639·53	40·502	26·270	0·3134	1·4423	15·0
16·0	102·41	375·51	102·43	537·82	640·25	40·671	24·732	0·3184	1·4322	16·0
17·0	104·14	377·24	104·18	536·76	640·94	40·831	23·369	0·3230	1·4228	17·0
18·0	105·79	378·89	105·84	535·75	641·59	40·981	22·155	0·3274	1·4140	18·0
19·0	107·36	380·46	107·43	534·79	642·22	41·124	21·062	0·3316	1·4057	19·0
20·0	108·87	381·97	108·95	533·87	642·82	41·260	20·075	0·3356	1·3977	20·0
21·0	110·32	383·42	110·42	532·97	643·39	11·389	19·180	0·3394	1·3901	21·0
22·0	111·71	384·81	111·83	532·09	643·92	41·514	18·365	0·3430	1·3823	22·0
23·0	113·05	386·15	113·19	531·24	644·43	41·632	17·617	0·3465	1·3758	23·0
24·0	114·34	387·44	114·50	530·43	644·93	41·745	16·929	0·3499	1·3690	24·0
25·0	115·59	388·69	115·75	529·64	645·39	11·855	16·294	0·3532	1·3625	25·0
26·0	116·80	389·90	116·98	528·87	645·85	41·961	15·707	0·3563	1·3563	26·0
27·0	117·97	391·07	118·17	528·15	646·32	42·061	15·162	0·3593	1·3501	27·0
28·0	119·11	392·21	119·32	527·42	646·74	42·159	14·656	0·3622	1·3447	28·0
29·0	120·21	393·31	120·43	526·72	647·15	42·254	14·183	0·3651	1·3392	29·0
30·0	121·28	394·38	121·51	526·01	647·52	42·345	13·740	0·3679	1·3337	30·0
31·0	122·33	395·43	122·58	525·34	647·92	42·435	13·326	0·3706	1·3286	31·0
32·0	123·35	396·45	123·63	524·67	648·30	42·522	12·937	0·3732	1·3234	32·0
33·0	124·33	397·43	124·63	524·03	648·66	42·603	12·572	0·3757	1·3186	33·0
34·0	125·31	398·41	125·68	523·39	649·02	42·684	12·224	0·3782	1·3137	34·0

TABLE 2. PROPERTIES OF DRY SATURATED STEAM.

Temperature. Centigrade.	Pressure. Lbs. per sq. inch.	Volume. Cubic feet per pound. Co-aggregation.	Volume. Water.	Volume. Dry saturated steam.	Energy. Lb.-calories per lb. External work.	Energy. Total energy of water.	Energy. Latent heat of steam.	Energy. Total energy of dry saturated steam.	Entropy per lb. Water.	Entropy per lb. Increase due to addition of latent heat.	Value of the constant G per lb.
t	p	c	w	v	$\frac{144p}{J}(v-w)$	I_w	L	I_s	ϕ_w	$\frac{L}{T}$	$T\phi_w - I_w$
0	0·0892	1·1917	0·01602	32[illegible]5·9	30·063	0·0	594·27	594·27	0·0	2·17602	0·0
10	0·1788	1·0572	0·01603	1693·8	31·156	9·98	589·03	599·01	0·03585	2·08064	0·18
20	0·3100	0·9417	0·01605	922·2	32·243	19·94	583·78	603·72	0·07046	1·99174	0·71
30	0·6162	0·8420	0·01609	525·8	33·324	29·91	578·48	608·39	0·10393	1·90855	1·59
40	1·0703	0·7557	0·01614	312·4	34·395	39·89	573·15	613·04	0·13631	1·83057	2·79
50	1·7888	0·6804	0·01621	192·7	35·455	49·88	567·75	617·63	0·16770	1·75720	4·30
60	2·887	0·6147	0·01629	122·90	36·498	59·87	562·29	622·16	0·19815	1·68805	6·18
70	4·516	0·5570	0·01638	80·80	37·523	69·88	556·73	626·61	0·22774	1·62265	8·26
80	6·863	0·5061	0·01648	54·60	38·526	79·90	551·05	630·95	0·25652	1·5[illegible]061	10·68
90	10·161	0·4611	0·01659	37·82	39·502	89·94	545·25	635·19	0·28454	1·50165	13·38
100	14·680	0·4212	0·01671	26·79	40·148	100·00	539·30	639·30	0·31186	1·44546	16·37
110	20·779	0·3857	0·01684	19·370	41·361	110·09	533·17	643·26	0·33853	1·39174	19·61
120	28·810	0·3540	0·01698	14·271	42·236	120·22	526·85	647·07	0·36460	1·34026	23·16
130	39·211	0·3255	0·01713	10·696	43·072	130·40	520·32	650·72	0·39011	1·29081	26·87
140	52·485	0·3000	0·01729	8·143	43·864	140·62	513·57	654·19	0·41511	1·24321	30·86
150	69·160	0·2770	0·01746	6·289	44·611	150·91	506·56	657·47	0·43963	1·19726	35·11
160	89·80	0·2562	0·01765	4·923	45·311	161·26	499·29	660·55	0·46373	1·15284	39·61
170	115·05	0·2375	0·01785	3·902	45·962	171·69	491·75	663·44	0·48743	1·10981	44·33
180	145·57	0·2204	0·01807	3·128	46·564	182·21	483·94	666·15	0·51078	1·06806	49·26
190	182·07	0·2050	0·01831	2·534	47·115	192·83	475·82	668·65	0·53381	1·02747	54·41
200	225·22	0·1909	0·01856	2·074	47·647	203·55	467·41	670·96	0·55654	0·98798	59·80
210	27[illegible]	0·1780	0·01885	1·713	48·070	214·40	458·70	673·10	0·57904	0·94947	65·36
220	[illegible]	0·1663	0·01914	1·428	48·474	225·37	449·69	675·06	0·60128	0·91199	71·14
230	401·[illegible]	0·1555	0·01946	1·200	48·831	236·49	440·39	676·48	0·62382	0·87536	77·12
240	47[illegible]·7	0·1456	0·01980	1·018	49·147	247·74	430·81	678·55	0·64517	0·83962	83·34
250	56[illegible]	0·1365	0·02016	0·869	49·419	259·16	420·96	680·12	0·66687	0·80474	89·72

FORMULÆ USED IN THE CALCULATION OF TABLE 2.

$$\log_{10}\frac{p}{p_0} = -4{\cdot}7172\log_{10}\frac{T}{T_0} + \frac{0{\cdot}40565(c-b)p}{T} - 0{\cdot}006474 + 7{\cdot}7817\frac{(t-100)}{T}$$ Eq. (21), page 180.

$$(v - 0{\cdot}016) = \frac{0{\cdot}11012JT}{P} - 1{\cdot}192\left(\frac{273{\cdot}1}{T}\right)^{\frac{10}{3}}$$ . . . . Eq. (8), page 172.

$$I = 0{\cdot}4772T - 0{\cdot}10286p\left\{\frac{13}{3}c - 0{\cdot}016\right\} + 464$$ . . . Eq. (5), page 175.

$$L = K_p^{\circ}T - \frac{1}{J}(n+1)Pc + \frac{1}{J}bP - sT - \frac{wL}{v-w} + 736{\cdot}181$$ Eq. (19), page 179.

$$I_w = 0{\cdot}9966t + \frac{wL}{v-w} - 0{\cdot}003$$ . . . . Eq. (15), page 177.

$$\phi_w = 0{\cdot}9966\log_e\frac{T}{273{\cdot}1} + \frac{wL}{T(v-w)}$$ . . . . Eq. (18), page 178.

$$v = \frac{2{\cdot}243(I-464)}{p} + 0{\cdot}0123$$ . . . . Eq. (8), page 176.

$$E = 0{\cdot}3671T - 0{\cdot}3427cp + 464$$ . . . . Eq. (10), page 176.

In these equations—

P = lbs. pressure per square foot. p = lbs. pressure per square inch.

T = the absolute temperature = $t + 273{\cdot}1$.

J = 1400 ft.-lbs. per mean pound-calorie.

R = 0·11012 mean pound-calorie.

I = total energy per pound of steam at the temperature T and pressure P.

L = latent heat per pound of steam.

I_w = total energy per pound of water.

ϕ_w = entropy per pound of water.

v = volume in cubic feet. b = 0·016. w = volume per pound of water.

c = co-aggregation volume per pound.

TABLE 3. TOTAL ENERGY OF SUPERHEATED STEAM.

$I = 0{\cdot}4772T - 0{\cdot}10286p\{(n+1)c - 0{\cdot}016\} + 464$ lb.-cals. per lb. (Eq. (5), page 175.)

p = pressure in lbs. per square inch.
T = absolute temperature, Cent.
c = the co-aggregation volume, see Table 2.
$n = \frac{10}{3}$.

Temperature to which Steam is Superheated.	Pressure of Formation in lbs. per sq. in. 0	50	100	150	200	250	300	Values of $0{\cdot}10286p\{(n+1)c-0{\cdot}016\}$ for 10 lb. steps.
400	785·1	783·9	782·7	781·5	780·2	779·0	777·8	0·25
390	780·4	779·1	777·[illegible]	776·5	775·2	773·4	772·6	0·26
380	775·6	774·2	772·9	771·5	770·1	768·8	767·4	0·27
370	770·9	769·4	768·0	766·5	765·1	763·6	762·2	0·29
360	766·1	764·6	763·0	761·5	760·0	758·4	756·9	0·30
350	761·3	759·7	758·1	756·4	754·8	753·2	751·6	0·32
340	756·6	754·9	753·2	751·5	749·8	748·0	746·3	0·34
330	751·8	750·0	748·2	746·4	744·5	742·7	740·9	0·36
320	747·0	745·1	743·2	741·2	739·3	737·4	735·5	0·38
310	742·2	740·2	738·2	736·1	734·1	732·1	730·0	0·41
300	737·5	735·3	733·1	730·9	728·8	726·6	724·4	0·43
290	732·7	730·4	728·1	725·8	723·5	721·2	718·9	0·46
280	727·9	725·5	723·0	720·6	718·1	715·7	713·3	0·49
270	723·1	720·5	717·9	715·3	712·7	710·1	707·5	0·52
260	718·4	715·6	712·8	710·0	707·3	704·5	701·7	0·55
250	713·6	710·6	707·6	704·7	701·7	698·8	695·8	0·59
240	708·8	705·7	702·5	699·3	696·2	693·0	689·8	0·63
230	704·1	700·7	697·3	693·9	690·5	687·1	683·8	0·67
220	699·3	695·7	692·0	688·4	684·8	681·2	677·5	0·72
210	694·5	690·6	686·7	682·9	679·0	675·1	(67[illegible]·9)	0·77
						(672·1)	(214·32)	
200	689·7	685·5	681·4	677·2	673·0	([illegible])		0·83
190	685·0	680·5	676·0	671·5	(669·7)			0·90
180	680·2	675·4	670·5	(666·5)	([illegible])			0·97
170	675·4	670·2	665·0	(181·31)				1·04
160	670·6	665·0	(661·8)					1·13
			(164·28)					
150	665·9	659·8						1·22
140	661·1	654·5						1·32
130	656·3	([illegible])						1·43
120	651·6	([illegible])						1·56
110	646·8							1·70
100	642·0							1·86

NOTE.—The smaller figures in brackets give the saturation temperature and the larger figures give corresponding values of I.

Expressions relating to the adiabatic expansion of superheated steam—

$\frac{P}{T^{\frac{10}{3}}}$ = a constant . . . . Eq. (6), page 193.

$p(v - b)^{1\cdot3}$ = a constant . . . . . Eq. (7), page 193.

$(v - b)T^{\frac{10}{3}}$ = a constant . . . . Eq. (8), page 193.

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